UNIVERSITY OF TECHNOLOGY
MATERIAL ENGINEERING DEPT.
MATHEMATICS

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Chapter 1

Lecture 1.

Preliminaries

- **OVERVIEW:** This chapter reviews the basic ideas you need to start calculus. The Greek alphabet, Algebra, The topics include the real number system, Intervals, Solving Inequalities, Cartesian coordinates in the plane, straight lines, functions, and trigonometry. We also give Conic Section(Circle, Parabolas, Ellipses, Hyperbola).

Greek Alphabet

Greek name	Greek letter	
	Lower case	Capital
Alpha	α	A
Beta	β	B
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ϵ	E
Zeta	ζ	Z
Eta	η	H
Theta	θ	Θ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M

Greek name	Greek letter	
	Lower case	Capital
Nu	ν	N
Xi	ξ	Ξ
Omicron	o	O
Pi	π	Π
Rho	ρ	P
Sigma	σ	Σ
Tau	τ	T
Upsilon	υ	Υ
Phi	ϕ	Φ
Chi	χ	X
Psi	ψ	Ψ
Omega	ω	Ω

Algebra

ARITHMETIC OPERATIONS

$$a(b + c) = ab + ac$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

FACTORING SPECIAL POLYNOMIALS

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

EXPONENTS AND RADICALS

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$$|x| = a \quad \text{means} \quad x = a \quad \text{or} \quad x = -a$$

$$|x| < a \quad \text{means} \quad -a < x < a$$

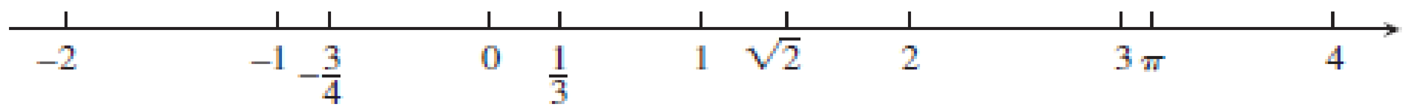
$$|x| > a \quad \text{means} \quad x > a \quad \text{or} \quad x < -a$$

Real Numbers and the Real Line

Much of calculus is based on properties of the real number system. **Real numbers** are numbers that can be expressed as decimals, such as

$$-\frac{3}{4} = -0.75000\ldots$$
$$\frac{1}{3} = 0.33333\ldots$$
$$\sqrt{2} = 1.4142\ldots$$

The real numbers can be represented geometrically as points on a number line called **the real line**.



The symbol $\mathbb{R}$ denotes either the real number system or, equivalently, the real line.

We distinguish three special subsets of real numbers.

1. The **natural numbers**, namely 1, 2, 3, 4, . . .
2. The **integers**, namely $0, \pm 1, \pm 2, \pm 3, \ldots$
3. The **rational numbers**, namely the numbers that can be expressed in the form of a fraction m/n, where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{3}, \quad -\frac{4}{9} = \frac{-4}{9} = \frac{4}{-9}, \quad \frac{200}{13}, \quad \text{and} \quad 57 = \frac{57}{1}.$$

1. Intervals

A subset of the real line is called an **interval** if it contains at least two numbers and contains all the real numbers lying between any two of its elements. For example, the set of all

real numbers *x* such that $x > 6$ is an interval, as is the set of all *x* such that $-2 \leq x \leq 5$ The set of all nonzero real numbers is not an interval; since 0 is absent, the set fails to contain

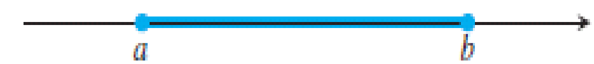
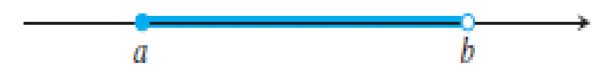
every real number between-1 and 1 (for example).

Geometrically, intervals correspond to rays and line segments on the real line, along with the real line itself. Intervals of numbers corresponding to line segments are **finite intervals**; intervals corresponding to rays and the real line are **infinite interval .**

- **Solving Inequalities:**

The process of finding the interval or intervals of numbers that satisfy an inequality in *x* is called **solving** the inequality.

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \le x \le b\}$	Closed	
	$[a, b)$	$\{x \mid a \le x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \le b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \ge a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \le b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open	

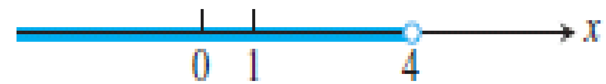
EXAMPLE 1: Solve the following inequalities and show their solution sets on the real line.

a. $2x - 1 < x + 3$ b. $-\frac{x}{3} < 2x + 1$ c. $\frac{6}{x-1} \geq 5$

Solution:

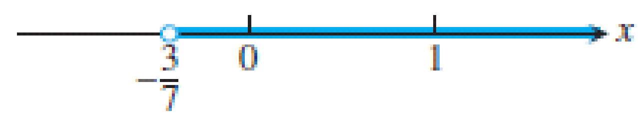
a. $2x - 1 < x + 3 \rightarrow 2x < x + 4 \rightarrow x < 4$

The solution set is the open interval $(-\infty, 4)$



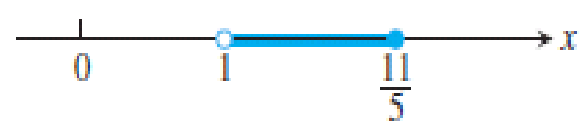
b . $-\frac{x}{3} < 2x + 1 \rightarrow -x < 6x + 3 \rightarrow 0 < 7x + 3$

$\rightarrow -3 < 7x \rightarrow -\frac{3}{7} < x$



The solution set is the open interval $(-\frac{3}{7}, \infty)$

C . $\frac{6}{x-1} \geq 5 \rightarrow 6 \geq 5x - 5 \rightarrow 11 \geq 5x \rightarrow \frac{11}{5} \geq x$



The inequality can hold only if $x > 1$,

Therefore, $(x - 1)$ is positive . solution set is the half-open interval

$(1, 11/5)$

- ***Absolute Value:***

The **absolute value** of a number *x*, denoted by $|x|$ is defined by the formula.

$$|x| = \begin{cases} x, & x \geq o \\ -x & x < 0 \end{cases}$$

EXAMPLE 2: Finding Absolute Values

$|3| = 3$, $|0| = 0$, $|-5| = -(-5) = 5$, $|-|a|| = |a|$

$|x| = \sqrt{x^2}$, $|a| = \sqrt{a^2}$

Absolute Values and Intervals

If a is any positive number, then

5. $|x| = a$ if and only if $x = \pm a$
6. $|x| < a$ if and only if $-a < x < a$
7. $|x| > a$ if and only if $x > a$ or $x < -a$
8. $|x| \leq a$ if and only if $-a \leq x \leq a$
9. $|x| \geq a$ if and only if $x \geq a$ or $x \leq -a$

Exercises 1.1

Solve the inequalities and show the solution sets on the real line.

1. $3(2 - x) > 2(3 + x)$
2. $\frac{4}{5}(x - 2) < \frac{1}{3}(x - 6)$
3. $-\frac{x+5}{2} \leq \frac{12+3x}{4}$
4. $\left|\frac{x}{5} - 1\right| \leq 1$
5. $\left|3 - \frac{1}{x}\right| < \frac{1}{2}$
6. $\left|\frac{3x}{5} - 1\right| > \frac{2}{5}$
7. $\left|\frac{2x+7}{3}\right| \leq 5$
8. $(x - 1)^2 < 4$
9. $x^2 - x < 0$
10. $x^2 - x - 2 \geq 0$

2. Lines

”This section reviews coordinates, lines, and distance.

● *Cartesian Coordinates in the Plane:*

draw two perpendicular coordinate lines that intersect at the 0-point of each line. These lines are called **coordinate axes** in the plane. On the horizontal x-axis, numbers are denoted by x increase to the right. On the vertical y-axis, numbers are denoted by y and increase upward *(Figure 1.5).* Thus “upward” and “to the right” are positive directions, whereas “downward” and “to the left” are considered as negative. The **origin** O, of the coordinate system is the point in the plane where x and y are both zero. If P is any point in the plane write $P(a, b)$. This coordinate system is called the **rectangular coordinate system** or **Cartesian coordinate system** this coordinate divide the plane into four regions called **quadrants**, as shown in *Figure 1.6*.

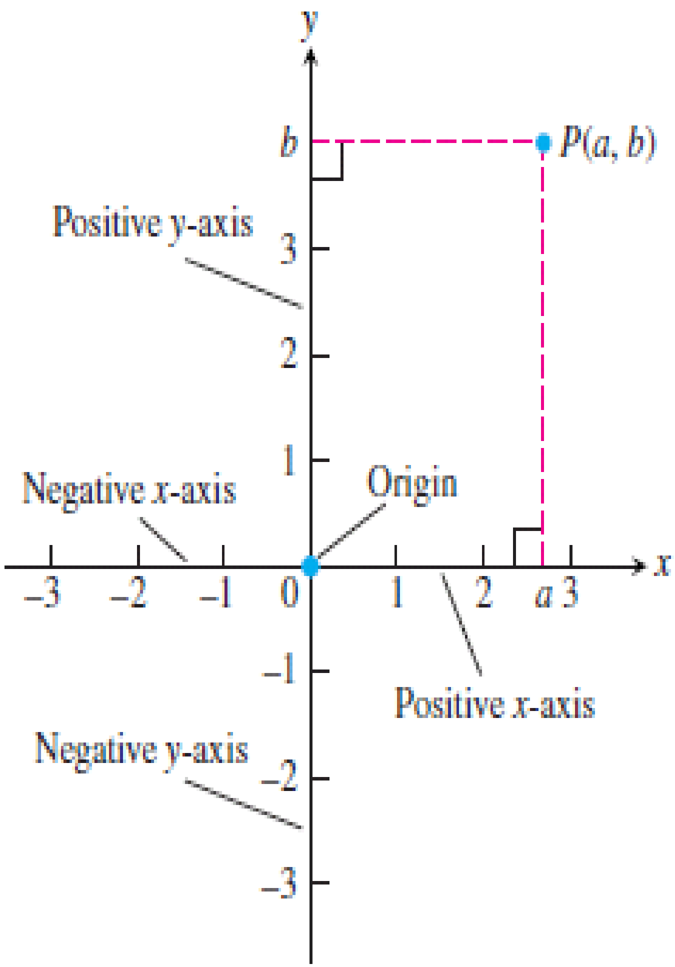


FIGURE 1.5 Cartesian coordinates in the plane are based on two perpendicular axes intersecting at the origin.

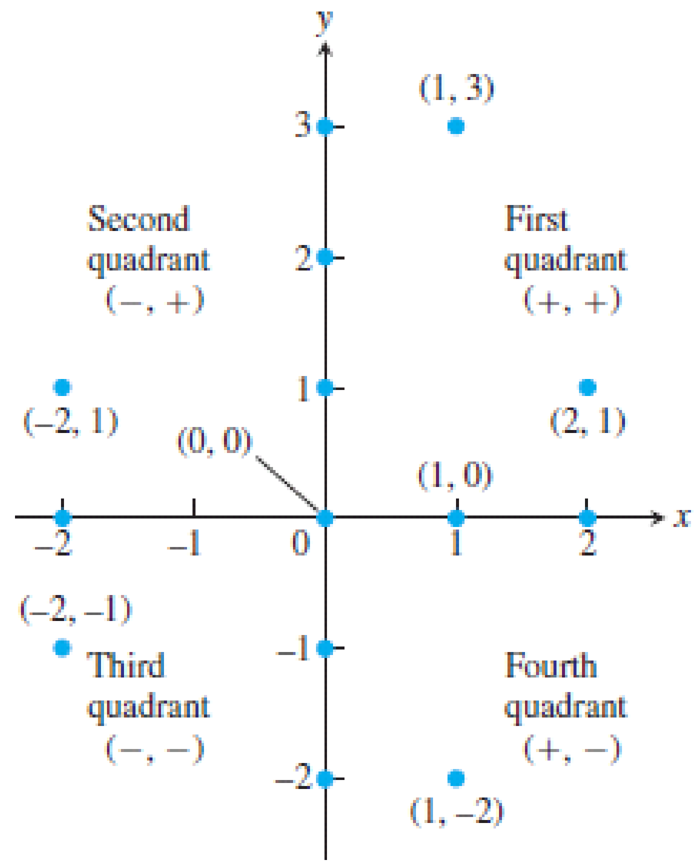


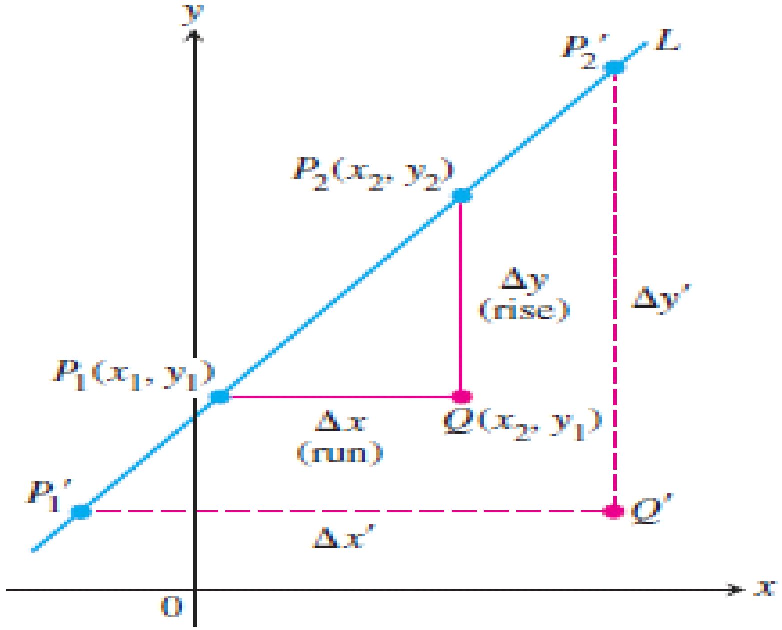
FIGURE 1.6 Points labeled in the xy-coordinate or Cartesian plane. The points on the axes all have coordinate pairs but are usually labeled with single real numbers, (so (1, 0) on the x-axis is labeled as 1). Notice the coordinate sign patterns of the quadrants.

• *Straight Lines (slop and equation):*

Slop: Given two points $P_1(x_1, y_1)$,and $P_2(x_2, y_2)$ in the plane, $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ the **run** and the **rise**, respectively, between P_1 and P_2 two such points straight line passing through them both. we call the line $P_1 P_2$. any nonvertical line in the plane has the property that the ratio .

the constant $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1}$

is the **slope** of the nonvertical line $P_1 P_2$.



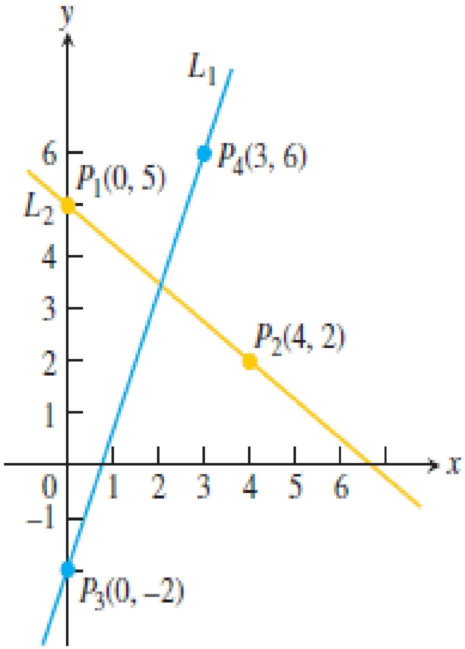
EXAMPLE 1: Find the slop of each line as shown.

solution: The slope of L1 is

$$m = \frac{\Delta y}{\Delta x} = \frac{6-(-2)}{3-0} = \frac{8}{3}$$

The slope of L2 is

$$m = \frac{\Delta y}{\Delta x} = \frac{2-5}{4-0} = \frac{-3}{4}$$



• Straight lines have relatively simple equations. All points on the *vertical line* through the point *a* on the *x*-axis have *x*-coordinates equal to *a*.Thus,x=a is an equation for the vertical line. y=b is an equation for the *horizontal line* meeting the *y*- axis at *b*.(see Figure 1.1)

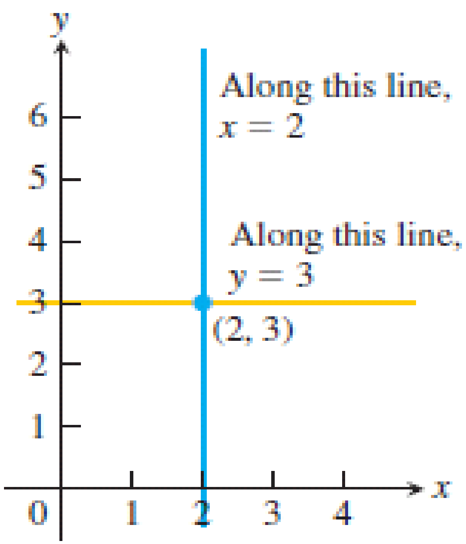


FIGURE 1.12 The standard equations for the vertical and horizontal lines through (2, 3) are $x = 2$ and $y = 3$.

- ***point-slope equation:***

We can write an equation for a nonvertical straight line *L* if we know its slope *m* and the coordinates of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is *any* other point on *L*, then we can use the two points P_1 and *P* to compute the slope,

$m = \frac{y-y1}{x-x1}$ so that $y - y_1 = m(x - x_1)$ or $y = y_1 + m(x - x_1)$

the equation $Y = Y_1 + m(X - X_1)$

is the **point-slope equation** of the line that passes through the point (X_1, Y_1) *and has slop m.*

EXAMPLE 2: Write an equation for the line through the point (2, 3) with slope -3⁄ 2 .

Solution: We substitute $x_1 = 2$, $y_1 = 3$, and m=-3⁄ 2 into the point-slope equation and obtain $y = 3 - \frac{3}{2}(x - 2)$ *or* $y = -\frac{3}{2}x + 6$

EXAMPLE 3: Write an equation for the line through(-2,-1) and (3,4).

Solution: The line's slope is

$$m = \frac{y2-y1}{x2-x1} \rightarrow m = \frac{4-(-1)}{3-(-2)} = \frac{5}{5} = 1$$

We can use this slope with either of the two given points in the point-slope equation: with $(x_1,y_1)=(-2,-1)$

$$y = y_1 + m(x - x_1) \rightarrow y = -1 + 1(x-(-2))$$

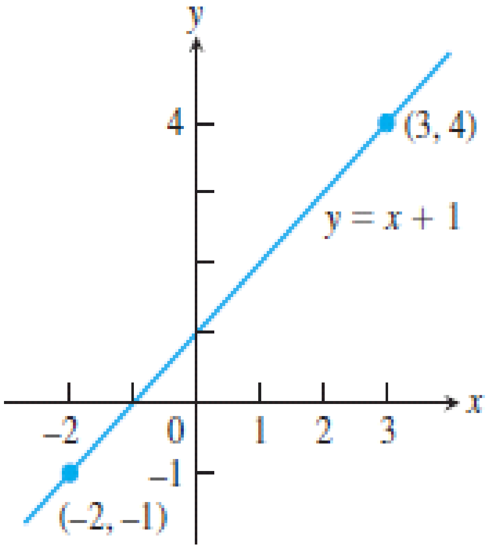
$$\rightarrow y = -1 + x + 2$$

$$\rightarrow y = x + 1$$

with $(x_1,y_1) = (3,4)$

$$y = 4 + 1(x - 3) \rightarrow y = 4 + x - 3$$

$y = x + 1$ same result



Either way, $y = x + 1$ is an equation for the line (Figure 1.13).

- ***Slope-intercept equation:***

The *y*-coordinate of the point where a nonvertical line intersects the *y*-axis is called the ***y*-intercept** of the line. Similarly, the ***x*-intercept** of the line (Figure 1.14). A line with slope *m* and *y*-intercept *b* passes through the point (0, *b*), so it has equation.

$$y = b + m(x - 0)$$

the equation $y = mx + b$ is called the slop-intercept equation of the line with slop m and y-intercept b.

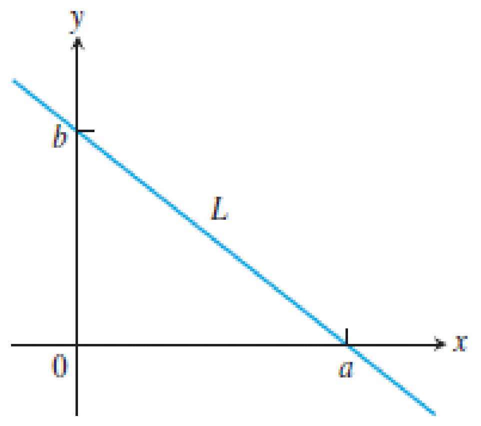


FIGURE 1.14 Line *L* has *x*-intercept *a* and *y*-intercept *b*.

Note: Lines with equations of the form y + mx have *y*-intercept 0 and so pass through the origin.

EXAMPLE 4:

Finding the Slope and *y*-Intercept Find the slope and *y*-intercept of the line $8x + 5y = 20$.

Solution: Solve the equation for *y* to put it in slope-intercept form:

$$8x + 5y = 20 \quad \rightarrow 5y = -8x + 20 \rightarrow y = -\frac{8}{5}x + 4$$

The slope is $m = -\frac{8}{5}$. The *y*-intercept is $b = 4$.

- **Parallel and Perpendicular Lines:**

Lines that are parallel, so they have the same slope. $m_1 = m_2$

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other:

$$m_1 = -\frac{1}{m_2} \quad , \quad m_2 = -\frac{1}{m_1}$$

- **Distance in the Plane**

The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem (Figure 1.16).

The distance between *P*(*x*1 , *y*1) and *Q(x2,y2) is*

d = $\sqrt{(\Delta x + \Delta y)^2} = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

EXAMPLE 5: Calculating Distance

(a) The distance between *p(-1,2)* and *Q*(3, 4) is $\sqrt{(3-(-1))^2 + (4-2)^2} = \sqrt{(4)^2 + (2)^2}$

$= \sqrt{20} = \sqrt{4.5} = 2\sqrt{5}$

(b) The distance from the origin to *P*(*x*, *y*) is

$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$.

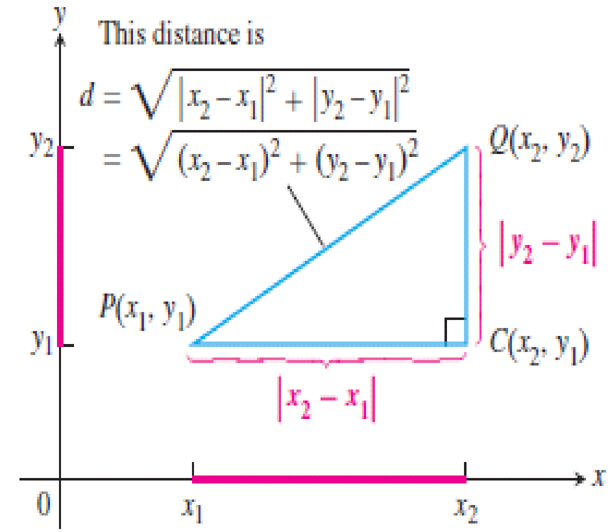


FIGURE 1.16 To calculate the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$, apply the Pythagorean theorem to triangle *PCQ*.

$P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Exercises1.2:

Write an equation for each line described.

1. Passes through (-8,0) and (-1,3) .
2. Passes through (2,-3)with slop $1/2$.
3. Has slope $-5/4$ and *y*-intercept 6 .
4. Passes through (-12, -9) and has slope 0 .
5. Passes through ($1/3$, 4), and has no slope .
6. Has *y*-intercept -6 and *x*-intercept 2 .
7. Passes through($-\sqrt{2}$, 2)parallel to the line $\sqrt{2}$ x+ 5y= $\sqrt{3}$.
8. Passes through(4,10) and is perpendicular to the line 6x - 3y=5 .
9. the vertical line through (0, -3).
10. the horizontal line through (0, 2).