Northern Technical University
College of Technical Kirkuk
Surveying Department

Fourth stage

Geodesy\ Practical part

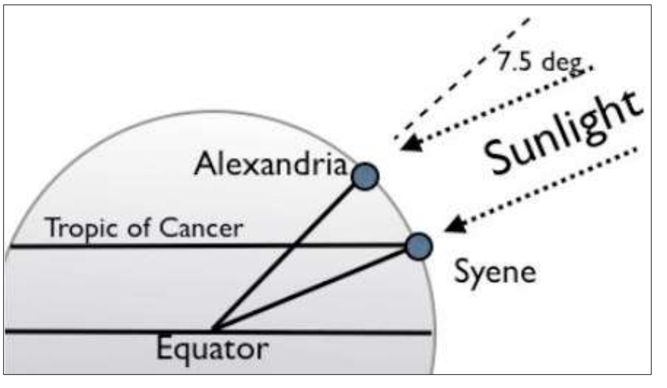
Topic one: Eratosthenes Finds Diameter of Earth

Objectives

- Estimate the diameter and circumference of the Earth by repeating Eratosthenes experiment.

The theory: A long time ago , but on this planet, a Greek philosopher created an experiment to measure the radius of the Earth. Really, it was a cool experiment. GREEK WAY

Eratosthenes estimated the radius of the Earth by looking at two shadows at two different locations on the Earth. This diagram should help.



So, by looking at the length of the shadow at Alexandria and knowing the distance between these two locations, the radius can be calculated. There is one trick. The size of a shadow changes during the day and during the year.

Instead of measuring the shadow at two different places at the same time, you measure at two different places on the same day (but a year later). So, if you know the day and the time, you can just repeat the experiment. The other trick is to use the local solar noon. This is when the sun is at the highest point in the sky. If you just move north-south, this time is the same for both locations. In the end, the Greeks obtained a fairly nice value for the radius of the Earth.

Materials

- Computer with Internet connection and e-mail access.
- Meter stick or pole of comparable length
- Measuring tape (or second meter stick)
- Scientific calculator

Background

Eratosthenes made a remarkably precise measurement of the size of the earth. He knew that at the summer solstice the sun shone directly into a well at Syene at noon. He found that at the same time, in Alexandria, Egypt, approximately ٧٨٧ km due north of Syene (now Aswan), the angle of inclination of the sun rays was about ٧،٢. With these measurements he computed the diameter and circumference of the earth as we will do.

Procedure:

١.Find someone that you can contact via e-mail that is at least ٥٠٠ km either due north or due south.. For example, from Kirkuk one could choose a Basra. Using an atlas to measure the distance between them, one finds they are … km apart.

٢.Drive a pole into the ground at a ٩٠ degree angle. Make sure it is in a sunny location. Measure the length of the pole from the ground to the topmost point. Write this number down.

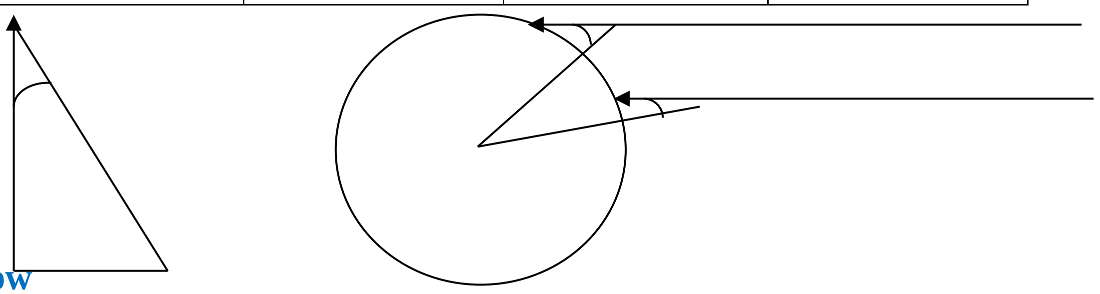
٣.Monitor the pole at local noon, that is, when the shadow is smallest. E-mail your partner to measure the sun's angle of inclination from the shadow cast by his/her pole and return the result. Make sure that your partner includes the length of the pole from ground to topmost point. See diagram below for measurements.

٤.Using the distance value and the measured angle, compute the circumference and diameter.

Write down the angles for each location.

Date	Rod length (m)	Shadow length (m)	Angle

rod



Shadow

tan θ = shadow length / rod length

Φ = θ٢ – θ١

Φ/٣٦٠° = distance between two place / circumference

Compare your results.

*The earth's average radius is usually accepted as ٦,٣٨ * ١٠^٦ m .*

Compute the percent of error for Eratosthenes result.______________%

Compute the percent of error for your result._____________%

Discussion questions:

Why do you think we need to choose two cities that lie roughly on the same longitudinal line?

How did Eratosthenes measure the distance between Alexandria and Syene over ٢٠٠٠ years ago?

Hint

To find the time that the sun is highest in the sky for any day, any location, look for the "time that sun transits" at US Naval Observatory website.

*You may have one partner for this activity, or you may do it alone. If you have a partner, please place both names on your answer sheet.

Write the date, the two locations, and show all work on your answer sheet.

Topic Two: Circumference of the earth

The aim: calculate of the radius of the earth

Al-Biruni's Classic Experiment.

Al-Biruni a pioneering Muslim scientist figured out a truly remarkable and ingenious method to calculate the radius of the earth (and subsequently its circumference etc.). This was very simple yet accurate requiring just four measurements in all to be taken and then applying a trigonometric equation to arrive at the solution. What Biruni figured out with unprecedented accuracy and precision in the ١٠th century was not known to the west until ١٦th century.

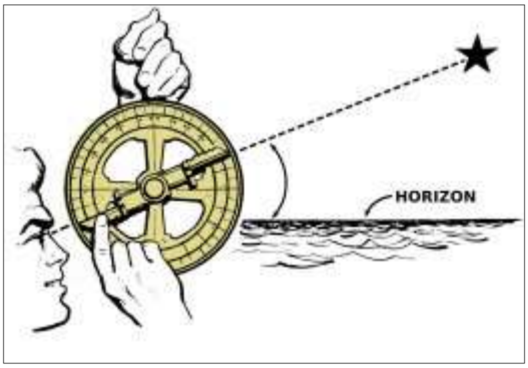
The need to calculate the size of earth was first felt when the Abbasid Caliphate spread far and wide from Spain till Indus river in modern day Pakistan. Muslims are required to pray facing the direction of the Kaaba and being far from Kaaba does not spares one from this obligation. So no matter how far Muslims were from the Kaaba they needed to determine its exact direction to pray. To do this accurately they needed to know the curvature of the earth and knowing this demanded that they know the size of the earth. By the way the Caliph was also curious to know the size of his empire!

Abbasid Caliph Al-Mamun thus employed a team of renowned scholars of that time and assigned them the task of calculating the size of the earth. They started by finding the distance over which the sun's angle at noon changed by ١ degree, multiply it by ٣٦٠ and you arrive at the circumference from which size can be deduced. They arrived at a value which was within ٤٪ of the actual value. The problem with this method was that it was cumbersome to measure large straight line distances between two points in the heat of the desert and perhaps they only had to count paces to measure it.

Al-Biruni devised a more sophisticated and reliable method to achieve this objective.

To carry out his method Biruni only needed three things.

١. An astrolabe.
٢. A suitable mountain with a flat horizon in front of it so that angle of depression of horizon could be accurately measured.
٣. Knowledge of trigonometry.

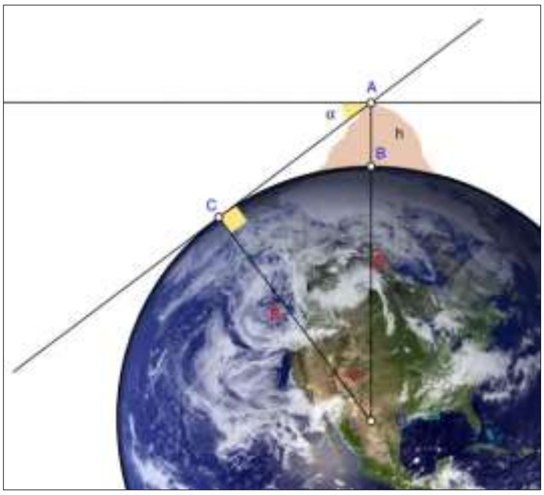


The astrolabe [Using an Astrolabe to measure angle of elevation]

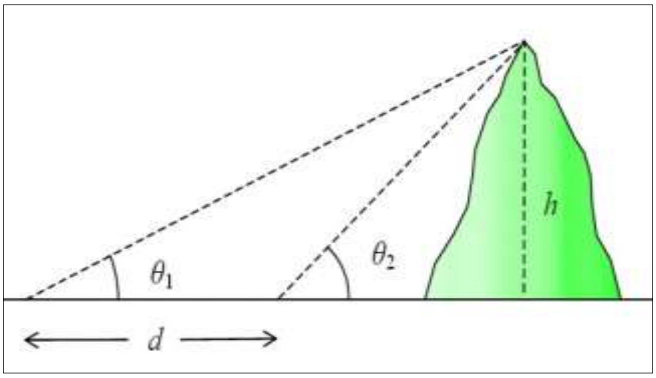
The first step: Calculation of the height of a mountain requires three measurements.

- Angle of elevation of the mountain top at two different points lying on a straight line were measured using an astrolabe. Biruni probably had a much larger astrolabe then that illustrated on the right to ensure maximum accuracy close to two decimal places of a degree.
- The third being the distance between these two points was perhaps found using paces.

These values were then computed with simple trigonometric techniques to find the height as shown in the figure below. This is a relatively simple and easy to understand problem. Method of determining height



$$h = \frac{d \tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1}$$



The second step: was to find the angle of dip or angle of depression of the flat horizon from the mountain top using the astrolabe in the same way, this being the fourth measurement. It can be further seen from the diagram that his line of sight from the mountain top to the horizon will make an angle of ٩٠° with the radius.

And finally we come to the useful bit, the ingenuity of this method lies in how Biruni figured out that that the figure linking the earth's center C, the mountaintop B, and the (sea or flat enough) horizon S was a huge right triangle on which the law of sines could be made to yield the earth’s radius.

Calculating radius of the Earth.

Now we can apply the law of sines to this triangle to find R.

$$\frac{AC}{\sin O} = \frac{R}{\sin A} = \frac{R+h}{\sin C}$$

$$\frac{AC}{\sin \alpha} = \frac{R}{\sin(90-\alpha)} = \frac{R+h}{\sin 90}$$

$$R = \frac{(R+h\,)sin(90-\alpha)}{\sin 90}$$

$$R = (R+h)\cos\alpha$$

This can be further simplified using trigonometry to arrive at the famous Biruni equation:

$$\boxed{R = \frac{h\cos\alpha}{1-\cos\alpha}}$$

With his formula Biruni arrived at the value of the circumference of the earth within ٢٠٠ miles of the actual value of ٢٤,٩٠٢ miles, that is less then ١٪ of error. Biruni's stated radius of ٦٣٣٥,٧٢٥ km is also very close to the original value.

Topic Three: minimum and maximum radii of curvature

Objectives:

To determine the minimum and maximum radii of curvature, and to prove that M and N increase monotonically from the equator to either pole.

Materials

- Computer with Internet connection and e-mail access or use world map to choose different values of longitude.
- Scientific calculator or computer

Background: Consider a curve on a surface, for example a meridian arc or a parallel circle on the ellipsoid, or any other arbitrary curve. The meridian arc and the parallel circle are examples of plane curves that are contained in a plane that intersects the surface. The amount by which the tangent to curve changes in direction as one moves along the curve indicates the curvature of the curve.

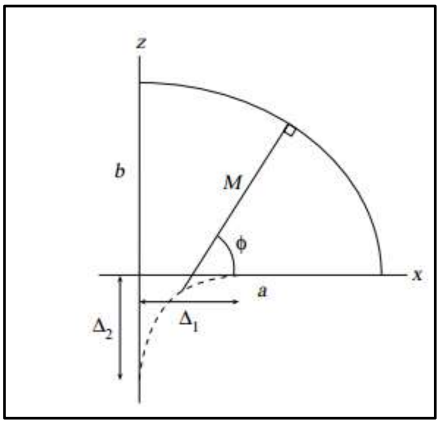
The curvature, χ , of a plane curve is the absolute rate of change of the tangent line to the curve with respect to arc length along the curve the reciprocal of the curvature is called the radius of curvature.

The curvature of the meridian ellipse is given by

$$= \frac{a}{b^2}\left(1 - e^2 \sin^2\phi\right)^{3/2}.$$

its reciprocal is the radius of curvature, denoted by M

$$M = \frac{a\left(1 - e^2\right)}{\left(1 - e^2 \sin^2\phi\right)^{3/2}},$$

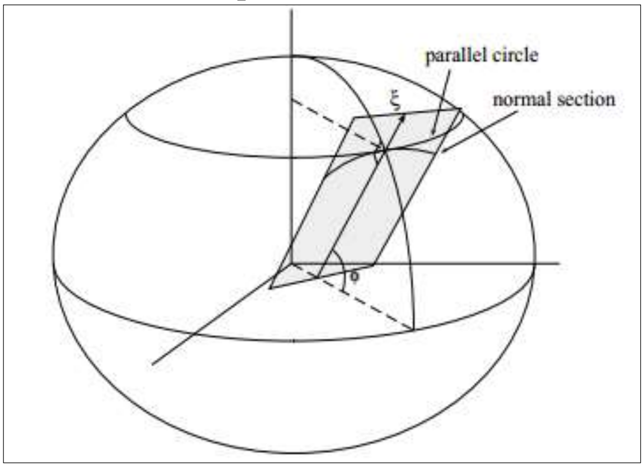


Note that M is a function of geodetic latitude where

$$\Delta_1 = a - M_{\text{equator}} = a - a\left(1 - e^2\right) = a\, e^2,$$

$$\Delta_2 = M_{\text{pole}} - b = \frac{a}{\frac{b}{a}} - b = b\, e'^2.$$

At any point on the ellipsoid, we may consider any other curve that passes through that point. At a point on the ellipsoid, let ζ be the unit vector defining the direction of the normal to the surface. By the symmetry of the ellipsoid, ζ lies in the meridian plane. Now consider any plane that contains ζ ; it intersects the ellipsoid in a curve known as a normal section ("normal" because the plane contains the normal to the ellipsoid at a point) (see Figure ٢). The meridian curve is a special case of a normal section; but the parallel circle is not a normal section;



The radius of curvature of the prime vertical normal section at the point of the ellipsoid normal is given by.

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2\phi}} \;.$$

we also find that the point of intersection of N with the minor axis is the following distance from the ellipsoid center:

$$\Delta = N \sin\phi - z = N e^2 \sin\phi \;.$$

M and N are known as the principal radii of curvature at a point of the ellipsoid.

<u>Procedure:</u>

١. Use different values of longitude to calculate M, N, χ, and Δ .
٢. Plot the relation between M,N and φ.

Latitude	M (m)	N (m)	Δ (m)	χ (١/m)	P (m)	X(m)	Z(m)

Discussion questions:

Why do you think that M is a function of geodetic latitude and not longitude?
Define Gaussian mean radius, use M and N to determine it, discuss the result.
Write the date, the two locations, and show all work on your answer sheet.

Topic Four: Triangulation

Introduction

The horizontal positions of points is a network developed to provide accurate control for topographic mapping, charting lakes, rivers and ocean coast lines, and for the surveys required for the design and construction of public and private works of large extent.

The horizontal positions of the points can be obtained in a number of different ways in addition to traversing. These methods are triangulation, trilateration, intersection, resection, and satellite positioning.

The method of surveying called *triangulation* is based on the trigonometric proposition that if one side and two angles of a triangle are known, the remaining sides can be computed. Furthermore, if the direction of one side is known, the directions of the remaining sides can be determined. A triangulation system consists of a series of joined or overlapping triangles in which an occasional side is measured and remaining sides are calculated from angles measured at the vertices of the triangles. The vertices of the triangles are known as *triangulation stations*. The side of the triangle whose length is predetermined, is called the *base line*. The lines of triangulation system form a network that ties together all the triangulation stations (Fig. ١,١).

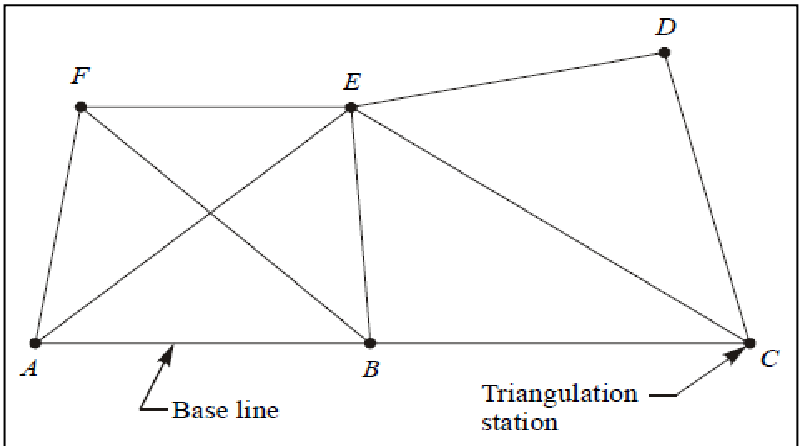


Fig. 1.1 Triangulation network

A **trilateration system** also consists of a series of joined or overlapping triangles. However, for trilateration the lengths of all the sides of the triangle are measured and few directions or angles are measured to establish azimuth. Trilateration has become feasible with the development of electronic distance measuring (EDM) equipment which has made possible the measurement of all lengths with high order of accuracy under almost all field conditions.

A combined triangulation and trilateration system consists of a network of triangles in which all the angles and all the lengths are measured. Such a combined system represents the strongest network for creating horizontal control.
Since a triangulation or trilateration system covers very large area, the curvature of the earth has to be taken into account. These surveys are, therefore, invariably geodetic. Triangulation surveys were first carried out by Snell, a Dutchman, in ١٦١٥.
Field procedures for the establishment of trilateration station are similar to the procedures used for triangulation, and therefore, henceforth in this chapter the term triangulation will only be used.

OBJECTIVE OF TRIANGULATION SURVEYS

The main objective of triangulation or trilateration surveys is to provide a number of stations whose relative and absolute positions, horizontal as well as vertical, are accurately established. More detailed location or engineering survey are then carried out from these stations.

The triangulation surveys are carried out

(i) to establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods,
(ii) to establish accurate control for photogrammetric surveys of large areas,
(iii) to assist in the determination of the size and shape of the earth by making observations for latitude, longitude and gravity, and
(iv) to determine accurate locations of points in engineering works such as :
(a) Fixing center line and abutments of long bridges over large rivers.
(b) Fixing center line, terminal points, and shafts for long tunnels.
(c) Transferring the control points across wide sea channels, large water bodies, etc.
(d) Detection of crustal movements, etc.
(e) Finding the direction of the movement of clouds

CLASSIFICATION OF TRIANGULATION SYSTEM

Based on the extent and purpose of the survey, and consequently on the degree of accuracy desired, triangulation surveys are classified as first-order or primary, second-order or secondary, and third-order or tertiary. First-order triangulation is used to determine the shape and size of the earth or to cover a vast area like a whole country with control points to which a second-order triangulation system can be connected. A second-order triangulation system consists of a network within a first-order triangulation. It is used to cover areas of the order of a region, small country, or province. A third-order triangulation is a framework fixed within and connected to a second-order triangulation system.

Table 1.1 Triangulation system

S.No.	Characteristics	First-order triangulation	Second-order triangulation	Third-order triangulation
1.	Length of base lines	8 to 12 km	2 to 5 km	100 to 500 m
2.	Lengths of sides	16 to 150 km	10 to 25 km	2 to 10 km
3.	Average triangular error (after correction for spherical excess)	less than 1″	3″	12″
4.	Maximum station closure	not more than 3″	8″	15″
5.	Actual error of base	1 in 50,000	1 in 25,000	1 in 10,000
6.	Probable error of base	1 in 10,00,000	1 in 500,000	1 in 250,000
7.	Discrepancy between two measures (k is distance in kilometre)	$5\sqrt{k}$ mm	$10\sqrt{k}$ mm	$25\sqrt{k}$ mm
8.	Probable error of the computed distances	1 in 50,000 to 1 in 250,000	1 in 20,000 to 1 in 50,000	1 in 5,000 to 1 in 20,000
9.	Probable error in astronomical azimuth	0.5″	5″	10″

Table 1.1 presents the general specifications for the three types of triangulation systems.

Topic Five: TRIANGULATION FIGURES AND LAYOUTS

The basic figures used in triangulation networks are the triangle, braced or geodetic quadilateral, and the polygon with a central station (Fig. ١,٣).

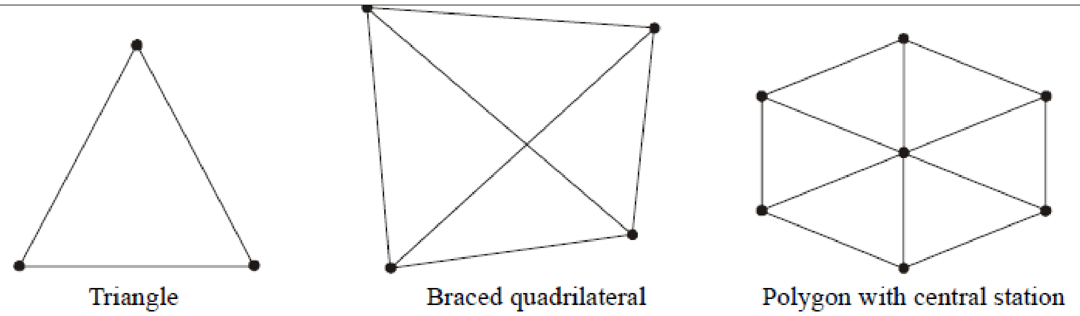


Fig. 1.3 Basic triangulation figures

The triangles in a triangulation system can be arranged in a number of ways. Some of the commonly used arrangements, also called layouts, are as follows:

١. Single chain of triangles
٢. Double chain of triangles
٣. Braced quadrilaterals
٤. Centered triangles and polygons
٥. A combination of above systems.

Single chain of triangles

When the control points are required to be established in a narrow strip of terrain such as a valley between ridges, a layout consisting of single chain of triangles is generally used as shown in Fig. ١,٤.

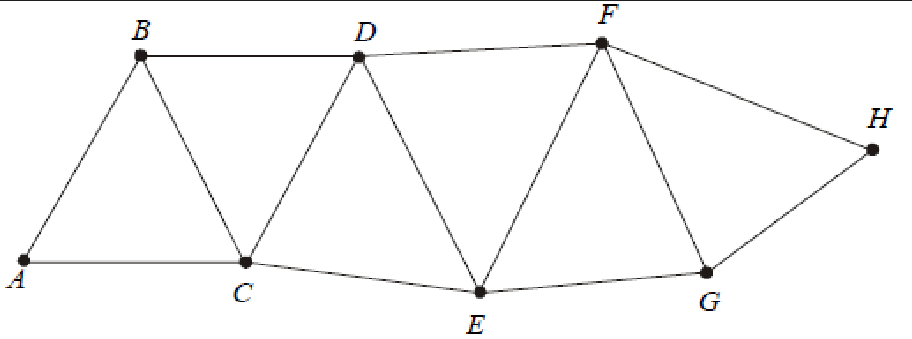


Fig. 1.4 Single of triangles

This system is rapid and economical due to its simplicity of sighting only four other stations, and does not involve observations of long diagonals.
This system does not provide any check on the accuracy of observations.

Double chain of triangles

A layout of double chain of triangles is shown in Fig. ١،٥. This arrangement is used for covering the larger width of a belt. This system also has disadvantages of single chain of triangles system

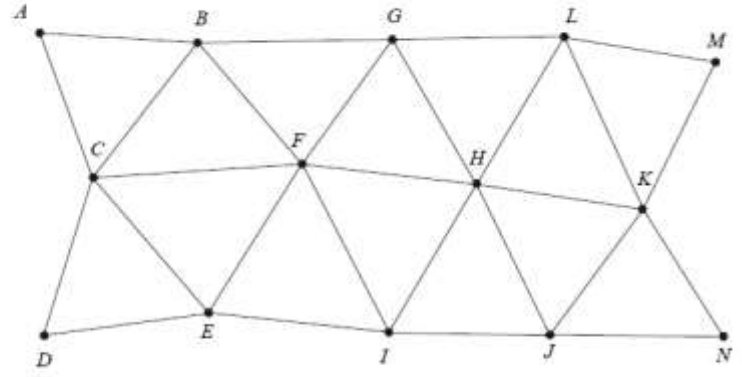


Fig. 1.5 Double chain of triangles

Braced quadrilaterals

A triangulation system consisting of figures containing four corner stations and observed diagonals shown in Fig. ١،٦, is known as a layout of braced quadrilaterals. In fact, braced quadrilateral consists of overlapping triangles. This system is treated to be the strongest and the best arrangement of triangles, and it provides a means of computing the lengths of the sides using different combinations of sides and angles. Most of the triangulation systems use this arrangement.

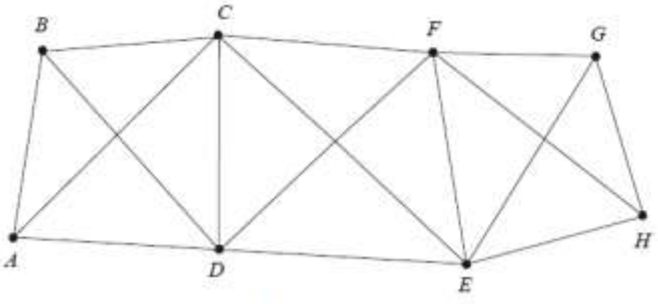


Fig. 1.6 Braced quadrilaterals

Centered triangles and polygons

A triangulation system which consists of figures containing interior stations in triangle and polygon as shown in Fig. ١،٧, is known as centered triangles and polygons.

This layout in a triangulation system is generally used when vast area in all directions is required to be covered. The centered figures generally are quadrilaterals, pentagons, or hexagons with central stations. Though this system provides checks on the accuracy of the work, generally it is not as strong as the braced quadrilateral arrangement. Moreover, the progress of work is quite slow due to the fact that more settings of

the instrument are required.

A combination of all above systems

Sometimes a combination of above systems may be used which may be according to the shape of the area and the accuracy requirements.

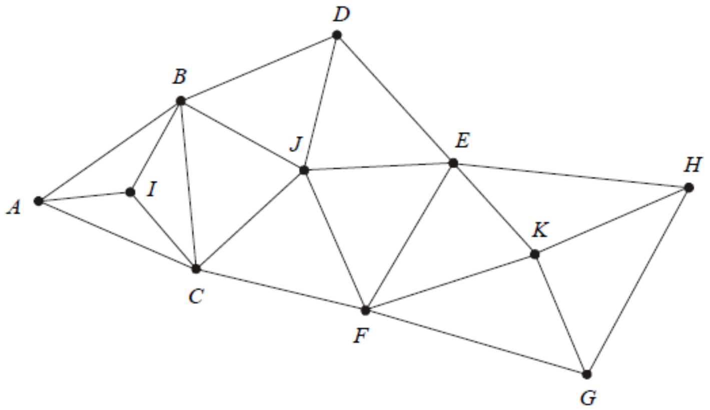


Fig. 1.7 Centered triangles and polygons

LAYOUT OF PRIMARY TRIANGULATION FOR LARGE COUNTRIES

The following two types of frameworks of primary triangulation are provided for a large country to cover the entire area.

١. Grid iron system

٢. Central system.

Grid iron system

In this system, the primary triangulation is laid usually in series of chains of triangles, which runs roughly along meridians (northsouth) and along perpendiculars to the meridians (east-west), throughout the country (Fig. ١،٨).

The distance between two such chains may vary from ١٥٠ to ٢٥٠ km.

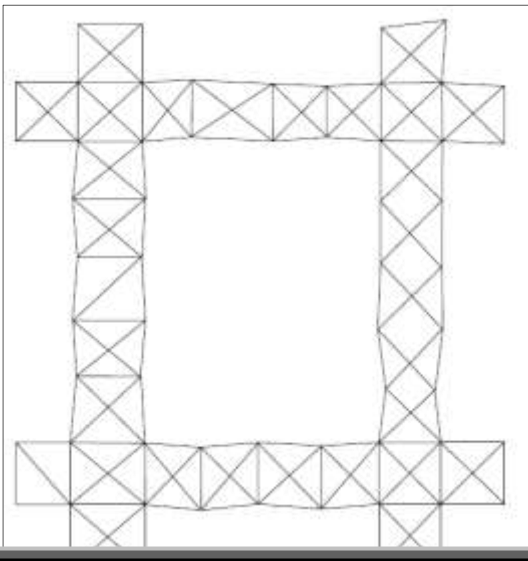


Fig. 1.8 Grid iron system of triangulation

The area between the parallel and perpendicular series of primary triangulation, are filled by the secondary and tertiary triangulation systems.

Grid iron system has been adopted in India and other countries like Austria, Spain, France, etc.

Central system

In this system, the whole area is covered by a network of primar triangulation extending in all directions from the initial triangulation figure ABC, which is generally laid at the centre of the country (Fig. ١,٩). This system is generally used for the survey of an area of moderate extent. It has been adopted in United Kingdom and various other countries.

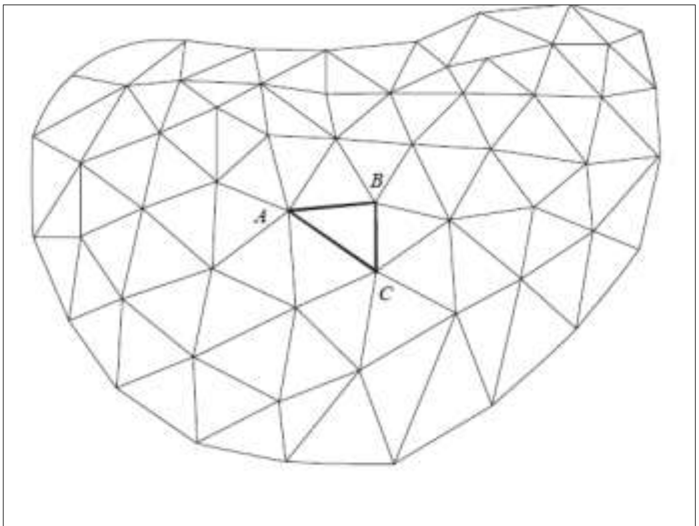


Fig. 1.9 Central system of triangulation

CRITERIA FOR SELECTION OF THE LAYOUT OF TRIANGLES

The under mentioned points should be considered while deciding and selecting a suitable layout of triangles.

١. Simple triangles should be preferably equilateral.

٢. Braced quadrilaterals should be preferably approximate squares.

٣. Centered polygons should be regular.

٤. The arrangement should be such that the computations can be done through two or more independent routes.

٥. The arrangement should be such that at least one route and preferably two routes form wellconditioned triangles.

٦. No angle of the figure, opposite a known side should be small, whichever end of the series is used for computation.

٧. Angles of simple triangles should not be less than ٤٥°, and in the case of quadrilaterals, no angle should be less than ٣٠°. In the case of centered polygons, no angle should be less than ٤٠°.

٨. The sides of the figures should be of comparable lengths. Very long lines and very short lines should be avoided.

٩. The layout should be such that it requires least work to achieve maximum progress.

١٠. As far as possible, complex figures should not involve more than ١٢ conditions.

It may be noted that if a very small angle of a triangle does not fall opposite the known side it does not affect the accuracy of triangulation.

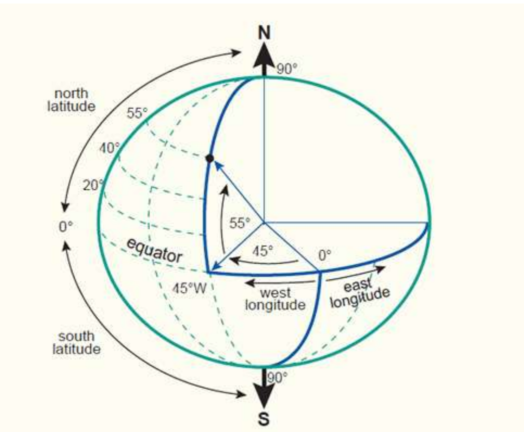
Topic Six: coordinate transformation

٢,٠ Introduction

Different kind of coordinates are used to position objects in a two- or three-dimensional space. ***Spatial coordinates*** (also known as global coordinates) are used to locate objects either on the Earth's surface in a ٣D space, or on the Earth's reference surface (ellipsoid or sphere) in a ٢D space. Specific examples are the geographic coordinates in a ٢D or ٣D space and the geocentric coordinates, also known as ٣D Cartesian coordinates. ***Planar coordinates*** on the other hand are used to locate objects on the flat surface of the map in a ٢D space. Examples are the ٢D Cartesian coordinates and the ٢D polar coordinates.

٢,١ ٢D geographic coordinates (ϕ,λ)

The most widely used global coordinate system consists of lines of geographic latitude (phi or ϕ or φ) and longitude (lambda or λ)Lines of equal latitude are called parallels. They form circles on the surface of the ellipsoid. Lines of equal longitude are called meridians and they form ellipses (meridian ellipses) on the ellipsoid. Both lines form the graticule when projected onto a map plane. Note that the concept of geographic coordinates can also be applied to a sphere as the reference surface.



latitude (ϕ) of a point P (figure section ٢,٢) is the angle between the ellipsoidal normal through P' and the equatorial plane. Latitude is zero on the equator (ϕ = ٠°), and increases towards the two poles to maximum values of ϕ = +٩٠ (٩٠°N) at the North Pole and ϕ = - ٩٠° (٩٠°S) at the South Pole.

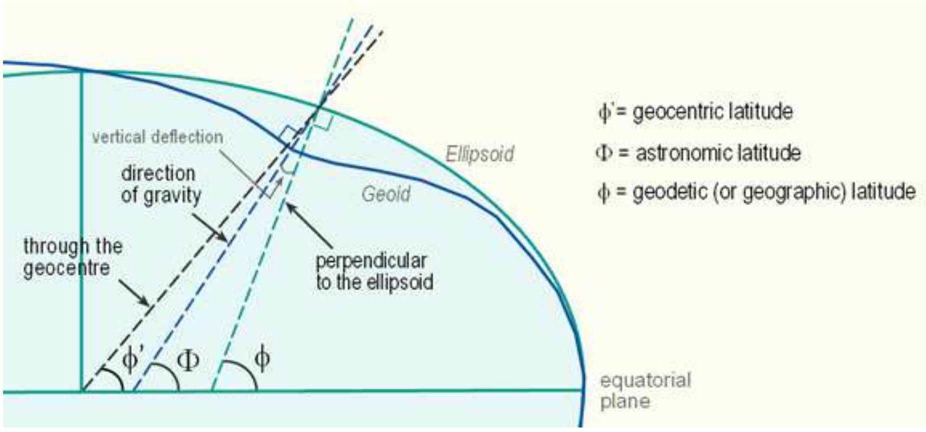
The longitude (λ) is the angle between the meridian ellipse which passes through Greenwich and the meridian ellipse containing the point in question. It is measured in the equatorial plane from the meridian of Greenwich (λ= ٠°) either eastwards through λ = + ١٨٠° (١٨٠°E) or westwards through λ = -١٨٠° (١٨٠°W).

Latitude and longitude represent **the geographic coordinates** (ϕ,λ)of a point P' (figure section ٢,٢) with respect to the selected reference surface. They are also called **geodetic**

Coordinates or ellipsoidal coordinates when an ellipsoid is used to approximate the shape of the Earth. Geographic coordinates are always given in angular units. An example, the coordinates for the City kirkuk are:

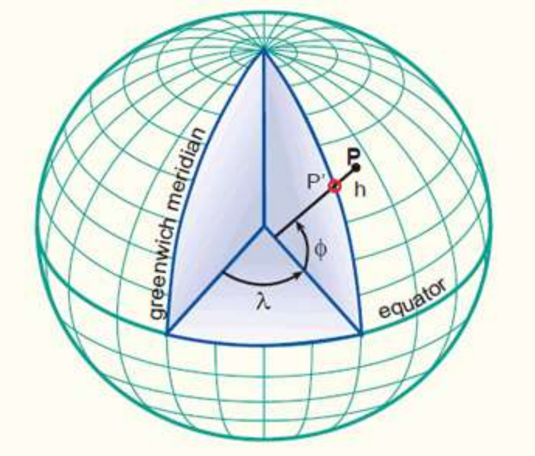
ϕ = ٣٥° ٢٤' ٠٠,٠٢٧٨٨" N, λ = ٤٤° ٢٠' ٣٨,٧٧١٤٠" E.

There are several formats for the angular units of geographic coordinates. The Degrees:Minutes:Seconds (٤٩°٣٠'٠٠"N, ١٢٣°٣٠'٠٠"W) is the most common format, another the Decimal Degrees (٤٩,٥٠٠٠°, -١٢٣,٥٠٠٠°), generally with ٤-٦ decimal numbers



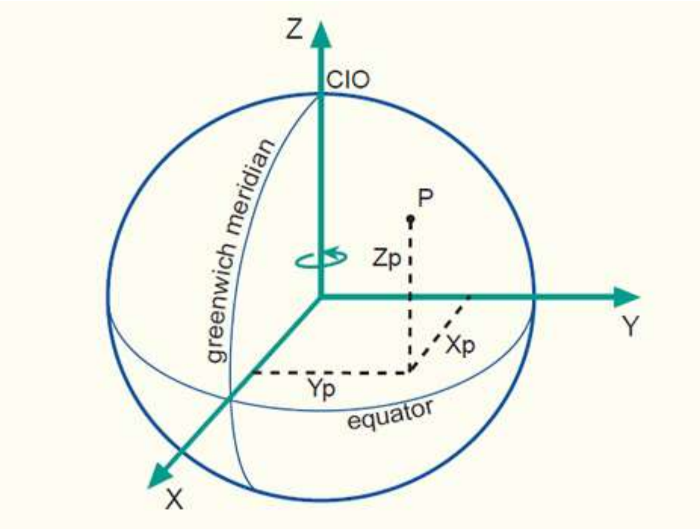
٢,٢ ٣D geographic coordinates (ϕ,λ, h)

٣D geographic coordinates (ϕ, λ,h) are obtained by introducing the ellipsoidal height h to the system. The ***ellipsoidal*** **height (h)** of a point is the vertical distance of the point in question above the ellipsoid. It is measured in distance units along the ellipsoidal normal from the point to the ellipsoid surface. ٣D geographic coordinates can be used to define a position on the surface of the Earth (point P in figure below).



٢,٣ Geocentric coordinates (X,Y,Z)

An alternative method of defining a ٣D position on the surface of the Earth is by means of ***geocentric coordinates*** (x,y,z), also known as ***٣D Cartesian coordinates***. The system has its origin at the mass-centre of the Earth with the X- and Y-axes in the plane of the equator. The X-axis passes through the meridian of Greenwich, and the Z-axis coincides with the Earth's axis of rotation. The three axes are mutually orthogonal and form a right-handed system. Geocentric coordinates can be used to define a position on the surface of the Earth (point P in figure below).



It should be noted that the rotational axis of the Earth changes its position over time (referred to as ***polar motion***). To compensate for this, the mean position of the pole in the year ١٩٠٣ (based on observations between ١٩٠٠ and ١٩٠٥) has been used to define the so-called 'Conventional International Origin' (CIO).

Topic Seven: The geoid undulation

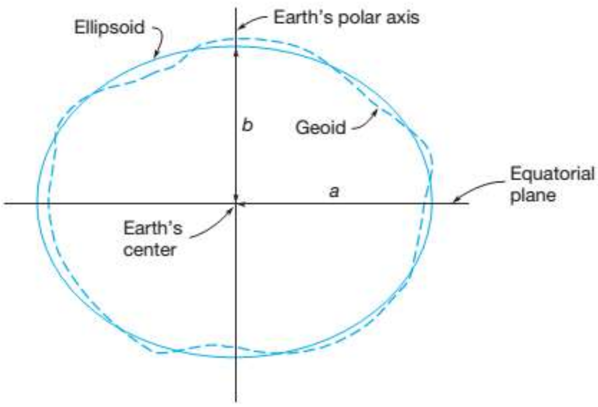
THE ELLIPSOID AND GEOID

The horizontal control surveys generally determine geodetic latitudes and geodetic longitudes of points. To explain geodetic latitude and longitude, it is necessary to first define the *geoid* and the ***ellipsoid.***

The geoid is an equipotential gravitational surface, which is everywhere perpendicular to the direction of gravity. Because of variations in the Earth's mass distribution and the rotation of the Earth, the geoid has an irregular shape.

The ellipsoid is a mathematical surface obtained by revolving an ellipse about the Earth's polar axis.

The dimensions of the ellipse are selected to give a good fit of the ellipsoid to the geoid over a large area and are based upon surveys made in the area.



Ellipsoids, which approximate the geoid and can be defined mathematically, are therefore used to compute positions of widely spaced points that are located through control surveys.

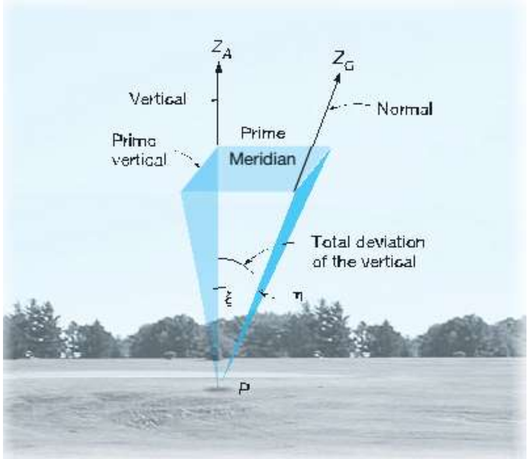
Currently, the *Geodetic Reference System of* ١٩٨٠ (GRS٨٠) and *World Geodetic System of* ١٩٨٤ (WGS٨٤) ellipsoids are commonly used in the United States because they provide a good worldwide fit to the geoid.

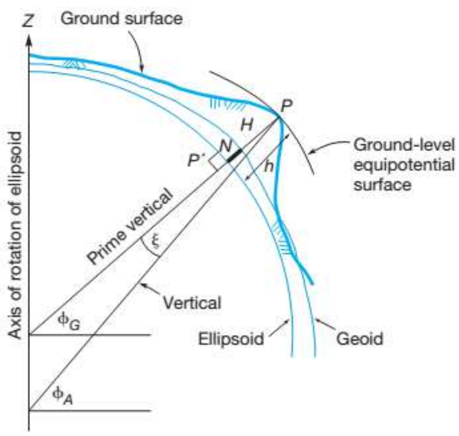
GEOID UNDULATION AND DEFLECTION OF THE VERTICAL

If the Earth was a perfect ellipsoid without internal density variations, the geoid would match the ellipsoid perfectly. However, this is not the case, and thus the geoid can depart from some ellipsoids by as much as ١٠٠ m or more in certain locations.

The separation between the geoid and the ellipsoid creates a difference between the height of a point above the ellipsoid (*geodetic height*) and its height above the geoid (*orthometric height*, which is commonly known as *elevation*). This difference, known as *geoid height* (***also called geoidal separation***),

The relationship between the orthometric height *H* and geodetic height *h* at any point is $h = H + N$ where *N* is the geoidal height.





The *deflection of the vertical* (also called *deviation of the vertical*) at any ground point *P* is the angle between the vertical (direction of gravity) and the normal to the ellipsoid. This angle is generally reported by giving two components: its orthogonal projections onto the meridian and normal planes. In the figure, the zenith of the ground-level equipotential surface is called the astronomical zenith ZA since it corresponds to the direction of gravity (zenith) of a leveled instrument during astronomical observations. Also ZG, is the normal at point *P*. The projected components of the total deviation of the vertical onto the meridian and normal planes are called ξ AND η, respectively.

$$\xi = \phi_A - \phi_G$$

$$\eta = (\lambda_A - \lambda_G) \cos \phi = (Az_A - Az_G) \cot \phi$$

Φ can be either the astronomic or geodetic latitude.
From this equation, the so-called *Laplace equation* can be derived as

$$Az_G = Az_A - (\lambda_A - \lambda_G) \sin \phi = Az_A - \eta \tan \phi$$

Stations at which the necessary parameters are known, such that above Equation can be formed are called *Laplace stations.* for points near the equator, latitude approaches ٠°, and the astronomic and geodetic azimuths become essentially the same.

REFERENCE FRAMES

Horizontal and vertical datums consist of a network of control monuments and benchmarks whose horizontal positions and/or elevations have been determined by precise geodetic control surveys. These monuments serve as reference points for originating subordinate surveys of all types and as such are known as *reference frames*. The horizontal and vertical reference systems used in the immediate past and at present in the United States are described in the following subsections.

North American Horizontal Datum of ١٩٢٧ (NAD٢٧)

In ١٩٢٧, a least-squares adjustment was performed, which incorporated all horizontal geodetic surveys that had been completed up to that date. This network of monumented points included in the adjustment, together with their adjusted geodetic latitudes and longitudes, was referred to as the *North American Datum of ١٩٢٧* (NAD٢٧).

The adjustment utilized the Clarke ellipsoid of ١٨٦٦ and held fixed the latitude and longitude of an "initial point," station *Meades Ranch* in Kansas, along with the azimuth to nearby station *Waldo*. The project yielded adjusted latitudes and longitudes for some ٢٥,٠٠٠ monuments existing at that time.

North American Horizontal Datum of ١٩٨٣ (NAD٨٣)

The National Geodetic Survey (NGS) began a new program in ١٩٧٤ to perform another general adjustment of the North American horizontal datum. The adjustment was deemed necessary because of the multitude of post-١٩٢٧ geodetic observations that existed and because many inconsistencies had been discovered in the NAD٢٧ network. The project was originally scheduled for completion in ١٩٨٣, hence its name *North American Datum of ١٩٨٣* (NAD٨٣), but it was not actually finished until ١٩٨٦. The adjustment was a huge undertaking, incorporating approximately ٢٧٠,٠٠٠ stations and all geodetic surveying observations on record—nearly ٢ million of them! About ٣٥٠ person-years of effort were required to accomplish the task.

The initial point in the new adjustment is not a single station such as Meades Ranch in Kansas; rather, the Earth's mass-center and numerous other points whose latitudes and longitudes had been precisely established using radio astronomy and satellite observations were used.

National Geodetic Vertical Datum of ١٩٢٩ (NGVD٢٩)

Vertical datums for referencing benchmark elevations are based on a single equipotential surface. The NGVD٢٩ was obtained from a best fit of mean sea level observations taken at ٢٦ tidal gauge stations in the United States and Canada, and thus is often referred to as "mean sea level (MSL)." Unfortunately, the use of the term "mean sea level" is still used today when expressing elevations of benchmarks.
the use of "mean sea level" to define the elevation of a station is incorrect since the current datum was arbitrarily defined using a single benchmark.

North American Vertical Datum of ١٩٨٨ (NAVD٨٨)

Between ١٩٢٩ and ١٩٨٨ more than ٦٢٥,٠٠٠ km of additional control leveling lines had been run. Furthermore, crustal movements and subsidence had changed the elevations of many benchmarks. To incorporate the additional leveling, and correct elevations of erroneous benchmarks, a general vertical adjustment was performed. This adjustment included the new observational data, as well as an additional ٨١,٥٠٠ km of releveled lines, and leveling observations from both Canada and Mexico. It was originally scheduled for completion in ١٩٨٨ and named the *North American Vertical Datum of ١٩٨٨* (NAVD٨٨), but it was not actually released to the public until ١٩٩١.

This adjustment shifted the position of the reference equipotential surface from the mean of the ٢٦ tidal gauge stations used in NGVD٢٩ to a single tidal gage benchmark known as *Father Point*, which is in Rimouski on the Saint Lawrence Seaway in Quebec, Canada. As a result of these changes, published elevations of benchmarks in NAVD٨٨ have shifted from their NGVD٢٩ values.

Topic Eight: Station Mark

The triangulation stations should be permanently marked on the ground so that the theodolite and signal may be centered accurately over them. The following points should be considered while marking the exact position of a triangulation station:

١- The station should be marked on perfectly stable foundation or rock. The station mark on a large size rock is generally preferred so that the theodolite and observer can stand on it. Generally, a hole ١٠ to ١٥ cm deep is made in the rock and a copper or iron bolt is fixed with cement.

٢- If no rock is available, a large stone is embeded about ١ m deep into the ground with a circle, and dot cut on it. A second stone with a circle and dot is placed vertically above the first stone.

٣- A G.I. pipe of about ٢٥ cm diameter driven vertically into ground up to a depth of one metre, also served as a good station mark.

٤- The mark may be set on a concrete monument. The station should be marked with a copper or bronze tablet. The name of the station and the date on which it was set, should be stamped on the tablet.

٥- In earth, generally two marks are set, one about ٧٥ cm below the surface of the ground, and the other extending a few centimeters above the surface of the ground. The underground mark may consist of a stone with a copper bolt in the centre, or a concrete monument with a tablet mark set on it (Fig. ١,٢٣).

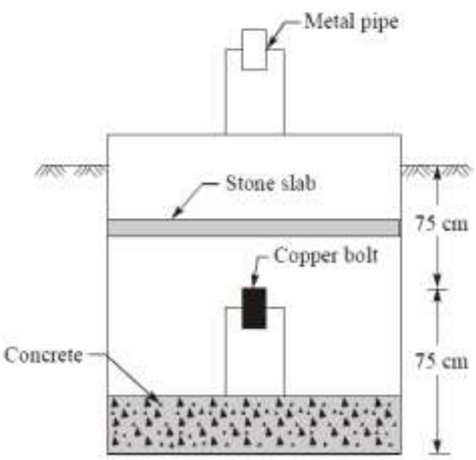


Fig. 1.23 Station mark

٦- The station mark with a vertical pole placed centrally should be covered with a conical heap of stones placed symmetrically. This arrangement of marking station, is known as placing a cairn (Fig. ١,٢٧).

٧- Three reference marks at some distances on fairly permanent features, should be established to locate the station mark, if it is disturbed or removed.

٨- Surrounding the station mark a platform ٣ m × ٣ m × ٠,٥ m should be built up of earth.

١,١٣ SIGNALS

Signals are centered vertically over the station mark, and the observations are made to these signals from other stations. The accuracy of triangulation is entirely dependent on the degree of accuracy of centering the signals. Therefore, it is very essential that the signals are truly vertical, and centered over the station mark Greatest care of centering the transit over the station mark will be useless, unless some degree of care in centering the signal is impressed upon.

A signal should fulfil the following requirements:

(i) It should be conspicuous and clearly visible against any background. To make the signal conspicuous, it should be kept at least ٧٥ cm above the station mark.

(ii) It should be capable of being accurately centered over the station mark.

(iii) It should be suitable for accurate bisection from other stations.

(iv) It should be free from phase, or should exhibit little phase (cf., Sec. ١,١٥).

١,١٣,١ Classification of signals

The signals may be classified as under:

(i) Non-luminous, opaque or daylight signals

(ii) Luminous signals.

(i) Non-luminous signals

Non-luminous signals are used during day time and for short distances. These are of various types, and the most commonly used are of following types.

(a) Pole signal (Fig. ١,٢٤) : It consists of a round pole painted black and white in alternate strips, and is supported vertically over the station mark, generally on a tripod. Pole signals are suitable upto a distance of about ٦ km.

(b) Target signal (Fig. ١,٢٥): It consists of a pole carrying two squares or rectangular targets placed at right angles to each other. The targets are generally made of cloth stretched on wooden frames.

Target signals are suitable upto a distance of ٣٠ km.

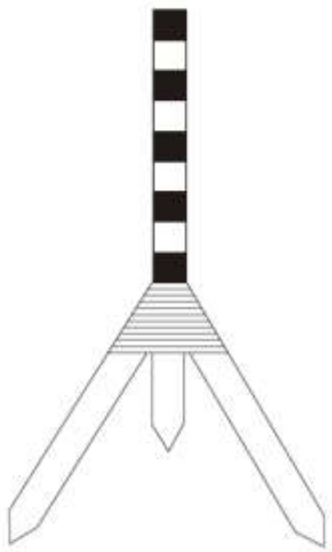


Fig. ١,٢٤ Pole signal

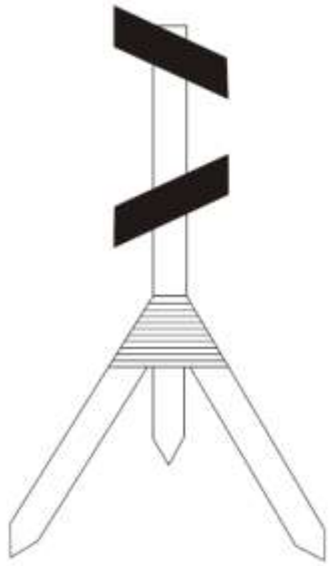


Fig. ١,٢٥ Target signal

(c) Pole and brush signal (Fig. ١,٢٦): It consists of a straight pole about ٢,٥ m long with a bunch of long grass tied symmetrically round the top making a cross. The signal is erected vertically over the station mark by heaping a pile of stones, upto ١,٧ m round the pole. A rough coat of white is given to make it more conspicuous to be seen against black background. These signals are very useful, and must be erected over every station of observation during reconnaissance.

(d) Stone cairn (Fig. ١,٢٧): A pile of stone heaped in a conical shape about ٣ m high with a cross shape signal erected over the stone heap, is stone cairn. This white washed opaque signal is very useful if the background is dark

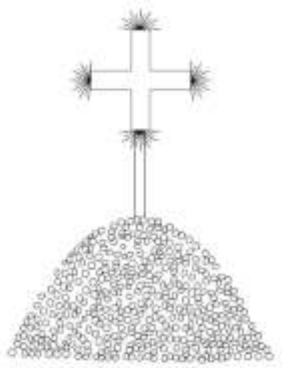


Fig. 1.26 Pole and brush signal

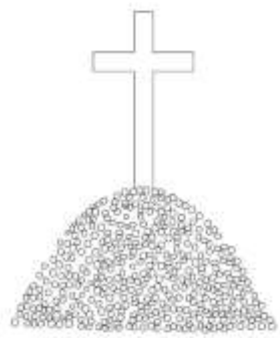


Fig. 1.27 Stone cairn

(e) Beacons (Fig. ١,٢٨): It consists of red and white cloth tied round the three straight poles. The beacon can easily be centered over the station mark. It is very useful for making simultaneous observations.

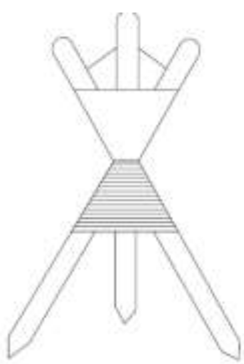


Fig. 1.28 Beacon

(ii) Luminous signals

Luminous signals may be classified into two types :

(i) Sun signals

(ii) Night signals.

(a) Sun signals (Fig. ١,٢٩): Sun signals reflect the rays of the sun towards the station of observation, and are also known as heliotropes. Such signals can be used only in day time in clear weather.

Heliotrope: It consists of a circular plane mirror with a small hole at its centre to reflect the sun rays, and a sight vane with an aperture carrying a cross-hairs. The circular mirror can be rotated horizontally as well as vertically through ٣٦٠°. The heliotrope is centered over the station mark, and the line of sight is directed towards the station of observation. The sight vane is adjusted looking through the hole till the flashes given from the station of observation fall at the centre of the cross of the sight vane. Once this is achieved, the heliotrope is disturbed. Now the heliotrope frame carrying the mirror is rotated in such a way that the black shadow of the small central hole of the plane mirror falls exactly at the cross of the sight vane. By doing so, the reflected beam of rays will be seen at the station of observation. Due to motion of the sun, this small shadow also moves, and it should be constantly ensured that the shadow always remains at the cross till the observations are over.

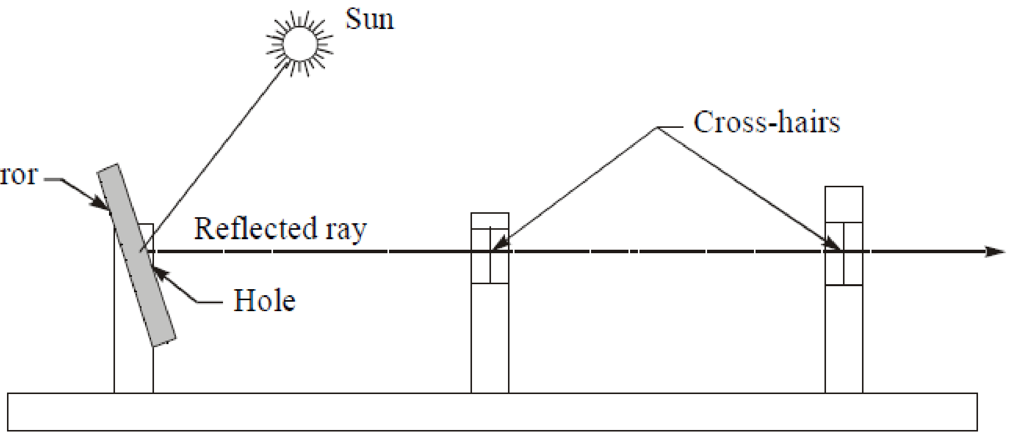


Fig. 1.29 Heliotrope

The heliotropes do not give better results compared to signals. These are useful when the signal station is in flat plane, and the station of observation is on elevated ground. When the distance between the stations exceed ٣٠ km, the heliotropes become very useful.

(b) Night signals: When the observations are required to be made at night, the night signals of following types may be used.

١. Various forms of oil lamps with parabolic reflectors for sights less than ٨٠ km.

٢. Acetylene lamp designed by Capt. McCaw for sights more than ٨٠ km.

٣. Magnesium lamp with parabolic reflectors for long sights.

٤. Drummond's light consisting of a small ball of lime placed at the focus of the parabolic reflector, and raised to a very high temperature by impinging on it a stream of oxygen.

٥. Electric lamps.

١,١٤ TOWERS

A tower is erected at the triangulation station when the station or the signal or both are to be elevated to make the observations possible form other stations in case of problem of intervisibility. The height of tower depends upon the character of the terrain and the length of the sight. The towers generally have two independent structures. The outer structure is for supporting the observer and the signal whereas the inner one is for supporting the instrument only. The two structures are made entirely independent of each other so that the movement of the observer does not disturb the instrument setting. The two towers may be made of masonary, timber or steel. For small heights, masonary towers are most suitable. Timber scaffolds are most commonly used, and have been constructed to heights over ٥٠ m. Steel towers made of light sections are very portable, and can be easily erected and dismantled. Bilby towers patented by J.S. Bilby of the U.S. Coast and Geodetic Survey, are popular for heights ranging from ٣٠ to ٤٠ m. This tower weighing about ٣ tonnes, can be easily erected by five persons in just ٥ hrs. A schematic of such a tower is shown in Fig. ١,٣٠.

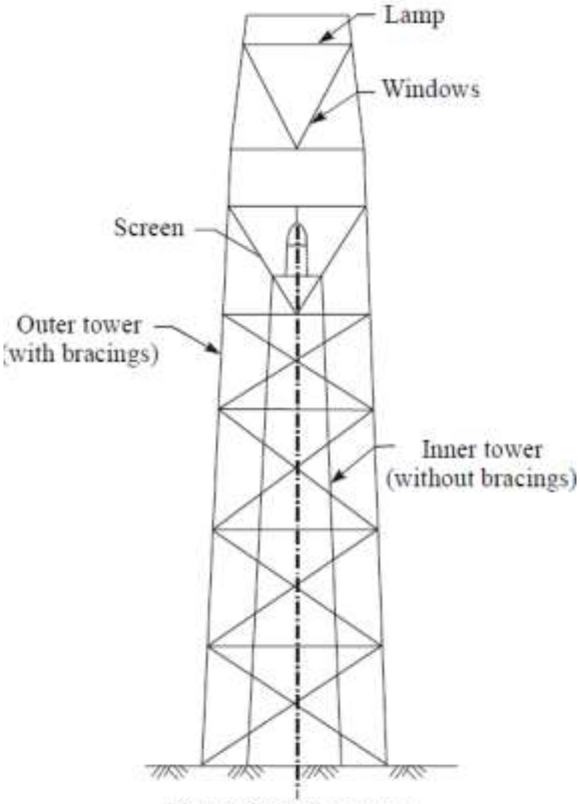


Fig. 1.30 Bilby tower