Northern Technical University

College of Technical Kirkuk

Surveying Department

Spherical trigonometry and astronomy

Third Stage

Introduction:

The science of field astronomy offers to surveyors a means of determining the absolute location of any point or absolute location and direction of any line on the surface of the earth, by making astronomical observations to celestial bodies.

To understanding the real and apparent motions of these celestial bodies surveyors must be familiar with the geometry of spheres and spherical triangles.Field astronomy has wide scope in geodetic surveying for determining of true meridian, latitude, longitude and time.

Application:

١. Determining the azimuth of the starting base of the triangulation series.
٢. The azimuth of starting and closing sides of precise traverses.
٣. Determining the latitude and longitude of at least one of the triangulation stations so as to locate its position on the earth.
٤. To check the accuracy of triangulation series as suitable intervals independently.
٥. Determining the international boundaries.

Target populations:

Undergraduate students (third year) – surveying engineering department

Course reading and references:

١. Advanced Surveying by R. Agur ٢٠٠٨
٢. Higher Surveying B. C. Punmia ٢٠٠٥
٣. Text Book of spherical Astronomy by W. M. Smart (sixth edition ١٩٧٧)
٤. المثلثات الكروية ليعقوب يوسف

After completing the subject the student will be able to:

١. the purpose and application of field astronomy in surveying.

٢. the importance properties of sphere.

٣. Introduction to elements and properties of spherical triangle.

٤. Using sine, cosine formula and Napier's rule to solve spherical triangle.

٥. determining the area of spherical triangle, the shortest distance between two points.

٦. Using different coordinate system to locate the position of heavenly bodies on the celestial sphere.

٧. Applying correction to the observed altitude of the celestial bodies for deducing their true altitude, at the time of observation.

٨. Introduction to the time, motion of the sun and earth, defining the systems used in measuring time.

٩. Determining the local sidereal time at upper transit from knowing the right ascension of the star, calculating the hour angle of star at the instant of elongation by knowing the value of latitude of the place and the declination of the circumpolar star

١٠. Studying the most generally used methods for determining the latitude of a place.

١١. using the difference in local times of two places to determine the difference in their longitudes, by various method.

١٢. use the star charts.

Syllabus

Week	Subject	Central ideas	Objective
١	Introduction to the field astronomy	Purpose of field astronomy	To introduce the student on the purpose and application of field astronomy in surveying
٢, ٣	Geometry of sphere	The important properties of sphere, zenith and nadir, celestial horizon, terrestrial poles and equator, vertical circle, the meridian, the latitude and longitude, azimuth	Introduction to the importance properties of sphere
٤	Spherical triangle	Elements of a spherical triangle, spherical angles, solution of a spherical triangle (sine and cosine formula)	Introduction to elements and properties of spherical triangle, Using sine and cosine formula to solve spherical triangle
٥	Solution of spherical triangle	Napier's rule	Solving right angled spherical triangle by Napier's rule
٦	The area of spherical triangle	Area pf spherical triangle, spherical excess	Determining the area of spherical triangle
٧, ٨	Astronomical	The right	Using different coordinate

	coordinate system	ascensions and declination system Altitude and azimuth system Declination and hour angles system Celestial latitude and longitude system Relationship between various coordinates	system to locate the position of heavenly bodies on the celestial sphere
٩, ١٠	The shortest distance between two point on the earth	The parallel latitude and the value of one degree latitude and longitude Nautical mile	Determining the shortest distance between two point on the earth
١١, ١٢	Different position of star with respect to the observers meridians	Star at elongation, star at culmination Star at horizon, circumpolar star	For calculation of the azimuth of the star at the time of the observation,
١٣, ١٤	Corrections to the observed altitudes of the celestial bodies	Observational corrections (refracted , dip, parallax, semi-diameter	Applying correction to the observed altitude of the celestial bodies for deducing their true altitude, at the time of observation

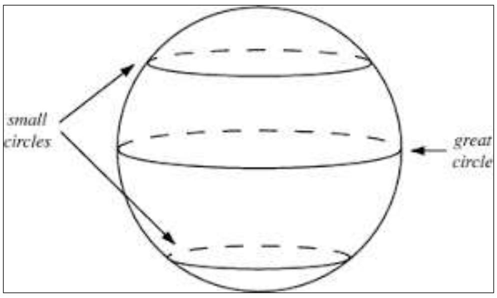
		correction) Instrumental correction (index error, bubble error, azimuth correction)	
١٥, ١٦	time	The earth and the sun, apparent motion of the heavenly bodies, classification of time (sidereal time, apparent solar time, mean solar time, standard time)	Introduction to the time, motion of the sun and earth, defining the systems used in measuring time.
١٧	Conversion between systems of time	Equation of time, conversion of time, conversion of standard time to local time	The difference between the mean and the apparent solar time.
١٨	=	Conversion of local mean time to local apparent time and vice versa	Conversion between different systems of time
١٩	=	Conversion of sidereal time interval to mean time interval	
٢٠	=	Conversion of local mean time at any	

		instant to local sidereal time if Greenwich sidereal time (G.S.T.) at Greenwich mean mid night (G.M.M.) is known	
٢١	=	Conversion of local sidereal time at any instant to local mean time if Greenwich sidereal time at Greenwich midnight (G.M.M or at G.M.N.)	
٢٢	=	Determination of the L.M.T. of the upper transit of a known star if G.S.T. of G.M.M. is known	Determining the local sidereal time at upper transit from knowing the right ascension of the star
٢٣	=	Determination of time of elongation of a circumpolar star	Calculating the hour angle of star at the instant of elongation by knowing the value of latitude of the place and the declination of the circumpolar star.
٢٤	Determination of latitude	By determination latitude by meridian altitude of a star	Studying the most generally used methods for determining the latitude of a place
٢٥	=	By determination latitude by equal meridian altitudes of two stars on either	Making observations upon two stars which culminate on the opposite sides of the observer's zenith, to reduce the error of observations

		side of zenith	
٢٦	=	By meridian altitudes of a circumpolar star at its upper and lower culminations	The altitude of a circumpolar star is measured both at its upper as well s the lower culmination, the knowledge of the declination of the star is not necessary.
٢٧	=	By ex meridian observations of star or sun	observe the altitude of the star at any position and the exact chronometer time is noted at the instant of observation.
٢٨	=	By altitude of a star on prime vertical	Measuring the time interval between east and west transits of the star, the altitude is not measured simply measure the interval of sidereal hours that elapses between the two transits.
٢٩	Determination of longitudes	By transportation of chronometers, by listening to radio signals, by observing the stars which culminate at the same time	using the difference in local times of two places to determine the difference in their longitudes, by various methods
٣٠	constellations	Zodiacal constellations star almanacs and star charts	Using the star charts by surveyors

Geometry of sphere

A sphere is a solid bounded by a surface whose every point is equidistant from a fixed point called Centre of the sphere



١- A section of a sphere is called a **great circle** if the section plane passes through the Centre of the sphere
٢- A section of a sphere is called a **small circle** when the plane cutting the sphere does not pass through the Centre of the sphere
٣- A diameter of a sphere perpendicular to a great circle is called the axis of the great circle

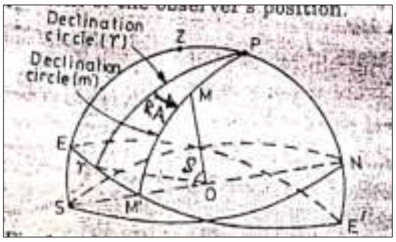
Astronomical Terms

١- **The celestial sphere** :The imaginary sphere on which heavenly bodies, i.e. stars, sun, moon, etc. appear to lie
٢- The Zenith: The point on the celestial sphere exactly above the observers station
٣- **The Nadir:** The point on the celestial sphere exactly below the observers station
٤- **The celestial Horizontal** : The great Circle of the celestial sphere obtained by a plane passing through the Centre of the earth and perpendicular to Zenith-Nadir line
٥- **The celestial equator**: The great circle of the celestial sphere, the plane of which is perpendicular to the axis of rotation of the earth and it is continuation
٦- **The celestial poles**: The points at which the earth's axis of rotation on prolongation on either side, meets the surface of the celestial sphere
٧- **Vertical circles:** The great circles of the celestial sphere which pass through the Zenith and Nadir of the station
٨- **The observer's meridian:** The Vertical circle which passes through the Zenith and Nadir of the station of observation as well as through the poles
٩- **The prime vertical:** The vertical circle which is perpendicular to the observer's meridian and passes through the east and west points of the horizon.

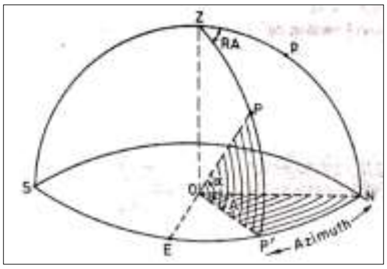
١٠- **North and south points:** The projected points of the elevated North Pole and depressed south poles on the horizon.
١١- **Ecliptic:** The great circle of the celestial sphere which the sun appears to describe with earth as centre during a period of one year.
١٢- **First points of Aries and Libra:** The first point of Aries(♈) is the point where the sun crosses the equator from south to north on or about ٢١ st March, when day and night are of equal duration. The first point of Libra (♎) is the point where the sun crosses the equator from north to south.

Astronomical Coordinate

١-Right ascension and declination system



٢-Altitude and Azimuth system



Declination (δ): The angular distance of the celestial body from the celestial equator along the great circle passing through the celestial poles and the celestial body.

Right Ascension: The equatorial angular distance measured eastward from the declination circle of the first point of Aries to the declination circle of the celestial body.

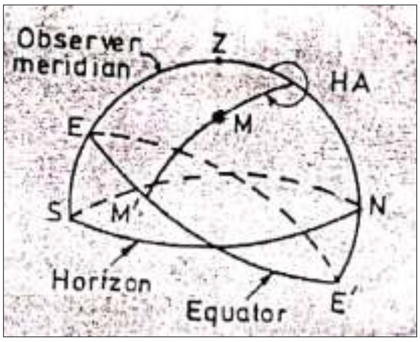
The Azimuth (A): the angle between the observer's meridian and the vertical circle passing through the celestial body and the zenith.

The Altitude (α): The angular distance of a heavenly body above the horizon, measured on the vertical circle passing through it.

Declination and hour angle:-

Hour angle (HA): The angular distance along the arc of the horizon measured from the observer's meridian westward to the declination circle of the body.

Relationships between Varian's coordinates



<Ez+Zp=٩٠°

<Np+pz=٩٠°

<Ez+Zp=<Np+pz

<Ez=<Np

θ+Zp= α+pz

θ= α

The altitude of the pole is always equal to the latitude of the observer position

The parallel of Latitude

A small circle through a point perpendicular to the axis of rotation of the earth, is known as the parallel of the point. (As the latitudes of various parallels increase, the radii of the parallels decrease.

The value of one degree of Latitude

The value of one degree of latitude is equal to

$\frac{2\pi * 6370}{360}$ = ١١١,١٢km

The value of a degree of latitude is a constant value everywhere.

The value of a degree of longitude is maximum, i.e. ١١١,١٢km. This value decreases as the latitude increases and finally its value becomes zero at the north and south poles.

The Nautical Mile

The angular distance along the great circle corresponding to an angle of one minute arc subtended at the Centre of the earth.

Nautical Mile=$\frac{2\pi * 6370}{360 * 60}$=١,٨٥٣km

Spherical Triangle

The triangle which formed upon the surface of a sphere by the intersection of three great circle is called spherical triangle

Properties of a spherical triangle

١-Angle opposite to equal sides is equal and vice versa

٢-Any angle is less than two right angles

Right angle =٩٠°

Two Right angles= ١٨٠°

٣-The sum of the three angles is always greater than two right angles but less than six right angles

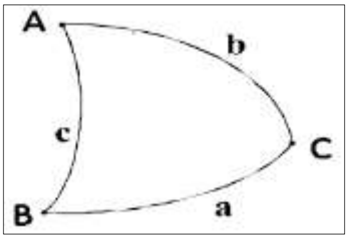
٤-The sum of any two sides is greater than the third

٥-The difference between two sides is less than the third

٦- The greater angle is opposite the greater side and vice versa

Solution of Spherical triangle

Knowing any three elements a,b,c,A,B,C of a spherical triangle ABC, the remaining three elements may be computed



Sine formula

$$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$$

Cosine formula

Cos a= cos b*cos c+ sin b *sin c *cos A

Cos b= cos a*cos c+ sin a *sin c *cos B

Cos c= cos a*cos b+ sin a *sin b*cos C

$$\text{Cos A}=\frac{\cos a-\cos \mathrm{b}*\cos \mathrm{c}}{\sin \mathrm{b} *\sin \mathrm{c}}$$

$$\text{Sin } \frac{A}{٢}=\sqrt{\frac{\sin(s-b)*\sin(s-c)}{\sin b*\sin c}}$$

$$\text{Cos } \frac{A}{٢}=\sqrt{\frac{\sin s*\sin(s-a)}{\sin b*\sin c}}$$

$$\text{Tan } \frac{A}{٢}=\sqrt{\frac{\sin(s-b)*\sin(s-c)}{\sin s*\sin(s-a)}}$$

Where $s=\frac{a+b+c}{٢}$

$$\text{Tan}\ \frac{١}{٢}\ (a+b) = \frac{\cos\frac{١}{٢}\ (A-B)}{\cos\frac{١}{٢}\ (A+B)} * \tan\frac{c}{٢}$$

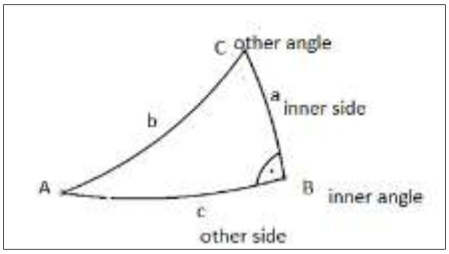
$$\text{Tan}\ \frac{١}{٢}\ (a-b) = \frac{\sin\frac{١}{٢}\ (A-B)}{\sin\frac{١}{٢}\ (A+B)} * \tan\frac{c}{٢}$$

$$\text{Tan}\ \frac{١}{٢}\ (A+B) = \frac{\cos\frac{١}{٢}\ (a-b)}{\cos\frac{١}{٢}\ (a+b)} * \cot\frac{c}{٢}$$

$$\text{Tan}\ \frac{١}{٢}\ (A-B) = \frac{\sin\frac{١}{٢}\ (a-b)}{\sin\frac{١}{٢}\ (a+b)} * \cot\frac{c}{٢}$$

The four parts formula

Cos a* cos B= sin a cot c – sin B * cot C

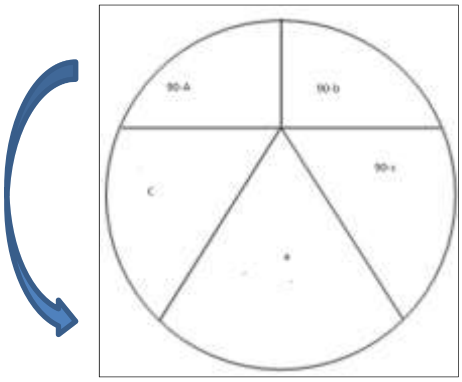


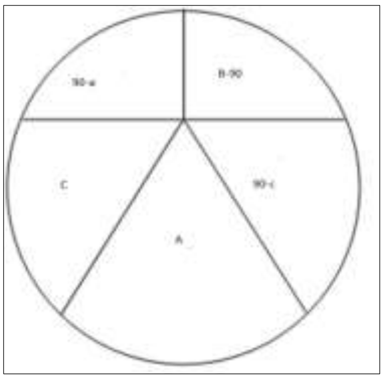
Solution of a right angle spherical triangle by Napier's Rule

Sin a = tan c tan(٩٠°-C)

sin a =cos (٩٠ –A)*cos (٩٠-b)

sin c =tan a tan (٩٠-A)





If b= ٩٠°

Spherical excess

The three angles of a spherical triangle don't sum up exactly to ١٨٠°

e=A+b+C-١٨٠°

Area $\Delta = \frac{\pi R^{٢} e}{١٨٠°}$

Geometry of an Astronomical triangle

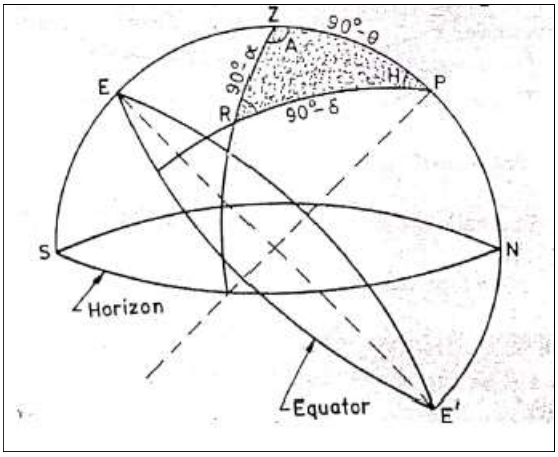
Az = Azimuth

HA= Hour angle

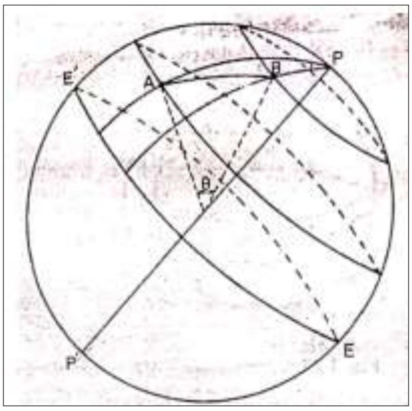
α = Altitude

δ= Declination

θ= Latitude



Example: - Calculated the shortest distance between two places A and B given that the latitudes of A and B ٢٨°٣٠´N and ٣٢°٤٢´N and longitudes are ٧٦°١٨´E and ٨٢°٥٤`E respectively?



Solution:-

AP= ٩٠°-٢٨°٣٠´=٦١°٣٠´

BP= ٩٠°-٣٢°٤٢´=٥٧°١٨´

Ø=٨٢°٥٤´-٧٦°١٨´=٦°٣٦´

Cos AB=Cos AP* Cos BP+ Sin AP* Sin BP * Cos P

= Cos ٦١°٣٠´* Cos ٥٧°١٨´+ Sin ٦١°٣٠´* Sin ٥٧°١٨´* Cos ٦°٣٦´

Cos AB =٠,٩٩٢

AB=$cos^{-١}$ ٠,٩٩٢ =٧°٠٣´٤٣˝.٣

The shorted in Km = R*angle *$\frac{\pi}{١٨٠°}$

=$\frac{٦٣٧٠*٧°٠٣´٤٣˝.٣*\pi}{١٨٠°}$

= ٧٨٥,١٣٨km

Example: - Calculate the distance in kilometer between two points A and B along the parallel of latitude given that

١) Lat of A=٢٨°٤٢´N Long of A =٣١°١٢´W

Lat of B=٢٨°٤٢´N Long of B =٤٧°٢٤´W

٢) Lat of A=١٢°٣٦´S Long of A =١١٥°٦´W

Lat of B=١٢°٣٦´S Long of B =١٥٠°٢٤´E

Solution

١) Difference of longitude between A and B

=٤٧°٢٤´W -٣١°١٢´W

=١٦°١٢= ٩٧٢´

Distance in Nautical miles = Difference of longitude *Cos latitude in minutes

= ٩٧٢ min * Cos ٢٨°٤٢´

=٨٥١,٧٢ N.M

Km=٨٥١,٧٢ * ١,٨٥٢ = **١٥٧٧,٣٤Km**

٢) Difference of longitude between A and B

=١١٥°٦´W-١٥٠°٢٤´E

=١١٥°٦´-(-١٥٠°٢٤´)

= ٢٦٥°٣٠´

=٣٦٠°-٢٦٥°٣٠´=٩٤°٣٠´=٥٦٧٠min

Distance in Nautical miles = Difference of longitude *Cos latitude in minutes

= ٥٦٧٠ min * Cos ١٢°٣٦´

=٥٥٣٣,٤٥ N.M

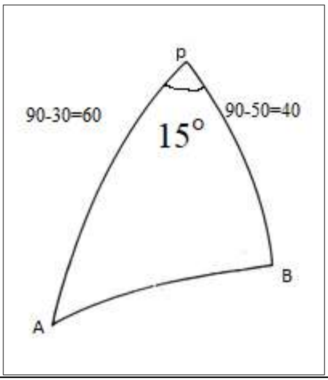
Km=٥٥٣٣,٤٥ * ١,٨٥٢ = **١٠٢٤٧,٢Km**

Example: - What is the geodetic area enclosed by the spherical triangle ABP on the earth's surface when the coordinate of the station are as follows?

A=٣٠°N ٤٥°E

B=٥٠°N ٦٠°E

R=٦٣٧٨ Km



Solution

<p=٦٠°-٤٥°=١٥°

Cos AB= cos ٦٠° cos ٤٠°+ sin ٦٠° sin ٤٠° cos ١٥°

Cos AB=٠,٩٢٠٧٢

AB= ٢٢,٩٦٧

$$\frac{\sin A}{\sin BP} = \frac{\sin P}{\sin AB}$$

Sin A= $\frac{\sin P}{\sin AB}$* $\sin BP$

=٠,٤٢٦٣

A= ٢٥°٢٣′٥٨″

$$\frac{\sin B}{\sin AP} = \frac{\sin P}{\sin AB}$$

Sin B = $\frac{\sin ١٥°}{\sin ٢٢.٩٦٧}$ *sin ٦٠°

Sin B = ٠,٥٧٤٥

<B=٣٥,٠٥٨

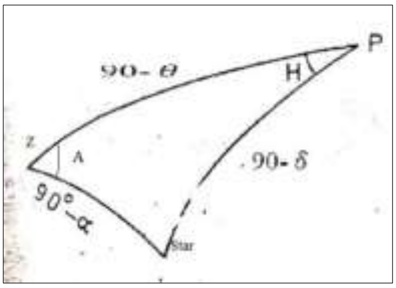
= ١٨٠°-٣٥,٠٥٨=١٤٤,٩١٥

e=A+B+P-١٨٠

=(٢٥,٢٣٥٨+١٤٤,٩١٥+١٥)-١٨٠ =١٨٥,١٧-١٨٠=٥,١٧

Area =$\frac{\pi * ٦٣٧٨^{٢}}{١٨٠°}$ * ٥,١٧ = **٣٦٨٧٤٦١,٦ Km^2**

Example: - Determine the hour angle and declination of a star from the following data



Latitude of place (θ)= ٤٨°٣٠′N

Azimuth of the star (A) = ٥٠° W

Altitude of the star (α) =٢٨°٢٤′

Solution

Cos(٩٠-δ)= Cos(٩٠- θ)* Cos(٩٠- α) + sin(٩٠- θ)*sin(٩٠- α) *Cos A

=Cos ٤١°٣٠´* Cos ٦١°٣٦´+ sin ٤١°٣٠´* sin ٦١°٣٦´*Cos ٥٠°

= ٠,٧٣٠٨

٩٠-δ= ٤٣°٠٢´٢١,٥˝

δ= ٤٦°٥٧´٣٨,٥˝

Sin H =$\frac{Sin\ A*Sin\ SZ}{Sin\ PS}$ = ٠,٩٨٧

H= ٨٠°٥٣´٠٥,٥˝

Example: - Find the Azimuth and the hour angle of the sun at sunset for a place of latitude ٤٩° N its declination being given to be ١٩° S?

Solution

δ=٩٠°-(-١٩°) = ١٠٩

Cos(٩٠-δ)= Cos(٩٠- θ)* Cos(٩٠- α) + sin(٩٠- θ)*sin(٩٠- α) *Cos A

Cos(٩٠-(-١٩°))= Cos(٩٠- ٤٩°)* Cos(٩٠- ٠) + sin(٩٠- ٤٩°)*sin(٩٠- ٠) *Cos A

Cos A = $\frac{-٠.٣٢٥}{٠.٦٥٦}$ = -٠,٤٩٦٢٤

A= ١١٩°٤٥´٧,٢˝

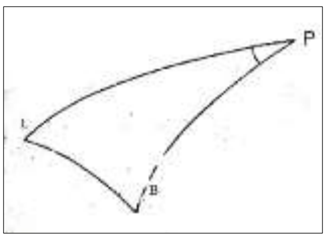
Cos H = $\frac{Cos\ sz-cos\ pz*cos\ ps}{sinpz*sinps}$

= ٠,٣٩٦١

H= ٦٦°٣٩´٥٤˝

Example: - Find the distance between London (UK) and Baghdad (Iraq)

L= (٥١,٣٠°N, ٠,١٠°W)



B= (٣٣,٢٠°N, ٤٤,٢٦°E)

Solution

Cos BL = Cos PL *Cos PB + Sin PL *Sin PB * Cos P

=Cos (٩٠-٥١,٣٠°) *Cos(٩٠- ٣٣,٢٠°)+ Sin(٩٠-٥١,٣٠°) *Sin(٩٠-٣٣,٢٠°)*Cos(٠,١٠°+٤٤,٢٦)

=٣٦,٧٤

BL $=\frac{R*\pi*٣٦.٧٤}{١٨٠}$ = **٤٠٨٢,٥ Km**

Example: - Find the distance between Chicago (UST) and Mexico city

Ch (٤١,٥٠°, ٨٧,٤٥°W)

M (١٩,٢٥°, ٩٩,١٠°W)

Example: - Find the distance between Buenos Aires and Athena

B(٣٤,٤٠°S, ٥٨,٣٠°W)

A (٣٦,٠٠°N, ٢٣,٤٤°E)

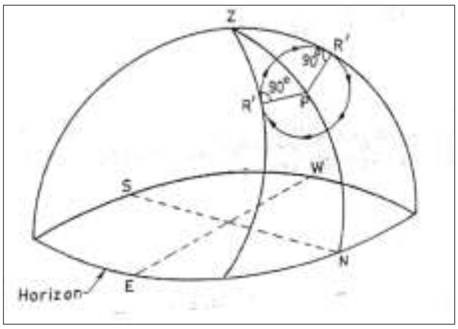
Different Positions of the Star with respect to the Observer's Meridian

The following positions of every star in the heaven are important to a surveyor.

١) Star at Elongation

A star is said to be at elongation when its distance east or west of the observer's meridian is the greatest

At elongation, the star does not move in azimuth, its motion being entirely in altitude



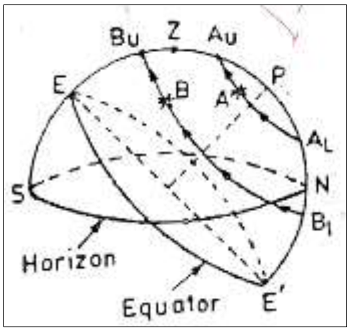
١-Star at eastern elongation: A star is said to be at eastern elongation when it is at its greatest distance to the east of the observer's meridian

٢-Star at western elongation: A star is said to be at western elongation when it is at its greatest distance to the west of the observer's meridian

٢) Star at Culmination

The diurnal circle or the path of the star crosses the observer's meridian twice during one revolution around the pole.

A star is said to be at Culmination, when it crosses the observer's meridian



a) Star at upper culmination: A star is said to be at upper culmination when it crosses the observer's meridian above the celestial pole.
b) Star at lower culmination: A star is said to be at lower culmination when it crosses the observer's meridian below the celestial pole

١) Upper culmination of Star A
The Zenith distance (Z) = ZA_u

ZA_u= ZP-PA_u

= (٩٠- θ)-(٩٠-δ)

ZA_u=δ- θ

٢) Upper culmination of Star B
ZB_u= PB_u-ZP

= (٩٠-δ)- (٩٠-θ)

= θ- δ

٣) Lower culmination of Star A
Z=١٨٠-($\theta + \delta$)

٤) Lower culmination of Star B

Z=١٨٠-($\theta + \delta$)

Example: - If the latitude of the observer's station is ٢٨°٣٠´N and declination of the star is ٦٢°٣٥´ N , calculate the zenith distances of the star at its upper and lower culminations.

Solution:

θ=٢٨°٣٠´N

δ =٦٢°٣٥´N

Z_u= δ- θ

= ٦٢°٣٥´-٢٨°٣٠´

$=٣٤°٠٥'$

$Z_l=١٨٠-(\theta + \delta)$

$=١٨٠-(٦٢°٣٥'+٢٨°٣٠') = ٨٨° ٥٥'$

Example: - Both culmination of a star occur on the north side of the zenith and its observed altitude at a place at upper and lower culmination are ٥٦°٣٠´ and ١٠°٣٠´respectively. Find the latitude of the place and declination of the star.

Solution:

$\alpha_u=٥٦°٣٠'$

$\alpha_l= ١٠°٣٠'$

$Z_u= \delta- \theta$

$(٩٠-\alpha_u)= \delta- \theta$

$٩٠-٥٦°٣٠'= \delta- \theta$

$٣٣°٣٠'= \delta- \theta$

$Z_l=١٨٠-(\delta + \theta)$

$(٩٠-\alpha_l) = ١٨٠-(\delta + \theta)$

$(٩٠-١٠°٣٠') = ١٨٠-(\delta + \theta)$

$١٠٠°٣٠'=(\delta + \theta)$

$٣٣°٣٠'= \delta- \theta$

$١٠٠°٣٠'= \delta + \theta$

$\boldsymbol{\theta} = ٣٣° ٣٠'$

$\boldsymbol{\delta} = ٦٧° ٠٠'$

Example: - If the upper culmination of a star (declination ٤٨°٣٨´N) is in the zenith of the observer's place, find the latitude of the place and altitude of the star at its lower culmination.

Solution:

$Z_u = \delta - \theta$

٠=٤٨°٣٨´- θ

θ =٤٨°٣٨´

Z_l=١٨٠-($\theta + \delta$)

=١٨٠-(٤٨°٣٨´+٤٨°٣٨´)

=٨٢°٤٤´

α= ٩٠-Z = ٧° ١٦´

Example: - Calculate the latitude of the place where a given star at its lower culmination remain at the horizon and its upper culmination occurs in zenith?

Solution:

Z_l=١٨٠°-($\theta + \delta$)

٩٠°- α=١٨٠°-($\theta + \delta$)

٩٠°=١٨٠°-($\theta + \delta$) $\delta = \theta$

٩٠°=١٨٠°- ٢ θ

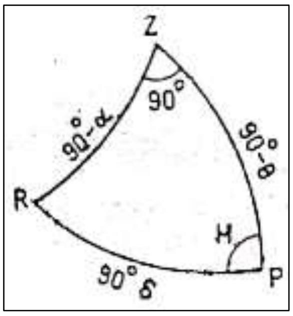
٢ θ=١٨٠°-٩٠°

٢ θ =٩٠°

θ = ٤٥°

٣) Star at prime vertical

A star is said to be at prime vertical when it occupies a position on the prime vertical.



٤) Star at horizon

A star is said to be at horizon when its altitude is zero.

α=٠

Cos A=$\frac{Sin\ \delta}{Cos\ \theta}$ = Sinδ * Sec θ

Cos HA = -Tan δ *Tan θ

٥) Circumpolar Stars

The stars which remain always above the horizon of the observer's position and do not set at any time.

δ> ٩٠- θ

Example: - Calculate the declination of the sun at a place of latitude ٢٨°٣٨´ if it rises on prime vertical.

Solution:

Az= ٩٠°

α=٠

δ=?

Cos (٩٠°- δ)=cos ٦١°٣٠`Cos ٩٠°+ Sin ٦١°٣٠´Sin ٩٠°Cos٩٠°

Cos (٩٠°- δ)=٠

δ= ٠

Example: - Show that on ٢١ march (when the sun's declination is ٠°) the sunrises exactly due to east in London and Nairobi and Sydney (Australia)?

London θ=٥١°٣٠´N

Nairobi θ =١,١٧° S

Sydney θ = ١٦°٤٠´

Solution:

Cos A $=\frac{Sin\ \delta}{Cos\ \theta} = \frac{Sin\ ٠}{Cos\ ٥١°٣٠´}$ =٠

A=$Cos^{-١}$ ٠ =٩٠°

Cos A $=\frac{Sin\ \delta}{Cos\ \theta} = \frac{Sin\ ٠}{Cos\ ١.١٧°}$ =٠

A=$Cos^{-١}$ ٠ **= ٩٠°**

Example: - Find the Sun rise position In Nairobi on ٢١ June and ٢١ December

δ In ٢١ June =٢٣,٥°N

δ In ٢١ December =٢٣,٥°S

θ of Nairobi = ١,١٧° S

Solution:

In ٢١ June

Cos A$=\frac{Sin\ \delta}{Cos\ \theta} = \frac{Sin\ ٢٣.٥°}{Cos\ ١.١٧°}$

Cos A = ٠,٣٩٨٨

A = ٦٦,٥٠°

In ٢١ December

Cos A$=\frac{Sin\ \delta}{Cos\ \theta} = \frac{Sin\ (-٢٣.٥°)}{Cos\ ١.١٧°}$

Cos A = -٠,٣٩٨٨ → **A= ١١٣,٥°**

Example: - What direction in Mecca (Saudi Arabia) from Jakarta (Indonesia)

Mecca (٢١,٢٦°N , ٣٩,٤٩°E)

Jakarta(٦,٠٨°S, ١٠٦,٤٥°E)

Solution:

P=١٠٦,٤٥°-٣٩,٤٩° =٦٦,٩٦°

Cos MJ=Cos MP Cos JP + Sin MP Sin JP Cos P

=٧١,٠٨°

$$\frac{Sin\ MP}{Sin\ J} = \frac{Sin\ MJ}{Cos\ P}$$

J= ٦٥,٠٤°

Corrections to the Observed Altitude of Celestial Bodies

The following corrections are generally applied to the observed altitudes of the celestial bodies for deducing their true altitude at the time of observation.

١) Observation correction

A) Refraction correction
B) Dip correction
C) Parallax correction
D) Semi-diameter correction

٢) Instrumental correction

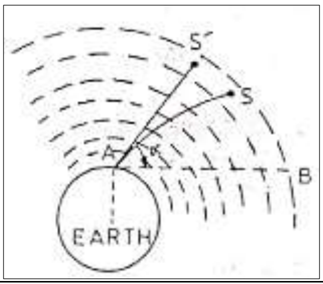
A) Index error correction
B) Bubble error correction
C) Azimuth error correction

١-Refraction correction

It is will established fact that the density of the air decreases as the distance from the earth surface increases, we also know that ray of light passing through layers of air of different densities get bent and thus their path is along a curve.

The magnitude of refraction correction depends upon the following factors:

١) Density of the air
٢) Temperature of the air
٣) Barometric pressure of the air



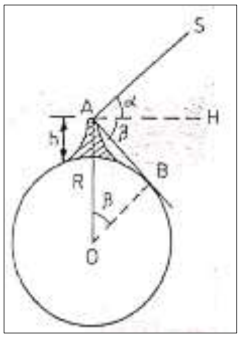
٤) Altitude of the celestial body

The apparent altitude greater than ٢٠°

Correction for refraction in seconds = ٥٨″cot α

= ٥٨″ tan Z

٢- Dip correction

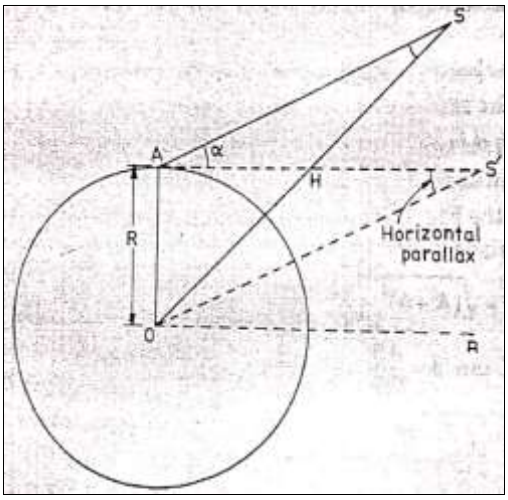


The angle between the sensible horizon and the visible horizon is called angle of Dip

Magnitude of dip depends upon the altitude of the observer's position above M.S.L.

Tan β=$\sqrt{\frac{٢h}{R}}$

٣- Parallax correction



The difference of altitude of the sun at a point on the surface of the earth and at the centre of the earth is known as sun's parallax in altitude.

The maximum horizontal parallax is ٨″.٩٥ on ٣١st December

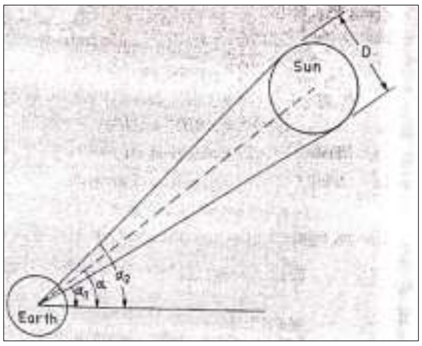
The minimum horizontal parallax is ٨″.٦٦ on ١st July.

Parallax in altitude = horizontal parallax* Cos α

=٨,٨″* Cos α

٤- Semi-diameter correction

The correction for semi-diameter is positive if the lower limb is observed and negative if the upper limb is observed.



Semi-diameter = $\alpha ١ + \frac{D}{٢}$

Semi-diameter = $\alpha ٢ - \frac{D}{٢}$

Example:- Find the true altitude of the sun's centre which gave an apparent altitude of ٥٥°٣٤′٢٣″ to the sun's lower limb. Diameter of the sun is ٣١′٤٦″

Solution:

١) Correction for refraction

r=٥٨″ cot α

=٥٨″ cot ٥٥°٣٤′٢٣″

=٣٩″.٨(-ve)

٢) Parallax in altitude = ٨,٨″* Cos α

=٨,٨″* Cos ٥٥°٣٤′٢٣″

=٥,١″(+ve)

٣) Correction for semi-diameter

Semi-diameter =+$\frac{٣١′٤٦″}{٢}$

=١٥′٥٣″(+ve)

Total correction

Apparent altitude = ٥٥°٣٤′٢٣″

r = - ٣٩″.٨

p=٥,١″

D=١٥′٥٣″

True Altitude of the sun =٥٥°٤٩′٤١″.٣

Example: - A force right observation on the sun's lower limb made and the altitude was found to be ٢٨°٣٦′٢٠″ the semi-diameter of the sun at the time of observation was ١٥′٥٩,٣٥″. Find the true Altitude of the sun

Solution:

١) Correction for refraction

r=٥٨″ cot α

=٥٨″ cot ٢٨°٣٦′٢٠″

=١′٤٦,٣٥″

٢) Parallax in altitude = ٨,٨″* Cos α

=٨,٨″* Cos ٢٨°٣٦′٢٠″

=٧,٧″(+ve)

٣) Correction for semi-diameter

Semi-diameter =١٥′٥٩,٣٥″

Total correction

Apparent altitude = ٢٨°٣٦′٢٠″

r = - ١´٤٦,٣٥″

p=٧,٧″

D=١٥´٥٩,٣٥″

True Altitude of the sun =٢٨°٥٠`٤٠``.٧

Time

Time: The interval which lapses between any two instants is termed as time.

Classification of Time

١-The sidereal time

٢-The apparent time

٣-The mean solar time

٤- The standard time

١-The sidereal time: The hour angle of the first point of Aeries (γ) measured west-ward ٠ to ٢٤ hours at any instant .The interval of time between two successive upper transits of the first point of Aeries (γ) is called the sidereal day.

The Local sidereal time (L.S.T): The interval of time which elapses since the upper transit of the first point of Aeries (γ) over observer's meridian.

L.S.T =H.A+R.A

٢- The apparent solar time: The measurement of time based on daily apparent motion of the sun around the earth, is known as apparent solar time.

The interval of time between two successive lower transits of the centre of the sun over the meridian of the place is called the apparent solar day.

٣- The mean solar time: The motion of the mean sun is the average of the motion of the true sun is right ascension.

The interval of time between two successive lower transits of the mean sun is called mean solar day or civil day.

The Local mean noon (L.M.N): The instant when the mean sun crosses the local meridian at its upper transit.

The Local mean time (L.M.T): The hour angle if the mean sun reckoned west-ward from ٠ to ٢٤ hours is known as Local mean time.

The mean solar day begins at the mid night and completes at the next mid night.

The difference in local mean times of two places is always equal to the difference of their longitude.

A civil day is divided into two periods, i.e. mid night to noon and noon to mid night.

٤-The standard time: As the local mean time at any meridian is reckoned from the lower transit of the mean sun at the meridian, the local mean time of each meridian will therefore, be different. The mean time of the central meridian of a country referred to as the standard time of the particular country.

The meridian, whose local mean time is used as the standard time of the country, is known as the standard of meridian of the country.

Standard time =L.M.T ∓ difference of longitude converted to time

Example: - Calculate the local mean time at a place whose longitude is ٩٢°٣٠´E, when the standard time is ٨ h ٤٠m ٣٠s .Assume the standard meridian of the country as ٨٢°٣٠´E.

Solution:-

Difference in longitudes =٩٢°٣٠´-٨٢°٣٠´ =١٠°

=١٠°*٤= ٤٠m

Standard time =L.M.T -difference of longitude in time

٨ h ٤٠m ٣٠s = L.M.T -٤٠m

L.M.T=٨ h ٤٠m ٣٠s+٤٠m

L.M.T=٩ h ٢٠m ٣٠s

٣٦٠° of longitude = ٢٤ hours of time

١٥° of longitude = ١ hours of time

١° of longitude = ٤ m of time

١٥´ of longitude = ١ m of time

١´ of longitude = ٤ sec of time

١٥″ of longitude = ١ sec of time

Example: -Convert the following difference in longitudes into interval of time

a) ٦٢°١٧´٤٢″

b) ١٧٦°٢٤´٥٧″

Solution:-

a) ٦٢°١٧´٤٢″

٦٢°=$\frac{٦٢}{١٥}$ h = ٤h ٠٨m ٠s

١٧´=$\frac{١٧}{١٥}$ m =٠h ١ m ٨s

٤٢´´=$\frac{٤٢}{١٥}$ s =٠h ٠m ٢,٨ s

Total =٤h ٩ m ١٠,٨ s

b) ١٧٦°٢٤´٥٧″

١٧٦°=$\frac{١٧٦}{١٥}$ h = ١١h ٤٤m ٠s

٢٤´=$\frac{٢٤}{١٥}$ m =٠h ١ m ٣٦s

٥٧´´=$\frac{٥٧}{١٥}$ s =٠h ٠m ٣.٨ s

Total =١١h ٤٥ m ٣٩.٨ s

Example: -Express the following interval of time into difference in longitudes.

a) ٥h ٣٠m ٤٥s
b) ١٠h ٢٤m ١٢s

Solution:-

a) ٥h ٣٠m ٤٥s

٥h= ٥* ١٥°=٧٥°٠´٠″

٣٠m=٣٠*١٥´=٧°٣٠´٠″

٤٥s=٤٥*١٥´=٠°١١´١٥″

Total =٨٢°٤١´١٥″

b) ١٠h ٢٤m ١٢s

١٠h= ١٠* ١٥°=١٥٠°٠´٠″

٢٤m=٢٤*١٥´=٦°٠´٠″

١٢s=١٢*١٥´=٠°٣´٠″

Total =١٥٦°٣´٠″

Equation of time

The difference between the apparent solar time and the mean solar time at any instant is known as the equation of time.

Equation of time= apparent solar time – mean solar time

Example: -If the standard time at a place in India is ١٨ h ١٨ m ١٨s corresponding to standard meridian ٨٢˚٣٠´E ,find the local mean time for the place whose longitudes are:

a) ٩٠˚E

b) ٤٨°W

Solution:-

Standard time =L.M.T ±difference of longitude in time

a) ٩٠˚E

Difference in longitude=٩٠˚-٨٢˚٣٠´=٧˚٣٠´

$\frac{٧°}{١٥}$ h= ٠h ٢٨m ٠s

$\frac{٣٠´}{١٥}$ m= ٠h ٢m ٠s

Total =٠h ٣٠m ٠s

Standard time =L.M.T ±difference of longitude in time

١٨ h ١٨ m ١٨s= L.M.T-٠h ٣٠m ٠s

L.M.T=١٨ h ١٨ m ١٨s+٠h ٣٠m ٠s

L.M.T=١٨ h ٤٨ m ١٨s

b) ٤٨°W

Difference in longitude=-٨٢°٣٠´-(-٤٨°) =١٣٠°٣٠´

$\frac{١٣٠°}{١٥}$ h= ٨h ٤٠m ٠s

$\frac{٣٠´}{١٥}$ m= ٠h ٢m ٠s

Total =٨h ٤٢m ٠s

Standard time =L.M.T ±difference of longitude in time

L.M.T=١٨ h ١٨ m ١٨s-٨h ٤٢m ٠s

L.M.T=٩ h ٣٦ m ١٨s

Example: - Find the G.M.T. corresponding to the following local mean time

a) ٨h ٣٦m ٤٨s A.M. at a place in longitude ٧٦°٤٥´E

b) ٨h ٣٦m ٤٨s P.M. at a place in longitude ٧٦°٤٥´W

Solution:-

a) ٨h ٣٦m ٤٨s A.M. at a place in longitude ٧٦°٤٥´E

Difference in longitude=٧٦°٤٥´E

$\frac{\text{٧٦°}}{\text{١٥}}$ h= ٥h ٤m ٠s

$\frac{\text{٤٥´}}{\text{١٥}}$ m= ٠h ٣m ٠s

Total =٥h ٧m ٠s

G.M.T =L.M.T −difference of longitude in time

G.M.T =٨h ٣٦m ٤٨s- ٥h ٧m ٠s

G.M.T =٣h ٢٩m ٤٨s

b) ٨h ٣٦m ٤٨s P.M. at a place in longitude ٧٦°٤٥´W

Difference in longitude=٧٦°٤٥´W

$\frac{\text{٧٦°}}{\text{١٥}}$ h= ٥h ٤m ٠s

$\frac{\text{٤٥´}}{\text{١٥}}$ m= ٠h ٣m ٠s

Total =٥h ٧m ٠s

G.M.T =L.M.T +difference of longitude in time

G.M.T =٨h ٣٦m ٤٨s+٥h ٧m ٠s

G.M.T =٢٠h ٣٦m ٤٨s+٥h ٧m ٠s

G.M.T =٢٥h ٤٣m ٤٨s

G.M.T =١h ٤٣m ٤٨s in the next day

Example: - The Greenwich civil time (G.C.T.) at the time of astronomical observations was known to be ٨h ٣٠m ٤٨s P.M. on January ٢١, ١٩٨٠. If the longitude of the place of observation is ٩٣°٤٥′٤٥″E, find the L.M.T. of the place.

Solution:-

Longitude of the place =٩٣°٤٥′٤٥″E

٩٣°=$\frac{٩٣°}{١٥}$ h=٦h ١٢m ٠٠s

٤٥′=$\frac{٤٥′}{١٥}$m =٠h ٣m ٠٠s

٤٥″=$\frac{٤٥″}{١٥}$m =٠h ٠m ٠٣s

Total= ٦h ١٥m ٣s

G.M.T =L.M.T ±difference of longitude in time

L.M.T= G.M.T + difference of longitude in time

=٢٠h ٣٠m ٤٨s +٦h ١٥m ٣s

=٢٦h ٤٥m ٥١s

L.M.T =٢h ٤٥m ٥١s A.M. on Jan.٢٢. ١٩٨٠

Example: - Find the local apparent time of observation of sun at a place in longitude ٧٢°٣٦′E, corresponding to local mean time ٩h ٢٥m ٢٠s. The equation of time at G.M.N. is ٤m ٣٤,٢٢s additive to the mean time, and decreases at the rate of ٠,٢٤s per hour

Solution:-

١-Calculate of G.M.T

G.M.T=L.M.T± difference of longitude in time

Difference of longitude in time =٧٢°٣٦′E =٤h ٥٠m ٢٤s

G.M.T=٩h ٢٥m ٢٠s -٤h ٥٠m ٢٤s

G.M.T=٤h ٣٤m ٥٦s

٢- Mean time interval before G.M.N.

=١٢h -٤h ٣٤m ٥٦s

= ٧h ٢٥m ٤s==٧,٤٢h

٣-it is given that equation of time decreases at the rate of ٠,٢٤s per hour at G.M.N

=٧,٤٢*٠,٢٤s\h = ٠h ٠m ١,٧٨s

٤-Equation of time at G.M.T = ٤m ٣٤,٢٢s +٠h ٠m ١,٧٨s

= ٤m ٣٦s

٥- G.A.T= E.T. +G.M.T.

=٤m ٣٦s +٤h ٣٤m ٥٦s

=٤h ٣٩m ٣٢s

٦-L.M.T= G.A.T. + difference of longitude in time

L.M.T=٤h ٣٩m ٣٢s +٤h ٥٠m ٢٤s
L.M.T. =٩h ٢٩m ٥٦s

Example: - Find the L.A.T. of an observation at place in longitude ٦٠٥١٨´E, corresponding to local mean time ١٠h ٢٠m ٣٠s. The equation of time at G.M.N. is ٥m ٤,٣٥s additive to the mean time, and decreases at the rate of ٠,٣٢s per hour

Solution:-

١-Calculate of G.M.T

G.M.T=L.M.T± difference of longitude in time

Difference of longitude in time =٦٠٥١٨´E =٤h ١m ١٢s

G.M.T=١٠h ٢٠m ٣٠s -٤h ١m ١٢s

G.M.T=٦h ١٩m ١٨s

٢- Mean time interval before G.M.N.

=١٢h -٦h ١٩m ١٨s

= ٥h ٤٠m ٤٢s==٥,٦٨h

٣-it is given that equation of time decreases at the rate of ٠،٢٤s per hour at G.M.N

=٥,٦٨h*٠,٣٢s\h = ٠h ٠m ١,٨٢s

٤-Equation of time at G.M.T = ٥m ٤,٣٥s +٠h ٠m ١,٨٢s

= ٥m ٦,١٧s

٥- G.A.T= E.T. +G.M.T.

=٥m ٦,١٧s+٦h ١٩m ١٨s

=٦h ٢٤m ٢٤,١٧s

٦-L.M.T= G.A.T. + difference of longitude in time

L.M.T=٦h ٢٤m ٢٤,١٧s +٤h ١m ١٢s

L.M.T. =١٠h ٢٥m ٣٦,١٧s

Example: - Find the L.M.T. of observation at place from the following data

L.A.T of observation = ١٥h ١٢m ٤٠s

Equation of time at G.M.N. =٥m ١٠,٦٥s additive to apparent time and increasing at ٠,٢٢ seconds per hour

Longitude of the place =٢٠°٣٠´W

Solution:-

١-Calculate of G.M.T

G.A.T=L.M.T± difference of longitude in time

Difference of longitude in time =٢٠°٣٠´W =١h ٢٢m

G.A.T =١٥h ١٢m ٤٠s +١h ٢٢m

G.A.T =١٦h ٣٤m ٤٠s

٢- Mean time interval before G.M.N.

=١٢h -١٦h ٣٤m ٤٠s = ٤h ٣٤m ٤٠s=٤,٥٧h

٣-it is given that equation of time decreases at the rate of ٠,٢٤s per hour at G.M.N

=٤,٥٧h *٠.٢٢s\h = ٠h ٠m ١.٠١s

٤-Equation of time at G.M.T = ٥m ١٠,٦٥s +٠h ٠m ١,٠١s

= ٥m ١١.٦٦s

٥- G.M.T.= G.A.T +E.T.

=١٦h ٣٤m ٤٠s +٥m ١١,٦٦s=١٦h ٣٩m ٥١,٦٦s

٦-L.M.T= G.M.T. - difference of longitude in time

L.M.T=١٦h ٣٩m ٥١,٦٦s -١h ٢٢m

L.M.T. =١٥h ١٧m ٥١,٦٦s

Conversion of Sidereal Time Interval to Mean Time Interval

In one tropical year, the mean sun apparently goes around the earth once with respect to the first point of Aeries (γ) in the same direction as that the earth rotation.

Total number of sidereal days in a tropical year should be equal to n, But actually the earth rotates only (n-١) time with respect to n sidereal days in tropical year.

٣٦٥,٢٤٢٢ mean solar days in a tropical year

١ sidereal day $=\frac{٣٦٥.٢٤٢٢}{٣٦٦.٢٤٢٢}$ mean solar day

١ sidereal day $=١-\frac{١}{٣٦٦.٢٤٢٢}$ mean solar day

١ sidereal day = ٢٣h ٥٦m ٤,٠٩s mean solar time

١h sidereal day = ١h -٩,٨٢٩٦s mean solar time

١m sidereal day=١m – ٠,١٦٣٨s mean solar time

١s sidereal day=١s – ٠,٠٠٢٧s mean solar time

Retardation

١ mean solar day = $١+\frac{١}{٣٦٥.٢٤٢٢}$ sidereal days

١ mean solar day = ٢٤h ٣m ٥٦,٥٦s sidereal time

١h mean solar time=١h + ٩,٨٥٦٥s sidereal time

١m mean solar time = ١m +٠,١٦٤٢s sidereal time

١s mean solar time =١s + ٠,٠٠٢٧s sidereal time

The mean solar day is ٣m ٥٦,٥٦s longer than the sidereal day.

The sidereal day is ٣m ٥٥,٩١s shorter than the mean solar day

Example: - Convert ٦h ٣٠m ٤٠s sidereal time to mean solar time interval.

Solution:-

Sidereal time=٦h ٣٠m ٤٠s

To convert sidereal time to mean solar time, the retardation @٩,٨٢٩٦ seconds per hour of sidereal time is applied.

٦h*٩,٨٢٩٦=٥٨,٩٧٧٦ seconds

٣٠m *٠,١٦٣٨=٤,٩١٤٠ seconds

٤٠s*٠,٠٠٢٧=٠,١٠٨٠ seconds

Total =٦٣,٩٩٩٦ seconds

=١m ٣,٩٩٩٦s

Mean solar time= sidereal time – Total retardation

=٦h ٣٠m ٤٠s-١m ٣,٩٩٩٦s

= ٦h ٢٩m ٣٦ sec

Example: - Convert ٦h ٢٩m ٣٦s mean solar time to sidereal time interval

Solution:-

To convert mean solar time to sidereal time, the acceleration @٩,٨٥٦٥ seconds per hour of mean solar time is applied.

Total acceleration

٦h *٩,٨٥٦٥=٥٩,١٣٩٠ seconds

٢٩m *٠,١٦٤٢=٤,٧٦١٨ seconds

٣٦s* ٠,٠٠٢٧=٠,٠٩٧٢ seconds

Total acceleration =٦٣,٩٩٨٠ sec = ١m ٣,٩٩٨s

Sidereal time interval = mean solar time + Total acceleration

=٦h ٢٩m ٣٦s+١m ٣,٩٩٨s

= ٦h ٣٠m ٣٩,٩٨٨sec

Conversion of local mean time at any instant to local sidereal time if Greenwich sidereal time (G.S.T.) at Greenwich mean mid-night (G.M.M) is known

Example: - Find L.S.T. at a place in longitude ٩٠° W of ١٠AM if G.S.T at G.M.M. is ١٣h ٥٨m ٤,١s

Solution:-

١- Calculate L.S.T at G.M.M

a) The longitude of the place

   ٩٠° W= ٦h

b) Calculate the total retardation or acceleration

For west total acceleration

٦h*٩,٨٥٦٥= ٥٩,١٣٩٠s

c) Obtain L.S.T at G.M.M

L.S.T at G.M.M=G.S.T at G.M.M. + acceleration

=١٣h ٥٨m ٤,١s+٥٩,١٣٩٠s

=١٣h ٥٩m ٣,٢٣٩s

٢- Calculate the mean time interval by the mean time interval between L.M.M. and L.M.T.
=١٠ *٩,٨٥٦٥sec = ٩٨,٥٦٥s
=١m ٣٨,٥٦٥s

٣- Convert mean time interval to sidereal time interval
Sidereal interval = Mean time interval + acceleration

=١٠h +١m ٣٨,٥٦٥s =١٠h ١m ٣٨,٥٦٥s

٤- Calculate local sidereal time to local mean time

L.S.T at local mean time = L .S.T at G.M.M +S.I.

=١٣h ٥٩m ٣,٢٣٩s+١٠h ١m ٣٨,٥٦٥s
=٢٤h ٠m ٤١,٨٠٤s
=٠h ٠m ٤١,٨٠٤s

Example: - Find the L.S.T. at a place in longitude ٧٦°٣٠´E at ٤h ٣٠m P.M., G.S.T. at G.M.N. being ٤h ٣٦m ١٨s.

Solution:-

١-Calculate L.S.T at G.M.N

a) The longitude of the place
٧٦°٣٠´= ٥h ٦m

b) Calculate the total retardation or acceleration
٥h *٩,٨٥٦٥s= ٤٩,٢٨٢٥

٦m*٠,١٦٤٢s= ٠,٩٨٥٢
Total = ٥٠,٢٦٧٧ sec

c) Obtain L.S.T at L.M.N

L.S.T at L.M.N= G.S.T. at G.M.N – retardation

=٤h ٣٦m ١٨s - ٥٠,٢٦٧٧ sec

=٤h ٣٥m ٢٧,٧٣٢٣s

٢- Calculate mean time interval

M.T.I= ٤h ٣٠m

٤h *٩,٨٥٦٥s =٣٩.٤٢٦٠s

٣٠m *٠,١٦٤٢=٤,٩٢٦٠s

Total = ٤٤,٣٥٢٠s

٣- Convert the mean time interval to Sidereal time interval

Sidereal time in interval = M.T.I+ acceleration

=٤h ٣٠m+٤٤,٣٥٢٠s

=٤h ٣٠m ٤٤,٣٥٢٠s

٤- Calculate L.S.T

L.S.T=L.S.T. at L.M.N. +S.T

=٤h ٣٥m ٢٧,٧٣٢٣s+٤h ٣٠m ٤٤,٣٥٢٠s

=٩h ٦m ١٢,٠٨٤sec

Conversion of local sidereal time at any instant to local mean time if Greenwich sidereal time at Greenwich mid night

Example: - Find L.M.T. at a place in longitude ٩٠° W if L.S.T. of the place is ٠h ٠m ٤١,٨s and G.S.T. at G.M.M. is ١٣h ٥٨m ٤,١s.

Solution:-

١- Calculate the sidereal time at local mid night

L.S.T. at L.M.M. = G.S.T at G.M.M. + acceleration

٩٠° W =٦h the place is west of Greenwich

Acceleration= ٦h * ٩,٨٥٦٥=٥٩,١٣٩٠s

L.S.T =G.S.T. + acceleration

=١٣h ٥٨m ٤,١s+٥٩,١٣٩٠s

=١٣h ٥٩m ٣,٢٣٩s

٢- Calculate the sidereal time interval between the L.S.T at local mean mid night and given sidereal time.
S.I = L.S.T. – L.S.T.at L.M.M.

=٠h ٠m ٤١,٨s-١٣h ٥٩m ٣,٢٣٩s

=٢٤h ٠m ٤١,٨s-١٣h ٥٩m ٣,٢٣٩s

=١٠h ١m ٣٨,٥٦١s

٣- Convert sidereal interval to mean time interval
Mean time interval = S.I - retardation

١٠h * ٩,٨٢٩٦ = ٩٨,٢٩٦s

١m*٠,١٦٣٨= ٠,١٦٣٨s

٣٨,٥٦١s*٠,٠٠٢٧=٠,١٠٤١s

Total = ٩٨,٥٦٣٩s= ١m ٣٨,٥٦٣٩s

Mean time interval = S.I. – retardation

=١٠h ١m ٣٨,٥٦s-١m ٣٨,٥٦s

=١٠ A.M

Example: - Find L.M.T. at a place in longitude ٧٦° ١٥´E if L.S.T. of the place is ٦h ٤٢m ١٨s and G.S.T. at G.M.M. is ٨h ١٦m ١,٥s.

Solution:-

١- Calculate the sidereal time at local mid night

L.S.T. at L.M.M. = G.S.T at G.M.M. - retardation

٧٦° ١٥´ =٥h ٥m

Retardation = ٥h * ٩,٨٥٦٥=٤٩,٢٨٢٥s

٥m*٠,١٦٤٢ = ٠,٨٢١٠s

Total =٥٠,١٠٣٥s

L.S.T =G.S.T. - retardation

= ٨h ١٦m ١,٥s.- ٥٠,١s

=٨h ١٥m ١١,٤s

٢- Calculate the sidereal time interval between the L.S.T at local mean mid night and given sidereal time.
S.I = L.S.T. – L.S.T.at L.M.M.

=٣٠h ٤٢m ١٨s-٨h ١٥m ١١,٤s

=٢٢h ٢٧m ٦.٦١s

٣- Convert sidereal interval to mean time interval
Mean time interval = S.I - retardation

٢٢h * ٩,٨٢٩٦ = ٣m ٣٦,٢٥١s

٢٧m*٠,١٦٣٨= ٤,٤٢٢٦s

٦,٦s*٠,٠٠٢٧=٠.٠١٧٨s

Total = ٣m ٤٠,٦٩s

Mean time interval = S.I. – retardation

=٢٢h ٢٧m ٦.٦١s -٣m ٤٠,٦٩s

=٢٢h ٢٣m ٢٥,٩ s

Determination of the L.M.T. of the upper Transit of a known star if G.S.T. of G.M.M. is known

The right ascension expressed in time of any star at its upper transit is equal to local sidereal time.

Example: - Calculate the L.M.T. of upper transit of a star at a place in longitude ٨٢° ٣٠´W, whose R.A. is ٢٠h ١٠m ٣٠s. Given : G.S.T. of previous G.M.N. as ٩h ٣٠m ٣٠s.

Solution:-

١- Calculate the local sidereal time at local mean time
L.S.T of L.M.N. =G.S.T. of G.M.N. ± retardation or acceleration

Longitude =٨٢° ٣٠´W=٥h ٣٠m

Acceleration

٥h *٩.٨٥٦٥s= ٤٩,٢٨٢٥s

٣٠m*٠,١٦٤٢s=٣,٢٨٤٠

Total = ٥٢,٥٦٦٥s

L.S.T of L.M.N. = ٩h ٣٠m ٣٠s+٥٢,٥٦٦٥s

=٩h ٣١m ٣٣,٥٦٦٥s

٢- Calculate the sidereal time interval of L.M.N.

S.I.=L.S.T.-L.S.T. at L.M.N.

=٢٠h ١٠m ٣٠s-٩h ٣١m ٣٣,٥٦٦٥s

=١٠h ٣٩m ٧,٤٣٣٥s

٣- Convert S.I. to mean interval

Mean time interval = S.I. – retardation

١٠h *٩,٨٢٩٦s= ٩٨,٢٨٦٠s

٣٩m*٠,١٦٣٨ = ٦,٣٨٨٢

٧,٤٣٣٥s*٠,٠٠٢٧=٠,٠١٢٨

Total = ١٠٤,٦٩٧٠s = ١m ٤٤,٦٩٧s

Mean time interval = S.I. – retardation

=١٠h ٣٩m ٧,٤٣٣٥s-١m ٤٤,٦٩٧s

=١٠h ٣٧m ٢٢.٧٣٦٥s P.M.

Example: - What will be L.M.T. of upper and following lower transit at a place in longitude ١٦٢° ٣٠′١٥″W of a star whose R.A. is ٢٢h ١١m ٣٠sec if the G.S.T. of previous G.M.N. is ١٠h ٣٠m ١٥sec

Solution:-

١- Calculate the local sidereal time at local mean time
L.S.T of L.M.N. =G.S.T. of G.M.N. ± retardation or acceleration

Longitude =١٦٢° ٣٠′١٥″W=١٠h ٥٠m ١٥s

١٠h *٩,٨٥٦٥s= ٩٨,٥٦٥s

٥٠m*٠,١٦٤٢s=٨,٢١

١٥*٠,٠٠٢٧=٠,٠٤٠٥

Total = ١٠٦,٨١٦s

L.S.T of L.M.N. = ١٠h ٣٠m ١٥sec +١٠٦,٨١٦s

=١٠h ٣٢m ١,٨٢s

٢- Calculate the sidereal time interval of L.M.N.

S.I.=L.S.T.-L.S.T. at L.M.N.

=٢٢h ١١m ٣٠sec -١٠h ٣٢m ١,٨٢s

=١١h ٣٩m ٢٨,٢s

٣- Convert S.I. to mean interval

Mean time interval = S.I. – retardation

١١h *٩,٨٢٩٦s

٣٩m*٠,١٦٣٨

٢٨,٢*٠,٠٠٢٧

Total = ١١٤,٥٩١s

Mean time interval = S.I. – retardation

=١١h ٣٩m ٢٨,٢s -١١٤,٥٩١s

=١١h ٣٧m ٣٣,٦٣٢s upper transit

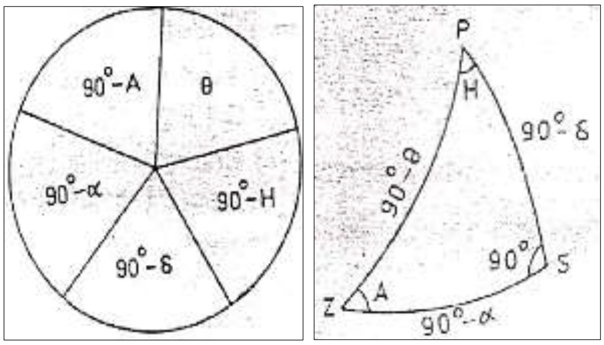
١٢h *٩,٨٢٩٦=١m ٥٧,٩٥٥sec

=١٢h-١m ٥٧,٩٥٥sec = ١١h ٥٨m ٢,٠٤sec

=١١h ٥٨m ٢,٠٤sec+١١h ٣٧m ٣٣,٦٣٢s =٢٣h ٣٥m ٣٥,٦٧٧s

=٢٣h ٣٥m ٣٥,٦٧٧s-١٢h=**١١h ٣٥m ٣٥,٦٧٧s A.M. following day**

Determine of Time of Elongation of a circumpolar star.



Let S be position of the star at elongation

At elongation angle ZSP is a right angle

Applying the Napier's sine rule

Sin (٩٠° –H) = tan θ * tan (٩٠°- δ)

Cos H = tan θ * cot δ

L.S.T. =R.A.(of star) + Hour angle of a star

Example: - Find the L.S.T. of western elongation of polar is in the evening at a place in latitude ٣٠°٢٢´١٥˝ given that the R.A. of the star is ١h ٥٢m ١٢s and its declination ٨٩°٣´٤٦˝

Solution:-

Latitude of the place θ = ٣٠°٢٢´١٥˝

Declination of the star δ =٨٩°٣´٤٦˝

Cos H = tan θ * cot δ

Cos H = tan ٣٠°٢٢´١٥″ * cot ٨٩°٣´٤٦″

H= ٨٩°٢٧´٢,٥٢″=٥h ٥٧m ٤٨,٢s

L.S.T. =R.A.(of star) + Hour angle of a star

=١h ٥٢m ١٢s+٥h ٥٧m ٤٨,٢s

=٧h ٥٠m ٠,٢s

Example: - Find the L.S.T. at which (β) ursae minor is will of elongate on the evening at a place in latitude ٥٠°٣٠´N given that the R.A. of the star is ١٤h ٥٠m ٥٢s and its declination ٧٤°٢٢´

Solution:-

Latitude of the place θ = ٥٠°٣٠´

Declination of the star δ =٧٤°٢٢´

Cos H = tan θ * cot δ

Cos H = tan ٥٠°٣٠´* cot ٧٤°٢٢´

H= ٧٠°٥٩´٢٠,٩″=٤h ٤٠m ٣٧,٣٩s

L.S.T. =R.A.(of star) + Hour angle of a star

=١٤h ٥٠m ٥٢s +٤h ٤٠m ٣٧,٣٩s

=١٩h ٣١m ٢٩,٤s

Time of rising and setting of heavenly body

The spherical triangle PMN is right angle of N using Napier’s law

Cos p = cot MP Tan PN

<p = ١٨٠-H

Hence -

Cos H = tan δ *tan θ

Knowing the declination of the star and the latitude of the place , its hour angle can be known then

L.S.T. of rising of star = R.A. of the star + Hour angle

The L.S.T. of the rising of the star can be known and this be converted into L.M.T.

Length of Day and night

Length of day = twice hour angle in time = ٢$(\frac{H}{١٥})$

Length of the night = ٢$(\frac{١٨٠-H}{١٥})$

The equation of hourangle

Cos H = -tan δ *tan θ can be used to determine the length at different place and different times

١- At a place at equator θ =٠
Cos H =٠ → H=٩٠°
Length of day or night =$(\frac{٢*٩٠}{١٥})$ =١٢
٢- At the time of equinox the sun's at equator δ=٠
Cos H=٠ → H=٩٠°
Length of day or night =$(\frac{٢*٩٠}{١٥})$ =١٢
٣- If δ =٩٠ - θ
Cos H =-١ → H=١٨٠°
Length of day =$(\frac{٢*١٨٠}{١٥})$ =٢٤h The sun does not set
٤- If δ=-(٩٠ – θ)
Cos H =١ → H=٠°
Length of day= ٠h the sun does not rise

The Duration of Twilight

The Twilight is the seduced light which separate night from day

To find the duration of twilight of particular place we must therefore find the time the sun takes to alter its zenith distance from ٩٠ to ١٨٠ in the evening from ١٨٠ to ٩٠ in the morning

Cos ١٠٨° =sin δsin θ+ cos δcos θcos H´

H´ hour angle of the end of twilight if H the hour angle of the sunset

We have

Cos H = -tan δ *tan θ

Hence the duration of twilight = H- H´

Determination of latitude

Knowledge of the latitude at different places on the surface of the earth , is very necessary for the land surveyors and civil engineering

١- Latitude by meridian altitude of star
٢- Declination time of the star δ
٣- Meridian altitude of the star α

Case ١:- star S١ between horizontal and equator

θ=$Z_{١}$-$\delta_{١}$

Case ٢

θ=$Z_{٢}$+$\delta_{٢}$

Case ٣

θ=$\delta_{٣}$-$Z_{٣}$

Case ٤

θ=١٨٠° $-$ ($Z_{٤}$+$\delta_{٤}$)

Example: - The meridian altitude of a star was observed to be ٦٥°٤٠´١٨″on a certain day the star lying between pole and zenith the distance declination of star was ٥٣°١٢´١٠″ find the latitude of the place observation

Solution:-

True altitude

Correction of refraction r=٥٨″ cot α

=٥٨″ cot ٦٥°٤٠´١٨″

=٢٦,٢٢″

True altitude= ٦٥°٤٠´١٨″-٢٦,٢٢″

=٦٥°٣٩´٥١,٧٨″

Z=٩٠-٦٥°٣٩´٥١,٧٨″=٢٤°٢٠´٨,٢٢″

θ= δ-Z

θ=٢٨°٥٢´١,٧٨″

Example: - The meridian altitude of a star was observed to be ٦٤°٣٦´٢٠″on a certain day the star lying between zenith and equator the distance declination of star was ٢٦°١٢´١٠″ find the latitude of the place observation

Solution:-

Correction of refraction r=٥٨″ cot α

=٥٨″ cot ٦٤°٣٦´٢٠″

=٢٧,٥٣″

True altitude= ٦٤°٣٦′٢٠″ -٢٧,٥٣″

=٦٤°٣٥′٥٢,٤٧″

Z=٩٠-٦٤°٣٥′٥٢,٤٧″ =٢٥°٢٤′٧,٥٣″

$\theta=\delta+Z$

θ=٥١°٣٦′١٧,٥٣″