وزارة التعليم العالي والبحث العلمي
الجامعة التقنية الشمالية الكلية التقنية – كركوك
قسم تقنيات هندسة الوقود و الطاقة
المرحلة الثالثة

Thermodynamics المفردات النظرية لمادة

Week	Item
1	Measuring units, examples, force, pressure, specific volume
2	Thermodynamics terms, state process, equilibrium in thermodynamics, classification of systems
3	Temperature, kind of temperature measuring and relation between them, pressure measurements and relation between theme
4	Work and kind of work, energy and forms of energy examples
5	The first law of thermodynamics
6	Enthalpy examples
7	Applying the first law on closed systems
8 – 9	Applying the first law on opened systems, examples
10	Specific heat, kinds of specific heat and relation between them, examples
11	Gas constant, the universal gas constant and specific, examples
12	Ideal gas boyle’s law, charl’s law, examples
13 - 14	Thermodynamics processes and applications
15	Reversible and irreversiblity
16	The second law of thermodynamics
17	Results of the second law of thermodynamics
18	Heat engine, heat pump
19	Entropy, changes on closed systems and temperature – entropy plane
20	The carnot cycle and examples

21	**Otto cycle (net work input), Efficiency of Otto cycle**
22	**Examples on Otto cycles**
23	**Diesel cycle, (net work input), Efficiency of Diesel cycle**
24	**Examples on diesel cycle**
25	**Dual cycle, examples**
26	**Comparing between fuel – air and the air standard cycles**
27 - 28	**The actual cycle, comparing between actual cycles and air standard cycles**
29	**Gas turbine**
30	**Example on gas Turbine**

Mechanical engineering thermodynamic:-
Energy transferred between heat and work.
Heat transferred to work by heat engine.
Work transferred to heat by heat pump or ref.

Force (F): - the force acting 1 kg mass moving with acceleration of m/sec^2.

Force S.I unit (N).

$N = (kg.\ m/sec^{2)}$.

Mass S.I unit (kg), symbol (m).

Acceleration S.I unit (m/sec^2), symbol (a).

Gravitational acceleration, symbol (g). $g = 9.81\ m/sec^2$.

$F = m*a$

$= kg * m/sec^2 = N$

For weight $(w) = m* g = N$

Pressure (p): - defined as the force per unit area, symbol.

$P= F/A = N/ m^2$, where A =area (m^2).

Absolute pressure (p_a):- weight of liquid column per unit area.

$$Pa = \frac{F}{A} = \frac{m * g}{A} = \frac{\rho * V * g}{A} = \frac{\rho * A* h * g}{A} = \rho * h * g$$

$Pa = \rho * h * g$

Where:-
ρ = density of liquid (kg /m^3).

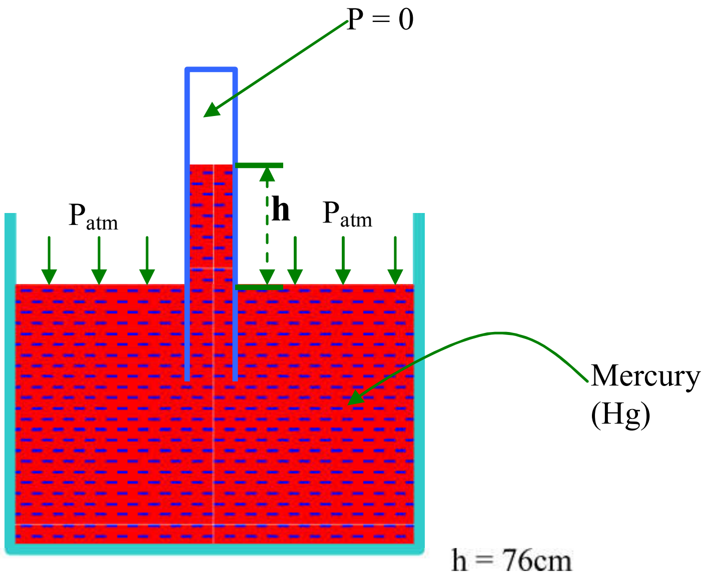
V = volume of liquid (m^3).

h = height of liquid (m).

$Pa = (kg /m^{3)}* m * (m/sec^2) = (kg)* (m/sec^2)/ m^2 = N / m^2$.

Atmospheric pressure (p_{atm}) :- weight of atmospheric per unit area of earth's surface.

The standard instrument for recording of (p_{atm}) is the barometer as shown in fig .



The density of mercury (Hg) = 13616 kg/m^3.

P_{atm} is measure in meters of mercury (m.Hg).

The conversion from (m.Hg) to (N/m^2) is given by: - for example,

$1\ (m.Hg) = \rho * h * g$

$= 13616 * 9.81*1$

$= 133660\ N/m^2$

$= 1.3366 * 10^5\ N/m^2 = 1.3366\ bar = 133.66\ KN/m^2$

$\underline{1\ bar} = \underline{10^5\ N/m^2} = \underline{10^2\ KN/m^2}$

$P_{atm} = 1$ atmosphere $= 1.01325$ bar

$= 101.325\ KN/m^2 = 76$ cm Hg $= 0.76$ m. Hg

$= 13616 * 0.76 * 9.81$

$= 101325\ N/m^2$

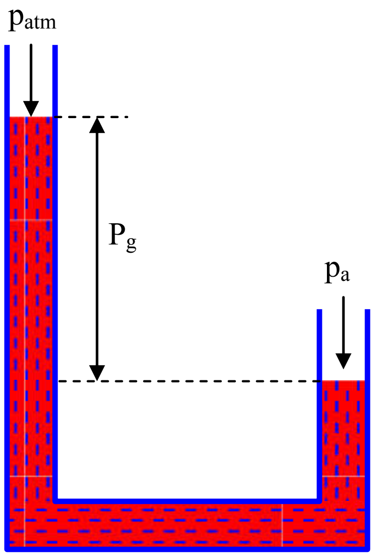
Gauge pressure (p_g):- it is defined as:-

$$p_g = Pa - P_{atm}$$

Where:-

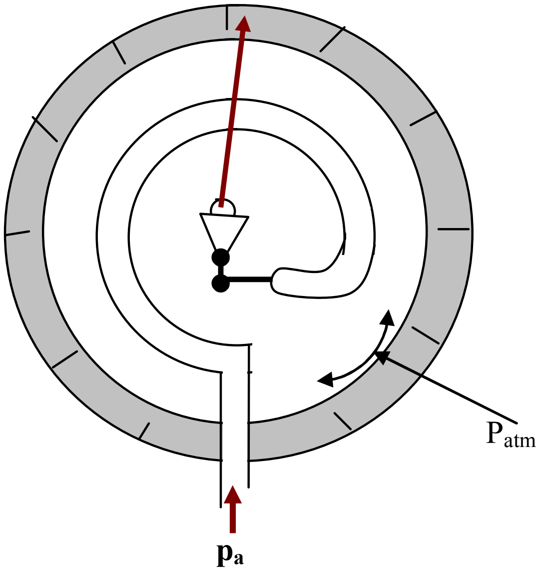
P_{atm} = 1.01325 bar

1- **p_g** is measured by differential manometer in which the P to be measured applied to one line of U tube containing barometer fluid to the other side as shown in fig.



(Differentional manometer)

2- **p_g** is measured with bourdon gauge :-



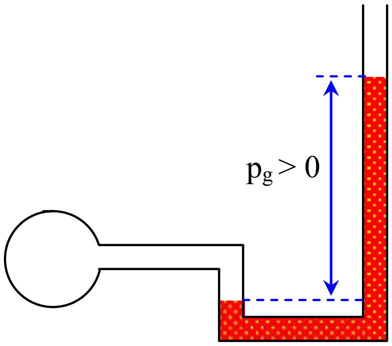
To convert the pressure reading from a bourdon gauge to a (p) measured we used,

$p_g = p_a - p_{atm}$

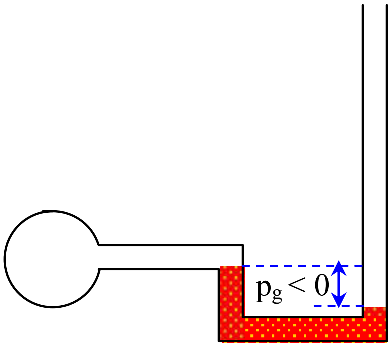
$p_g = +$ vie when $p_a - p_{atm} > 0$

$p_g = -$ vie when $p_a - p_{atm} < 0$ or $p_{atm} - p_a > 0$

Both bourdon gauge & manometers can be read either positive or negative (p_g) as shown in fig.



+ ve



- ve

The (-ve) value of p_g is called vacuum which given by,

$p_g = p_{atm} - p_a$

Vacuum $= p_{atm} - p_a$

EX: - A steam turbine exhausted in to a condenser the gauge on which read (0.65 m.Hg) vacuum. Express this in absolute pressure, assuming the atmosphere pressure is 1.013bar.

Sol:-
Vacuum $= p_{atm} - p_a$
Vacuum $= 0.65 * 13616 * 9.81$

$= 86822.4 \text{N/m}^2$

$= 0.868224$ bar

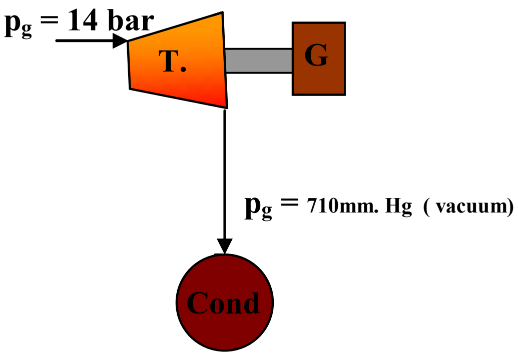
$p_a = p_{atm} -$ Vacuum

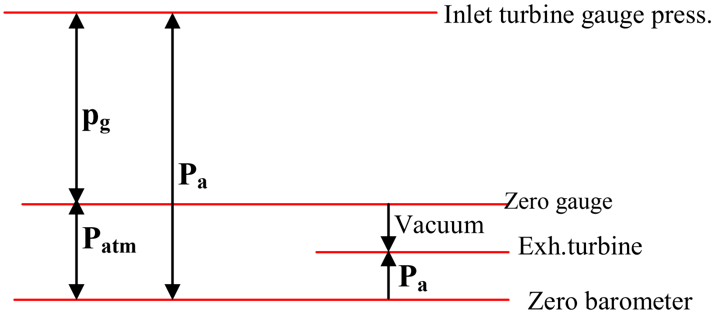
$= 1.013 - 0.868224$

$= 0.145$ bar

EX: - A turbine is supplying with steam at a gauge pressure of 14 bar, after expansion in the turbine the steam passed to a condenser which is maintained at a vacuum of 710mm.Hg by means of pumps the barometer pressure is 770 mm.Hg. Express the inlet & exhaust steam absolute pressure in N/m^2.

Sol:-





Inlet turbine

$p_g = p_a - p_{atm}$

$p_{atm} = 770 \text{ mm.Hg} = 0.77 \text{ m.Hg} * 13616 * 9.81$

$= 102851 \text{ N/m}^2$

$= 1.02851 * 10^5 \text{ N/m}^2 = 1.02851 \text{ bar}$

$p_a = p_g + p_{atm}$

$= 14 + 1.02851 = 15.0285 \text{ bar} = 15.0285 * 10^5 \text{ N/m}^2$

Exhausted from turbine

$\text{Vacuum} = 710 \text{ mm.Hg} = 0.71 \text{ m.Hg} * 13616 * 9.81$

$= 94836 \text{N/m}^2 = 0.94836 \text{bar}$

$p_a = p_{atm} - \text{vacuum}$

$= 1.02851 - 0.94836 = 0.08015 \text{ bar} = 0.08015 * 10^5 \text{ N/m}^2 = 8.015 * 10^3 \text{ N/m}^2$

EX:- If the vacuum in the condenser is 720mm.Hg and barometer reading is 763mm.Hg find the condenser absolute pressure and atmosphere pressure in bar.

Sol:-

$\text{Vacuum} = 720 \text{ mm.Hg} = 0.72 \text{ m.Hg} * 13616 * 9.81$

$= 0.961725312 \text{bar}$

$p_{atm} = 763 \text{ mm.Hg} = 0.763 \text{ m.Hg} * 13616 * 9.81$

$= 102851 \text{ N/m}^2$

$= 1.019161685 * 10^5 \text{ N/m}^2 = 1.019161685 \text{ bar}$

$p_a = p_{atm} - \text{vacuum} \quad = 1.019161685 - 0.961725312 = 0.0574336373 \text{ bar}$

EX: - Find the pressure in bar for an oil of density is $0.8 kg/m^3$ and height of 30cm.

Sol:

$P = \rho * h * g$

$= 0.8 * 0.3 * 9.81 = 2.3544\ N/m^2 = 0.000023544$ bar

EX:- A vessel is connected to tube manometer the difference in the liquid level is 20 mm find the pressure in the vessel if the liquid is: - (a) water, (b) Hg

Sol:-

(a) Water
$P = \rho * h * g$

$= 1000 * 0.02 * 9.81 = 196.2 N/m^2 = 0.001962$ bar

(b) Hg.

$P = \rho * h * g$

$= 13616 * 0.02 * 9.81 = 2671.4592 N/m^2 = 0.026714592$bar

Temperature

Temperature: - Is the property which determines the ability of the system to transfer heat.

Temperature scale: - Is an arbitrary set of number and method for assigning each number to a definition level of temperature for example the melting point of ice and boiling point of water at standard atmosphere pressure

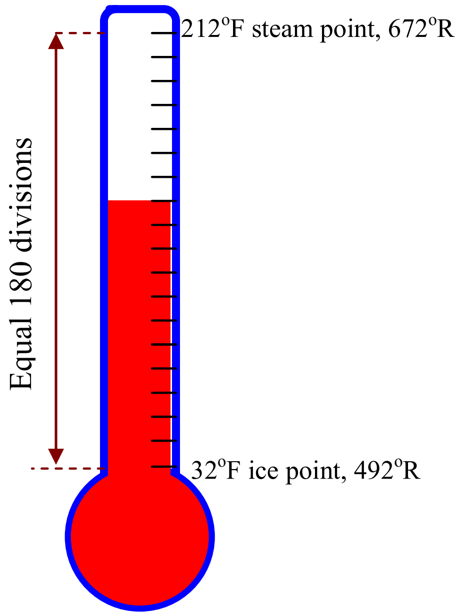
Thermometer: - is a measuring device yielding a number at each temperature level the number is functionally related to the temperature.

Actual thermometer is based on changes of certain property with temperature. For example:-

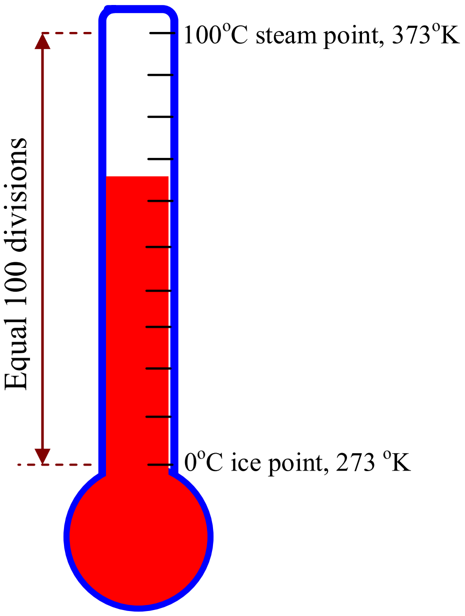
a- Volumetric expansion of gases, liquid, and solids.
b- Pressure exerted by gases.
c- Electrical resistance of solids.
d- Vapor pressure of liquids.
e- Thermo electricity.

- The mercury in glass thermometer provides an example of the observed quantity and height of the column of mercury dependent not only on temperature but also the external pressure acting on the bulb and stem.
- The ice point or freezing point of water occurs at 273.15 $^oK = 0\ ^oC$ and the steam point of boiling point of water occurs at 373.15 $^oK = 100\ ^oC$. The interval is100 ^oC and this basis of Celsius scale whose determine ice point 0 ^oC.

- On the Fahrenheit scale interval between the ice and steam point is divided in to 180^o while the datum is the freezing point at 32^oF and steam point at212^oF.
- The absolute scale corresponding to the Fahrenheit scale is the Rankine scale.



Fahrenheit thermometer



Celsius thermometer

$$\frac{{}^oC - 0}{100} = \frac{{}^oF - 32}{180} = \frac{{}^oK - 273}{100} = \frac{R - 492}{180}$$

$${}^oF = \frac{9}{5}\,{}^oC + 32 \longrightarrow {}^oF = {}^oR - 460$$

$${}^oC = \frac{5}{9}\left({}^oF - 32\right) \longrightarrow {}^oC = {}^oK - 273$$

EX: - At p_{atm} the boiling point of water is 100^oC and for helium 15 ^oK. Express these in^oF.

Sol:-

For water

$^oF = 9/5\ ^oC + 32 = 9/5 * 100 + 32 = 212\ ^oF$ boiling point of water.

For helium

$^{o}C = {}^{o}K - 273$

$= 15 - 273 = -258\,^{o}C$

$^{o}F = 9/5\,^{o}C + 32$

$= 9/5 * (-258) + 32 = -432.4\,^{o}F$ boiling point of helium

EX: - The following fixed point at p_{atm} are given in ^{o}F & ^{o}R. Express them to degree Celsius. Oxygen point 297.32 ^{o}F, gold point 2400^{o}R, sulfur point 832.28 ^{o}F , Antimony 1620 ^{o}R.

Sol:-
Oxygen point

$$\frac{^{o}C - 0}{100} = \frac{^{o}F - 32}{180}$$

$$\frac{^{o}C}{100} = \frac{297.32 - 32}{180}$$

$^{o}C = 147.4^{o}$

gold point

$$\frac{^{o}C - 0}{100} = \frac{^{o}R - 492}{180}$$

$$\frac{^{o}C}{100} = \frac{2400 - 492}{180}$$

$^{o}C = 1060^{o}$

Sulfur point

$$\frac{^{o}C - 0}{100} = \frac{^{o}F - 32}{180}$$

$$\frac{^{o}C}{100} = \frac{832.28 - 32}{180}$$

$^{o}C = 444.6^{o}$

For Antimony

$$\frac{{}^\circ C - 0}{100} = \frac{{}^\circ R - 492}{180}$$

$$\frac{{}^\circ C}{100} = \frac{1620 - 32}{180}$$

$${}^\circ C = 882.222^\circ$$

Thermal Equilibrium: - If a temperature of two bodies is equal when heat flows between them, two bodies is called bodies at thermal equilibrium.

Zero law of thermodynamics: - when there are two bodies in thermal equilibrium with third body, the two bodies is become in state of thermal equilibrium and this defined as (Zero Law of thermodynamic).

Energy

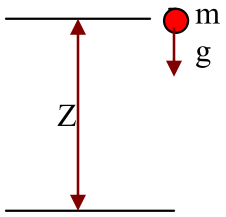
Energy: - Is the capacity either latent or apparent to exert a force through the distance.

$$\boxed{\mathbf{E = F * S}}$$ …….. Joule (N.m)

Where:-
E = Energy
F = Force
S = distance

Potential Energy "pot. E.":- It is energy stored in a mass by virtue of its position in gravitational.

$$\boxed{\mathbf{Pot.\ E = m.\ g\ .Z}}$$ ... Joule (N.m)



Where:-
Z = height

Kinetic energy "K.E":- it is energy wanted to move a mass with velocity.

$$\boxed{\mathbf{K.E = \tfrac{1}{2}\, m\, V^2}}$$ …….. Joule (N.m)

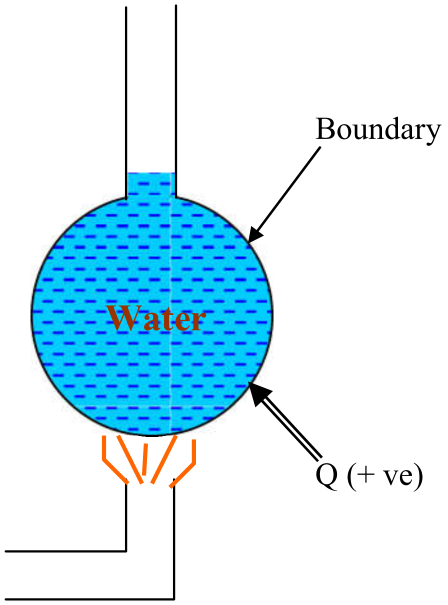
Where:-

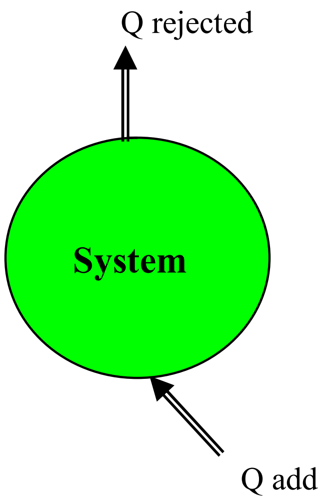
V= velocity m/sec

Heat "Q":- is regarded as energy in thermo. If it crossing the boundary of the system by virtue of temperature difference.

Q = KJ or Kcal

q = KJ /kg or Kcal / kg





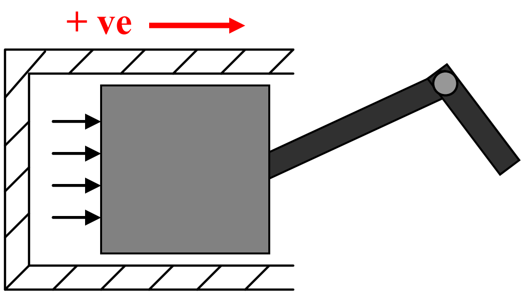
Q = (+ ve) when added to the system.
Q = (- ve) when rejected from system.

System: - Is a region neither necessary of constant volume nor fixed in space, where transfer of mass & energy are to be studied.

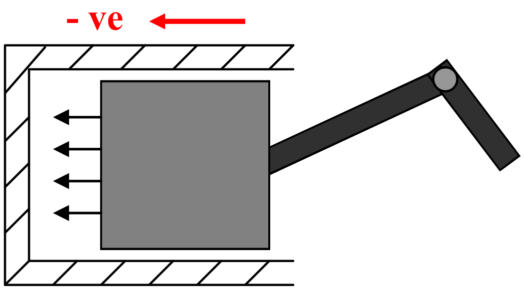
Boundary: - The actual envelope in closing the system is defined as boundary.

Surrounding: - The region out side the system is defined as surrounding.

Work: - It is the mechanical energy crossing the boundary of system and the only effect of which is rising of a weight out side or inside the system.



W = kJ
w = kJ /kg



Work is (+ ve) if it is done by the system on the surrounding.
Work is (- ve) if it is done on the system from the surrounding.

Internal energy (U):- It is energy stored in the mass of substance in the system.

$U = kJ$
$u = kJ/kg$

$$\frac{du}{dT} = Cv$$

$$du = Cv \,.\, dT$$

$$\int_{u_1}^{u_2} du = \int_{T_1}^{T_2} Cv \,.\, dT$$

$u_2 - u_1 = Cv\,(T_2 - T_1)$ or

$$\boxed{\Delta u = Cv.\, \Delta T}$$

$U_2 - U_1 = m.\, Cv\,(T_2 - T_1)$ or

$$\boxed{\Delta u = m.Cv.\, \Delta T}$$

Flow energy (F.E):- It is energy necessary to flow or to move a fluid at steady rate without changing its state.
The force necessary to move the piston without changing P, V, T of the gas.

$F = p * A$

The energy necessary to move the gas without changing its state = the energy necessary to move piston a distance "L".

$F.E = p*A*L = p*V = m*p*v$ (Joule)

Where (v) = specific volume $m^3/kg = 1/\rho$, where (ρ) = density kg/m^3.

Flow energy per unit mass of the gas (F.E /m) = p.v (J /kg)

F.E/mass = 10^2 *p * v (kJ / kg)

Where:-
(p)= pressure (bar)
(v) = specific volume m^3 /kg

Power: - is the rate of doing work, and is measured in (J/sec) or (N.m / sec) = watt,

1metric (h.P) = 75 kg

F.m = 75 * 9.81 = 735.75 watt ≈ 740 watt

1 h.P = 0.74 kW.

1 British h.P = 0.745 kW = 745 watt.

1Cal = 4.1868 joule or 1kCal = 4.1868 kJ

Enthalpy (H):- is the sum of internal energy plus flow energy of a moving gas.

$$\mathbf{H = U + p.V \longrightarrow kJ}$$

$$\mathbf{h = u + p.v \longrightarrow kJ/kg}$$

As Uis property and p,V are properties of the gas therefore H is a property.

EX: - Express the following quantities in term of kJ and kCal.
a- h.P. hour (h.P.h). **b- kW .hour (kW .h).**

Sol:-
a-
h.p = 0.7457 kW = 0.7457 kJ /sec

h.p.h = 0.7457 kJ /sec * 3600 sec = $\underline{2685\ \text{kJ}}$

1kCal = 4.1868 kJ

2685 / 4.186 = $\underline{640.2\ \text{kCal}}$

b-
kW.h = 1 kJ /sec *3600 sec = $\underline{3600\ \text{kJ}}$

1kCal = 4.1868 kJ

3600/ 4.186 = $\underline{859.845\ \text{kCal}}$

EX: - Nitrous Oxide of density 31.084 kg/m^3 is carried in a pipe line 60.96m above sea level the gas is at temperature 148.9°C. It flows along a pipe at the rate of 0.096m/sec and its specific at constant volume (Cv) is 0.161kCal / kg.K, taking sea level as datum for heights

and 0^oC as datum for energy involving temperature evaluate the potential energy, kinetic energy, internal energy and total energy of the gas.

Sol:-

$\rho = 31.084\ kg/m^3$

$h\ (Z) = 60.96m$

$T = 148.9^oC$

$C = 0.096\ m/sec$

$Cv = 0.161\ kCal\ /kg\ .K$

$Cv = 0.161 * 4.186 = 0.673946\ kJ\ /kg.K$

$P.E = m.g.Z(J)$

$= g.Z\ (J/kg)$

$= 9.81 * 60.96 = 598.0176\ kJ/kg$

$K.E = ½\ m * C^2\ (J)$

$= ½\ C^2\ (J/kg)$

$= ½\ (0.096)^2 = 0.004608\ kJ/kg$

$U = m.\ Cv\ .T\ (J)$

$u = Cv.T(J/kg)$

$= 0.673946 * (148.9 +273) = 284.337\ kJ/kg$

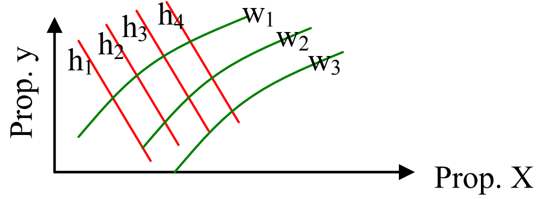
$T.E = Pot\ .E + K.E +u$

$= 598.0176 + 0.004608 + 284.337 = 882.3592\ kJ/kg$

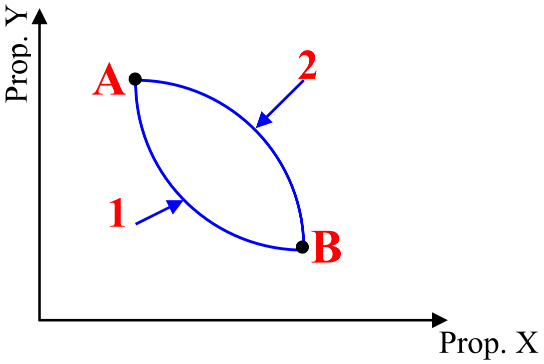
Property: - It is an observable characteristic of a system or part of a system.

State: - It is the condition of the system or part of system, at any constant of time, described by its properties.

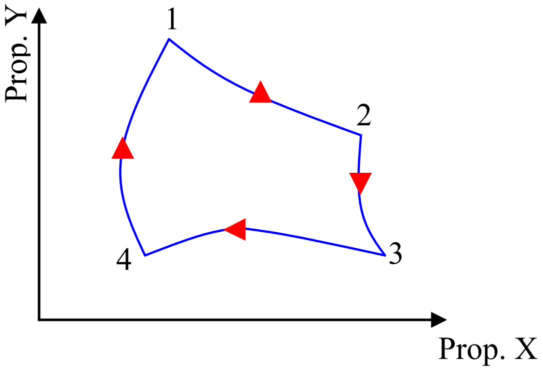
Property diagram: - It is a plane described by two coordinates representing two in depended properties; a point in the plane represents state families of curves of other properties such as T, W and H can be drawn on the diagram and curves of some families could not intersect.



Process: - It is locus of state represented on property diagram at which gas take to the same point from primary state or inlet state to the end of final state either through the process A (1) B or A (2) B.



Cycle: - It is sum of different processes such that the final process brings the substance to the original state.



Problems

1-1 **A turbine is supplied with steam at a gauge pressure of 14bar. After expansion in the turbine the steam passes to a condenser which is maintained at a vacuum of 710mm Hg by means of pumps. The barometric pressure 772mm Hg. Express the inlet and exhaust steam (absolute) pressures N/m^2. Take the density of mercury as 13.6*10^{3}kg/m^3**

(1502999, 8271.8N/m^2)

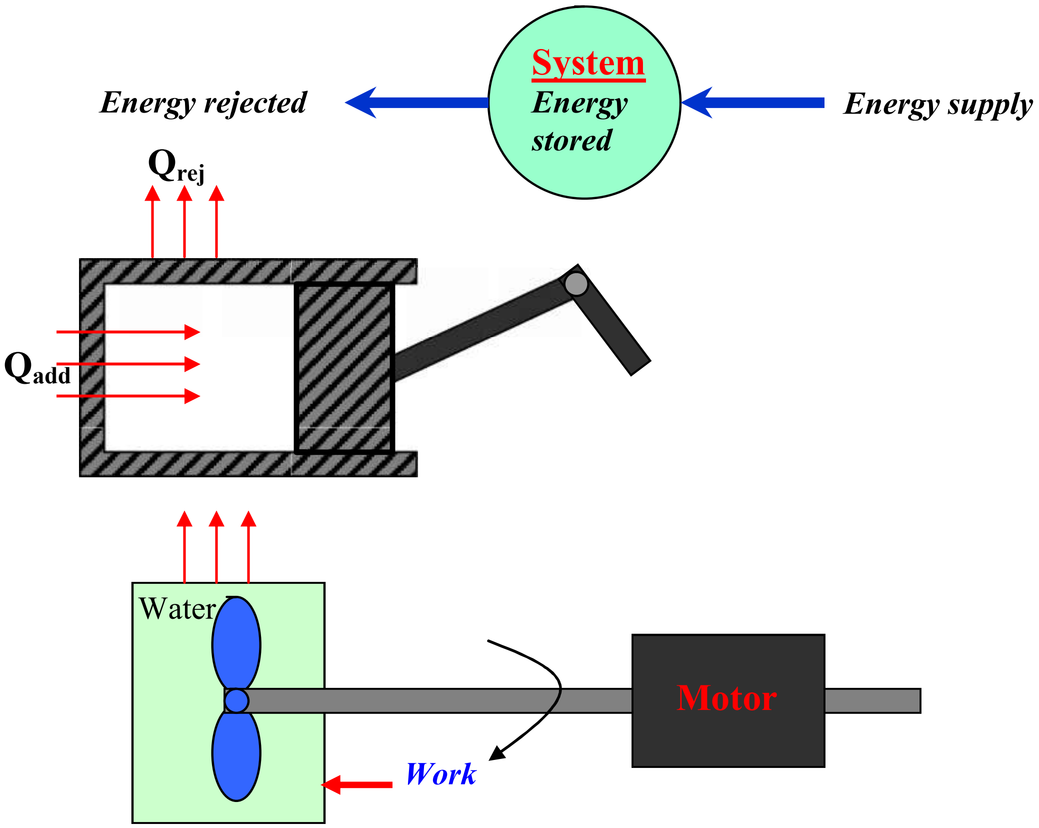
1-2 **A mercury manometer measuring the pressure in a steam plant condenser reads 63cm vacuum. The barometer in the plant room reads 76cm of mercury. What is the absolute pressure in the condenser in N/m^2?**

(17344N/m^2)

The first law of thermodynamics

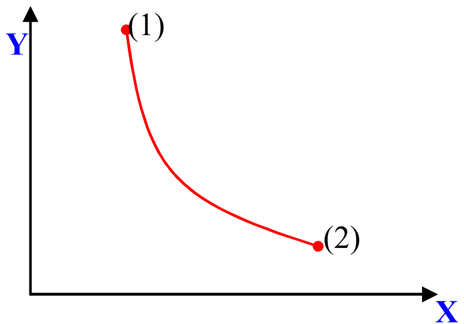
The first law of thermodynamics means that Energy can neither be created nor dissipated it can be transferred from a type to another.

Energy supplied = energy rejected + stored energy



Application to close system process

For the process shown on the property diagram the system changes its state from (1) to (2) through the shown process while heat energy “Q_{12}” is supplied to it,



Work “W_{12}” done by the system & increase or decrease the stored energy will appear as a change in pot. E, K.E, and U.

$Q_{12} = W_{12}$ + stored energy

$$Q_{12} = W_{12} + \Delta\, Pot.E_{12} + \Delta\, K.E_{12} + \Delta\, U_{12}$$

$$Q_{12} = W_{12} + \Delta E_{12} \text{ ——— } \textbf{(1) energy equation}$$

Where:-

$$\Delta E_{12} = \Delta\, Pot.E_{12} + \Delta\, K.E_{12} + \Delta\, U_{12}$$

Usually in closed system processes the change in Pot.E and change in K.E is negligible. Thus the Equ. (1) Reduced to $Q_{12} = W_{12} + \Delta U_{12}$

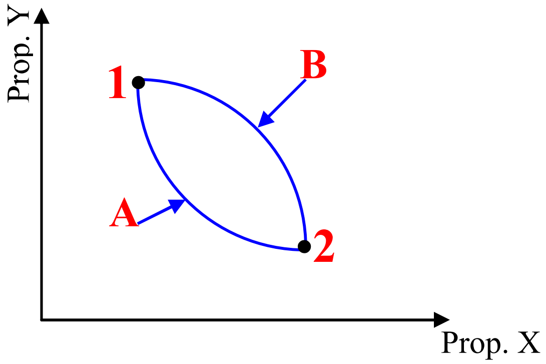
$$\boxed{Q_{12} = W_{12} + \Delta U_{12}}$$ ——— for closed system process

Cyclic process:-

In cyclic process the total change in stored energy should equal at sum of changes of stored energy during the different processes constituting the cycle as substance is returned back to the original state then there is no change in the stored energy and thus :-

$$\boxed{\oint dQ = \oint dw}$$

The symbol $\oint$, which is called the cyclic integral

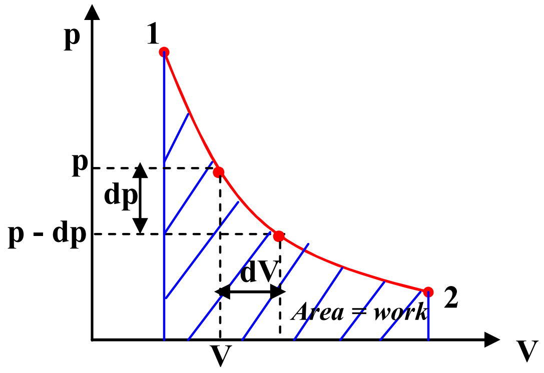


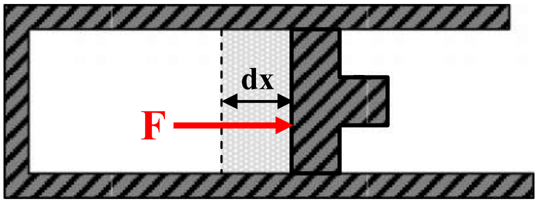
Working done at the moving boundary a closed system:-

For the shown system in diagram the cylinder is filled with a certain gas and fitted with a friction less piston against the certain resistance. The force necessary to move the piston is:-

F = P * area of the piston = P* A

Assume that the piston moves every small distance dx, such that the pressure between V & V + dV will be assumed not to change.





The energy – work done to move the piston the infinitesimal distance should equal to:-

Work done = F * dx = p*A*dx

$$dw = p * dV$$

The total work done developed by the gas on the piston a sit moves from (1) to (2),

$\int_1^2 dw = \int_1^2 p.dV$, there fore,

$$\boxed{w_{12} = \int_1^2 p.dV}$$

w_{12} can be calculated by measuring the area under the process represented on the p.V diagram.

EX: - The energy of system increases 120 kJ while 150 kJ work is transferred to surrounding. Is heat added or taken a way from the system.

Sol:-

$\Delta E_{12} = + 120$ kJ

$w_{12} = + 150$ kJ

$Q_{12} = w_{12} + \Delta E_{12}$

$= 150 + 120 = 270$ kJ added

EX: - During an expansion process. The work done and the heat received by the system are respectively 20 kJ and 50000 J. Calculate the change of energy for the system.

Sol:-

$w_{12} = +20$ kJ

$Q_{12} = +50000$ J = 50 kJ

$Q_{12} = w_{12} + \Delta E_{12}$

$\Delta E_{12} = Q_{12} - w_{12} = 50 - 20 = +30$ kJ

EX: - A closed system consists of a cylinder of water stirred by a paddle wheel. For the process the work was 34 kJ the initial internal energy was 120 kJ and the final internal energy after (1) hour of stirring was 144 kJ. Find the heat transferred in kJ /hour. Is the temperature of the system raising or falling.

Sol:-

$w_{12} = -34$ kJ

$U_1 = 120$ kJ

$U_2 = 144$ kJ

$\Delta U_{12} = 144 - 120 = 24$ kJ

$Q_{12} = w_{12} + \Delta U_{12} = -34 + 24 = -10$ kJ / h the temperature is rising

Problems

2-1 During the expansion of 1kg of a fluid the internal energy changes from an initial value of 1162kJ/kg to a final value of 1025kJ/kg of shaft work are obtained, calculate the quantity and direction of exchange between the fluid and surroundings. Assume expansion is frictionless. (52kJ out wards)

2-2 In a certain non – flow process with air as the working fluid, 1000kJ is removed from the system and the internal energy decreases by 200kJ. Is the process an expansion or compression? What work is performed? (800kJ compression work)

Ideal (perfect) gas

Ideal gas it is the gas which has the following properties:-

1- It follows the general gas equation.
2- Its specific heat at constant pressure and specific heat at constant volume are constant.

Ideal gas laws:-

1- Boyle's law: - The pressure of the gas is inversely proportional to the volume at constant temperature.

$$p \ \alpha \ \frac{1}{V} \quad \text{at } T = \text{constant} \ , \ p = \frac{C}{V} \qquad \text{or} \qquad pV = \text{constant} \ ,$$

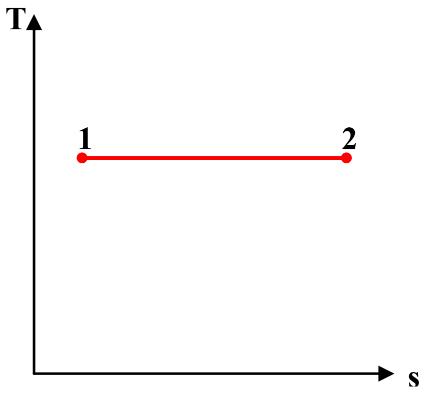
$$\boxed{p_1V_1 = p_2V_2} \quad \text{——} \quad p.V = C$$

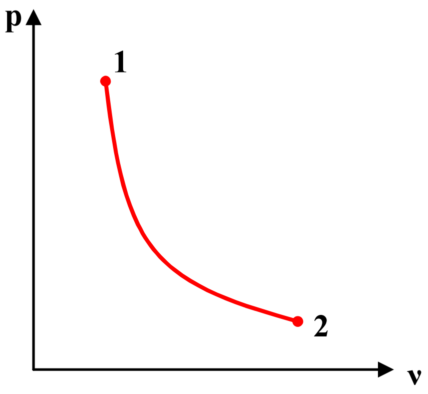
$$V = m \, . \, \nu \qquad \text{or} \qquad \nu = \frac{V}{m}$$

$$p_1.m \, .\nu_1 = p_2 \, . \, m \, .\nu_2$$

$$p_1.\nu_1 = p_2 \, .\nu_2$$

$$\boxed{\frac{p_1}{p_2} = \frac{\nu_2}{\nu_1} = \frac{V_2}{V_1}} \quad \text{———} \quad \text{at } T = \text{constant}$$

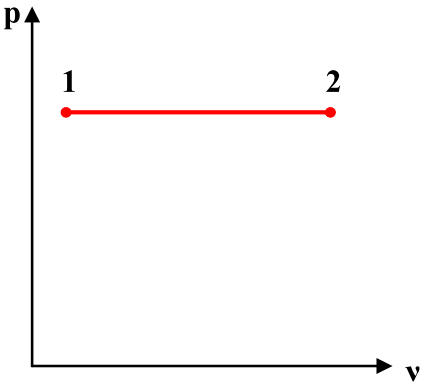




2- Charles law: - The volume of the gas is directly proportional to the temperature at constant pressure.

$V \ \alpha \ T$ at p = constant, therefore $V = C. \, T$ or, $V/T = \text{const.}$

$$\boxed{\frac{T_2}{T_1} = \frac{\nu_2}{\nu_1} = \frac{V_2}{V_1}} \quad \text{———} \quad \text{at } p = \text{constant}$$

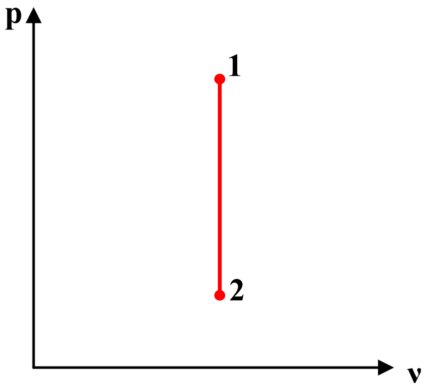


3- Gay – Lussac law: - The pressure of the gas is directly proportional to the temperature at constant volume.

p α T at V = constant, therefore p = C .T or,

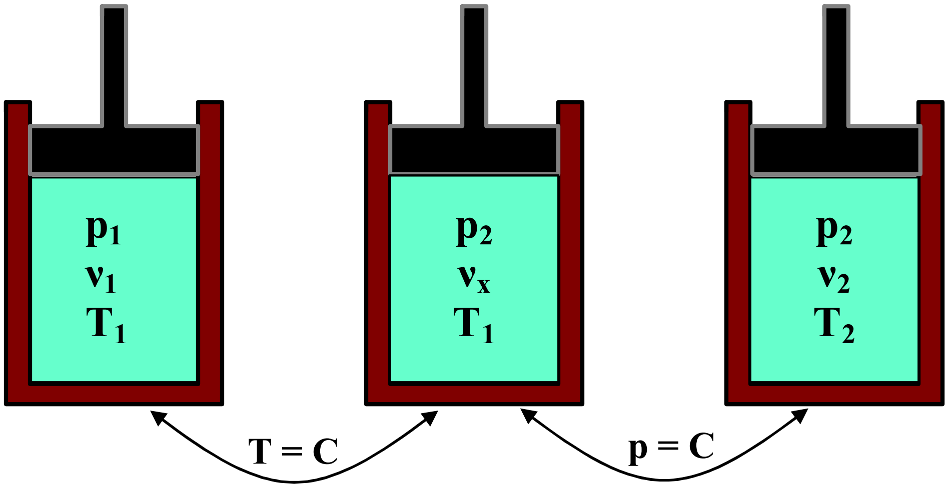
$$\frac{p}{T} = C$$

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$ ——— at V = constant



Equation of State of an Ideal Gas:-

A relationship between p, V& T



Boyle's law

$$p_1v_1 = p_2v_x$$

$$v_x = \frac{p_1}{p_2} * v_1$$

Charles law

$$\frac{v_x}{T_1} = \frac{v_2}{T_2}$$

$$v_x = \frac{T_1}{T_2} * v_2$$

$$\frac{p_1}{p_2} * v_1 = \frac{T_1}{T_2} * v_2$$

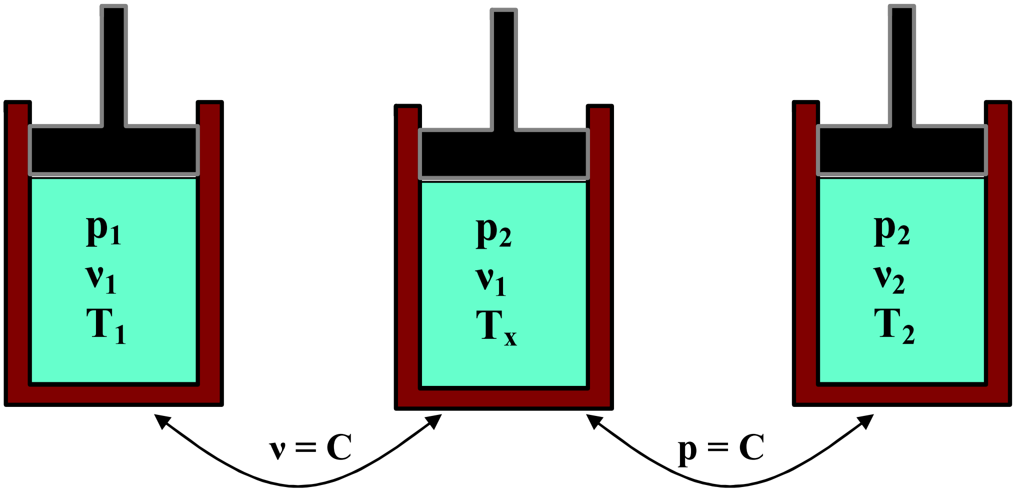
$$\boxed{\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}}$$

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2} = \frac{pv}{T} = R$$

Where (R) is the gas constant kJ/kg.K

$Pv = RT$

$\boxed{pV = mRT}$ — Is called general of state of an ideal gas (general equation of gas).



Gay – Lussac law

$$\frac{p_1}{T_1} = \frac{p_2}{T_x}$$

$$T_x = \frac{T_1}{p_1} * p_2$$

Charles law

$$\frac{v_1}{T_x} = \frac{v_2}{T_2}$$

$$T_x = \frac{T_2}{v_2} * v_1$$

$$\frac{T_1}{p_1} * p_2 = \frac{T_2}{v_2} * v_1$$

$$\boxed{\frac{v_2 * p_2}{T_2} = \frac{v_1 * p_1}{T_1}}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{pv}{T} = R$$

Where (R) is the gas constant kJ/kg.K

Pv = RT

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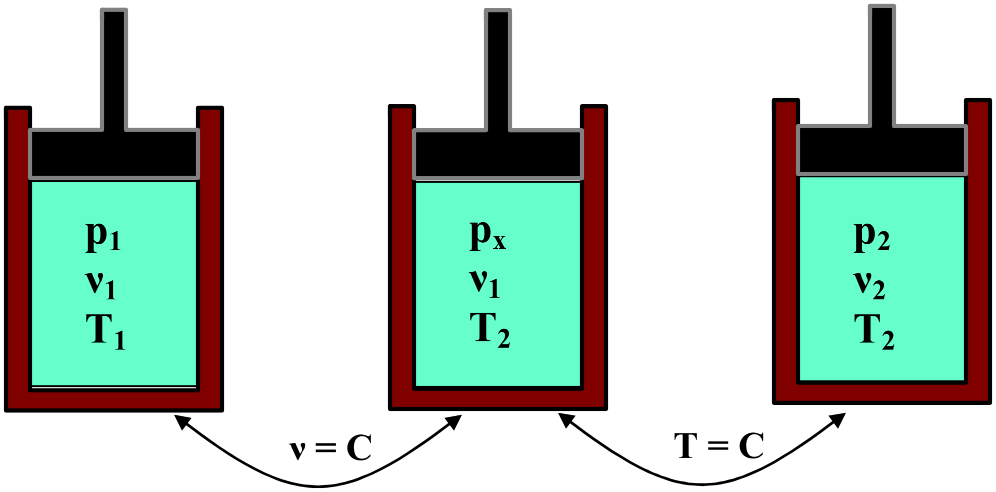


Gay – Lussac law

$$\frac{p_1}{T_1} = \frac{p_x}{T_2}$$

$$p_x = \frac{T_2}{T_1} * p_1$$

Boyle's law

$$p_x v_1 = p_2 v_2$$

$$p_x = \frac{p_2}{v_1} * v_2$$

$$\frac{T_2}{T_1} * p_1 = \frac{p_2}{v_1} * v_2$$

$$\boxed{\frac{v_2 * p_2}{T_2} = \frac{v_1 * p_1}{T_1}} = R$$

$T_1 \quad T_2 \quad T$

Where (R) is the gas constant kJ/kg.K

$Pv = RT$

$\boxed{pV = mRT}$ — Is called general of state of an ideal gas (general equation of gas).

EX: - An ideal gas occupies a volume of 0.105 m^3 at temperature 20 C^o and the pressure of 1.5 kg f /cm^2 absolute. Find the final temperature if the gas compressed to pressure7.5 kg f / cm^2 and occupies a volume of 0.04 m^3 (note 1kg f / cm^2 = 1bar).

Sol:-

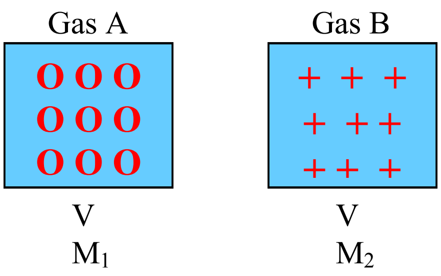
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

$$T2 = \frac{p_2\, v_2}{p_1\, v_1} * T_1$$

$T_1 = 20 + 273 = 293\ ^{o}K$

$$T2 = \frac{7.5 * 0.04}{1.5 * 0.105} * 293 = 558\ ^{o}K$$

Avogadro law: - All ideal gases have the same amount of molecular in equal volume at the same pressure and temperature.



Where:-
M = molecular weight.

n = is the number of moles.

$m_1 = M_1.n \longrightarrow$ gas A

$m_2 = M_2.n \longrightarrow$ gas B

$$v_1 = \frac{V}{m_1} = \frac{V}{M_1.n} \longrightarrow v_1\, M_1 = \frac{V}{n}$$

$$v_2 = \frac{V}{m_2} = \frac{V}{M_2.n} \longrightarrow v_2\, M_2 = \frac{V}{n}$$

Therefore,
$v_1\, M_1 = v_2\, M_2 = v_3\, M_3 = v\, M$

At the standard condition (i .e $p = p_o = 1$ bar, & $T = T_o = 273\ ^{o}K$). The const. (M. v) is obtained for 1 kg. mol of any gas and found to be, $M\, v_o = 22.4\ m^3$ / mol

1kg mol of O_2 = 32 kg

N_2 = 28 kg

$H_2 = 2kg$

$CO = 28\ kg$

$CO_2 = 44kg$

$CH_4 = 16kg$

$$\boxed{\mathbf{v_o = \frac{22.4}{M}\ (m^3 /kg)}}$$

$p.V = m.R.T$

$10^2\ p.V = m.R.T$, where p = bar

$$10^2\ p * \frac{V}{m} = R.T$$

$10^2\ p.v = R.T$

$10^2\ p_o.v_o = R_o.T_o$ …. At standard condition

$$10^2 * 1 * \frac{22.4}{M} = R * 273$$

$$M.R = \frac{10^2 * 22.4}{273} = 8.314\ kJ / mol\ .K$$

Therefore MR = G = 8.314 kJ / mol .^{o}K which is called the universal gas constant and from which you find the specific gas constant. (R) For each gas as following.

$$\boxed{\mathbf{R = \frac{G}{M} = \frac{8.314}{M}}}$$

For example:-

$$R_{O2} = \frac{8.314}{32} = 0.2598\ kJ/kg\ .K$$

$$R_{N2} = \frac{8.314}{28} = 0.2969\ kJ/kg\ .K$$

$$R_{CO2} = \frac{8.314}{44} = 0.1889 \text{ kJ/kg.K}$$

Note:-

The general equation of gas (10^2 p .V = m R T) where p = bar, V =m^3, m = kg, R = kJ/kg .K

EX: - A vessel of volume $0.2m^3$contains nitrogen at 1.013 bar and 15°C if 0.2kg of nitrogen is now pumped in to vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molecular weight of nitrogen is 28, and it may be assumed to be a perfect gas.

Sol:-

$$\text{Gas constant, } R = \frac{8.314}{M} = \frac{8.314}{28} = 0.2969 \text{kJ/kg.K}$$

$p_1\ V_1 = m_1\ R\ T_1$

$$m_1 = \frac{p_1\ V_1}{R\ T_1} = \frac{10^2 * 1.013 * 0.2}{0.2969 * 288} = 0.237 \text{ kg}$$

(Where $T_1 = 15 + 273 = 288^oK$).

0.2 kg of nitrogen are added, hence $m_2 = 0.2 + 0.237 = 0.437$kg. then for the final condition,

$10^2\ P_2V_2= m_2\ R\ T_2$

But $V_2 = V_1$, & $T_2=T_1$,

$$p_2 = \frac{m_2\ R\ T_2}{V_2} = \frac{0.437 * 0.2969 * 288}{10^2 * 0.2} = 1.87 \text{bar}$$

Specific heat: - "C"

Specific heat it is it is the amount of heat add to unit mass of substance (1kg) to increase its temperature 1°C unit kJ /kg .K.
For solid and liquid, there is one value of "C".
But for gases, we have two value of "C" called Cp and Cv ,

Where:-
Cv is the specific heat at constant volume (kJ/kg.K).
Cp is the specific heat at constant pressure (kJ/kg.K).

Specific heat at constant volume “Cv”:-

For a close system the energy equation is,

$Q_{12} = w_{12} + \Delta U_{12}$

For constant volume Cv = constant

$\Delta V = 0$,$w_{12} = 0$ therefore,

$Q_{12} = \Delta U_{12} = m\ Cv\ \Delta T$

Also,
$q_{12} = \Delta u_{12} = Cv\ \Delta T$

$$\boxed{\mathbf{Cv = \frac{\Delta u}{\Delta T}}}$$

Specific heat at constant pressure “Cp”

$q_{12} = w_{12} + \Delta u_{12}$

$\quad = (p_2 .V_2 - p_1 .V_1) + u_2 - u_1$

$\quad = (u_2 + p_2 .V_2) - (u_1 + p_1 .V_1)$

$q_{12} = h_2 - h_1 = \Delta h = Cp\ \Delta T$

$$\boxed{\mathbf{Cp = \frac{\Delta h}{\Delta T}}}$$

The relation between Cp & Cv:-

$H = p .V + U$

$\Delta H = \Delta\ p.V + \Delta\ U$

For a perfect gas, from equation pV = mRT .Also for a perfect gas, from Joule's law, U = m Cv T. Hence, substituting,

$$\left(m .Cp .\Delta T = m . R .\Delta T + m .Cv .\Delta T \right) \div m\ \ \Delta T$$

$\boxed{\mathbf{Cp = R + Cv}}$ ….. Therefore Cp> Cv

Ratio of specific heats

The ratio of the specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma) or gas index.

$$\boxed{\gamma = \frac{Cp}{Cv}}$$

Relationships between Cp, Cv, R and γ can be derived from equation,

$Cp = R + Cv$

$R = Cp – Cv$

Dividing through by Cv

$$\frac{Cp}{Cv} - 1 = \frac{R}{Cv}$$

Therefore using equation, $\gamma = Cp / Cv$, then ,

$$\gamma - 1 = \frac{R}{Cv}$$

$$Cv = \frac{R}{(\gamma - 1)}$$

$$\left(Cv = \frac{R}{(\gamma - 1)}\right) * \gamma$$

$$Cv\, \gamma = \frac{\gamma R}{(\gamma - 1)}$$

Also from equation, $Cp = \gamma\, Cv$, substituting in equation ,

$$Cp = \frac{\gamma R}{(\gamma - 1)}$$

EX: - A certain perfect has specific heats as follows, Cp = 0.846kJ/kg.K, Cv = 0.657 kJ/kg.K. Find the gas constant and the molecular weight of the gas

Sol:-

$Cp - Cv = R$

$R = Cp - Cv$

$= 0.846 - 0.657 = 0.189$ kJ/kg.K

$R = 8.314 / M$

$M = 8.314 / 0.189$

$= 44$

EX:- A perfect gas has a molecular weight of 26 and a value of $\gamma = 1.26$. calculate Specific heat at constant pressure & the heat rejected per kg of gas , when the gas is contained in a riged vessel at 3 bar and 315 ^{o}C and is then cooled until the pressure falls to 1.5 bar.

Sol:-

$R = 8.314 / M = 8.314 / 26 = 0.3198$ kJ/kg.K

$$Cv = \frac{R}{(\gamma - 1)} = \frac{0.3198}{(1.26 - 1)} = 1.229 \text{kJ/kg.K}$$

$Cp / Cv = \gamma$

$Cp = \gamma * Cv = 1.26 * 1.229 = 1.548$ kJ/kg.K

The volume remains constant for the mass of gas present, and hence the specific volume remains constant

$p_1 v_1 = R T_1$ & $p_2 v_2 = R T_2$

Therefore since $v_1 = v_2$

$T_2 = T_1 (p_2 / p_1)$

$= 588 (1.5 / 3) = 294$ K

Where $T_1 = 315 + 273 = 588$K

Heat rejected per kg of gas $= Cv (T_2 - T_1)$

$= 1.229 (588 - 294)$

$= 361$kJ/kg

Problems

3 -1 The molecular weight of carbon dioxide, CO_2, is 44. In an experiment the value γ for CO_2 was found to be 1.3. Assuming that CO_2 is a perfect gas, calculate the gas constant, R, and the specific heats at constant pressure and constant volume, Cp, Cv
(0.189 kJ/kg.K; 0.63kJ/kg.K; 0.819kJ/kg.K)

3-2 Oxygen, O_2, at 200 bar is to be stored in a steel vessel at 20°C the capacity of the vessel is $0.04m^3$. Assuming that O_2 is a perfect gas, calculate the mass of oxygen that can be stored in the vessel. The vessel is protected against excessive pressure by a fusible plug which will melt if the temperature rises too high. At what temperature must the plug melt to limit the pressure in the vessel to 240bar? The molecular weight of oxygen is 32
(10.5 kg; 78.6°C)

3-3 A quantity of a certain perfect gas is compressed from an initial state of $0.085m^3$, 1 bar to a final state of $0.034m^3$, 3.9 bar. The specific heats at constant volume are 0.724 kJ/kg.K, and the specific heats at constant pressure are 1.02kJ/kg.K. The observed temperature rise is 146K. Calculate the gas constant, R, the mass of gas present, and the increase of internal energy of the gas. (0.296kJ/kg.K; 0.11kg; 11.63kJ)

Particular closed system process

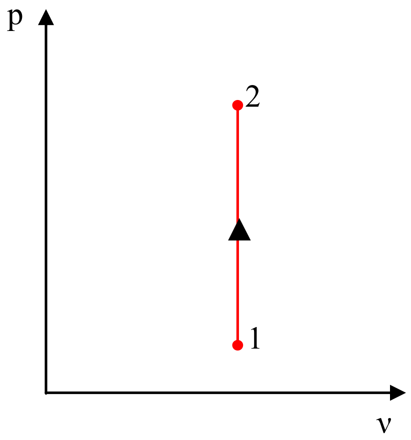
1- The isochoric process (constant volume process):- In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle wheel work input. It will be assumed that constant volume implies zero work unless stated otherwise.

From the non- flow energy equation,

$$Q_{12} = U_{12} + W_{12}$$

Since no work is done, we therefore have

$$Q_{12} = U_2 - U_1$$



$W_{12} = 0$ (Area under curve p.v diagram = 0)

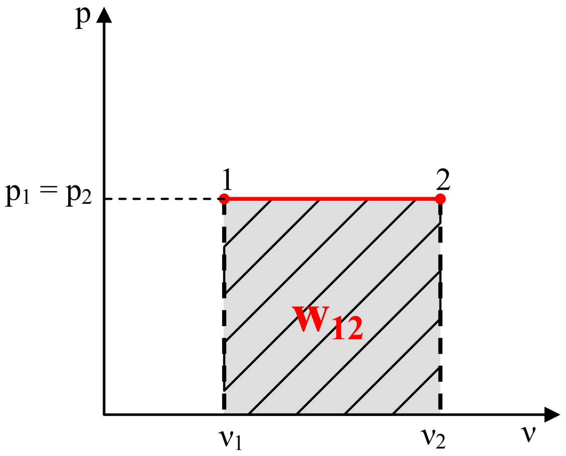
$Q_{12} = m\,Cv\,(T_2 - T_1)$

$v_1 = v_2$

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$ ------ p / T = constant

2- Isobaric process (constant pressure process):- for a constant pressure process the boundary must move against an external resistance as heat is supplied ; for instance a fluid in a cylinder behind a piston can be made to undergo a constant pressure process. Since the piston is pushed through a certain distance by the force exerted by the fluid, then work is done by the fluid on its surroundings.

$$W = \int_{v_1}^{v_2} p\,dv$$



Therefore, since p is constant,

$w = p\,(v_2 - v_1)$

From the non -flow energy equation,

$Q_{12} = W_{12} + \Delta U_{12}$

$q_{12} = w_{12} + \Delta u_{12}$

$= p\,(v_2 - v_1) + u_2 - u_1$

$= (p\,v_2 + u_2) - (p\,v_1 + u_1)$

Enthalpy, $h = p\,v_1 + u_1$, hence,

$q_{12} = h_2 - h_1$

or for mass ,m, of a fluid ,

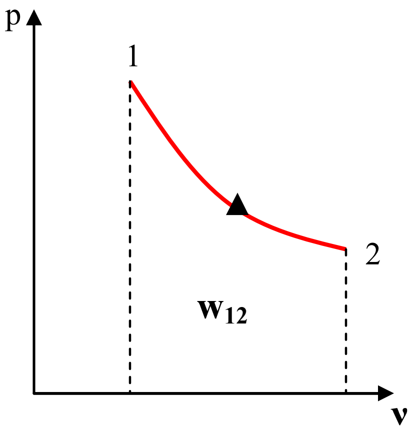
$Q_{12} = H_2 - H_1$
$Q_{12} = m\,Cp\,(T_2 - T_1)$

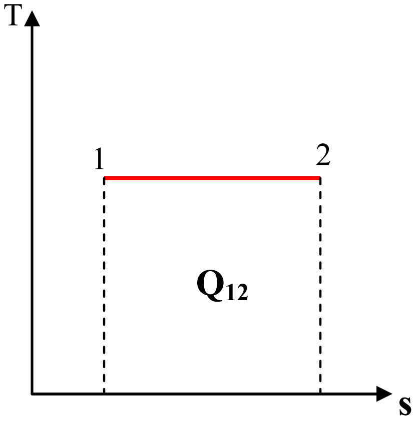
$q_{12} = Cp\,(T_2 - T_1)$

$$\frac{V}{T} = C = \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$ ------ V/ T = constant

3- The isothermal process (constant temperature process):- in an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the fluid continuously during the process.





$V_2 > V_1$ & $p_1 > p_2$

$T_2 = T_1$

When the temperature is constant as an isothermal process then we have,

$pV = mRT = \text{constant}$

Therefore for an isothermal process for a perfect gas,

$p.V = \text{constant}$ or $p.v = \text{constant}$

$\boxed{p_1.V_1 = p_2.V_2}$ ---- $p.V = \text{constant}$

$Q_{12} = W_{12} + \Delta U_{12}$

$\Delta U_{12} = m\,Cv(T_2 - T_1) = 0$

$Q_{12} = W_{12}$

$W_{12} = \int_1^2 p\,dV$

In this case, $pV = \text{constant}$, or $p = c / V = m R T / V$ (where $c = \text{constant}$).

$W_{12} = \int_{V1}^{V2} (m R T / V)\,dV$

$W_{12} = m R T \int_{V1}^{V2} (dV / V)$

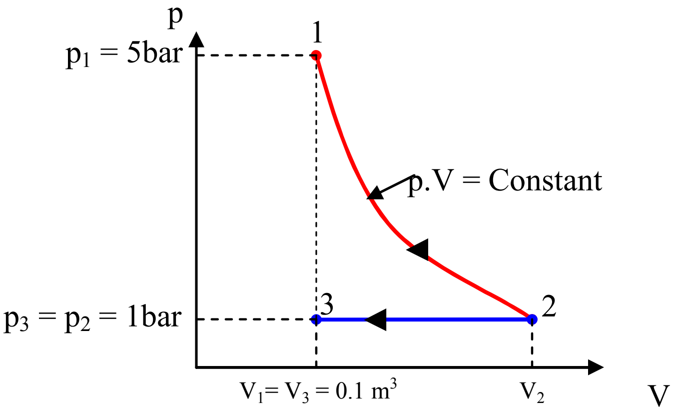
$W_{12} = m R T (\ln V_2 - \ln V_1)$

$= m\ R\ T\ \ln (V_2/V_1) = pV \ln (V_2/V_1)$

$$\boxed{W_{12} = m\ R\ T\ \ln (V_2/V_1)}$$

EX: - O_2 gas at a pressure of 5 bar, volume is (0.1) m^3 and temperature (1227C°) expand isothermally until its pressure is 1 bar it is then cooled isobarically to original volume. Calculate the change of internal energy, work done and heat transferred during each process.

Sol:-



p_1 = 5bar, $V_1 = 0.1\ m^3$, $T_1 = 1227 + 273 = 1500^oK$, p_2 = 1 bar

$T_2 = T_1 = 1500^oK$ (isothermal process)

$p_3 = p_2$ = 1bar (isobaric process)

$V_3 = V_1$ (cooled isobarically to original volume)

V_2 =? , T_3 =? , Q_{12} =? , W_{12} =? , ΔU_{12} =? , Q_{23} =? , W_{23} =? , ΔU_{23} =? ,

Process (1-2) isothermal process

$$Q12 = w_{12} = m\ .R\ .\ T \ln \frac{V_2}{V_1}$$

$$p_1.V_1 = p_2.V_2$$

$$V_2 = \frac{p_1.V_1}{p_2} = \frac{5 * 0.1}{1} = 0.5\ m^3$$

$$R = \frac{8.314}{MO_2} = \frac{8.314}{32} = 0.259\ kJ/kg\ .K$$

$10^2 . p_1 . V_1 = m . R . T_1$

$$m = \frac{10^2 . p_1 . V_1}{R . T_1} = \frac{10^2 * 5 * 0.1}{0.259 * 1500} \quad 0.128$$

$Q_{12} = W_{12} = 0.259 * 0.128 * 1500 * \ln (0.5 / 0.1)$

$= 800$ kJ

Process (2-3) isobaric process

$\Delta U_{23} = m . Cv . (T_3 - T_1)$

$Cp = R + Cv$

$Cv = Cp - R = 1 - 0.259 = 0.741$ kJ /kg .K

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$T_3 = \frac{V_3}{V_2} * T_2 = \frac{0.1}{0.5} * 1500 = 300\ ^oK$$

$\Delta U_{23} = 0.128 * 0.791(300 - 1500)$

$= - 113.8$ kJ

$Q_{23} = \Delta H_{23} = m. Cp (T_3 - T_2)$

$= 0 .128 * 1 (300 - 1500)$

$= - 153.8$ kJ

$W_{23} = Q_{23} - \Delta U_{23}$

$= - 153.8 - (- 113.8) = - 40$ kJ

Or

$W_{23} = p_3 V_3 - p_2 V_2$

$= 10^2 * p_2 (V_3 - V_2)$

$=10^2 * 1 (0.1 - 0.5)$

$= -40$ kJ

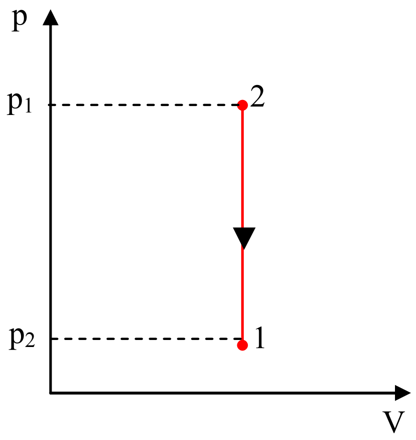
$Q_{23} = W_{23} + \Delta U_{23}$

$= -40 - 113.8$

$= -153.8$ kJ

EX: - Air in a closed vessel of fixed volume 0.15m^3, its pressure is 10 bar and at temperature 250^{o}C. If the vessel is cooled so that the pressure falls to 2.5 bar, determine the final temperature, and the heat transferred. Take (Cv = 0.71kJ/kg .K, R = 0.286 kJ / kg.K).

Sol:-
Closed vessel = constant volume
$V_1 = 0.15m^3 = V_2$, p_1= 10 bar, T_1= 250+273 = 523^{o}K, p_2= 2.5bar, T_2 =? , Q_{12}=?



$$\frac{p_2}{T_2} = \frac{p_1}{T_1}$$

$$T_2 = \frac{p_2}{p_1} * T_1 = \frac{2.5}{10} * 523 = 130.75^oK = -142.25\ ^oC$$

$Q_{12} = m\, Cv\, (T_2 - T_1)$

$10^2 .p_1.V_1 = m .R .T_1$

$$m = \frac{10^2 * 10 * 0.15}{0.286 * 523} = 1.003kg$$

$Q_{12} = 1.003 * 0.717\,(130.75 - 523) = -282.08$ kJ

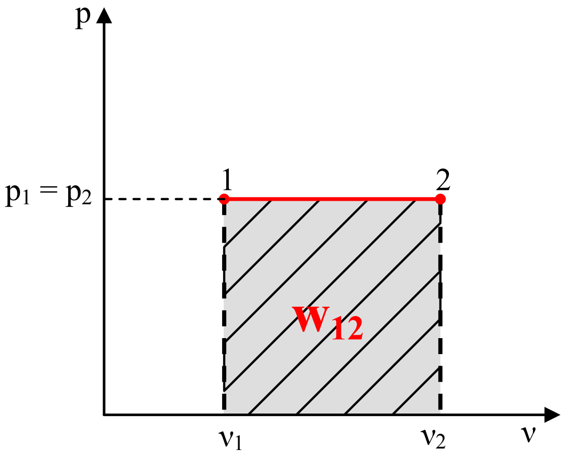
EX: - 2 kg of air initially at 150^{o}C expand at constant pressure at 0.6bar until the volume is doubled. Take Cp = 1.005 kJ /kg.K, γ =1.4, Find,

- **a- the final temperature**
- **b- the work transferred**
- **c- the heat transferred**

Sol:-

$m = 2kg$

$T_1 = 150+273 = 423\ K$, $p = 0.6bar$, $V_2 = 2V_1$



$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$T_2 = \frac{V_2 * T1}{V_1}$$

$$T_2 = \frac{2V_1 * 423}{V_1} = 846\ K - 273 = 573\ ^oC$$

$$W_{12} = 10^2 *p\ (V_2 - V_1)$$

$$Cp = \frac{\gamma\ R}{\gamma - 1}$$

$$R = \frac{Cp\ (\gamma - 1)}{\gamma}$$

$$R = \frac{1.005\ (1.4 - 1)}{1.4} = 0.287\ kJ / kg\ .K$$

$$10^2 *p *V_1 = m * R * T_1$$

$$10^2 *0.6 *V_1 = 2*0.287 * 423$$

$V_1 = 4.0467\ m^3,\ V_2 = 8.0934 m^3$

$W_{12} = 10^2 * 0.6\ (8.0934 - 4.0467) = +\ 242.802\ kJ$

$Q_{12} = m * Cp\ (T_2 - T_1)$

$= 2 * 1.005 * (846 - 423) = +850.23\ kJ$

EX: - Air initially at 127°C and 1bar is compressed isothermally to 8bar. Find for unit mass of air,

a- change of internal energy
b- heat transferred
c- work done

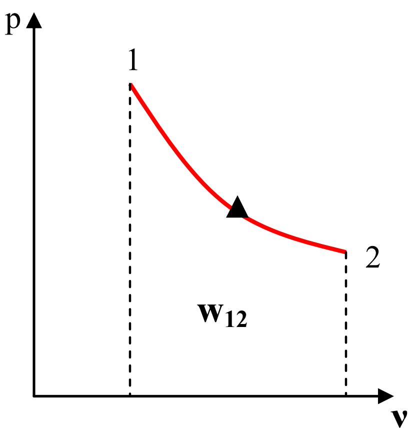
Sol: -

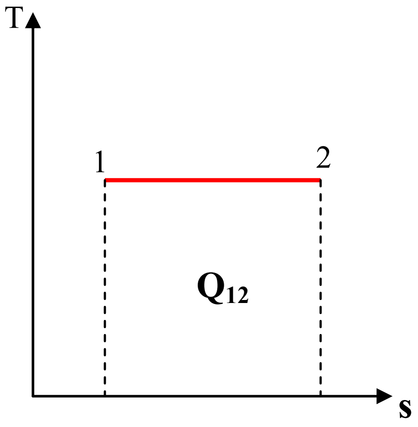
$T_1 = 127+273 = 400^oC,\ p_1 = 1bar$, isothermally T= constant, $p_2 = 8bar$

$\Delta u_{12} = 0$

$$q_{12} = w_{12} = R\ .T\ \ln\frac{v_2}{v_1} \qquad \ldots\ldots\ (v_2 / v_1 = p_1 / p_2)$$

$$= R\ .T\ \ln\frac{p_1}{p_2} = 0.287 * 400 * \ln\frac{1}{8} = -238.719\ kJ/kg$$



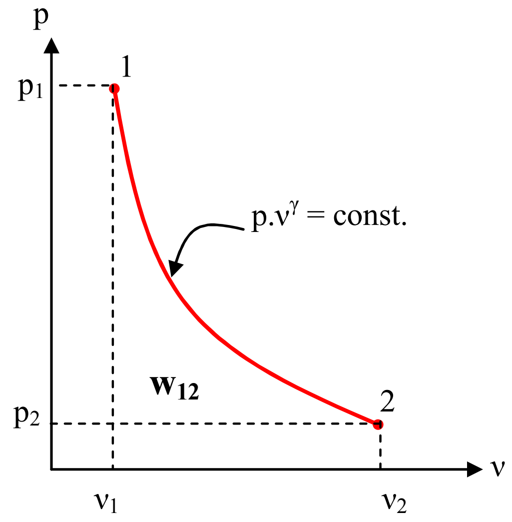


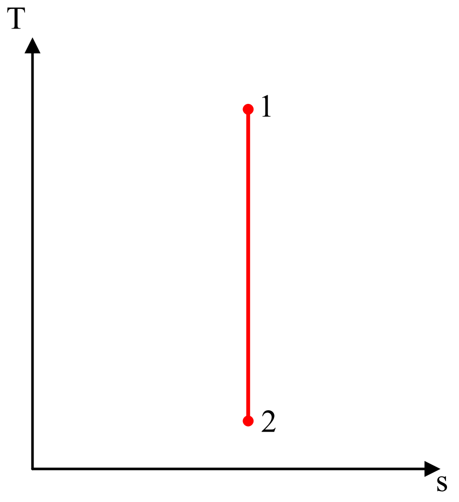
4- The adiabatic process: - is one in which no heat is transferred to or from the fluid during the process.

$Q_{12} = 0$ ……. (An adiabatic process)

$Q_{12} = \Delta\ u_{12} + w_{12}$

$w_{12} = -\ \Delta\ u_{12}$





$$\int_1^2 p.dV = -m\,Cv \int_1^2 dT$$

$p.V = m.R.T$

$$p = \frac{m.R.T}{V} \qquad \&\ Cv = \frac{R}{\gamma -1}$$

$$\int_1^2 \frac{m.R.T}{V}.dV = -m\frac{R}{\gamma -1}\int_1^2 dT$$

$$-(\gamma -1)\int_1^2 \frac{dV}{V} = \int_1^2 \frac{dT}{T}$$

$$-(\gamma -1)\,[\ln V_2 - \ln V_1] = \ln T_2 - \ln T_1$$

$$-(\gamma -1)\ln\frac{V_2}{V_1} = \ln\frac{T_2}{T_1}$$

$$\ln\left(\frac{V_2}{V_1}\right)^{-(\gamma -1)} = \ln\frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \frac{T_2}{T_1}$$

$$\boxed{\mathbf{T_1.V_1^{\gamma -1} = T_2.V_2^{\gamma -1} = const.}} \quad \text{..........a}$$

$$\frac{p_1 . V_1}{T_1} = \frac{p_2 . V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{p_2 . V_2}{p_1 . V_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \frac{p_2 . V_2}{p_1 . V_1}$$

$$\frac{V_1^{\gamma -1} * V_1}{V_2^{\gamma -1} * V_2} = \frac{p_2}{p_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

$$\mathbf{p_1 . V_1^{\gamma} = p_2 . V_2^{\gamma} = const.} \quad \text{……….b}$$

$$\frac{p_1 . V_1}{T_1} = \frac{p_2 . V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{p_2 . T_1}{p_1 . T_2}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \left(\frac{p_2 . T_1}{p_1 . T_2}\right)^{\gamma -1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2 . T_1}{p_1 . T_2}\right)^{\gamma -1}$$

$$\frac{T_2 . T_2^{\gamma-1}}{T_1 . T_1^{\gamma-1}} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1}\right)^{\gamma} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{T_2}{p_2^{\frac{\gamma-1}{\gamma}}} = \frac{T_1}{p_1^{\frac{\gamma-1}{\gamma}}} = \text{const.}} \quad \ldots\ldots\ldots\ldots\ldots\text{c}$$

$p.V^{\gamma} = \text{constant}$

$T.\ V^{\gamma-1} = \text{constant}$

$$\frac{T}{p^{\gamma-1/1}} = \text{constant}$$

$w_{12} = -\Delta\ U_{12}$

$$w_{12} = {}_1\int^2 p.dV$$

$p.V^{\gamma} = \text{constant}$

$$p = \frac{\text{Const}}{V^{\gamma}}$$

$$w_{12} = {}_1\int^2 \frac{\text{const.}}{V^{\gamma}} .dV$$

$$= \text{const.}\ {}_1\int^2 V^{-\gamma}\ .dV$$

$$= \text{const.} \left(\frac{V^{1-\gamma}}{1-\gamma}\right)_1^2$$

$$= \text{const.}\left(\frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma}\right)$$

$$= \frac{\text{const.}V_2^{1-\gamma} - \text{const.}\ V_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{p.V_2^{\gamma}.V_2^{1-\gamma} - p.V_1^{\gamma}\ V_1^{1-\gamma}}{1-\gamma}$$

$$\boxed{\mathbf{w_{12} = \frac{p_2.V_2 - p_1.V_1}{1-\gamma}}}$$

$$w_{12} = \frac{p_2.V_2 - p_1.V_1}{1-\gamma} = \frac{m..R\ .T_2 - m..R\ .T_1}{1-\gamma} = \frac{m..R\ (T_2 - T_1)}{1-\gamma}$$

$$w_{12} = \frac{m..R\ (T_1 - T_2)}{\gamma - 1} = m\ Cv\ (T_1 - T_2) = -\ m\ Cv\ (T_2 - T_1) = -\Delta\ U_{12}$$

We can call this process isentropic process because $s_1 = s_2$

5- The polytropic process: -

$$\boxed{p.V^n = \text{constant}}$$

The index n depends only on the heat and work quantities during the process. For example,

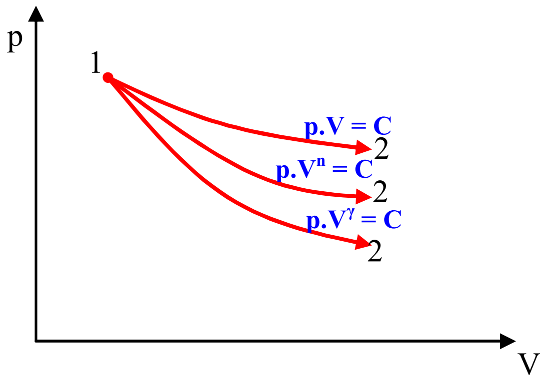
When $n = 0$ $p.V^0 = \text{constant}$, i.e. $p = \text{constant}$

When $n = \infty$ $p.V^{\infty} = \text{constant}$

or $p^{1/\infty}.V = \text{constant}$, i.e. $V = \text{constant}$

When $n = 1$ $p.V = \text{constant}$, i.e. $T = \text{constant}$

When $n = \gamma$ $p.V^{\gamma} = \text{constant}$, i.e. reversible adiabatic



Generally $(1 < n < \gamma)$

As $p.V^n = \text{constant}$

$\therefore$ From general equation of gases

$$\boxed{\mathbf{T.V^{n-1} = constant}} \quad \text{..... a}$$

$$\boxed{\mathbf{\frac{T}{p^{n-1/n}} = constant}} \quad \text{..... b}$$

$$\boxed{\mathbf{W_{12} = \frac{p.V_1 - p_2\, V_2}{n\text{-}1}}} \quad \text{....c}$$

For calculation of heat,

$Q_{12} = w_{12} + \Delta U_{12}$

$$= \frac{p_1.V_1 - p_2.V_2}{n\text{-}1} + m\ .Cv\ (T_2 - T_1)$$

$$= \frac{m\ R\ (\ T_1 - T_2)}{n\text{-}1} - m\ \frac{R}{\gamma\text{-}\ 1}\ (T_1 - T_2)$$

$$= m\ R\ (T_1 - T_2)\left(\frac{1}{n\text{-}1} - \frac{1}{\gamma\text{-}1}\right)$$

$$= m\ R\ (T_1 - T_2)\left(\frac{(\gamma\text{-}1) - (n\text{-}1)}{(\ n\text{-}1\ )\ (\gamma\text{-}1)}\right)$$

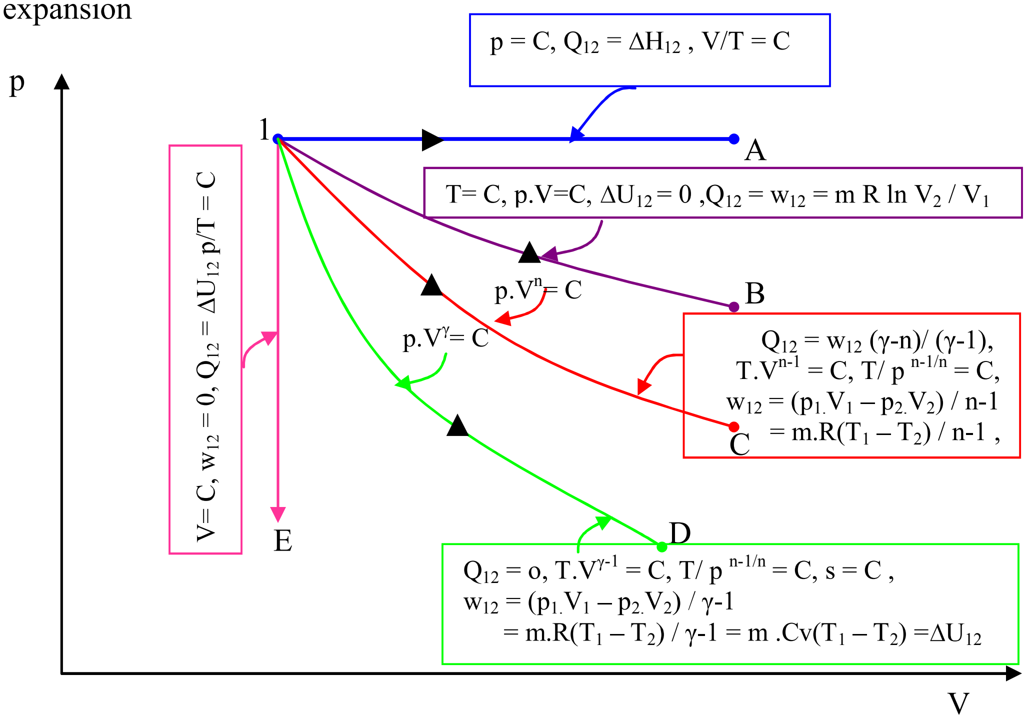
$$= m\ R\ (T_1 - T_2)\left(\frac{(\gamma - n)}{(\ n\text{-}1\ )\ (\gamma\text{-}1)}\right)$$

$$Q_{12} = \frac{m\ R\ (T_1 - T_2)}{n\text{-}1} - \frac{\gamma - n}{\gamma\text{-}1}$$

$$Q_{12} = w_{12} - \frac{\gamma - n}{\gamma\text{-}1}$$

$$\boxed{Q_{12} = w_{12} - \frac{\gamma - n}{\gamma - 1}}$$ ….. d

n = is called the index of expansion



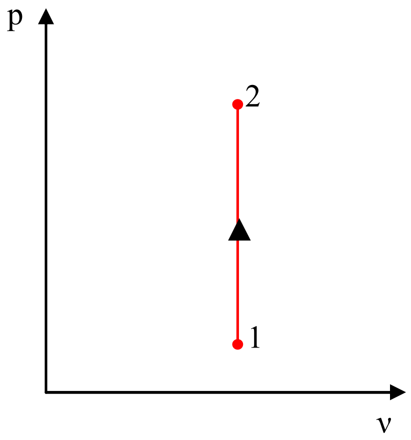
State 1 to state A is constant pressure heating,
State 1 to state B is isothermal expansion,
State 1 to state C is polytropic expansion,
State 1 to state D is reversible adiabatic expansion,
State 1 to state E is constant Volume cooling,

EX: - 2kg of gas, occupying 0.7m^3, had an original temperature of 15^{o}C. It was then heated at constant volume until its temperature became 135^{o}C. How much heat transferred to the gas? & what was its final pressure? [Take Cv = 0.75 kJ / kg .K, R= 0.29kJ /kg .K].

Sol:-

$m = 2kg$, $V_1 = 0.7m^3$, $T_1 = 15 + 273 = 288\ ^oK$, $T_2 = 135 + 273 = 408\ ^oK$,

$Q_{12} =?$, $p_2 =?$,



$Q_{12} = m .Cv (T_2 – T_1)$

$= 2 * 0.75 (408 – 288)$

$= 172.8$ kJ

$p_1/ T_1 = p_2/ T_2$

$10^2 p_1.V_1 = m.R.T_1$

$$p_1 = \frac{m. R.T_1}{10^2V_1} = \frac{2 * 0.29 * 288}{10^2 * 0.7} = 2.38 \text{ bar}$$

$p_2 = (p_1/ T_1) *T_2 = (2.38/ 288) *408 = 3.38$ bar

EX: - A gas whose pressure, volume and temperature are 275kN / m^2, 0.09 m^3 and 185^{o}C, respectively, has its state changed at constant pressure until its becomes15^{o}C. How much heat is transferred and how much work is done during the process? [Take R= 0.29 kJ/ kg .K, Cp = 1.005 kJ/kg.K] .

Sol:-

$p_1 = 275$ kN / m^2 = 2.75 bar = p_2 at constant pressure, $V_1 = 0.09$ m^3,

$T_1 = 185 +273 = 458$ ^{o}K,

$Q_{12} = \Delta H_{12}$

$= m. Cp. (T_2-T_1)$

$10^2 p_1.V_1 = m.R.T_1$

$$m= \frac{10^2 p_1.V_1}{R.T_1} = \frac{10^2* 2.75* 0.09}{0.29 * 458} = 0.186 \text{ kg}$$

$Q_{12} = m. Cp. (T_2-T_1) = 0.186 * 1.005 (288- 458) = -31.7781$kJ

$W_{12} = 10^2. p_1 (V_2-V_1)$

$V_2/ T_2 = V_1/ T_1$

$V_2= (T_2 / T_1)* V_1$

$= (288 / 458) * 0.09 = 0.0565$ m^3

$W_{12} = 10^2. p_1 (V_2-V_1)$

$= 10^2* 2.75(0.0565- 0.09) = - 9.2125$ kJ

Or

$\Delta U_{12} = m \cdot Cv\ (T_2 - T_1)$

$R = Cp - Cv$

$Cv = Cp - R = 1.005 - 0.29 = 0.715$ kJ/kg .K

$\Delta U_{12} = 0.186 * 0.715(288 - 458) = -22.6083$ kJ

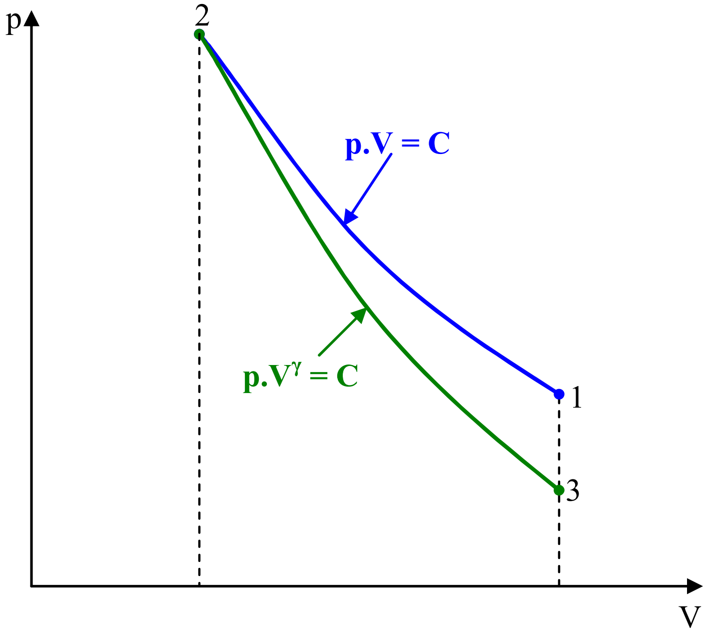
$w_{12} = Q_{12} - \Delta U_{12}$

$= -31.7781 - (-22.6083) = -9.1698$kJ

EX: - A quantity of gas occupies a volume of 0.3m^3 at a pressure of 100kN / m^2 and a temperature of 20^{o}C. The gas is compressed isothermally to a pressure of 500kN/m^2 and then expanded adiabatically to its initial volume. Determine, a) the heat received or rejected during compression, b) the change of internal energy during the expansion. [Take R= 0.29 kJ/ kg .K, Cp = 1.005 kJ/kg.K] .

Sol:-

$V_1 = 0.3m^3$, $p_1 = 100kN/m^2 = 1bar$, $T_1 = 20 + 273 = 293K$, $p_2 = 500\ kN/m^2$



Process (1-2) isothermal compression

$Q_{12} = m\ R\ T_1 \ln (V_2/V_1)$

$$m = \frac{10^2\ p_1.V_1}{R.T_1} = \frac{10^2 * 1 * 0.3}{0.29 * 293} = 0.35\ kg$$

$p_1.V_1 = p_2.V_2$

$$V_2 = \frac{p_1.V_1}{p_2} = \frac{1 * 0.3}{5} = 0.06\ m^3$$

$Q_{12} = 0.35 * 0.29 * 293 \ln (0.06/0.3)$

$= - 47.86\ kJ$

Process (2-3) adiabatic expansion

$$w_{23} = \frac{10^2(p_3.V_3 - p_2.V_2)}{1 - \gamma}$$

$\gamma = Cp / Cv$

$Cv = Cp - R = 1.005 - 0.29 = 0.715\ kJ/kg.K$

$\gamma = 1.005 / 0.715 = 1.4$

$$p_3 = \frac{p_2 . V_2^{\gamma}}{V_3^{\gamma}} = \frac{5 * (0.06)^{1.4}}{(0.3)^{1.4}} = 0.52\ bar$$

$$w_{12} = \frac{(5 * 0.06 - 0.52 * 0.3) * 10^2}{1 - 1.4} = 0.36 * 10^2\ kJ$$

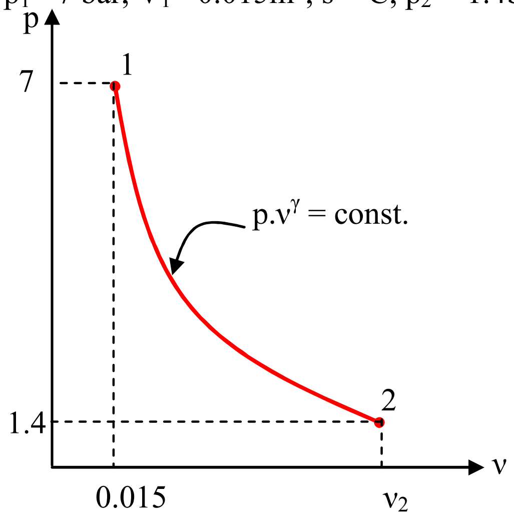
$\Delta U_{12} = - w_{12} = - 0.36 * 10^2\ kJ$

EX: - A gas expands adiabatically from a pressure and volume of 7 bars &0.015 m^3, respectively, to a pressure of 1.4 bar. Determine the final volume and the work done by the gas. What is the change of internal energy in this case?

[Take Cp= 1.046 kJ/kg.K , Cv = 0.752 kJ/kg .K].

Sol:-

$p_1 = 7$ bar, $V_1 = 0.015 m^3$, s = C, $p_2 = 1.4$bar



$$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$$

$$V_2^{\gamma} = \frac{p_1.\ V_1^{\ \gamma}}{p_2}$$

$$\gamma = Cp/Cv = 1.046\ /\ 0.752 = 1.39$$

$$V_2^{1.39} = \frac{7*\ (0.015)^{\ 1.39}}{1.4} = 0.014579$$

$$V_2^{1.39} = 0.014579$$

$$V_2 = 0.014579^{1/1.39} = 0.0477m^3$$

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2)*10^2}{\gamma -1}$$

$$w_{12} = \frac{(7*0.015 - 1.4*0.0477)*10^2}{1.39 - 1} = 9.555kJ$$

$$w_{12} = -\Delta U_{12} = -\ 9.555\ kJ$$

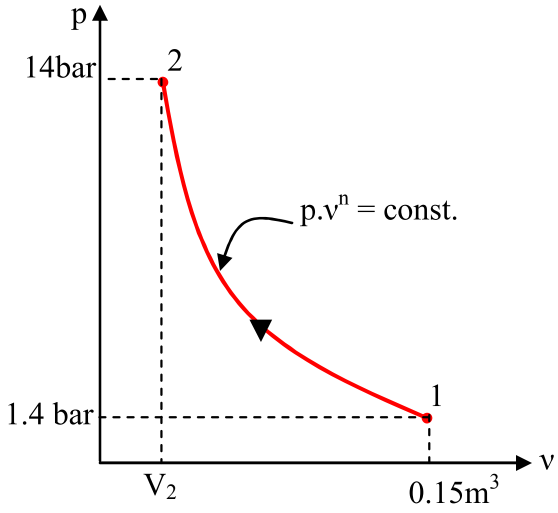
EX: - 0.25kg of air at a pressure of 1.4 bar occupies 0.15 m^3 and from this condition it is compressed to 14 bar according to the law $p.V^{1.25} = C$. Determine,

- **a- the change of internal energy ,**
- **b- the work done,**
- **c- The heat transferred, [Take Cp= 1.005 kJ/kg.K , Cv = 0.718 kJ/kg .K].**

Sol:-

$m = 0.25$ kg, $p_1 = 1.4$ bar, $V_1 = 0.15\ m^3$

$p_1 = 7$ bar, $V_1 = 0.015m^3$, s = C, $p_2 = 1.4$bar



$\Delta U_{12} = m.\ Cv\ (T_2 - T_1)$

$10^2\ p_1.V_1 = m.\ R.T_1$

$R = Cp - Cv$

$= 1.005 - 0.718 = 0.287$ kJ/kg.K

$$T_1 = \frac{10^2\ p_1.V_1}{m\ .R} = \frac{10^2 * 1.4 * 0.15}{0.25 * 0.287} = 292.68\ K$$

$$\frac{T_1}{p_1^{n-1/n}} = \frac{T_2}{p_2^{n-1/n}}$$

$$\frac{292.68}{1.4^{(1.25-1)/1.25}} = \frac{T_2}{14^{(1.25-1)/1.25}}$$

$T_2 = 463.867\ K$

$\Delta U_{12} = 0.25 * 0.718\ (463.867 - 292.68) = 30\ .73$ kJ

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2) * 10^2}{n - 1}$$

$$w_{12} = \frac{m.\ R\ (T_1 - T_2)}{n - 1} = \frac{0.25 * 0.287\ (292.68 - 463.867)}{1.25 - 1} = -\ 49\ .13kJ$$

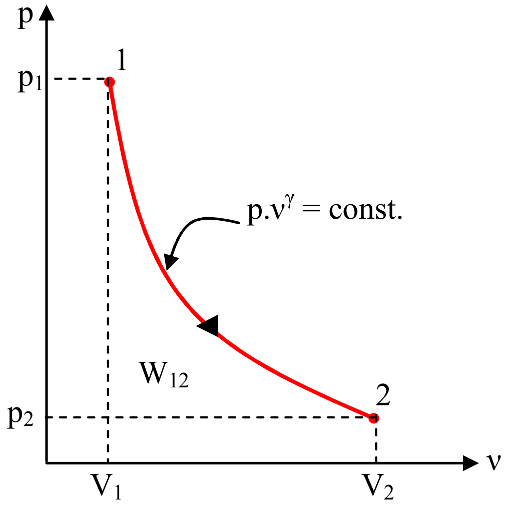
$Q_{12} = w_{12} + \Delta U_{12}$

$= -\ 49\ .13 + 30\ .73 = -18.4kJ$

EX: - If 1 kg of gas expands adiabatically, its temperature falls from 240 °Cto116 °C, while the volume is doubled. The gas does 19.5 kJ of work in this process. Find the values of the specific heats Cv, Cp and the molecular weight of the gas.

Sol:-

$m = 1kg,\ T_1 = 240 + 273 = 513\ K,\ T_2 = 116 + 273 = 389\ K,\ w_{12} = +\ 91.5$ kJ



$Q_{12} = 0$

$w_{12} = - m \,.Cv\,(T_2-T_1)$

$10^2 \,.p_1 \,.V_1 = m.\,R.T_1$

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2}$$ the volume is doubled when it expand

$(2)^{\gamma-1} = 513/\,389$

$(\gamma -1)\ln 2 = \ln (513/\,389)$

$\gamma = 1.399$

$w_{12} = - m.\,(R/\gamma -1)\,(T_2-T_1)$

$$R = \frac{w_{12} * (\gamma -1)}{m\,(T_1 - T_2)} = \frac{91.5 * (1.399 -1)}{1 * (513- 389)} = 0.294 \text{kJ/kg.K}$$

$Cv = R/\,(\gamma -1)$

$= 0.294 \,/\, (1.399 -1)$

$= 0.738$ kJ/kg.K

$Cp = R + Cv$

$= 0.294 + 0.738 = 1.032$ kJ/kg.K

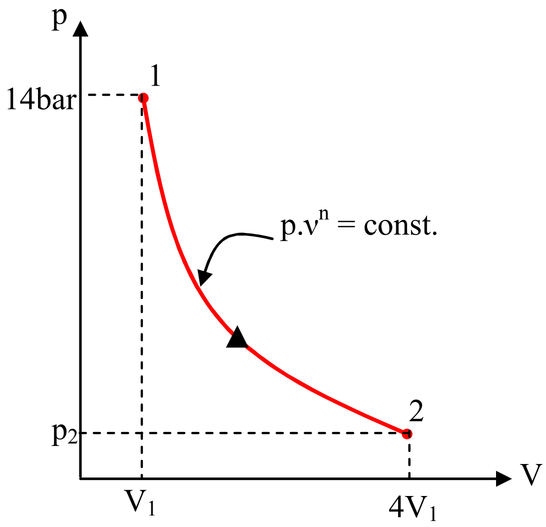
$M = 8.314 \,/\, R = 8.314 \,/0.294 = 28 .27$ kg/mol.

EX: - 0.675 kg of gas at 14 bar & 280°C is expanded to four times the original volume according to the law $p.V^{1.3} = C$. Determine

- **a- the original and final volume of the gas ,**
- **b- the final pressure**
- **c- The final temperature [Take R= 0.287 kJ/kg.K].**

Sol:-

$m = 0.675$ kg, $p_1 = 14$ bar, $T_1 = 280 + 273 = 553\ ^oK$, $n = 1.3$, $V_2 = 4 V_1$



$10^2 .p_1 .V_1 = m. R.T_1$

$$V_1 = \frac{m.R.T_1}{10^2 p_1} = \frac{0.675 * 0.287 * 553}{10^2 * 14} = 0.0765\ m^3 \text{ the original volume}$$

$V_2 = 4V_1 = 4* 0.0765 = 0.306\ m^3$ the final volume

$p_1.V_1^n = p_2.V_2^n$

$p_2 = (V_1/ V_2)^n\ p_1 = (V_1/ 4V_1)^{1.3} * 14 = 2.309$ bar $= 0.2309\ MN/m^3$ the final pressure

$T_1 .V_1^{n-1} = T_2 .V_2^{n-1}$

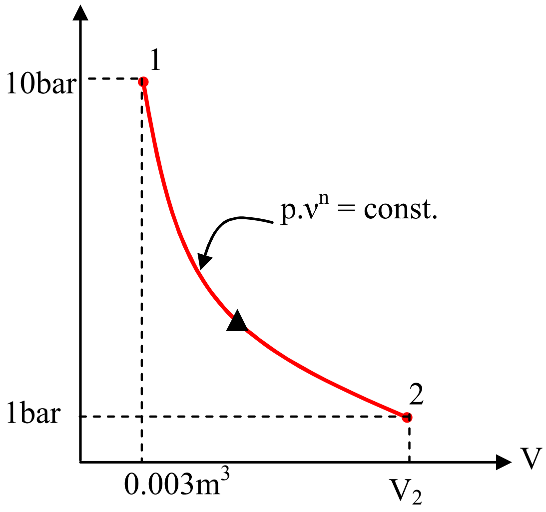
$T_2 = (V_1/ V_2)^{n-1} * T_1 =(V_1/ 4V_1)^{1.3-1} * 553 = 364 . 84\ ^oK = 91.84\ ^oC$ final temperature

EX: - A gas expand according to the law $p.V^{1.3} = C$ from a pressure of 10 bar & a volume $0.003m^3$ to a pressure 1 bar. How much heat was transferred?

[Take $\gamma = 1.4$, Cv = 0.718 kJ/kg .K]

Sol:-

$p_1 = 10$ bar, $V_1 = 0.003m^3$, $p_2 = 1$ bar



$$Q_{12} = w_{12} \frac{\gamma - n}{\gamma - 1}$$

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2) * 10^2}{n - 1}$$

$$p_1.V_1^n = p_2.V_2^n$$

$$V_2 = (p_1/ p_2)^{1/n} V_1 = (10 /1)^{1/1.3} * 0.003 = 0.0176 \text{ m}^3$$

$$w_{12} = \frac{(10*0.003 - 1*0.0176) * 10^2}{1.3 - 1} = 4.122 \text{ kJ}$$

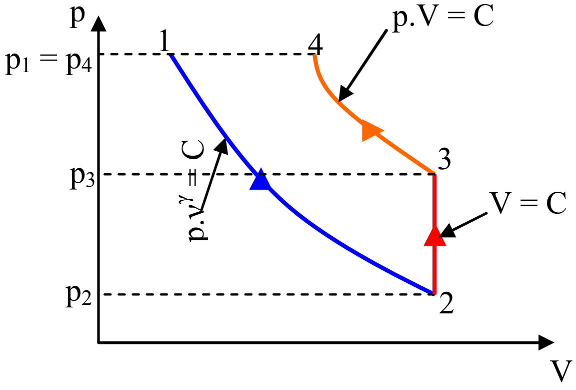
$$Q_{12} = 4.122* \frac{1.4 - 1.3}{1.4 - 1} = 1.03 \text{ kJ}$$

EX: - A gas at a pressure of 14 bar and a temperature of 360 °C is expanded adiabatically to a pressure of 1 bar the gas is then heated at constant volume until it again attains 360°C when its pressure is found to be 2.2 bar and finally it is compressed isothermally until the original pressure of 14 bar is attained. Find for until mass.

a- the value of the adiabatic index γ ,

b- The change in internal energy during each process. [Take Cp = 1.005 kJ/kg .K]

Sol:-



p_1 = 14 bar, T_1 = 360 + 273= 633 ^oK, p_2 = 1 bar, p_3 = 2.2 bar, m=1kg

$V_3 = V_2$ heated at constant volume

$T_4 = T_3$ = 633 ^oK isothermal compression

$p_4 = p_1$ = 14 bar compressed to original pressure

$p_2 / T_2 = p_3 / T_3$

$T_2 = (p_2 / p_3) * T_3 = (1 / 2.2) * 633 = 287.73\ ^oK$

$$\frac{T_1}{p_1^{\gamma-1/\gamma}} = \frac{T_2}{p_2^{\gamma-1/\gamma}}$$

$$(p_2 / p_1)^{\gamma-1/\gamma} = T_2 / T_1$$

$$\frac{\gamma - 1}{\gamma} \ln (p_2 / p_1) = \ln (T_2 / T_1)$$

$$\frac{\gamma - 1}{\gamma} \ln (1 / 14) = \ln (287.73 / 633)$$

$$\frac{\gamma - 1}{\gamma} (- 2.64) = -0.788$$

$$[1- (1/\gamma)](- 2.64) = -0.788$$

$$[1- (1/\gamma)] = 0.2986$$

$$0.7013 = (1/\gamma)$$

$$\gamma = 1.425$$

$\Delta U_{12} = m.\ Cv\ (T_2 - T_1)$

$\Delta U_{12} = m.\ (Cp\ /\ \gamma)\ (T_2 - T_1)$

$= 1 * (1.005\ /\ 1.\ 425)\ (287.73 - 633)$

$= - 243.36$ kJ

$\Delta U_{23} = m.\ (Cp\ /\ \gamma)\ (T_3 - T_2)$

$= 1 * (1.005\ /\ 1.\ 425)\ (633 - 287.73)$

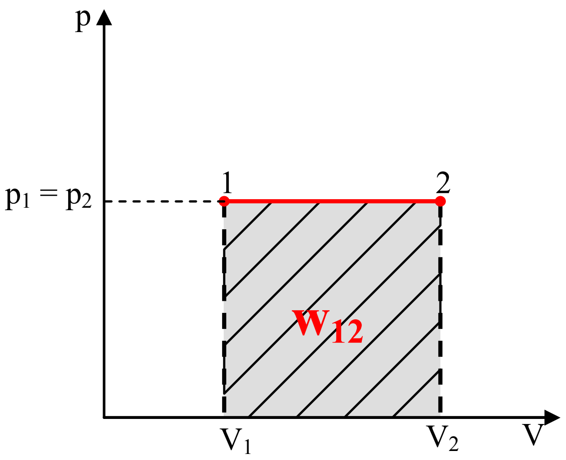
$= 243.36$ kJ

$\Delta U_{34} = 0$

EX: - A certain gas has a density of 1.875 kg /m^3 at 1bar and 15 ^{o}C. Calculate the characteristic gas constant. When 0.9 kg of this gas is heated from 15 ^{o}C to 250^{o}C at constant pressure, the heat required is 175 kJ. Calculate the specific heat capacity of the gas at constant pressure and the specific heat at constant volume. Calculate also, the change of internal energy and the external work done during the heating.

Sol:-

$\rho_1 = 1.875$ kg/m^3, $p_1 = 1$ bar, $T_1 = 15 + 273 = 288\ ^oK$, $m = 0.9$ kg, $T_2 = 250 + 273 = 523^oK$
$p_1 = p_2 = 1$ bar at constant pressure, $Q_{12} = 175$ kJ



$Q_{12} = \Delta H_{12}$

$= m.\ Cp\ (T_2 - T_1)$

$175 = 0.9 * Cp * (523 - 288)$

$Cp = 0.827$ kJ/kg.K

$\rho_1 = m\ /\ V_1$

$V_1 = m / \rho_1 = 0.9 / 1.875 = 0.48\ m^3$

$10^2 .p_1 .V_1 = m. R.T_1$

$$R= \frac{10^2 .p_1 .V_1}{m. T_1} = \frac{10^2 *1 *0.48}{0.9 * 288} = 0.185\ kJ/kg.K$$

$Cv = Cp - R$

$= 0.827 - 0.185 = 0.642\ kJ/kg.K$

$\Delta U_{12} = m .Cv (T_2 - T_1)$

$= 0.9 * 0.642 * (523 - 288)$

$= 135.78\ kJ$

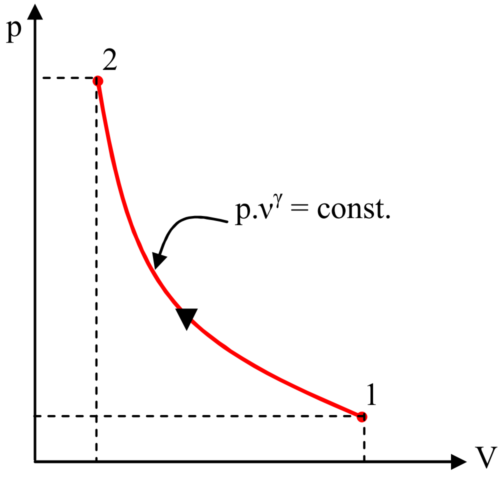
$w_{12} = Q_{12} - \Delta U_{12}$

$= 175 - 135.78 = 39 .22\ kJ /kg.K$

EX: - A gas whose original pressure and temperature were 300 kN /m^2 and 25 ^{o}C. Respectively, is compressed according to the law p.V$^{1.4}$ = C until its temperature becomes 180 ^{o}C. Determine the new pressure of the gas.

Sol:-

$p_1 = 300\ kN /m^2$, $T_1 = 25 + 273 = 298\ ^oK$, $p.V^{\ 1.4}= C$, $T_2 = 180 + 273 = 453\ ^oK$



$$\frac{T_2}{p_2^{\gamma-1/\gamma}} = \frac{T_1}{p_1^{\gamma-1/\gamma}}$$

$$\frac{T_2^{\gamma/\gamma-1}}{p_2} = \frac{T_1^{\gamma/\gamma-1}}{p_1}$$

$$\frac{(453)^{1.4/1.4-1}}{p_2} = \frac{(298)^{1.4/1.4-1}}{300 * 10^{-2}}$$

$p_2 = 12.99$ bar

EX: - A certain mass of air initially at a pressure of 4.8 bar is expanded adiabatically to a pressure of 0.94 bar. It is then heated at constant volume until it attains its initial temperature, when the pressure is found to be 1.50 bar. State the type of compression necessary to bring the air back to its original pressure and volume. Using the information calculate the value of γ. If the initial temperature of the air is 190 °C, determine the work done per kg of air during the adiabatic expansion [take R = 0.29 kJ /kg .K].

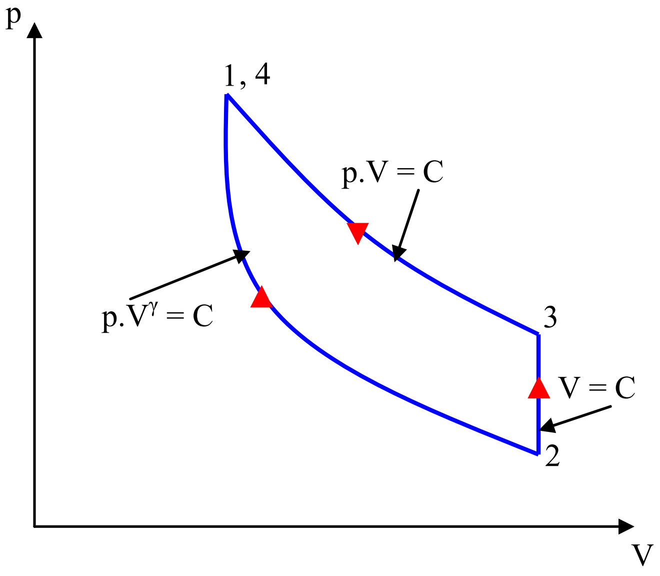
Sol:-

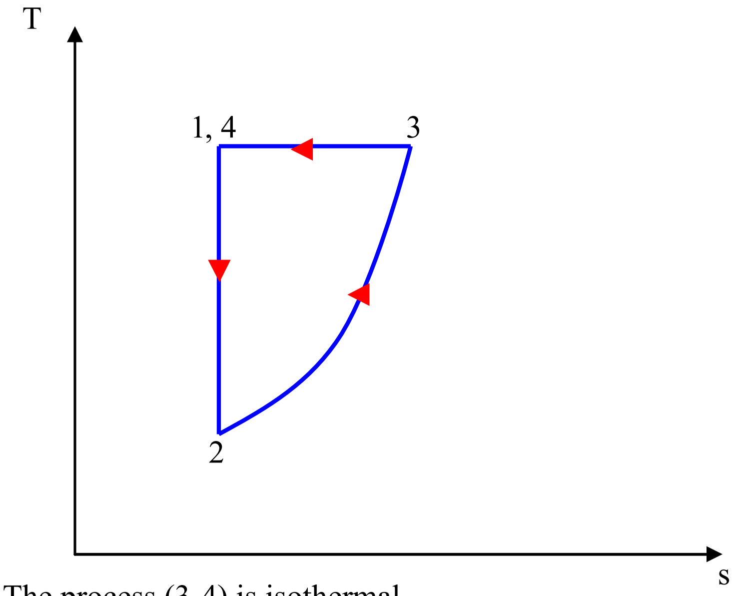
$p_1 = 4.8$ bar, $T_1 = 190 + 273 = 463$ °K,

s = C adiabatic exp. exp. $p_2 = 0.94$ bar,

V = C heated at constant volume to initial temp. $T_3 = T_1 = 463$ °K, $p_3 = 1.5$ bar

$p_4 = p_1 = 4.8$ bar $V_4 = V_1$ (air back to its original pressure and volume).





The process (3-4) is isothermal

$$\frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}}$$

$$\frac{T_1}{T_2} = \frac{p_1^{(\gamma-1)/\gamma}}{p_2^{(\gamma-1)/\gamma}}$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$$\frac{0.94}{T_2} = \frac{1.5}{463}, \quad T_2 = 290\ {}^{o}K$$

$$\ln\frac{T_1}{T_2} = \ln\left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

$$\mathrm{Ln}\,\frac{T_1}{T_2} = [(\gamma-1)/\gamma]\ln\frac{p_1}{p_2}$$

$$\mathrm{Ln}\,\frac{463}{290} = [(\gamma-1)/\gamma]\ln\frac{4.8}{0.94}$$

$\gamma = 1.402$

$$w_{12} = \frac{10^2 (p_1 V_1 - p_2 V_2)}{\gamma - 1} = Cv\,(T_1 - T_2)$$

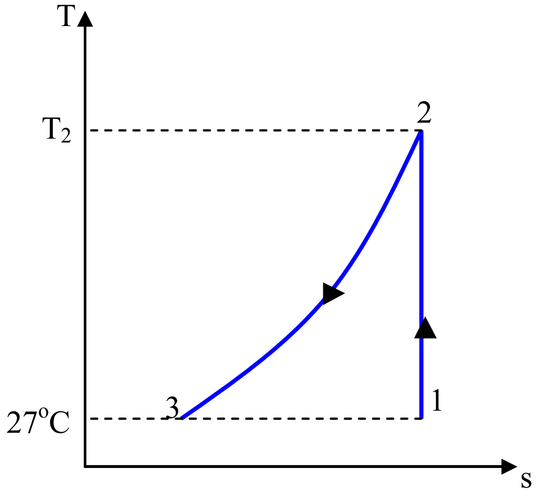
$$Cv = \frac{R}{\gamma - 1} = \frac{0.29}{1.4 - 1} = 0.725 \text{ kJ//kg.K}$$

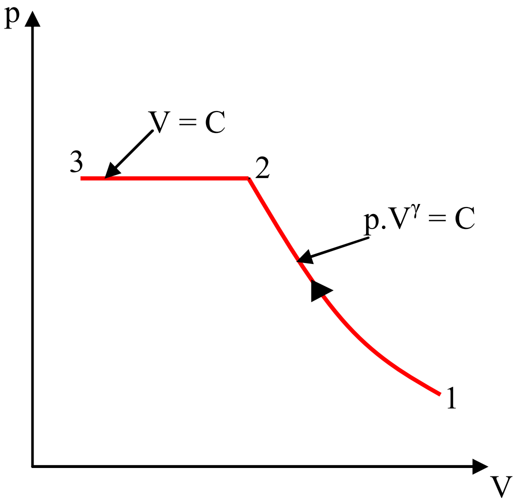
$$w_{12} = 0.725\,(463 - 290) = 125.425 \text{ kJ /kg}$$

EX: - 1 kg of air at 1 bar and 27 °C is compressed to 6 bar adiabatically. Find the final temperature and work done. If the air is then cooled at constant pressure to original temperature of 27°C, what amount of heat is rejected and, what further work of compression is done? The final state could have been reached by a reversible isothermal compression, instead of by the two other processes. What would be the work done and heat transferred in this case? [Take Cp = 1005 kJ/kg.K, Cv = 0.717 kJ/kg.K]

Sol:-

m=1kg, p_1 = 1 bar, $T_1 = 27 + 273 = 300$ °K, p_2 = 6 bar, T_2 =? , $T_3 = T_1$ =300 °K, $p_3 = p_2$, Q_{23} =? , w_{12} =?





Process (1-2) adiabatic compression
Process (2-3) cooled at constant pressure

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

$$\gamma = Cp/Cv = 1.005 / 0.717 = 1.4$$

$$\frac{300}{T_2} = \left(\frac{1}{6}\right)^{(1.4-1)/1.4}$$

$T_2 = 500.55$ °K = 227.55°C

$w_{12} = m.C_v\,(T_1 - T_2)$

$= 1 * 0.717\,(300 - 500.55) = -\,143.79$ kJ

$Q_{23} = m.\,C_p\,(T_3 - T_2)$

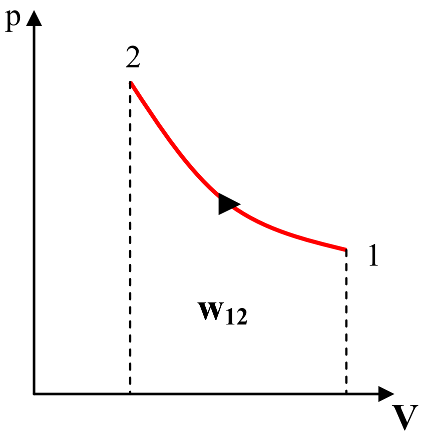
$= 1 * 1.005\,(300 - 500.55) = -\,201.55$ kJ

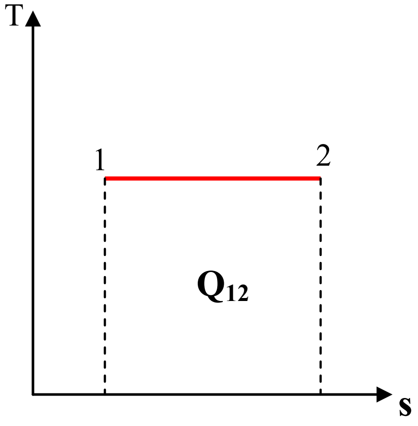
- Instead of by the two processes the final state could have been reached by reversible isothermal compression process

$$Q_{12} = w_{12} = m.\,R\,.T \ln\frac{V_2}{V_1} \qquad \ldots\ldots\ (V_2 / V_1 = p_1/p_2)$$

$R = C_p - C_v = 1.005 - 0.717 = 0.288$ kJ/kg.K

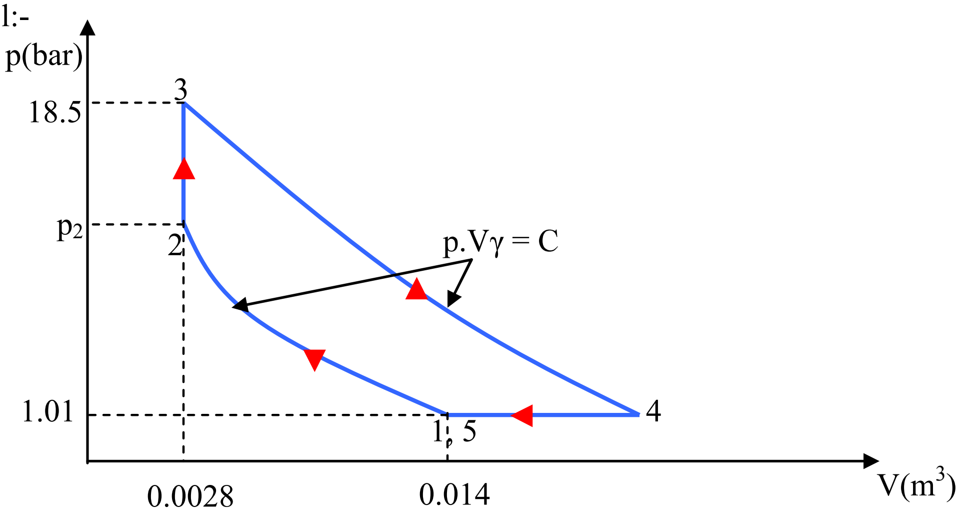
$$= m.R\,.T_1 \ln\frac{p_1}{p_2} = 1 * 0.288 * 300 * \ln\frac{1}{6} = -154\,.8 \text{ kJ}$$





EX: - Air occupying 0.014m^3 at 1.01 bar and 15^{o}C is compressed adiabatically to a volume of 0.0028 m^3. Heat is then supplied at constant volume until the pressure becomes 18.5 bar. Adiabatic expansion returns the air to the initial pressure of 1.01 bar, and then it is cooled at constant pressure until it attains the original volume of 0.014m^3. Calculate for each process, the heat transferred, the work done and change in internal energy. [Take $\gamma = 1.4$, $R = 0.29$ kJ/kg.K]

Sol:-



$V_1 = 0.014\ m^3$, $p_1 = 1.01$ bar, $T_1 = 15 + 273 = 288\ ^oC$, $V_2 = 0.0028\ m^3$, $p_3 = 18.5$

$V_3 = V_2 = 0.0028\ m^3$, $p_4 = p_1 = 1.01$ bar $= p_5$, $V_5 = V_1$

Process (1-2) adiabatic compression

Process (2-3) heat at constant volume

Process (3-4) adiabatic expansion

Process (4-1, 5) cooled at constant pressure

$Q_{12} = 0$

$w_{12} = -\ \Delta\ U_{12}$

$= -\ m\ Cv\ (T_2 - T_1)$

$10^2\ p_{1.}\ V_1 = m\ .R\ .T_1$

$$m = \frac{10^2\ p_{1.}\ V_1}{R\ .T_1} = \frac{10^2 * 1.01 * 0.014}{0.29 * 288} = 0.0169\ kg$$

$Cv = R / (\gamma-1) = 0.29 / (1.4 - 1) = 0.725$ kJ/kg.K

$T_1 .\ V_1^{\gamma-1} = T_2 .\ V_2^{\gamma-1}$

$288 * 0.014^{1.4-1} = T_2 * 0.0028^{1.4-1}$

$T_2 = 548.25\ ^oK$

$w_{12} = -\ m\ Cv\ (T_2 - T_1)$

$= -\ 0.0169 * 0.725 * (548.25 - 288) = -\ 3.1887$ kJ

$\Delta\ U_{12} = -\ w_{12} = 3.1887$

$Q_{23} = \Delta\ U_{23} = m\ Cv\ (T_3 - T_2)$

$p_2 / T_2 = p_3 / T_3$

$p_1 .V_1^{\gamma} = p_2 .V_2^{\gamma}$

$$p_2 = \frac{p_1 .V_1^{\gamma}}{V_2^{\gamma}} = \frac{1.01 * (0.014)^{1.4}}{(0.0028)^{1.4}} = 9.613\ bar$$

$9.613 / 548.25 = 18.5 / T_3$

$T_3 = 1124\ .38\ ^oK$

$Q_{23} = 0.0169 * 0.725 (1124 .38 – 548.25) = 7.059\ kJ = \Delta U_{23}$

$Q_{34} = 0$

$W_{34} = - \Delta U_{34}$

$= - m\ Cv (T_4 – T_3)$

$T_3. V_3^{\gamma-1} = T_4. V_4^{\gamma-1}$

$p_3 .V_3^{\gamma} = p_4 .V_4^{\gamma}$

$18.5 *(0.0028)^{1.4 =} 1.01 *V_4^{\gamma}$

$V_4 = 0.0223\ m^3$

$1124 .38*(0.0028)^{1.4 -1} = T_4*(0.0223)^{1.4 -1}$

$T_4 = 490\ ^oK$

$W_{34} = - 0.0169 * 0.725 (490 – 1124 .38) = 7.773\ kJ$

$W_{34} = - \Delta U_{34} = - 7.773\ kJ$

$Q_{41} = \Delta H_{41}$

$= m .Cp (T1 – T_4)$

$$Cp = \frac{\gamma R}{\gamma -1} = \frac{1.4 * 0.29}{1.4 – 1} = 1.015\ kJ/kg.K$$

$Q_{41}= 0.0169 *1.015 (288 – 490) = - 3.342\ kJ$

$\Delta U_{41} = m\ Cv (T_1 – T_4)$

$= 0.0169*0.725 (288 – 490) = - 2.475\ kJ$

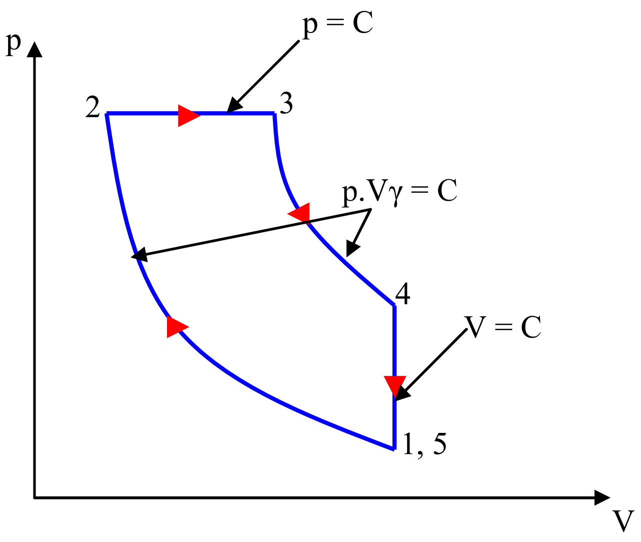
$w_{41} = Q_{41} - \Delta U_{41}$

$=- 3.342 – (- 2.475) = - 0.867kJ$

EX: - 1 kg of air is initially at 0.9 bar and 40 ^{o}C is compressed adiabatically till the volume is reduced to one – sixteenth of its initial value. Heat is then added at constant pressure till the temperature is 1400^{o}C. The air is then expanded adiabatically until the initial volume is reached and finally it is cooled at constant volume until the original state of 0.9 bar and 40^{o}C is Attained. Calculate the work done, internal energy and heat transferred for each process. [Take $\gamma = 1.4$, Cp = 1.004kJ/kg.K]

Sol:-

m = 1 kg, $p_1 = 0.9$ bar, $T_1 = 40 + 273 = 313\ ^oK$, $V_2 = V_1/16$, $T_3 = 1400 + 273 = 1673\ ^oK$



(1-2) Process adiabatic compression, s = C

(2 – 3) Process isobaric heat add at constant pressure process, $p_2 = p_3$

(3-4) Process adiabatic expansion, s = C

(4-1) Process isochoric cooling at constant volume process $V_4 = V_1$

$V_5 = V_1$ cooled at constant volume to the original state

$p_5 = p_1$ cooled at constant volume to the original state

$w_{12} = m\,.Cv\,(T_1 - T_2) = -\Delta U_{12}$

$\gamma = Cp\,/Cv$

$Cv = Cp\,/\,\gamma = 1.004\,/\,1.4 = 0.717$ kJ/kg.K

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_1/16}\right)^{1.4-1} = \frac{T_2}{313}$$

$T_2 = 948.84\ ^oK$

$w_{12} = 1*0.717\,(313 - 948.84) = -455.896$ kJ

$\Delta U_{12} = 455.896$ kJ

$Q_{12} = 0$

$Q_{23} = m.\ Cp\ (T_3 - T_2)$

$= 1 * 1.004\ (1673 - 948.84) = 727.1$ kJ

$\Delta U_{23} = m\ .Cv\ (T_3 - T_2)$

$= 1 * 0.717\ (1673 - 948.84) = 519\ .222$ kJ

$w_{23} = Q_{23} - \Delta U_{23}$

$= 727.1 - 519.222 = 207.88$ kJ

$w_{34} = m\ .Cv\ (T_3 - T_4)$

$10^2\ p_{1.}\ V_1 = m\ .R\ .T_1$

$R = Cp - Cv = 1.004 - 0.717 = 0.287$ kJ/kg.K

$10^2 * 0.9 * V_1 = 1 * 0.287 * 313$

$V_1 = 0.998\ m^3$

$V_2 = V_1 / 16 = 0.998 / 16 = 0.0624\ m^3$

$V_2 / T_2 = V_3 / T_3$

$0.0624 / 948.84 = V_3 / 1673 \quad , V_3 = 0.11 m^3$

$$\left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_3}{T_4}$$

$$\left(\frac{0.998}{0.11}\right)^{1.4-1} = \frac{1673}{T_4}$$

$T_4 = 692.47\ ^oK$

$w_{34} = 1 * 0.717 * (1673 - 692.47) = 703.04$ kJ

$\Delta U_{34} = -\ 703.04$ kJ/kg.K

$Q_{41} = \Delta U_{41} = m\ .Cv\ (T_1 - T_4) = 1 * 0.717 * (313 - 692.47) = -\ 272.08$ kJ

$w_{41} = 0$

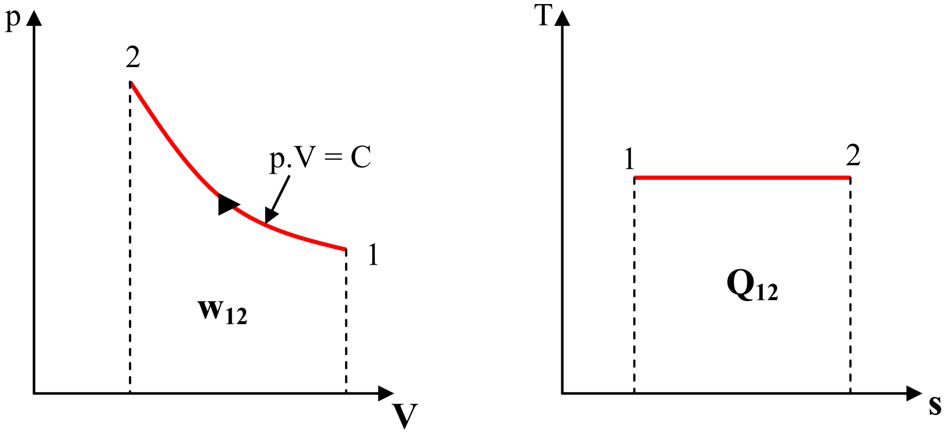
EX: - One kilogram of a gas is at 1.1 bar and 15°C. It is compressed until its volume is $0.1 m^3$. Calculate the final pressure and temperature if the compression is a- isothermal, b- adiabatic. Calculate also the work done, change in internal energy and heat transferred in each case. [Take Cp = 0.92 kJ/kg.K, Cv = 0.66 kJ/kg.K] .

Sol:-

$m = 1kg$, $Cp = 0.92$ kJ/kg.K , $p_1 = 1.1$ bar, $Cv = 0.66$ kJ/kg.K , $T_1 = 15 + 273 = 288\ ^oK$,

$V_2 = 0.1m^3$,

$p_2 =?$, $T_2 =?$, $w_{12} =?$, $\Delta U_{12} =?$, $Q_{12} =?$



a-

$$10^2\ p_1 . V_1 = m .R .T_1$$

$$R = Cp - Cv = 0.92 - 0.66 = 0.26 \text{ kJ/kg.K}$$

$$V_1 = \frac{m .R .T_1}{10^2\ p_1} = \frac{1*0.26 * 288}{10^2 * 1.1} = 0.68 \text{ m}^3$$

$$p_1.V_1 = p_2.V_2$$

$$p_2 = \frac{p_1.V_1}{V_2} = \frac{1.1*0.68}{0.1} = 7.488 \text{ bar} = 0.7488 \text{ MN/m}^2$$

$$10^2\ p_2 . V_2 = m .R .T_2$$

$$T_2 = \frac{10^2\ p_2 . V_2}{m .R} = \frac{10^2*7.488*0.1}{1 * 0.26} = 288^oK = 15^oC$$

Or $T_1 = T_2$ isothermal compression process is at constant temperature.

$$Q_{12} = w_{12} = m.\ R\ .T\ \ln\frac{V_2}{V_1}$$

$$= 1 * 0.26 * 288 \ln (0.1/ 0.68) = - 143.54 \text{ kJ}$$

$\Delta U_{12} = 0$

b-

$10^2\ p_1.\ V_1 = m\ .R\ .T_1$

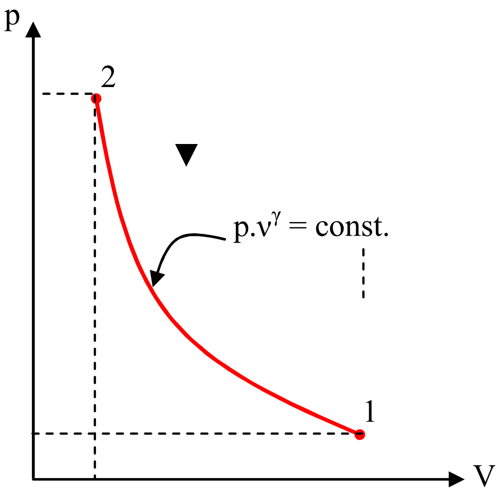
$R = Cp - Cv = 0.92 - 0.66 = 0.26$ kJ/kg.K

$$V_1 = \frac{m\ .R\ .T_1}{10^2\ p_1} = \frac{1*0.26 * 288}{10^2 * 1.1} = 0.68\ m^3$$

$\gamma = Cp / Cv = 0.92 / 0.66 = 1.393$

$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$$p_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} * p_1 = \left(\frac{0.68}{0.1}\right)^{1.393} * 1.1 = 15.89 \text{ bar} = 1.589\ MN/m^2$$



$$T_2 = \frac{10^2\ p_2.\ V_2}{m\ .R} = \frac{10^2*15.89*0.1}{1 * 0.26} = 611.1^oK = 338.1^oC$$

$w_{12} = m\ .Cv\ (T_1 - T_2) = - \Delta U_{12}$

$w_{12} = 1*0.66\ (288 - 611.1) = -213.25$ kJ

$\Delta U_{12} = 213.25$ kJ

$Q_{12} = 0$

Problems

4 -1 0.1m^3 of gas is compressed from a pressure of 120 kN / m^2 and temperature 25°C to a pressure of 12 bar according to the law $p.V^{1.2}$ = constant. Calculate;

1- The work done on the gas
2- The change in internal energy
3- The quantity heat transferred, Assume, Cv = 0.72 kJ/kg.K , R = 0.285kJ/kg.K

(-28.2kJ, 14.2kJ, -14kJ)

4 -2 1 kg of air enclosed in a rigid container is initially at 4.8 bar and 150°C. The container is heated until the temperature is 200°C. Calculate the pressure of the air finally and the heat supplied during the process (5.37 bar ; 35.9kJ/kg)

4 -3 Oxygen (molecular weight 32) expands reversibly in a cylinder behind a piston at a constant pressure of 3 bar. The volume initially is 0.01m^3; the initial temperature is 17°C. Calculate the work done by the oxygen and the heat flow to or from the cylinder walls during the expansion. Assume oxygen to be a perfect gas and take Cp = 0.917kJ/kg.K (6kJ; 21.16kJ)

4 -4 0.05 m^3 of a perfect gas at 6.3 bar undergoes a reversible isothermal process to a pressure of 1.05 bar. Calculate the heat flow to or from the gas (56.4kJ)

4-5 1kg of air is compressed isothermally & reversibly from 1 bar and 30°C to 5 bar. Calculate the work done on the air and the heat flow to or from the air.

(140kJ/kg; -140kJ/kg)

4 -6 1kg of air at 1 bar, 15°C is compressed reversibly and adiabatically to a pressure of 4 bar. Calculate the final temperature and the work done on the air. (155°C; 100.5kJ/kg)

4 -7 Nitrogen (molecular weight 28) expands reversibly in perfectly thermally insulated cylinder from 3.5bar, 200°C to a volume of 0.09m^3. If the initial volume occupied was 0.03m^3, calculate the work done during the expansion. Assume nitrogen to be perfect gas and take Cv = 0.741kJ/kg.K (9.31kJ)

4 -8 1kg of air at 1.02bar, 20°C is compressed reversibly according to a law $p.V^{1.3}$ = const., to a pressure of 5.5bar. Calculate the work done on the air and the heat flow to or from the cylinder walls during the compression. (133.5kJ/kg; -33.38kJ/kg)

4 -9 Oxygen (molecular weight 32) is compressed reversibly and polytropically in a cylinder from 1.05 bar, 15°C to 4.2bar in such a way that one- third of the work input is rejected as heat to the cylinder walls . calculate the final temperature of the oxygen. Assume oxygen to be aperfect gas and take Cv = 0.649kJ/kg.K (113°C)

4 -10 0.05 kg of carbon dioxide (molecular weight 44), occupying a volume of 0.03m^3 at 1.025bar , is compressed reversibly until the pressure is 6.15bar. calculate the final temperature the work done on the CO_2 and the heat flow to or from the cylinder walls ,

1- when the process is according to the law $p.v^{1.4}$ = constant ,
2- when the process is isothermal ,

assume carbon dioxide to be a perfect gas , and take $\gamma = 1.3$

(270°C ; 5.138kJ/kg ; 1.713kJ; 52.6°C ; 5.51kJ/kg ; -5.51kJ)

FIRST LAW OF THERMODYNAMICS

- Whenever a system undergoes a cyclic change, the algebraic sum of work transfer is proportional to the algebraic sum of heat transfer as work and heat are mutually convertible from one form into the other.

- For a closed system, a change in the energy content is the algebraic difference between the heat supply, *Q*, and the work done, *W*, during any change in the state. Mathematically,

$$dE = \delta Q - \delta W \tag{2.10}$$

- The energy *E* may include many types of energies like kinetic energy, potential energy, electrical energy, surface tension etc., but from the thermodynamic point of view these energies are ignored and the energy due to rise in temperature alone is considered. It is denoted by *U* and the first law is written as:

$$dU = \delta Q - \delta W \tag{2.11}$$

or during a process 1-2, *dU* can be denoted as

$$U_2 - U_1 = \int_1^2 \left(\delta Q - \delta W\right) \tag{2.12}$$

2.6 PROCESS

- A change in the condition or state of a substance is called a process.

- The process may consist of heating, flow from one place to another, expansion etc.

- In general, a process may be divided into non-flow or flow processes.

2.6.1 Non-flow Processes

- If there is no flow of material into or out of a system during a process, it is called a non-flow process.

- This is the simplest kind of process, and much can be learned about it by applying the principle of conservation of energy.

2.7 ANALYSIS OF NON-FLOW PROCESSES

- The purpose of the analysis is to apply the First Law of Thermodynamics to process in which a non-flow system changes from one state to the other and also to develop some useful relations.

2.7.1 Constant Volume or, Isochoric Process

- This process is usually encountered in the analysis of air-standard Otto, Diesel and Dual cycles.

- Figure 2.1 shows the constant volume process on a p-V diagram.

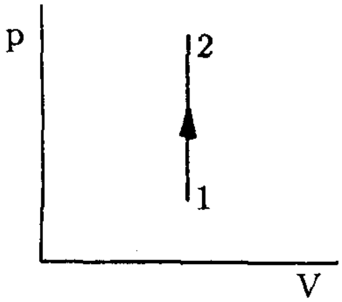


Fig. 2.1 Constant Volume Process

- As there is no change in volume, the work $\int(pdV)$ is zero.

- Hence, according to the first law for the constant volume process the change in internal energy is equal to the heat transfer, i.e.,

$$dU = \delta Q = mC_v dT = mC_v(T_2 - T_1) \qquad (2.13)$$

- For unit mass,

$$du = \delta q = C_v dT$$

$$C_v = \left(\frac{du}{dT}\right)_V \qquad (2.14)$$

i.e., specific heat at constant volume is the rate of change of internal energy with respect to absolute temperature.

2.7.2 Constant Pressure or Isobaric Process

- Figure 2.2 shows a system that changes from state 1 to state 2 at constant pressure.

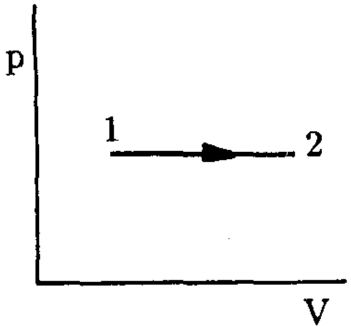


Fig. 2.2 Constant Pressure Process

- Application of first law yields,

$$\delta Q = dU + pdV = d(U + pV) = dH \qquad (2.15)$$

where H is known as the enthalpy.

- Thus, during constant pressure process, heat transfer is equal to change in enthalpy or

$$dH = \delta Q = mC_p dT \qquad (2.16)$$

For unit mass,

$$\delta q = dh = C_p dT \qquad (2.17)$$

$$C_p = \left(\frac{dh}{dT}\right)_p \qquad (2.18)$$

i.e., specific heat at constant pressure is the rate of change of specific enthalpy with respect to absolute temperature.

2.7.3 Constant Temperature or Isothermal Process

- The isothermal process on a p-V diagram is illustrated in Fig.2.3.
- As there is no temperature change during this process, there will not be any change in internal energy i.e., $dU = 0$, then according to the first law

$$\delta Q = \delta W \qquad (2.19)$$

or

$$Q_{1\text{-}2} = \int_1^2 pdV = p_1 V_1 \log_e\left(\frac{V_2}{V_1}\right) \qquad (2.20)$$

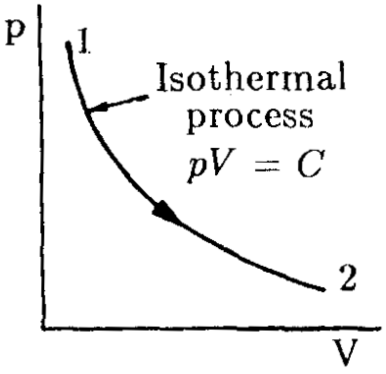


Fig. 2.3 Constant Temperature Process

2.7.4 Reversible Adiabatic or Isentropic Process

- If a process occurs in such a way that there is no heat transfer between the surroundings and the system, but the boundary of the system moves giving displacement work, the process is said to be adiabatic.

- Such a process is possible if the system is thermally insulated from the surroundings.

- Hence, $\delta Q = 0$, therefore,

$$\delta W = -\ \delta u = -mC_v dT \qquad (2.21)$$

- Reversible adiabatic process is also known as isentropic process.

- Let $pV^{\gamma} = C$ be the law of the isentropic process. For unit mass flow,

$$q_{1\text{-}2} = 0 = w_{1\text{-}2} + u_2 \text{ ï } u_1 \qquad (2.22)$$

or

$$w_{1\text{-}2} = -(u_2 \text{ ï } u_1)$$

- In other words, work is done at the expense of internal energy

$$W_{1\text{-}2} = \int_1^2 pdV = \int_1^2 \frac{C}{V^{\gamma}} dV \qquad (2.23)$$

$$= \frac{\left[CV^{1-\gamma}\right]_{V1}^{V2}}{1-\gamma}$$

$$= \frac{CV_2^{1-\gamma} - CV_1^{1-\gamma}}{1-\gamma}$$

when $C = p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$,

$$W_{1-2} = \frac{p_2 V_2^{\gamma} V_2^{1-\lambda} - p_1 V_2^{\gamma} V^{1-\gamma}{}_2}{1-\gamma}$$

$$= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

- Using $pV = RT$ for unit mass flow, so

$$w_{1\text{-}2} = \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$= C_v (T_1 - T_2) = -(u_2 - u_1)$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

therefore,

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \left(\frac{V_2}{V_1}\right)$$

$$= \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} \tag{2.24}$$

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} \left(\frac{p_2}{p_1}\right)$$

$$= \left(\frac{p_2}{p_1}\right)^{\left(1-\frac{1}{\gamma}\right)} = \left(\frac{p2}{p1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \tag{2.25}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \tag{2.26}$$

2.7.5 Reversible Polytropic Process

- In polytropic process, both heat and work transfers take place.

- It is denoted by the general equation $pV^n = C$, where n is the polytropic index.

- The following equations can be written by analogy to the equations for the reversible adiabatic process which is only a special case of polytropic process with $n = \gamma$.

- Hence, for a polytropic process

$$p_1V_1^n = p_2V_2^n \tag{2.27}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$$

$$\frac{T1}{T2} = \left(\frac{p1}{p2}\right)^{\left(\frac{n-1}{n}\right)} \tag{2.28}$$

and

$$w_{1-2} = \frac{p_1V_1 - p_2V_2}{n-1} \tag{2.29}$$

2.7.6 Heat Transfer During Polytropic Process

- Heat transfer per unit mass,

$$q_{1-2} = (u_2 - u_1) + \int_1^2 pdV \tag{2.30}$$

$$= C_v(T_2 - T_1) + \frac{R(T_2 - T_1)}{1-n}$$

$$= \left(C_v + \frac{R}{1-n}\right)(T_2 - T_1)$$

$$= \left(C_v + \frac{C_p - C_v}{1-n}\right)(T_2 - T_1)$$

$$= \left(\frac{C_p - nC_v}{1-n}\right)(T_2 - T_1)$$

$$= \left(\frac{C_v}{1-n}\right)\left(\frac{C_p}{C_v} - n\right)(T_2 - T_1)$$

$$= \left(\frac{C_v}{1-n}\right)(\gamma - n)(T_2 - T_1)$$

$$= \left(\frac{\gamma - n}{1-n}\right)C_v(T_2 - T_1) \qquad (2.31)$$

- Hence,

$$q_{1-2} = C_n(T_2 - T_1)$$

where,

$$C_n = \left(\frac{\gamma - n}{1-n}\right)C_v \qquad (2.32)$$

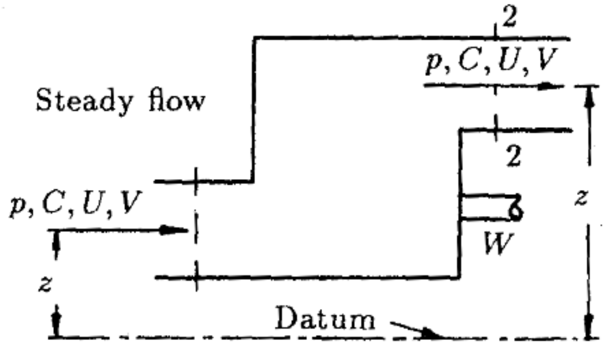
- Table 2.1 gives the formulae for various process relations for easy reference.

Table 2.1 *Summary of Process Relations for a Perfect Gas*

Process	Index n	p, V, T relation	Heat Transfer	Work $\int p\, dV$	Work $\int V\, dp$	ΔU
Constant Volume	∞	$T_1/T_2 = p_1/p_2$	$mC_v(T_2 - T_1)$	0	$V(p_2 - p_1)$	$mC_v(T_2 - T_1)$
Constant Pressure	0	$T_1/T_2 = V_1/V_2$	$mC_p(T_2 - T_1)$	$p(V_2 - V_1)$	0	$mC_v(T_2 - T_1)$
Isothermal	1	$p_1V_1 = p_2V_2$	$p_1V_1 \log_e\left(\frac{V_2}{V_1}\right)$	$p_1V_1 \log_e\left(\frac{V_2}{V_1}\right)$	$p_1V_1 \log_e\left(\frac{V_1}{V_2}\right)$	0
Isentropic	γ	$p_1V_1^\gamma = p_2V_2^\gamma$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$	0	$\frac{p_1V_1 - p_2V_2}{\gamma - 1}$	$\frac{\gamma}{\gamma-1}(p_2V_2 - p_1V_1)$	$mC_v(T_2 - T_1)$
Polytropic	n	$p_1V_1^n = p_2V_2^n$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}}$	$\left(\frac{\gamma-n}{1-n}\right)mC_v(T_2 - T_1)$	$\frac{p_1V_1 - p_2V_2}{n - 1}$	$\frac{n}{n-1}(p_2V_2 - p_1V_1)$	$mC_v(T_2 - T_1)$

2.8 ANALYSIS OF FLOW PROCESS

- Consider a device shown in the Fig.2.4 through which a fluid flows at uniform rate and which absorbs heat and does work, also at a uniform rate. In engines, the gas flow, heat flow and work output vary throughout each cycle, but if a sufficiently long time interval (such as a minute) be chosen, then, even an engine can be considered to be operating und-r steady-flow conditions.



$p \rightarrow$ Pressure
$V \rightarrow$ Volume
$C \rightarrow$ Velocity
$U \rightarrow$ Internal Energy
$H \rightarrow$ Enthalpy
$z \rightarrow$ Datum Height

Fig. 2.4 Illustration of Steady Flow Process

- Applying the principle of conservation of energy to such a system during the chosen time interval, the total energy in the mass of fluid which enters the machine across boundary 1-1 plus the heat added to the fluid through the walls of the machine, minus the work done by the machine, must equal the total energy left in the equal mass of fluid crossing boundary 2-2 and leaving the machine.

- Expressed in the mathematical form,

$$Q - W_x = m\left[\left(h_2 + \frac{C_2^2}{2} + gz_2\right) - \left(h_1 + \frac{C_1^2}{2} + gz_1\right)\right]$$

or

$$\frac{Q - W_x}{m} = \Delta\left(h + \frac{C^2}{2} + gz\right) \qquad (2.33)$$

- In the above equation, *m* donates mass. It is known a, the general energy equation and is the one which illustrates the first law of thermodynamics.

- Here, *W*, is the external work and given by

$$W = W_x - mp_1V_1 + mp_2V_2 \qquad (2.34)$$

where *W* is the net work and *V* stands for volume.

2.8.1 Flow Work

- The term *pV* in the above equation is called flow work and must be considered whenever flow takes place.
- It is the, work necessary either to push certain quantity of mass into or out of the system.

2.9 WORK, POWER AND EFFICIENCY

2.9.1 Work

- Work might be defined as the energy expended when motion takes place in opposition to a force.
- Work thus represents energy being transferred from one place to another.

2.9.2 Power

- Power is the rate of doing work.
- It is generally useless to do a certain piece of work unless it can be done within a reasonable length of time.
- When work is done at the rate of 736 Nm/s or 75 kgf m/s, the quantity of power being developed is known as one horse power.
- In SI units power is expressed in kW = 1 kN m/s.

2.9.3 Efficiency

- It is common practice in engineering to establish a figure of merit for a device by comparing the actual performance of the device with the performance it would have had under some arbitrary set of ideal conditions.
- The ratio of actual performance to the ideal performance is called the efficiency of the device.
- Since the ideal performance is usually unattainable, the efficiency is usually less than 1.
- In internal combustion engines, one of the most important efficiencies is the thermal efficiency, which is defined as

$$\text{Thermal efficiency} = \eta_{\text{th}} = \frac{\text{Work delivered by the engine}}{\text{Chemical energy in the fuel}}$$

$$= \frac{\text{Work output}}{\text{Fuel energy input}}$$

- Obviously, the output and input may be measured over any convenient interval such as one cycle or one minute.

- The output and input energies must be expressed in same kind of units, since η_{th} is a number without units.

- Let W be the work delivered by the engine (Nm) and m_f be the mass of fuel (kg) required and CV be the calorific value of the fuel (kJ/kg), then the efficiency η_{th} is given by

$$\eta_{th} = \frac{W}{m_f CV} \tag{2.35}$$

- If W is the indicated work then η_{th} is called the indicated thermal efficiency, η_{ith}, and if W is the work delivered at the output shaft then it is called the brake thermal efficiency, η_{bth}.

- The ratio of the brake thermal efficiency to the indicated thermal efficiency is called the mechanical efficiency, η_{m}.

AIR-STANDARD CYCLES AND THEIR ANALYSIS

3.1 INTRODUCTION

- The internal combustion engine does not operate on a thermodynamic cycle as it involves an open system.

- To simplified the analysis of the engine system, the following assumptions have been made:

  i) A closed system; the working fluid at the exit conditions back to the condition of the starting point.

  ii) A perfect gas, $pV = mRT$ as the working medium.

  iii) No change of mass of the working medium.

  iv) All processes that constitute the cycle are reversible.

  v) Heat is supplied from a constant high temperature source and not from chemical reactions during the cycle.

  vi) Some heat is rejected to a constant low temperature sink during cycle.

  vii) There are no heat losses from the system to the surroundings.

  viii) The working medium has constant specific heats throughout the cycle.

  ix) The physical constant viz. C_p, C_v, γ and M of working medium are the same as those of the air at standard at atmospheric conditions.

C_p = 1.005 kJ/kg K
C_v = 0.717 kJ/kg K
γ = 29 kg/kmol
M = 1.4

- Due to these assumptions, the analysis becomes over-simplified and the results do not agree with those of the actual engine.

- Work output, peak pressure, peak temperature and thermal efficiency based on air-standard cycles will be the maximum that can be attained and will differ considerably from those of the actual engine.

- It is often used in getting the approximately answers to the complicated processes in internal combustion engines.

- In this chapter various cycles will be reviewed and also the equations for work output, mean effective pressure, efficiency etc.

- Comparison will be made between Otto, Dual and Diesel cycle to see which cycle is more efficient under a set of given operating conditions.

3.2 THE CARNOT CYCLE

- The cycle consists of two isothermal and two reversible adiabatic processes as shown in Fig 3.1.

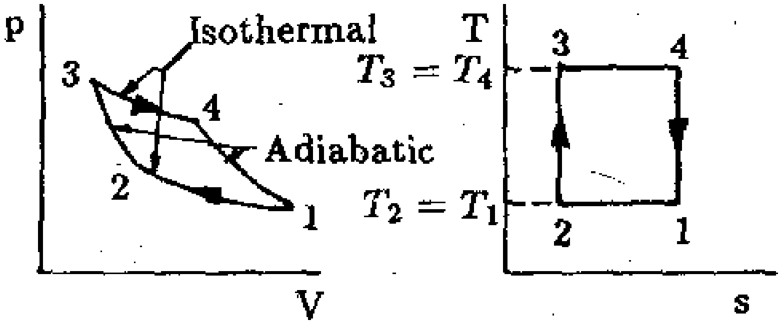


Fig. 3.1 Carnot Engine

- It is represented as a standard of perfection and engines can be compared with it to judge the degree of perfection.

- The efficiency of the cycle can be written as:

$$\eta_{Carnot} = \frac{\text{Work done by the system during the cycle } (W)}{\text{Heat supplied to the system during the cycle } (Q_S)}$$

According to the 1st law of thermodynamics,

Work done = Heat supplied ï Heat rejected

$$W = Q_S - Q_R \qquad (3.1)$$

Considering the isothermal processes 1→2 and 3→4,

$$Q_R = mRT_1 \log_e \frac{V_1}{V_2} \qquad (3.2)$$

$$Q_S = mRT_3 \log_e \frac{V_4}{V_3} \qquad (3.3)$$

Considering the adiabatic processes 2→3 and 4→1

$$\frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\left(\frac{1}{\gamma-1}\right)} \qquad (3.4)$$

and

$$\frac{V_4}{V_1} = \left(\frac{T_1}{T_4}\right)^{\left(\frac{1}{\gamma-1}\right)} \qquad (3.5)$$

Since $T_1 = T_2$ and $T_4 = T_3$,

$$\frac{V_4}{V_1} = \frac{V_3}{V_2}$$

or

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} = r \qquad (3.6)$$

then

$$\eta_{carnot} = \frac{mRT_3 \log_e r - mRT_1 \log_e r}{mRT_3 \log_e r} \qquad (3.7)$$

$$= \frac{T_3 - T_1}{T_3} = 1 - \frac{T_1}{T_3} \qquad (3.8)$$

- Mean effective pressure, pm, is defined as that hypothetical constant pressure acting on the piston during its expansion stroke producing the same work output as that from the actual cycle:

$$p_{\mathrm{m}} = \frac{\text{Work Output}}{\text{Swept Volume}} \qquad (3.9)$$

It can be shown that

$$p_m = \frac{\text{Area of indicator diagram}}{\text{Length of diagram}} \times \text{constant} \tag{3.10}$$

3.3 THE STIRLING CYCLE

- One of the modified Carnot cycle to produce higher mean effective pressure.

- It consists of two isothermal and two constant volume processes as shown in Fig. 3.3.

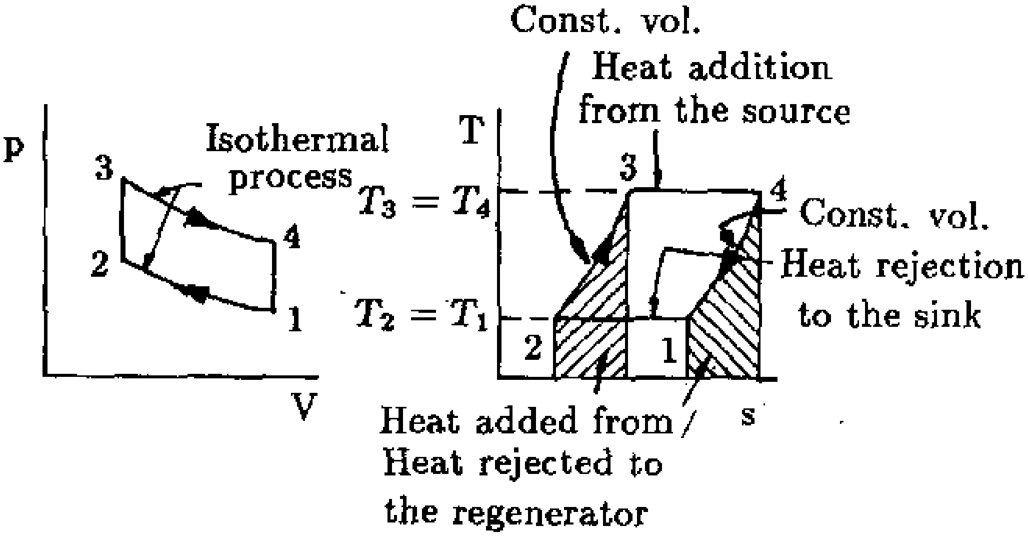


Fig 3.3 Stirling Cycle

- The amount of heat addition and rejection during constant volume processes is same.

- The efficiency of the cycle is

$$\eta_{Stirling} = \frac{RT_3 \log_e\left(\frac{V_4}{V_3}\right) - RT_1 \log_e\left(\frac{V_1}{V_2}\right)}{RT_3 \log_e\left(\frac{V_4}{V_3}\right)} \tag{3.11}$$

but $V_3 = V_2$ and $V_4 = V_1$

$$\eta_{Stirling} = \frac{T_3 - T_1}{T_3} \tag{3.12}$$

- The stirling cycle was used earlier for hot air engine before Otto and Diesel cycles came into use but with the development in metallurgy and intensive research in this type of engine has made it been used in the design of heat exchanger.

- In practice, the heat exchangers efficiency cannot be 100%. Hence the Stirling cycle efficiency will be less than Carnot efficiency;

$$\eta = \frac{R(T_3 - T_1)\log_e r}{RT_3 \log_e r + (1-\varepsilon)C_V(T_3 - T_1)} \tag{3.13}$$

where ε is the heat exchanger effectiveness.

3.4 THE ERICSSON CYCLE

- It is also modified from the Carnot cycle for higher mean effective pressure.

- It consists of two isothermal processes and two constant pressure processes as shown in Fig. 3.4.

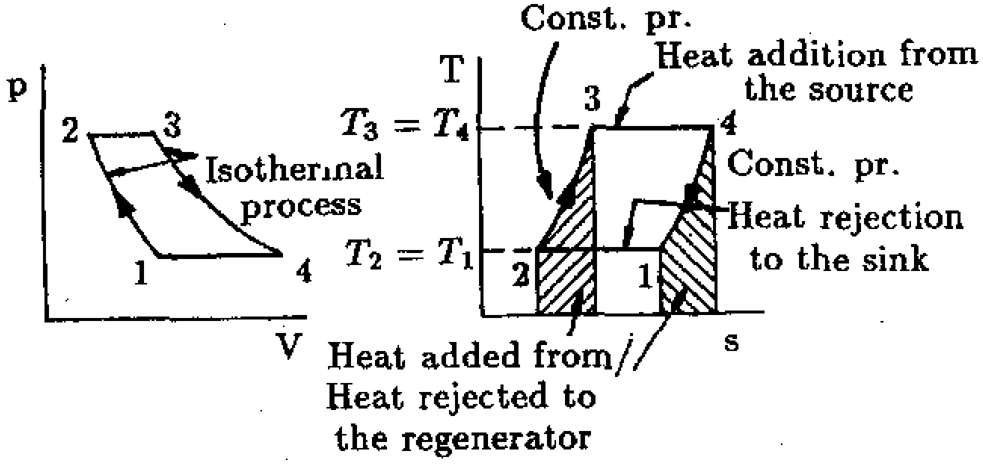


Fig. 3.4 Ericsson Cycle

- The heat addition and rejection take place at constant pressure and at the same source and sink temperature.

- The advantage of the cycle is its smaller pressure ratio for a given ratio of maximum to minimum specific volume.

- This cycle does not find practical application in piston engines but is approached by a gas turbine employing a large number of stages with heat exchangers, insulators and reheaters.

3.5 THE OTTO CYCLE

- This cycle is a constant-volume heat addition cycle (as shown in Fig. 3.5), which forms the basic for the working of today's spark-ignition engines.

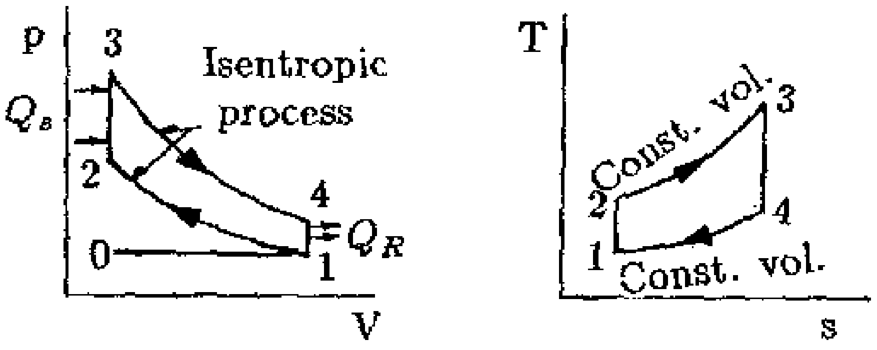


Fig. 3.5 Otto Cycle

0→1	Suction process
1→2	Isentropic compression of the air when piston moves from BDC to TDC.
2→3	Heat is supplied reversibly at constant volume (correspond to spark ignition and combustion in the actual engine)
3→4	Isentropic expansion and constant volume heat rejection.
4→1	Isentropic expansion and constant volume heat rejection.
1→0	Exhaust process.

- The thermal efficiency of Otto cycle

$$\eta_{Otto} = \frac{Q_S - Q_R}{Q_S} \tag{3.14}$$

Considering constant volume process 2→3 and 4→1, the heat supplied and rejected of air are

$$Q_S = mC_v(T_3 - T_2) \tag{3.15}$$

$$Q_R = mC_V\left(T_4 - T_i\right) \tag{3.16}$$

$$\eta_{Otto} = \frac{m\left(T_3 - T_2\right) - m(T_4 - T_1)}{m\left(T_3 - T_2\right)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \tag{3.17}$$

Considering isentropic process 1→2 and 3→4, so

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \tag{3.18}$$

and

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{(\gamma-1)} \tag{3.19}$$

But the volume ratios V_1/V_2 and V_3/V_4 are equal to the compression ratio, r. therefore

$$\frac{V_1}{V_2} = \frac{V_4}{V_3} = r \tag{3.20}$$

therefore,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \tag{3.21}$$

From Eq. (3.21), it can be easily shown that

$$\frac{T_4}{T_3} = \frac{T_1}{T_2} = \frac{T_4 - T_1}{T_3 - T_2} \tag{3.22}$$

$$\eta_{Otto} = 1 - \frac{T_1}{T_2} \tag{3.23}$$

$$= 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{(\gamma-1)}} \tag{3.24}$$

$$= 1 - \frac{1}{R^{(\gamma-1)}} \tag{3.25}$$

Note: The thermal efficiency of Otto cycle is a function of compression ratio *r* since γ is assumed to be constant for any working fluid. Further, the efficiency is independent of heat supplied and pressure ratio. However the use of gasses with higher γ values would increase efficiency of Otto cycle.

- The work output for an Otto cycle can be expressed as

$$W = \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1} \tag{3.26}$$

Also

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = r^{\gamma}$$

$$\frac{p_3}{p_2} = \frac{p_3}{p_4} = r_p \tag{3.27}$$

$$V_1 = rV_2 \qquad \text{and} \qquad V_4 = rV_3$$

therefore

$$W = \frac{p_1V_1}{\gamma - 1}\left(\frac{p_3V_3}{p_1V_1} - \frac{p_4V_4}{p_1V_1} - \frac{p_2V_2}{p_1V_1} + 1\right) \tag{3.28}$$

$$= \frac{p_1V_1}{\gamma - 1}\left(\frac{r_p r^{\gamma}}{r} - r_p - \frac{r^{\gamma}}{r} + 1\right)$$

$$= \frac{p_1V_1}{\gamma - 1}\left(r_p r^{\gamma - 1} - r_p - r^{\gamma - 1} + 1\right)$$

$$= \frac{p_1V_1}{\gamma - 1}\left(r_p - 1\right)\left(r^{\gamma - 1} - 1\right) \tag{3.29}$$

- The mean effective pressure of the cycle is given by

$$p_m = \frac{\text{Work output}}{\text{Swept volume}} \tag{3.30}$$

Swept volume = V_1 ï V_2 = $V_2\,(r\text{-}1)$

$$p_m = \frac{\dfrac{1}{\gamma - 1}\, p_1V_1\left(r_p - 1\right)\left(r^{(\gamma - 1)} - 1\right)}{V_2\left(r - 1\right)}$$

$$= \frac{p_1 r\left(r_p - 1\right)\left(r^{(\gamma - 1)} - 1\right)}{\left(\gamma - 1\right)\left(r - 1\right)} \tag{3.31}$$

3.6 DIESEL CYCLE

- This cycle is used in the compression-ignition engines, which work on heavy liquid fuels.

- In this engine, the fuel is injected into the cylinder, which contains compressed air at a higher temperature than the self-ignition temperature of the fuel. Hence the fuel ignites on its own accord and requires no special device like an ignition system in a spark-ignition engine.

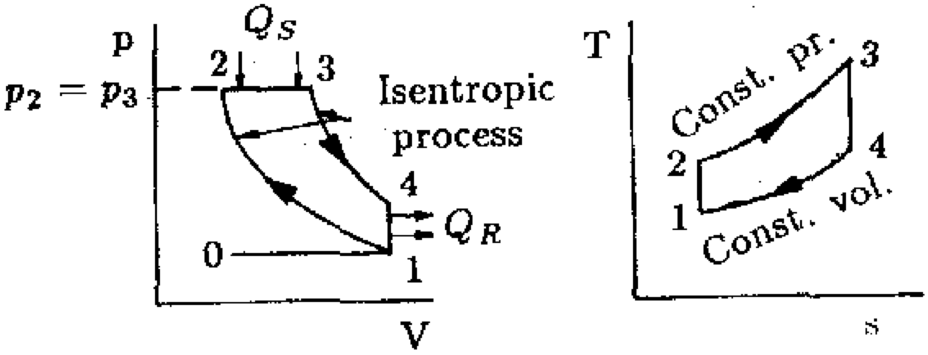


Fig. 3.7 Diesel Cycle

- In Otto cycle the heat addition takes place at constant volume whereas in the Diesel cycle it is at constant pressure.

- The thermal efficiency of the Diesel cycle is

$$\eta_{Diesel} = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{mC_p(T_4 - T_2) - mC_v(T_4 - T_1)}{mC_p(T_3 - T_2)} \qquad (3.32)$$

$$= 1 + \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma}\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \qquad (3.33)$$

Considering the process 1→2

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = T_1 r^{(\gamma-1)} \qquad (3.34)$$

Considering the constant pressure process 2→3

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

$$T_3 = T_2 r_c \tag{3.35}$$

The volume ratio $\frac{V_3}{V_2}$ is called the cut-off ratio r_c.

- Eq. (3.39) is different from the Otto cycle only in the bracketed factor and it is always greater than unity. Hence for given compression, the Otto cycle ratio is more efficient.
- Therefore unlike the Otto cycle the air-standard efficiency of the diesel cycle depends on output as the fuel cut-off ratio, r_c depends on it, being maximum for maximum output.
- However, in practice the operating compression ratios of diesel engine are much higher compared to spark-ignition engines working on Otto cycle. (Compression ratio of diesel engines are 16-20 whereas 6-10 for spark-ignition engines).
- So, the efficiency of a diesel engine is more than that of gasoline engine.
- The net work output for a diesel cycle is given:

$$W = p_2(V_3 - V_2) + \frac{p_3 v_3 - p_4 r V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} \tag{3.40}$$

$$= p_2 V_2 (r_c - 1) + \frac{p_3 r_c V_2 - p_4 r V_2}{\gamma - 1} - \frac{p_2 V_2 - p_1 r V_2}{\gamma - 1}$$

$$= V_2 \left[\frac{p_2 (r_c - 1)(\gamma - 1) + p_3 r_c - p_4 r - (p_2 - p_1 r)}{\gamma - 1} \right]$$

$$= V_2 \left[\frac{p_2 (r_c - 1)(\gamma - 1) + p_3 \left(r_c - \frac{p_4}{p_3} r \right) - p_2 \left(1 - \frac{p_i}{p_2} r \right)}{\gamma - 1} \right]$$

$$= p_2 V_2 \left[\frac{(r_c - 1)(\gamma - 1) + \left(r_c - r_c{}^{\gamma (r_r - 1)}) - (1 - r^{(1-\gamma)}) \right)}{\gamma - 1} \right]$$

$$= \frac{p_1 V_1 r^{(\gamma-1)}\left[\gamma(r_c - 1) - r^{(1-\gamma)}\left(r_c^{\gamma} - 1\right)\right]}{\gamma - 1} \tag{3.41}$$

- The expression for mean effective pressure can be shown as

$$p_m = \frac{p_1 V_1\left[r^{(\gamma-1)}\gamma(r_c - 1) - \left(r_c^{\gamma} - 1\right)\right]}{(\gamma - 1)V_1\left(\frac{r-1}{r}\right)} \tag{3.42}$$

$$= \frac{p1\left[\gamma r^{\gamma}(r_c - 1) - r\left(r_c^{\gamma} - 1\right)\right]}{(\gamma - 1)(r - 1)} \tag{3.43}$$

3.7 THE DUAL CYCLE

- Since some time interval is required for the chemical reactions during combustion process, the combustion cannot take place at constant value. Similarly, due to the rapid uncontrolled combustion in diesel engines, combustion does not occur at constant pressure.

- Therefore, a cycle that is a compromise between Otto and Diesel cycles (also call mixed cycle or limited pressure cycle) as shown in Fig. 3.8, is introduced.

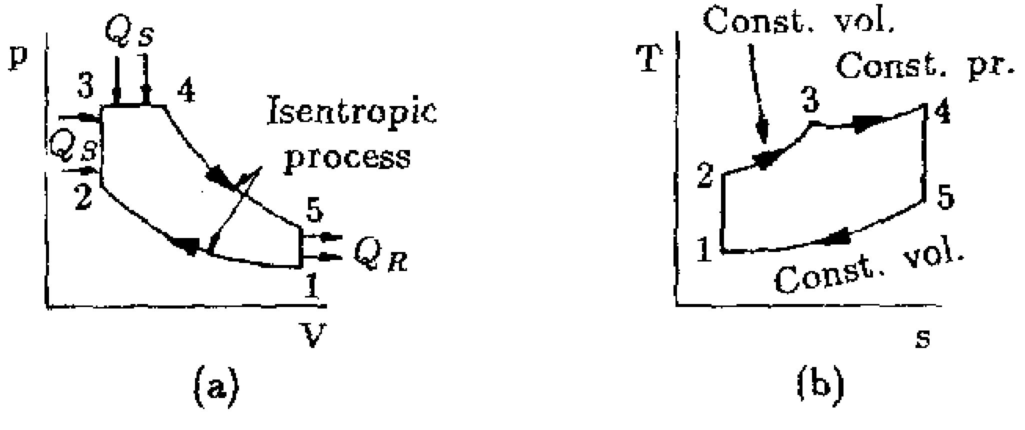


Fig. 3.8 Dual Cycle

- In this cycle, the heat is supplied to the system at constant volume and then, the remaining part at constant pressure.

- The efficiency of the cycle may be written as

$$\eta_{Dual} = \frac{Q_S - Q_R}{Q_S} \tag{3.44}$$

$$= \frac{mC_v(T_3 - T_2) + mC_p(T_4 - T_3) - mC_v(T_5 - T_1)}{mC_v(T_3 - T_2) + mC_p(T_4 - T_3)}$$

$$= 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)} \tag{3.45}$$

now

$$T_2 - T_1\left(\frac{V_i}{V_2}\right)^{\gamma-1} = T_1 r^{(\gamma-1)} \tag{3.46}$$

$$T_3 = T_2\left(\frac{p_3}{p_2}\right) = T_1 r_p r^{(\gamma-1)} \tag{3.47}$$

where r_p is the pressure ratio in constant volume heat addition process and is equal to $\frac{p_3}{p_2}$ and the cut off ratio r_c is given by $\left(\frac{V_4}{V_3}\right)$

$$T_4 = T_3\frac{V_4}{V_3} = T_3 r_c$$

Substituting for T_3 from Eq. 3.47

$$T_4 = T_1 r_c r_p r^{(\gamma-1)} \tag{3.48}$$

and

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{(\gamma-1)} \tag{3.49}$$

$$= T_1 r_p r_c r^{(\gamma-1)}\left(\frac{V_4}{V_5}\right)^{(\gamma-1)} \tag{3.50}$$

Now

$$\frac{V_4}{V_5} = \frac{V_4}{V_1} = \frac{V_4}{V_3} \times \frac{V_3}{V_1} \tag{3.51}$$

$$= \frac{V_4}{V_3} \times \frac{V_2}{V_1} \qquad \text{(since } V_2 = V_3\text{)}$$

Therefore,

$$\frac{V_4}{V_5} = \frac{r_c}{r} \tag{3.52}$$

where $\frac{V_4}{V_5}$ is the expansion ratio. Now,

$$
\begin{aligned}
T_5 &= T_1 r_p r_c r^{\gamma-1}\left(\frac{r_c}{r}\right)^{\gamma-1} \\
&= T_1 r_p r_c^{\gamma} \qquad (3.53)
\end{aligned}
$$

Substituting for T_2, T_3, T_4 and T_5 into Eq. 3.45 and simplifying

$$
\eta = 1 - \frac{1}{r^{(\gamma-1)}}\left[\frac{r_p r_c^{\gamma} - 1}{(r_p - 1) + r_p\gamma(r_c - 1)}\right] \qquad (3.54)
$$

- It can be seen that a value of $r_p > 1$results in an increased in efficiency for a give value r_c and γ. Thus the efficiency of this cycle lies between that of Otto cycle and the Diesel cycle having the same compression ratio.

- The work output of the cycle is given by;

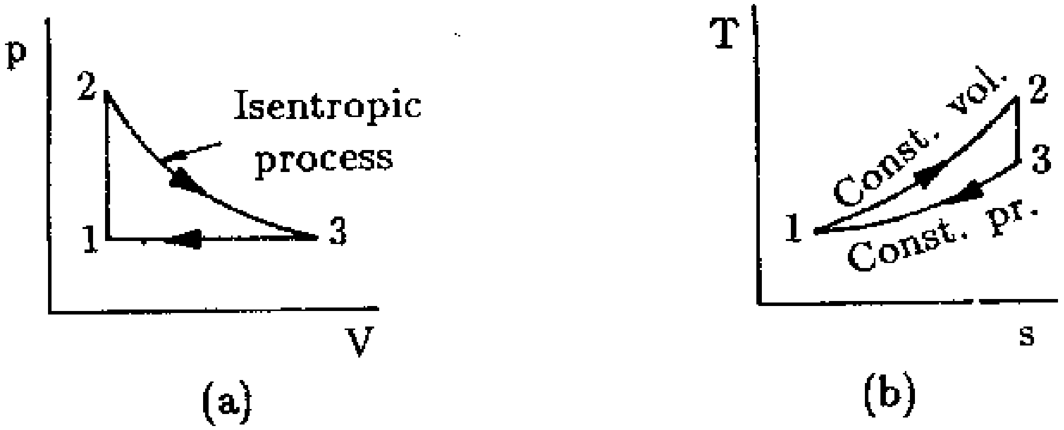
$$
\begin{aligned}
W &= p_3(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1} \\
&= \frac{p_1V_1}{\gamma - 1}\left[(\gamma - 1)\left(\frac{p_4V_4}{p_1V_1} - \frac{p_3V_3}{p_1V_1}\right) + \frac{p_4V_4}{p_1V_1} - \frac{p_5V_5}{p_1V_1} - \frac{p_2V_2}{p_1V_1} + 1\right] \\
&= \frac{p_1V_1}{\gamma - 1}\left[(\gamma - 1)\left(r_c r_p^{\gamma-1} - r_p r^{\gamma-1}\right) + r_c r_p r^{\gamma-1} - r_p r_c^{\gamma} - r^{\gamma-1} + 1\right] \\
&= \frac{p_1V_1}{\gamma - 1}\left[\gamma r_c r_p r^{\gamma-1} - \gamma r_p r^{\gamma-1} + r_p r^{\gamma-1} - r_p r_c^{\gamma} - r^{\gamma-1} + 1\right] \\
&= \frac{p_1V_1}{\gamma - 1}\left[\gamma r_p r^{\gamma-1}(r_c - 1) + r_{\gamma} - 1(r_p - 1) - (r_p r_c^{\gamma} - 1)\right] \qquad (3.55)
\end{aligned}
$$

- The mean effective pressure is

$$
\begin{aligned}
p_m &= \frac{\text{Work output}}{\text{Swept volume}} = \frac{W}{V_s} \\
&= \frac{1}{V_1 - V_2}\frac{p_1V_1}{\gamma - 1}\left[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\right] \\
&= \frac{1}{\left(1 - \dfrac{V_2}{V_1}\right)}\frac{p_1}{(\gamma - 1)}\left[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\right] \\
&= p_1\frac{\left[\gamma r_p r^{\gamma}(r_c - 1) + r^{\gamma}(r_p - 1) - r(r_p r_c^{\gamma} - 1)\right]}{(\gamma - 1)(r - 1)} \qquad (3.56)
\end{aligned}
$$

3.9 THE LENOIR CYCLE

- Consists of the following processes:



(a) (b)

Fig. 3.13 Lenoir Cycle

1→2	Constant volume heat addition
2→3	Isentropic expansions
3→1	Constant pressure heat rejection

- It is used for pulse jet engines.

- The efficiency of the cycle is given as

$$\eta_{Lenior} = \frac{Q_S - Q_F}{Q_S}$$

$$Q_S = mC_v(T_2 - T_1) \tag{3.57}$$

$$Q_R = mC_p(T_3 - T_1) \tag{3.58}$$

$$\eta_{Lenoir} = \frac{mC_v(T_2 - T_1) - mC_p(T_3 - T_1)}{mC_v(T_2 - T_1)} \tag{3.59}$$

$$= 1 - \gamma\left(\frac{T_3 - T_1}{T_2 - T_1}\right)$$

Taking $p_2 / p_1 = r_p$ and $T_2 = T_1 r_p$

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \tag{3.60}$$

$$T_3 = T_2\left(\frac{1}{r_p}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_1 r_p\left(\frac{1}{r_p}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_1 r_p^{(1/\gamma)}$$

$$\eta = 1-\gamma\left(\frac{T_1 r_p^{(1/\gamma)} - T_1}{T_1 r_p - T_1}\right) \tag{3.61}$$

$$= 1-\gamma\left(\frac{r_p^{(1/\gamma)} - 1}{r_p - 1}\right) \tag{3.62}$$

- Thus the efficiency of this cycle depends upon the pressure ratio as well as the ratio of specific heats, γ.

3.10 THE ATKINSON CYCLE

- It is an ideal cycle for Otto engine exhausting to a gas turbine.

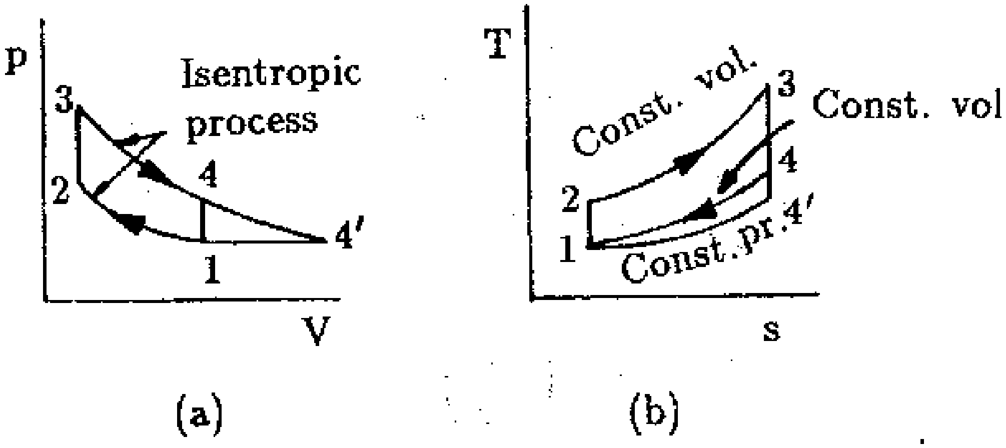


Fig. 3.14 Atkinson Cycle

- In this cycle the isentropic expansion (3→4) of an Otto cycle is further allowed to proceed to the lowest cycle pressure so as to increase the work output.

- The efficiency of the cycle is given as

$$\eta_{Atkinson} = \frac{Q_S - Q_R}{Q_S} \tag{3.63}$$

$$= \frac{mC_v(T_3 - T_2) - mC_p(T_{4'} - T_1)}{mC_v(T_3 - T_2)} \qquad (3.64)$$

$$= 1 - \gamma\left(\frac{T_{4'} - T_1}{T_3 - T_2}\right) \qquad (3.65)$$

the compression ratio, $r = \frac{V_1}{V_2}$ and the expansion ratio $e = \frac{V_{4'}}{V_3}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = r^{(\gamma-1)} \qquad (3.66)$$

Therefore

$$T_2 = T_1 r^{(\gamma-1)} \qquad (3.67)$$

$$\frac{T_2}{T_1} = \frac{p_3}{p_2} = \left(\frac{p_3}{p_{4'}} \times \frac{p_{4'}}{p_2}\right)$$

$$= \left(\frac{p_3}{p_{4'}} \times \frac{p_1}{p_2}\right) \qquad (3.68)$$

$$\frac{p_3}{p_{4'}} = \left(\frac{V_{4'}}{V_3}\right)^{\gamma} = e^{\gamma} \qquad (3.69)$$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{1}{r^{\gamma}} \qquad (3.70)$$

Substituting Eq. 3.69 and 3.70 in Eq. 3.68

$$\frac{T_3}{T_2} = \frac{e^{\gamma}}{r^{\gamma}} \qquad (3.71)$$

$$T_3 = T_2 \frac{e^{\gamma}}{r^{\gamma}} = T_1 r^{(\gamma-1)} \frac{e^{\gamma}}{r^{\gamma}} = T_1 \frac{e^{\gamma}}{r} \qquad (3.72)$$

$$\frac{T_{4'}}{T_3} = \left(\frac{V_3}{V_{4'}}\right)^{(\gamma-1)} = \frac{1}{e^{(\gamma-1)}}$$

$$T_{4'} = T_3 \frac{1}{e^{(\gamma-1)}} \tag{3.73}$$

$$= T1\left(\frac{e^{\gamma}}{r}\right)\left(\frac{1}{e^{(\gamma-1)}}\right)$$

$$T_{4'} = T_1 \frac{e}{r} \tag{3.74}$$

Substituting the values of T_2, T_3, $T_{4'}$ in Eq. 3.65

$$\eta_{Atkinson} = 1-\gamma\left[\frac{T_1 e / r - T_1}{T_1 e^{\gamma} / r - T_1 r^{(\gamma-1)}}\right]$$

$$= 1-\gamma\left[\frac{e-r}{e^{\gamma}-r^{\gamma}}\right] \tag{3.75}$$

3.11 THE BRAYTON CYCLE

- This cycle consists of two reversible adiabatic or isentropic processes and two constant pressure processes (Fig. 3.15).

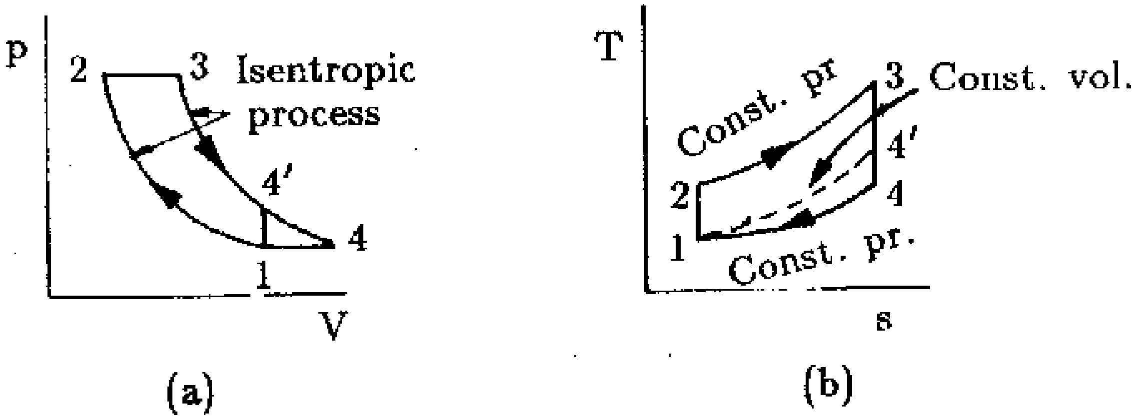


Fig. 3.15 Brayton Cycle

- It is a theoretical cycle for gas turbine.
- This cycle is similar to the Diesel cycle in compression and heat addition but the isentropic expansion is further extended followed by constant pressure heat rejection.

$$\eta_{Brayton} = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{mC_p(T_3 - T_2) - mC_p(T_4 - T_1)}{mC_p(T_3 - T_2)} \tag{3.76}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

If *r* is compression ratio i.e., (V_1/V_2) and r_p is the pressure ratio i.e., (p_2/p_1) then,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \tag{3.77}$$

$$= \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = r^{(\gamma-1)} \tag{3.78}$$

$$T_4 = \frac{T_3}{r^{(\gamma-1)}} \tag{3.79}$$

$$T_1 = \frac{T_2}{r^{(\gamma-1)}} \tag{3.80}$$

$$\eta_{Brayton} = 1 - \frac{\left(T_3 / r^{(\gamma-1)}\right) - \left(T_2 / r^{(\gamma-1)}\right)}{T_3 - T_2}$$

$$= 1 - \frac{1}{r^{(\gamma-1)}} \tag{3.81}$$

$$r = \frac{V_1}{V_2} = \left(\frac{p2}{p1}\right)^{\frac{1}{\gamma}} = r_p^{\frac{1}{\gamma}} \tag{3.82}$$

$$r = 1 - \frac{1}{\left(r_p^{\frac{1}{\gamma}}\right)^{\gamma-1}} = 1 - \frac{1}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)}} \tag{3.83}$$

- From Eq. 3.83, it is seen that the efficiency of the Brayton cycle depends only on the pressure ratio and the ratio of specific heat, γ.

Network output = Expansion work ï Compression work

$$= C_p\left(T_3 - T_4\right) - C_p\left(T_2 - T_1\right)$$

$$= C_p T_1\left(\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1\right)$$

$$\frac{W}{C_p T_1} = \frac{T_3}{T_1} - \frac{T_4}{T_3}\frac{T_3}{T_1} - \frac{T_2}{T_1} + 1 \qquad (3.84)$$

- It can be easily seen from the Eqn.3.84 (work output) that, the work output of the cycle depends on initial temperature, T_1, the ratio of the maximum to minimum temperature, $\frac{T_3}{T_1}$, pressure ratio, r_p and γ which are used in the calculation of $\frac{T_2}{T_1}$.
- Therefore, for the same pressure ratio and initial conditions work output depends on the maximum temperature of the cycle.

ACTUAL CYCLES AND THEIR ANALYSIS

5.1 INTRODUCTION

- The actual cycle efficiency is much lower than the air-standard efficiency due to various losses occurring in the actual engine operation such as:

    i. Variation of specific heats with temperature
    ii. Dissociation of the combustion products
    iii. Progressive combustion
    iv. Incomplete combustion of fuel
    v. Heat transfer into the walls of the exhaust process
    vi. Blowdown at the end of the exhaust process
    vii. Gas exchange process

- An estimation of these losses can be made from previous experience and some simple tests on the engines in evaluating the performance of an engine.

5.2 COMPARISON OF THE THERMODYNAMIC AND ACTUAL CYCLES

- The actual cycles for internal combustion engines differ from thermodynamic cycles in many respect due to:

    i. The working substance is the mixture of air and fuel vapour combined with the products of combustion left from the previous cycle.
    ii. The change in chemical composition of the working substance.
    iii. The variation of specific heats with temperature.
    iv. The change in the composition, temperature and actual amount of fresh charge because of the residual gases.
    v. The progressive combustion rather than the instantaneous combustion.
    vi. The heat transfer to and from the working medium.
    vii. The substantial exhaust blowdown loss, i.e., loss of work on the expansion stroke due to early opening of the exhaust valve.
    viii. Gas leakage, fluid friction etc., in actual engines.

- Most of the factors listed above tend to decrease the thermal efficiency and power output of the actual engines.

- On the other hand, the analysis of the cycles while taking these factors into account clearly indicates that the estimated thermal efficiencies are not very different from those of the actual cycles.

- Out of all the above factors, major influence is exercised by:

  i. Time loss factor i.e. loss due to time required for mixing of fuel and air and also for combustion.
  ii. Heat loss factor i.e. loss of heat from gases to cylinder walls.
  iii. Exhaust blowdown factor i.e. loss of work on the expansion stroke due to early opening of the exhaust valve.

These major losses are discussed in the following sections.

5.3 TIME LOSS FACTOR

- In thermodynamic cycles, the heat addition is assumed to be an instantaneous process whereas in an actual cycle it is over a definite period of time.

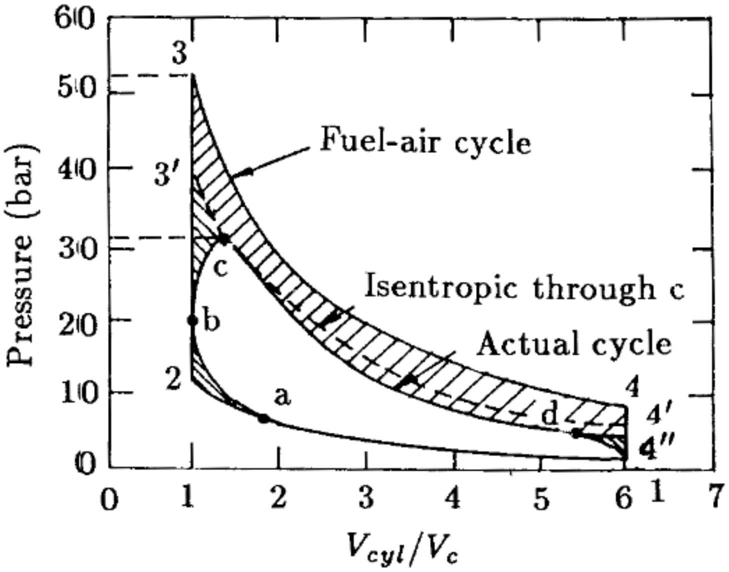


Fig. 5.1 The Effect of Time Losses shown on *p*-*V* Diagram

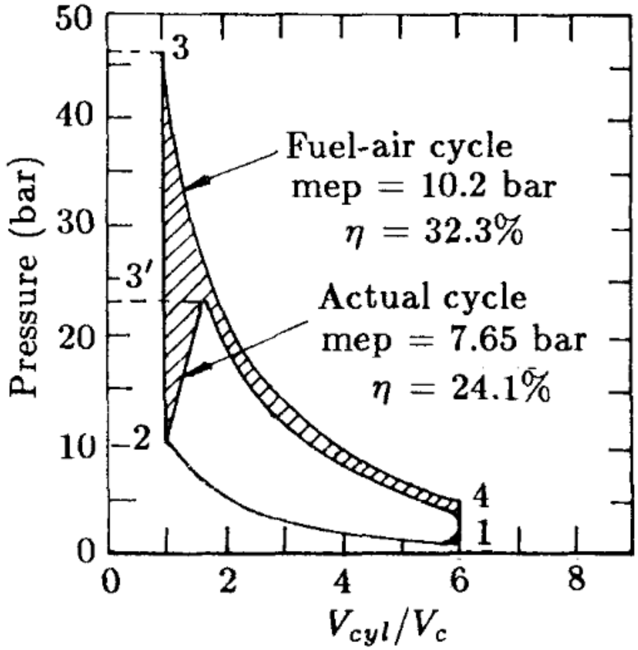
- The time required for the combustion is such that under all circumstances some change in volume takes place while it is in progress. The crankshaft will usually turn about 30 to 40° between the time the spark occurs and the time the charge is completely burnt or when the peak pressure in the cycle is reached.

- The consequence of the finite time of combustion is that the peak pressure will not occur when the volume is minimum i.e., when the piston is at TDC; but will occur some time after TDC. The pressure, therefore, rises in the first part of the working stroke from *b* to *c* as shown in Fig. 5.1.

- The point 3 represents the state of gases had the combustion been instantaneous and an additional amount of work equal to area shown hatched would have been done. This loss of work reduces the efficiency and is called time loss due to progressive combustion or merely time losses.

- The time taken for the burning depends upon the flame velocity which in turn depends upon the:

i) type of fuel
ii) the fuel-air ratio
iii) the shape and size of the combustion chamber
iv) the distance from the point of ignition to the opposite side of the combustion space.



v) **Fig. 5.2** Spark at TDC, Advance 0°

- In order that the peak pressure is not reached too late in the expansion stroke, the time at which the combustion starts is varied by varying the spark timing or spark advance. Figures 5.2 and 5.3 show the effect of spark timing on *p* - *V* diagram from a typical trial.
- With spark at TDC (Fig.5.2) the peak pressure is low due to the expansion of gases. If the spark is advanced to achieve complete combustion close to TDC (Fig.5.3), a direct loss, which is caused by additional work that is required to compress the burning gases.

- Therefore, a moderate or optimum spark advance (Fig.5.4) is the best compromise resulting in minimum losses on both the compression and expansion strokes.

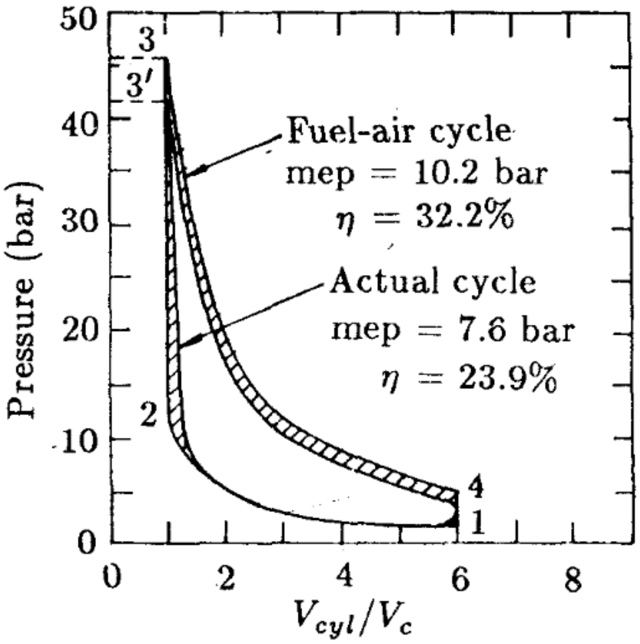


Fig. 5.3 Combustion Completed at TDC, Advance 35°

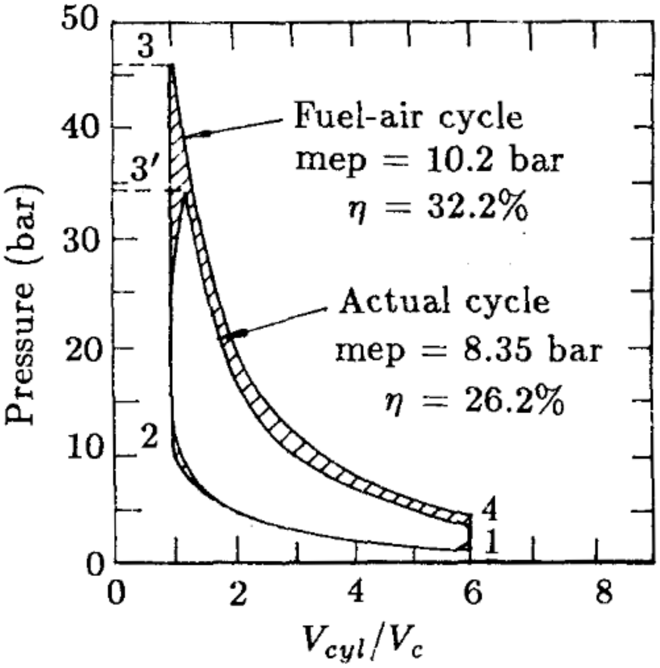


Fig. 5.4 Optimum Advance 15°- 30°

- Table 5.1 compares the engine performance for various ignition timings.

Table 5.1 Cycle Performance for Various Ignition Timings for $r = 6$

Cycle	Ignition advance	Max. cycle pressure bar	*mep* bar	efficiency %	$\frac{\text{Actual } \eta}{\text{Fuel cycle } \eta}$
Fuel-air cycle	0°	44	10.20	32.2	1.00
Actual cycle	0°	23	7.50	24.1	0.75
"	17°	34	8.35	26.3	0.81
"	35°	41	7.60	23.9	0.74

- Figure 5.5 shows the effect of spark advance on the power output by means of the p - V diagram.

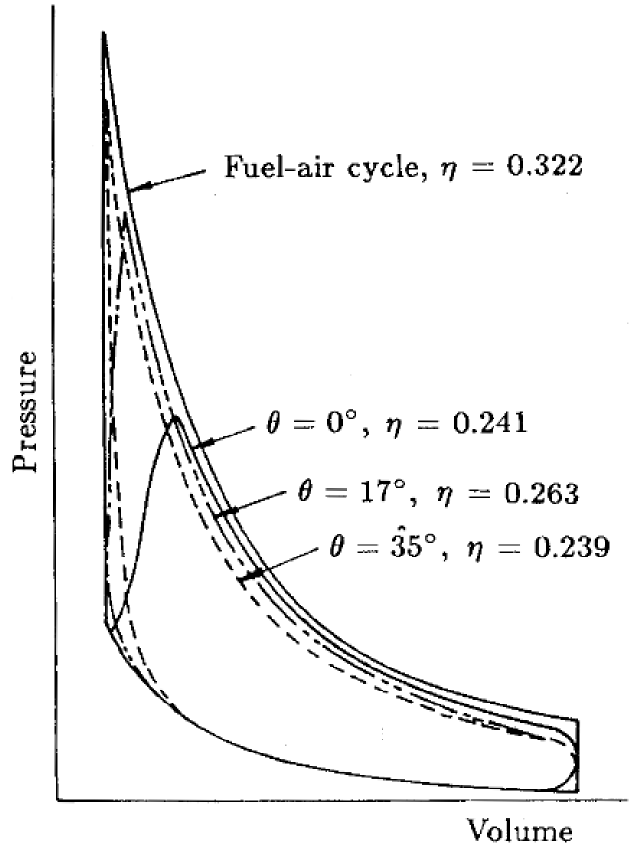


Fig. 5.5 *p-V* Diagram showing Power Loss due to Ignition Advance

- As seen from Fig.5.6, when the ignition advance is increased there is a drastic reduction in the *imep* and the consequent loss of power however some times a deliberate spark retardation from optimum may be necessary in actual practice in order to simultaneously reduce exhaust emissions of hydrocarbons and CO.

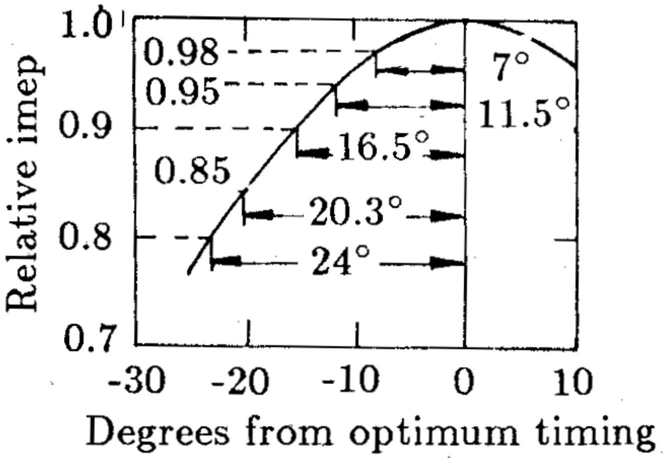


Fig. 5.6 Power Loss due to Ignition Advance

- It is impossible to obtain a perfect homogeneous mixture with fuel-vapour and air, since:

i) residual gases from the previous cycle are present in the clearance volume of the cylinder.

ii) only every limited time is available between the mixture preparation and ignition.

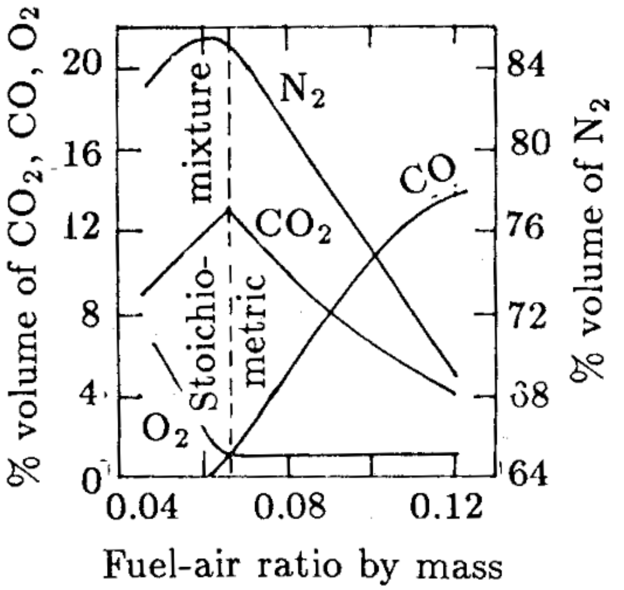


Fig. 5.7 The Comparison of Exhaust Gases for Various Fuel-Air Ratios

- Under these circumstances, it is possible that a pocket of excess oxygen is present in one part of the cylinder and a pocket of excess fuel in another part.

- Therefore, some fuel does not burn or burns partially to CO and the unused 0_2 appears in the exhaust as shown in Fig.5.7.

- It should be noted that it is necessary to use a lean mixture to eliminate wastage of fuel, while a rich mixture is required to utilize all the oxygen.

- Slightly leaner mixture would give maximum efficiency but too lean a mixture will burn slowly increasing the time losses or will not burn at all causing total wastage of fuel. In a rich mixture a part of the fuel will not get the necessary oxygen and will be completely lost.

- Also the flame speed in mixtures more than 10% richer is low, thereby, increasing the time losses and lowering the efficiency.

- Even if this unused fuel and oxygen eventually combine during the exhaust stroke and burn, the energy which is released at such a late stage cannot be utilized.

- Imperfect mixing of fuel and air may give different fuel-air ratios on the following suction strokes or certain cylinders may get continuously leaner mixtures than others.

5.4 HEAT LOSS FACTOR

- It is caused by

  i) Heat flows from the cylinder gases through the cylinder walls and cylinder head into the water jacket or cooling fins during the combustion process and the subsequent expansion stroke.

  ii) Some heat enters the piston head and flows through the piston rings into the cylinder wall or is carried away by the engine lubricating oil, which splashes on the underside of the piston.

- The heat loss, along with other losses is shown on the *p -V* diagram in Fig. 5.8.

- Heat loss during combustion will naturally have the maximum effect on the cycle efficiency while heat loss just before the end of the expansion stroke can have very little effect because of its contribution to the useful work is very little.

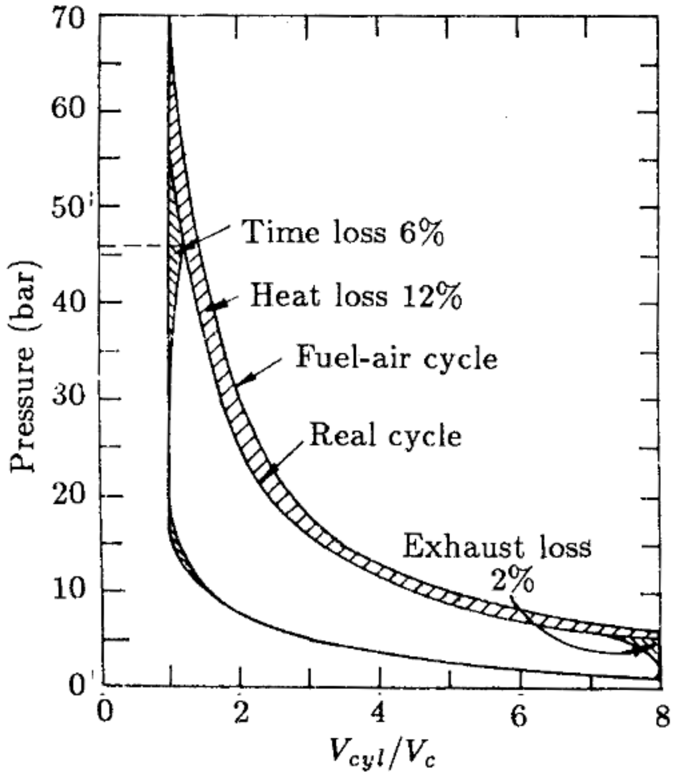


Fig. 5.8 Time Loss, Heat Loss and Exhaust Loss in Petrol Engines

- The heat lost during the combustion does not represent a complete loss because, even under ideal conditions assumed for air-standard cycle, only a part of this heat could be converted into work (equal to Q x η_{th}) and the rest would be rejected during the exhaust stroke.

- About, 15% of the total heat is lost during combustion and expansion. Of this, however, much is lost so late in the cycle to have contributed to useful work. If all the heat, loss is recovered only about 20% of it may appear as useful work.

- The effect of loss of heat during combustion is to reduce the maximum temperature and therefore, the specific heats are lower. It may be noted from the Fig.5.8 that of the various losses, heat loss factor contributes around 12%.

5.5 EXHAUST BLOWDOWN

- If the exhaust valve is opened at the BDC, the piston has to do work against high cylinder pressures during the early part of the exhaust stroke.

If the exhaust valve is opened too early, a part of the expansion stroke is lost.

- The best compromise is to open the exhaust valve 40° to 70° before BDC thereby reducing the cylinder pressure to halfway to atmospheric before the exhaust stroke begins as shown in Fig.5.9 by the roundness at the end of the diagram.

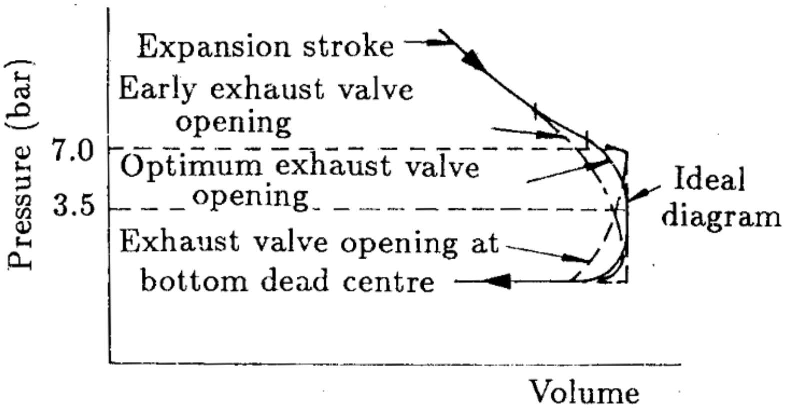


Fig. 5.9 Effect of Exhaust Valve Opening Time on Blowdown

5.5.1 Loss Due to Gas Exchange Processes

- The difference of work done in expelling the exhaust gases and the work done by the fresh charge during the suction stroke is called the pumping work.

- Loss due to the gas exchange process (pumping loss) is due to pumping gas from lower inlet pressure p_i to higher exhaust pressure p_e.

- The pumping loss increases at part throttle because throttling reduces the suction pressure. Pumping loss also increases with speed.

- The gas exchange processes affect the volumetric efficiency of the engine. The performance of the engine, to a great deal, depends on the volumetric efficiency.

5.5.2 Volumetric Efficiency

- Volumetric efficiency is an indication of the breathing ability of the engine and is defined as the ratio of the volume of air actually inducted at ambient condition to swept volume.

- However, it may also be defined on mass basis as the ratio of the actual mass of air drawn into the engine during a given period of time to the theoretical mass which should have been drawn in during that same period of time, based upon the total piston displacement of the engine, and the temperature and pressure of the surrounding atmosphere.

- The above definition is applicable only to the naturally aspirated engine. In the case of the supercharged engine, however, the theoretical mass of air should be calculated at the conditions of pressure and temperature prevailing in the intake manifold.

- The volumetric efficiency is affected by many variables, some of the important ones are:

  i) The density of the fresh charge

     Heat transfer from the hot chamber walls and the hot residual exhaust gases to the fresh charge arrives in the hot cylinder (raising its temperature) results in a decrease in the mass of fresh charge admitted and causes a reduction in volumetric efficiency.

     The volumetric efficiency is increased by low temperatures (provided there are no heat transfer effects) and high pressure of the fresh charge, since density is thereby increased, and more mass of charge can be inducted into a given volume.

  ii) The exhaust gas in the clearance volume

     As the piston moves from TDC to BDC on the intake stroke, the exhaust products tend to expand and occupy a portion of the piston displacement greater than the clearance volume, thus reducing the space available to the incoming charge. In addition, these exhaust products tend to raise the temperature of the fresh charge, thereby decreasing its density and further reducing volumetric efficiency.

  iii) The design of the intake and exhaust manifolds

     The exhaust manifold should be so designed as to enable the exhaust products to escape readily, while the intake manifold should be designed to as to bring in the maximum possible fresh charge. This implies minimum restriction is offered to the fresh charge flowing into the cylinder, as well as to the exhaust products being forced out.

  iv) The timing of the intake and exhaust valves

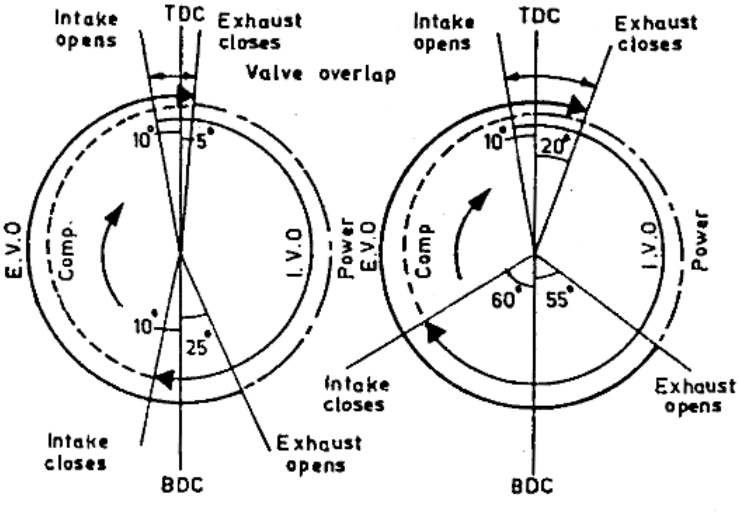
Valve timing is the regulation of the points in the cycle at which the valves are set to open and close. Since, the valves require a finite period of time to open or close for smooth operation, a slight "lead" time is necessary for proper opening and closing. The design of the valve operating cam provides for the smooth transition from one position to the other, while the cam setting determines the timing of the valve.

- The effect of the intake valve timing on the engine air capacity is indicated by its effect on the air inducted per cylinder per cycle, i.e., the mass of air taken into one cylinder during one suction stroke.

- Figure 5.10 shows representative intake valve timing for both a low speed and high speed SI engine.

- The valve timing during intake process;
  i) An intake valve opens a few degrees before TDC on the exhaust stroke to ensure that the valve will be fully open and the fresh charge starts to flow into the cylinder as soon as the piston reaches TDC.

  ii) The intake valve is closed relatively early after BDC for a slow speed engine to avoid the up moving piston on the compression stroke from forcing some of the charge already which is moving relatively slowly and its inertia is relatively low in the cylinder back into the intake manifold which can resulted in the reduction of the volumetric efficiency.



(a) Low Speed Engine (b) High Speed Engine

Fig. 5.10 Valve Timing Diagram of Four-Stroke Engines

iii) In the high speed engine, the intake valve closing is delayed for a greater period of time after BDC in order to take advantage of greater speed and inertia of the charge through the intake manifold and the "ram" effect produced by the incoming mixture which tends to pack more charge into the cylinder s the piston moves up on the compression stroke in order to induct the maximum quantity of charge.

- The exhaust valve timing

  i) The exhaust valve usually opens prior to the time when the piston reaches BDC on the expansion stroke although the work done by the expanding gases during the power stroke is reduced but the work necessary to expel the burned products during the exhaust stroke is decreased and results in an overall gain in output.

  ii) Consequently, the exhaust valve is often set to close a few degrees after TDC on the exhaust stroke. If the closing of the exhaust valve is delayed beyond TDC, the inertia of the high velocity exhaust gases during the exhaust stroke tends to scavenge the cylinder better by carrying out a greater mass of the gas left in the clearance volume, and results in increased volumetric efficiency.

- It should be noted that it quite possible for both the intake and exhaust valves remain open, or partially open, at the same time. This is termed overlap. This overlap must be excessive enough to allow the burned gases to be sucked into the intake manifold, or the fresh charge to escape through the exhaust valve.

- The valve cannot be lifted instantaneously to a desired height, but must be opened gradually in order to avoid the cam follower from losing contact with the cam and then forced back to close contact by valve spring, resulting in a bow against the cam when there is sudden change of acceleration.

5.6 LOSS DUE TO RUBBING FRICTION

- These losses are due to

  i) friction between the piston and the cylinder walls
  ii) friction in various bearings
  iii) the energy spent in operating the auxiliary equipment such as cooling water pump, ignition system, fan, etc.

- These frictions increase rapidly with engine speed. However the piston ring friction also increases to a small extent with increase in mean effective pressure.

- The efficiency of an engine is maximum at full load and decreases at part loads because the percentage of direct heat loss, pumping loss and rubbing friction loss increase at part loads.

5.7 ACTUAL AND FUEL-AIR CYCLES OF CI ENGINES

- In the diesel cycle the losses are less than in the Otto cycle.

- The main loss is due to incomplete combustion and is the cause of main difference between fuel-air cycle and actual cycle of a diesel engine.

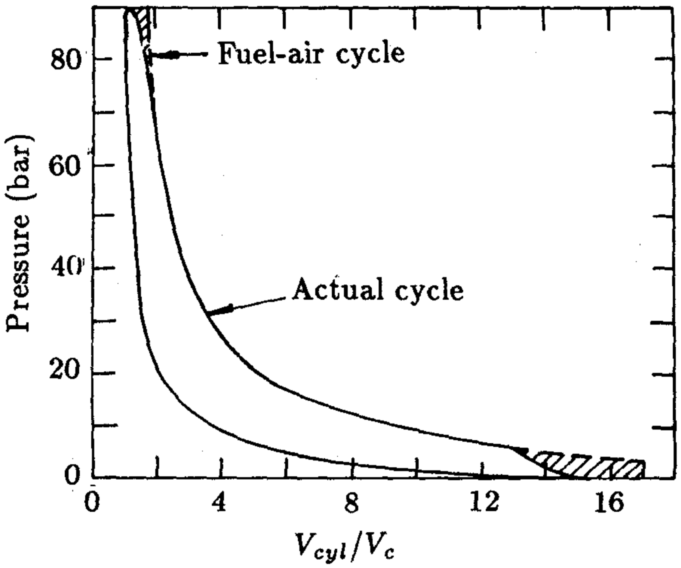


Fig. 5.11 Actual Diesel Cycle Vs. Equivalent Fuel Combustion Limited Pressure Cycle for Two-Stroke Diesel Engine

- In a fuel-air cycle the combustion is supposed to be completed at the end of the constant pressure burning whereas in actual practice after burning continues up to half of the expansion stroke.

- The ratio between the actual efficiency and the fuel-air cycle efficiency is about 0.85 in the diesel engines.

- In fuel-air cycles, when allowance is made for the presence of fuel and combustion products, there is reduction in cycle efficiency.

- In actual cycles, allowances are also made for the losses due to phenomena such as heat transfer and finite combustion time.

- This reduces the cycle efficiency further.

- For complete analysis of actual cycles, computer models are being developed nowadays.