Tutorial / Lec-1- / DSP مرحلة الثالثة / الهندسة أجهزة

Signal Manipulation

- Time
  - Shifting
  - Reversal
  - Scaling
- Amplitude
  - Addition
  - Multiplication
  - Scaling

Signal Manipulation of Time:

1- Time Shifting:

$$x(t) \longrightarrow x(t-t_0)$$

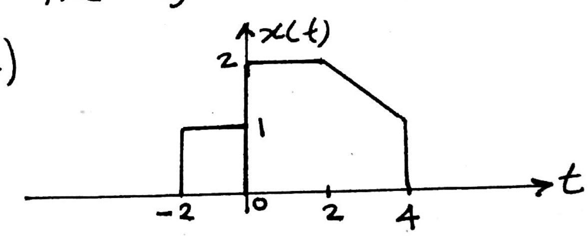
$$x[n] \longrightarrow x[n-n_0]$$

if $t_0\ (n_0) > 0 \longrightarrow$ delay

$t_0\ (n_0) < 0 \longrightarrow$ Advance

Example// For the signal $x(t)$ shown in Fig below:

Determine $x(t-2)$



Solution:

$x(t-2) \rightsquigarrow t_0 = +2$

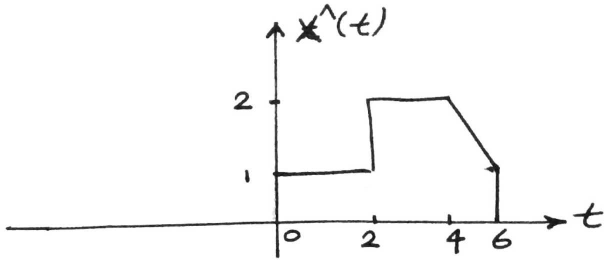
$x(-2) \longrightarrow \hat{x}(-2+2) = \hat{x}(0)$

$x(0) \longrightarrow \hat{x}(0+2) = \hat{x}(2)$

$x(2) \longrightarrow \hat{x}(2+2) = \hat{x}(4)$

$x(4) \longrightarrow \hat{x}(4+2) = \hat{x}(6)$

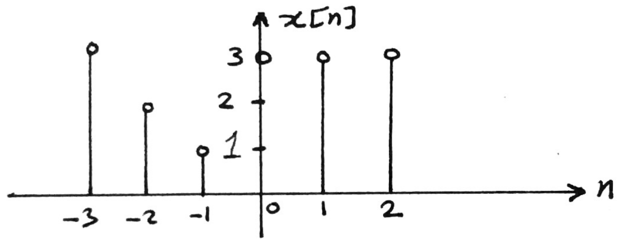
$\therefore$ the signal $x(t-2)$ will be :



Ex: For the discrete signal :

$$x[n] = [3 \quad 2 \quad 1 \quad 3 \quad 3 \quad 3]$$
$$n = [-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2]$$

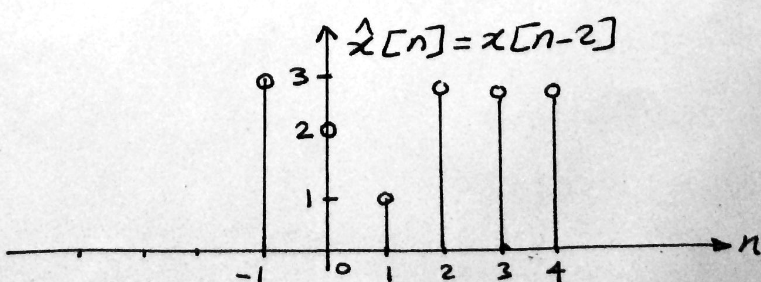
Find $x[n-2]$, $x[n+1]$ (sketch the signals in all cases)



1- $x[n-2] \rightsquigarrow n_0 = +2$

$x[n-n_0]$

$$x[-3] \rightsquigarrow \hat{x}[-3+2] = \hat{x}[-1]$$
$$x[-2] \rightsquigarrow \hat{x}[-2+2] = \hat{x}[0]$$
$$x[-1] \rightsquigarrow \hat{x}[-1+2] = \hat{x}[1]$$
$$x[0] \rightsquigarrow \hat{x}[0+2] = \hat{x}[2]$$
$$x[1] \rightsquigarrow \hat{x}[1+2] = \hat{x}[3]$$
$$x[2] \rightsquigarrow \hat{x}[2+2] = \hat{x}[4]$$



$x[n+1]$

$x[n-n_0]$ $\rightsquigarrow n_0 = -1$

$x[-3] \rightsquigarrow \hat{x}[-3-1] = \hat{x}[-4]$

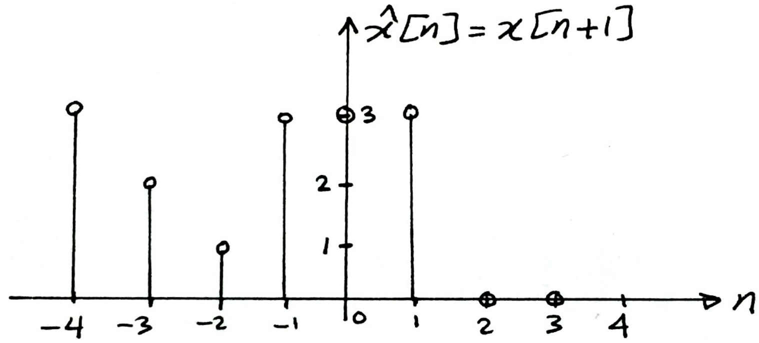
$x[-2] \rightsquigarrow \hat{x}[-2-1] = \hat{x}[-3]$

$x[-1] \rightsquigarrow \hat{x}[-1-1] = \hat{x}[-2]$

$x[0] \rightsquigarrow \hat{x}[0-1] = \hat{x}[-1]$

$x[1] \rightsquigarrow \hat{x}[1-1] = \hat{x}[0]$

$x[2] \rightsquigarrow \hat{x}[2-1] = \hat{x}[1]$



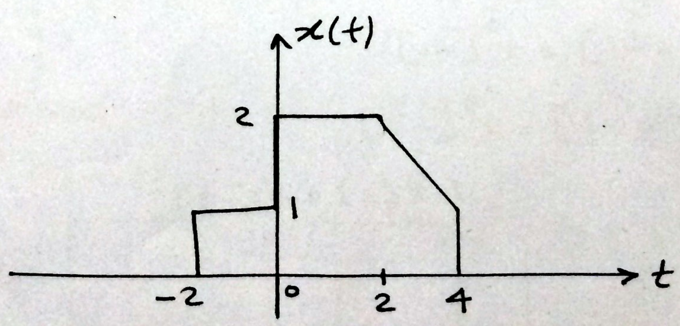
2- Time Reversal:

$$x(t) \rightsquigarrow x(-t)$$

$$x[n] \rightsquigarrow x[-n]$$

Ex: For the signal $x(t)$ shown below:

Find $x(-t)$



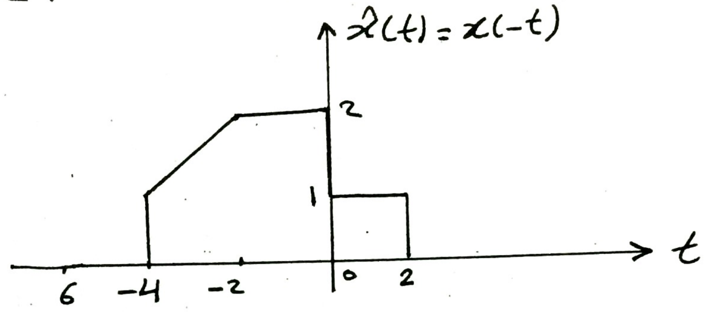
Sol.

$x(-2) \longrightarrow \hat{x}[-2 * -1] = \hat{x}[2]$

$x[0] \longrightarrow \hat{x}[0 * -1] = \hat{x}[0]$

$x[2] \longrightarrow \hat{x}[2 * -1] = \hat{x}[-2]$

$x[4] \longrightarrow \hat{x}[4 * -1] = \hat{x}[-4]$



Ex: For the signal $x[n]$ shown below:

$$x[n] = [3 \quad 2 \quad 1 \quad \underset{\uparrow}{3} \quad 3 \quad 3]$$

Find $x[-n]$

Sol.

$x[-3] \longrightarrow \hat{x}[-3 * -1] = \hat{x}[3]$

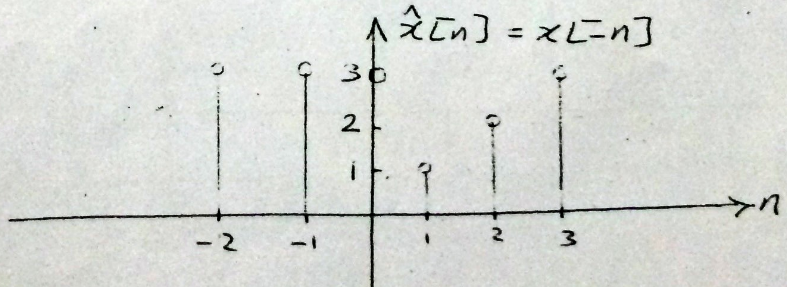
$x[-2] \longrightarrow \hat{x}[-2 * -1] = \hat{x}[2]$

$x[-1] \longrightarrow \hat{x}[-1 * -1] = \hat{x}[1]$

$x[0] \longrightarrow \hat{x}[0 * -1] = \hat{x}[0]$

$x[1] \longrightarrow \hat{x}[1 * -1] = \hat{x}[-1]$

$x[2] \longrightarrow \hat{x}[2 * -1] = \hat{x}[-2]$



Time Scaling :

$$x(t) \longrightarrow x(at)$$

$$x[n] \longrightarrow x[an]$$

$a > 1 \rightarrow$ Decimation, Downsampling

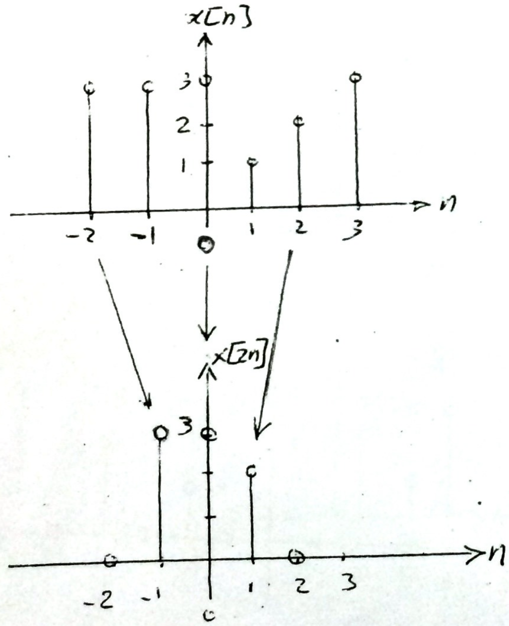
$0 < a < 1 \rightarrow$ Expansion, Up Sampling

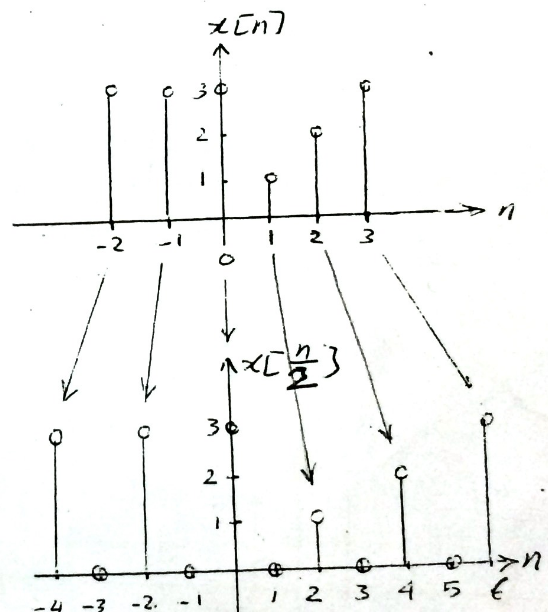
Ex: For the discrete Signal

~~$x[n] = [3\ 2\ 1\ 3\ 3\ 3]$~~ $\quad x[n] = [3\ 3\ 3\ 1\ 2\ 3]$

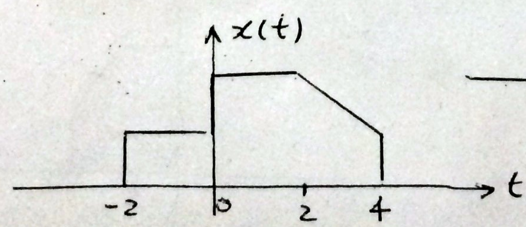
~~$n = [-3\ -2\ -1\ 0\ 1\ 2]$~~ $\quad n = [-2\ -1\ 0\ 1\ 2\ 3]$

Find $x[2n]$, $x[\frac{n}{2}]$

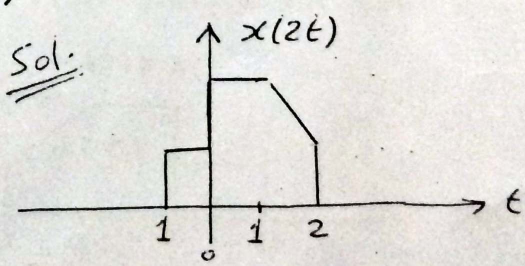




Ex : For the signal $x(t)$, Find $x(2t)$



Sol.



Amplitude Manipulation

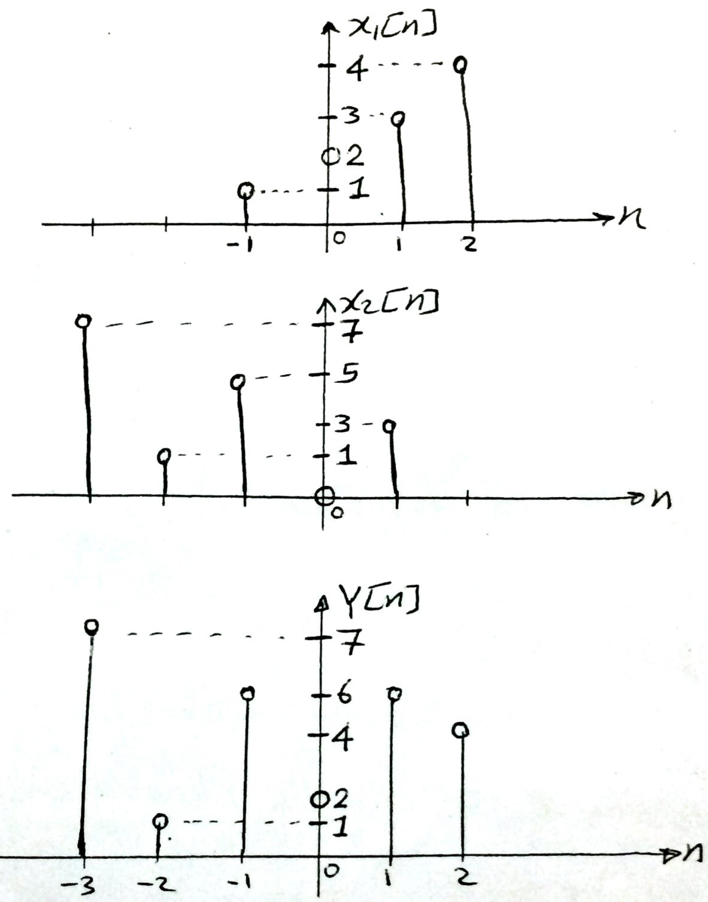
1- Addition :

Ex: Find and sketch $Y[n] = x_1[n] + x_2[n]$, if

$$x_1[n] = [1 \;\; \underset{\uparrow}{2} \;\; 3 \;\; 4]$$

$$x_2[n] = [7 \;\; 1 \;\; 5 \;\; \underset{\uparrow}{0} \;\; 3]$$

Sol.



2- Multiplication:

H.W. For the signals shown below:

$$x_1[n] = [1 \;\; 0 \;\; 2 \;\; 0 \;\; 1]$$

$$x_2[n] = 2\delta[n+1] - \delta[n] + 4\delta[n-2] + \delta[n-3]$$

Find $Y[n] = x_1[n] \cdot x_2[n]$

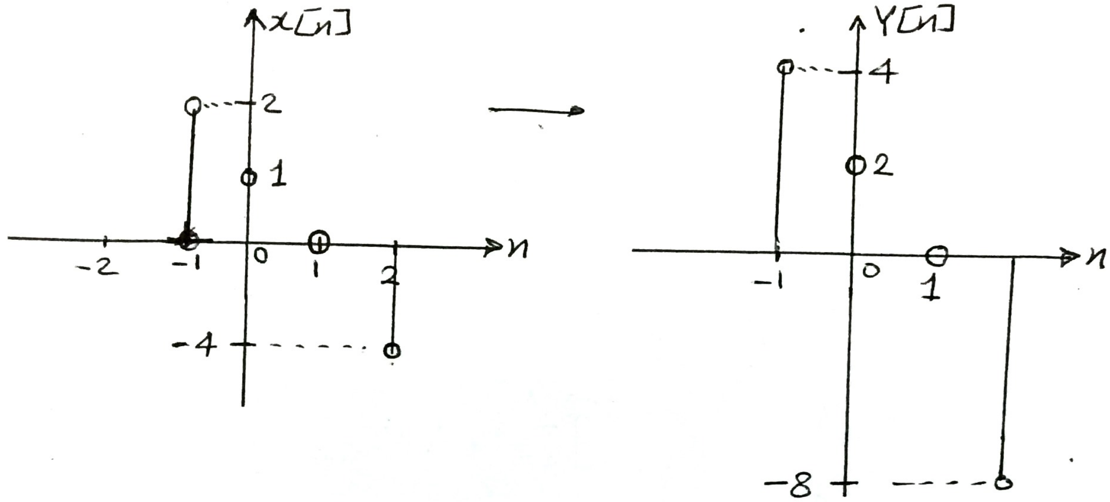
3. Scaling:

$x[n] = a\,x[n]$

Ex: For the signal $x[n] = \delta[n] + 2\delta[n+1] - 4\delta[n-2]$

Find $Y[n] = 2x[n]$

Sol.



H.W: If you have the following Signals:

$$x_1[n] = [3 \quad 7.5 \quad \underset{\uparrow}{3} \quad 0 \quad 1]$$

$$x_2[n] = -2\delta[n] + 2\delta[n-2] + 1.5\delta[n+1]$$

Determine and sketch $Y[n]$ for the following cases:

1. $Y[n] = x_1[n] + x_2[n]$

2. $Y[n] = -3\,x_2[n]$

3. $Y[n] = x_1[n] \cdot \frac{1}{2} x_2[n]$

4. $Y[n] = 2x_1[n] - x_2[n]$

H.W: sketch the following Signals:

1- $x(t) = e^{-3t}$ within the time $t(-3:+3)$

2- $x[n] = \sin(5\pi n)$ within the time $n[0:4]$

H.W: If you have the following Signal:

$$x[n] = [1 \ 2 \ 4 \ 5 \ 3 \ -1]$$
$$n = [-1 \ 0 \ 1 \ 2 \ 3 \ 4]$$

Find and sketch the following cases:

1- $Y[n] = 2x[-n]$

2- $Y[n] = x[3n]$

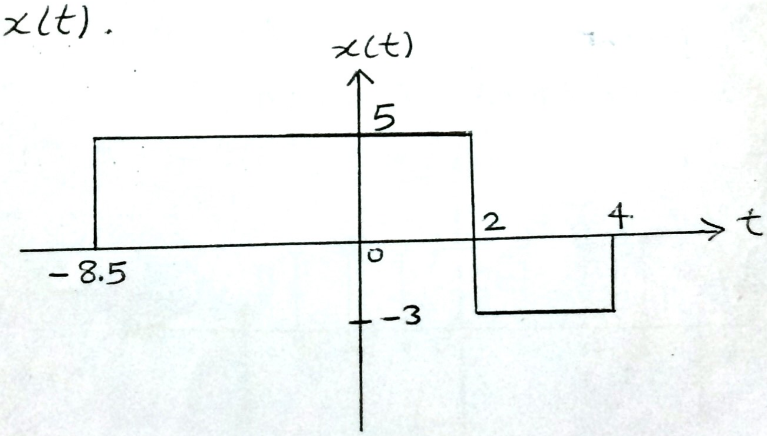
3- $Y[n] = x[\frac{n}{3}]$

H.W1 Determine and sketch the following signals:

1- $Y[n] = 2r[n-2]$

2- $Y[n] = -u[n+3]$

Ex: Find the mathematical expression of the following Signal $x(t)$.



Sol.

1- a step Signal started at $t=-8.5$ with amplitude $=5$

$$x(t) = 5u(t+8.5)$$

2- another step started at $t=2$ with amplitude $=-8$

$$x(t) = -8\,u(t-2)$$

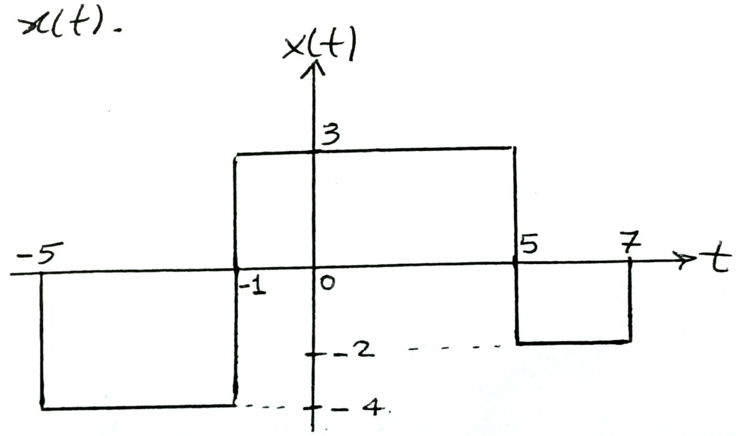
3. another step started at $t=4$ with amplitude $= 3$

$$x(t) = 3\,u(t-4)$$

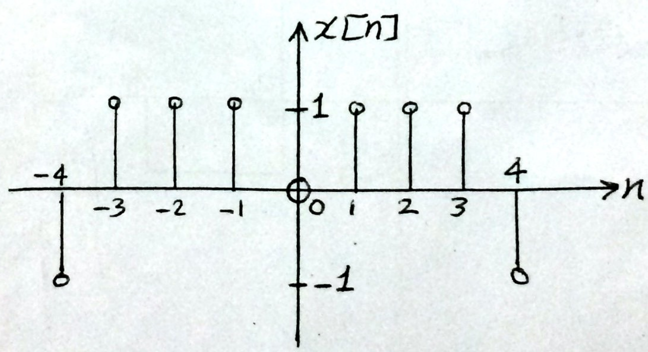
$\therefore$ The mathematical expression of the over all signal

$$x(t) = 5u(t+8.5) - 8u(t-2) + 3u(t-4)$$

H-W: Find the mathematical expression of the following signal $x(t)$.



Ex: Find the mathematical expression of the following signal



Sol.

1- a step signal started at $n=-4$ with amplitude $= -1$

$$x[n] = -u[n+4]$$

2- another step signal started at $n=-3$ with amplitude $= +2$

$$x[n] = 2u[n+3]$$

3. another step signal started at $n=0$ with amplitude $= -2$

$$x[n] = -u[n]$$

4. another step signal started at $n=1$ with amplitude $= 1$

$$x[n] = u[n-1]$$

5. another step signal started at $n=4$ with amplitude $= -2$

$$x[n] = -2u[n-4]$$

6. another step signal started at $n=5$ with amplitude $= -1$

$$x[n] = u[n-5]$$

$\therefore$ the overall signal is

$$x[n] = -u[n+4] + 2u[n+3] - u[n] + u[n-1] - 2u[n-4] + u[n-5]$$

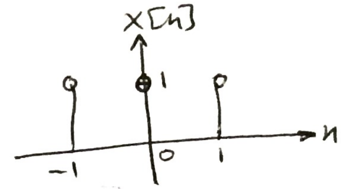
H.W/ sketch the discrete signal that expressed by the following equation:

$$x[n] = \delta[n+1] + 2\delta[n+2] + u[n+2] - u[n-3]$$

Even and Odd Signals :

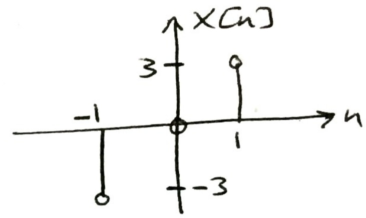
if $x[n] = x[-n] \Rightarrow$ even signal

ex:



$x[n] = -x[-n] \Rightarrow$ odd signal

ex:



For random signals,

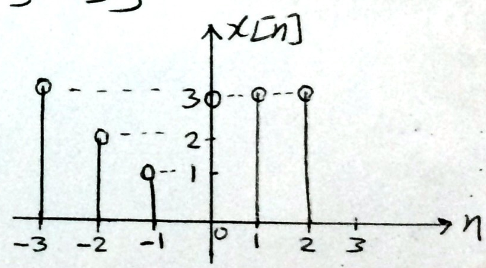
$x[n] = x_e[n] + x_o[n]$ (it consists of even and odd components)

$x_e[n] = \frac{1}{2}[x[n] + x[-n])$

$x_o[n] = \frac{1}{2}(x[n] - x[-n])$

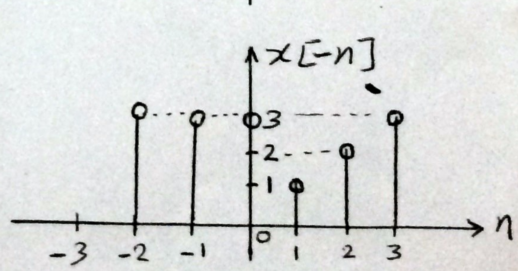
Ex: Find the even and odd parts of the following signal

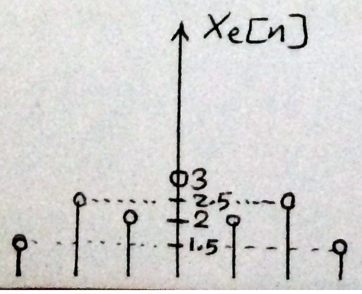
$x[n] = [3 \ \ 2 \ \ 1 \ \ \underset{\uparrow}{3} \ \ 3 \ \ 3]$



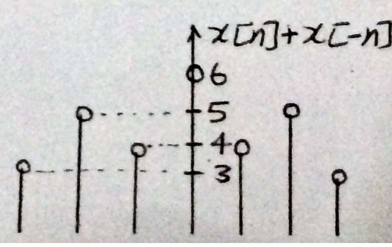
Sol

$x_e[n] = \frac{1}{2}(x[n] + x[-n])$

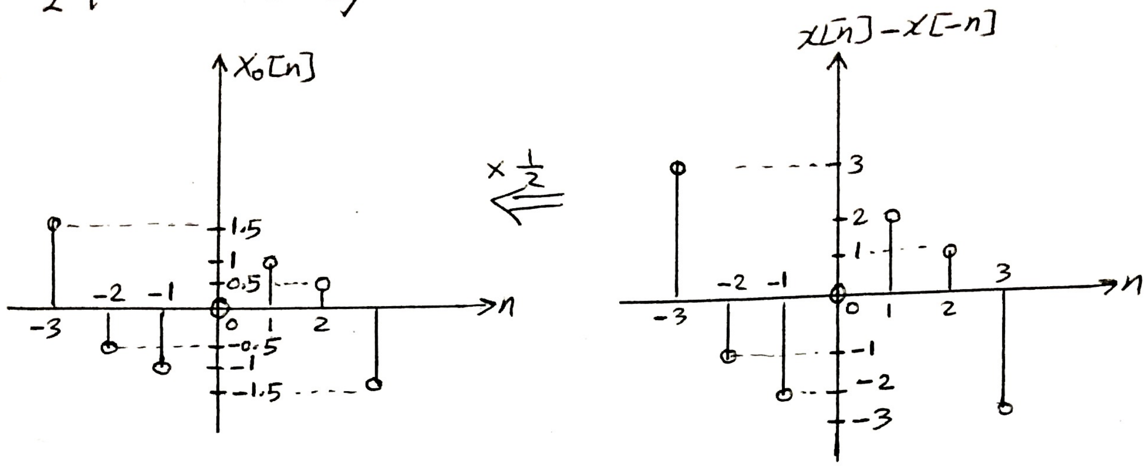




$\overset{\times \frac{1}{2}}{\Longleftarrow}$



$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$



HW: Find the even and odd Components of the following Signal:

$$x[n] = 2u[n+5] - u[n-1] - u[n-4]$$

H.W: Find the even and odd components of the following Signal:

$$x[n] = -2.5\,\delta[n+2] + \delta[n+1] + \delta[n-1] - 2.5\,\delta[n-2]$$

H.W: Find the even and odd parts of the following Signal

$$x[n] = [-2 \quad 2 \quad 0 \quad 1 \quad 2 \quad -2]$$

$$n = [-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4]$$

H.W: If you have the following Signal:

$$x[n] = 1.5\,u[n+2] - u[n-2] - 0.5\,u[n-6]$$

Answer the following:

1- Sketch the Signal.

2- Find and sketch $x[-n]$

3- Find and sketch $x[\frac{n}{3}]$

4- Find the even and parts component of the signal $x[n]$.

Linear Time Invariant System (LTIS)

$$x[n] \longrightarrow \boxed{T\{\cdot\}} \longrightarrow Y[n] = T\{x[n]\}$$

T = Transformation

LTIS Properties:

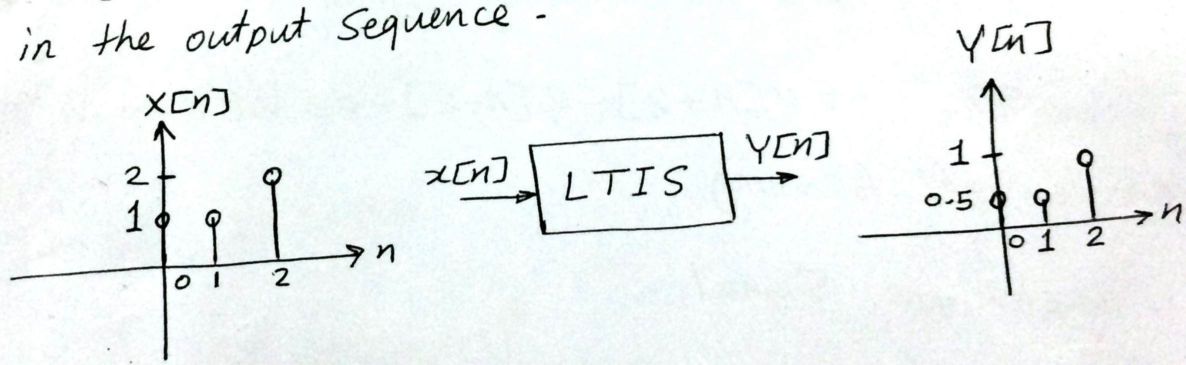
1- Linear System

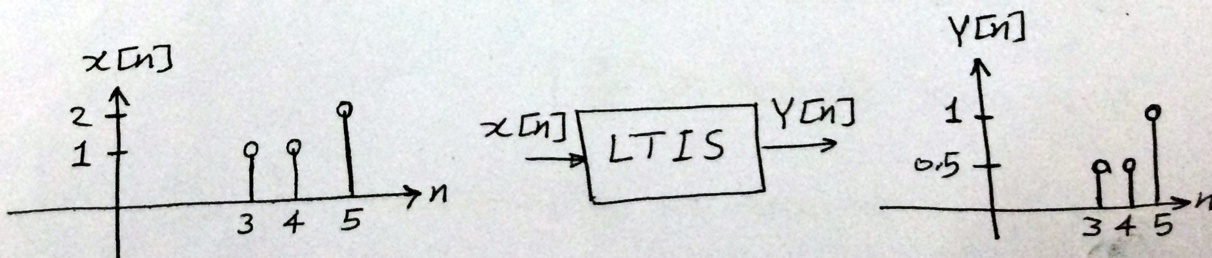
The system is linear if and only if

$$T\{x_1[n]\} + T\{x_2[n]\} = T\{x_1[n] + x_2[n]\}$$

2- Time Invariant

The system is time invariant if a time shift or delays of the input sequence causes a corresponding in the output sequence.





Ex: Determine if the system described by the following input-output equation is a linear or non linear.

$$Y[n] = \sqrt{x[n]}$$

Sol.

let $Y_1[n] = \sqrt{x_1[n]}$

$Y_2[n] = \sqrt{x_2[n]}$

$$\sqrt{x_1[n]} + \sqrt{x_2[n]} \neq \sqrt{x_1[n] + x_2[n]}$$

$\therefore$ The system is non linear.

H.W: Determine if the following systems described below are linear or non-linear.

a- $Y[n] = (x[n])^3$

b- $Y[n] = \log_2(x[n])$

c- $Y[n] = \ln(x[n])$.

d- $Y[n] = (\log(x[n]))^2$.

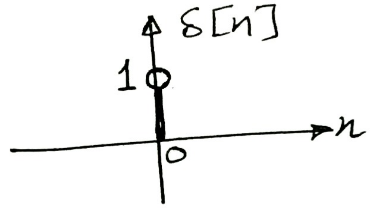
Ex: Determine if the following system is time invariant or not?

$$Y[n] = 2.x[2-n] - x[n]$$

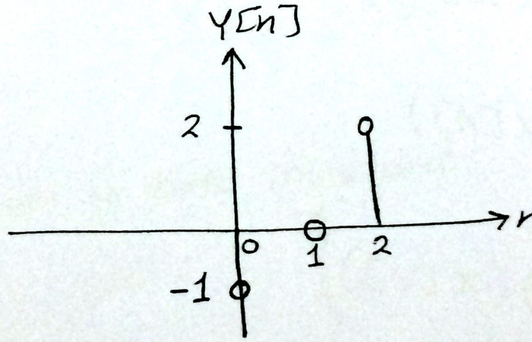
Sol.

let the input signal is unit impulse

$$x[n] = \delta[n]$$



$$Y[0] = 2.x[2] - x[0]$$
$$= 2.0 - 1 = -1$$
$$Y[1] = 2.x[1] - x[1]$$
$$= 2.0 - 0 = 0$$
$$Y[2] = 2.x[0] - x[2]$$
$$= 2.1 - 0 = 2$$



Then, let the input is shifted by 2 unit

$$Y[n] = 2.x\boxed{[2-n]} - x[n]$$

$$Y[n] = 2.x[2-(n-2)] - x[n-2]$$

$Y[n] = 2 \cdot x[4-n] - x[n-2]$

$Y[0] = 2 \cdot x[4] - x[-2]$

$= 2 \cdot 0 - 0 = 0$

$Y[1] = 2 \cdot x[3] - x[-1]$

$= 2 \cdot 0 - 0 = 0$

$Y[2] = 2 \cdot x[2] - x[0]$

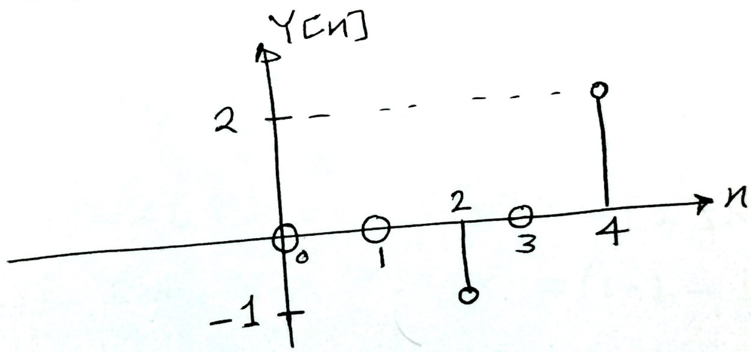
$= 2 \cdot 0 - 1 = -1$

$Y[3] = 2 \cdot x[1] - x[1]$

$= 2 \cdot 0 - 0 = 0$

$Y[4] = 2 \cdot x[0] - x[2]$

$= 2 \cdot 1 - 0 = 2$



$\therefore$ The system is time invariant

Ex: Determine if the following system is time invariant or not?

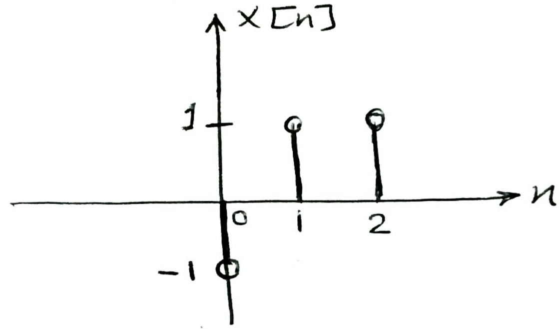
$$Y[n] = 2 \cdot x[2-n] - x[n]$$

and the input signal $x[n]$ is

$$x[n] = [-1 \quad 1 \quad 1]$$
$$n = [\ 0 \quad 1 \quad 2\ ]$$

Sol.

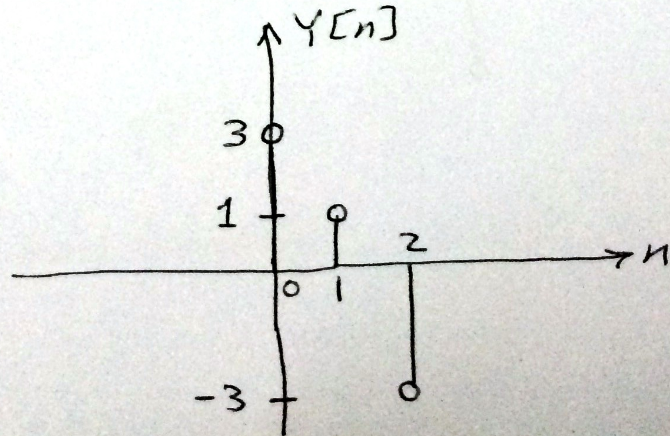
the input signal is shown below:



$$Y[0] = 2 \cdot x[2] - x[0]$$
$$= 2 \cdot 1 - (-1) = 3$$

$$Y[1] = 2 \cdot x[1] - x[1]$$
$$= 2 \cdot 1 - 1 = 1$$

$$Y[2] = 2 \cdot x[0] - x[2]$$
$$= 2 \cdot (-1) - 1 = -3$$



let the input is shifted by 2

$Y[n] = 2 \cdot x[2-n] - x[n]$

$Y[n] = 2 \cdot x[2-(n-2)] - x[n-2]$

$Y[n] = 2 \cdot x[4-n] - x[n-2]$

$Y[0] = 2 \cdot x[4] - x[-2]$

$= 2 \cdot (0) - 0 = 0$

$Y[1] = 2 \cdot x[3] - x[-1]$

$= 2 \cdot (0) - 0 = 0$

$Y[2] = 2 \cdot x[2] - x[0]$

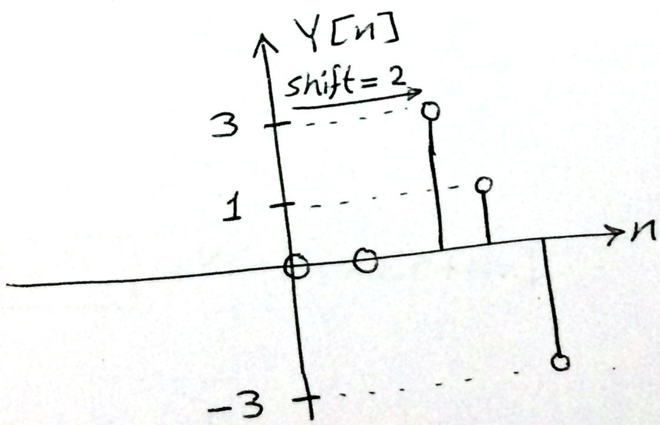
$= 2 \cdot (1) - (-1) = 3$

$Y[3] = 2 \cdot x[1] - x[1]$

$= 2 \cdot (1) - (1) = 1$

$Y[4] = 2 \cdot x[0] - x[2]$

$= 2 \cdot (-1) - 1 = -3$



$\therefore$ The system is time invariant.

H.W: Determine if the following System is time invariant or not?

$$Y[n] = x[n^2 - 1] + 2x[n]$$

H.W: Determine if the following system is time invariant or not?

$$Y[n] = 2x[n-2] - x[n-1] - 1.5x[n]$$

using the input Signal $x[n]$ as described below:

$$x[n] = 3\delta[n+3] + 2\delta[n+2] + 1\delta[n+1] + 1\delta$$