ENGINEERING MECHANICS

LECTURES

FOR

FIRST STAGE

Force Vectors

First we define scalars and vectors:

Scalar: is a quantity that characterized by a positive or negative number. For example: mass, length.

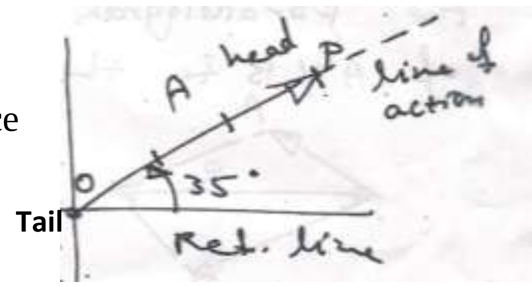
Vector: is a quantity that has both magnitude and direction. For example: force, velocity.

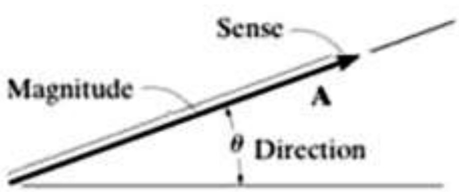
A vector is represented graphically by an arrow. The length of arrow represent the magnitude, and the angle between the arrow line of action and a reference axis represents the direction.

From the figure shown:

The vector A has a magnitude of 3 units and a direction equals 35° measured counterclockwise from the reference line (horizontal here)

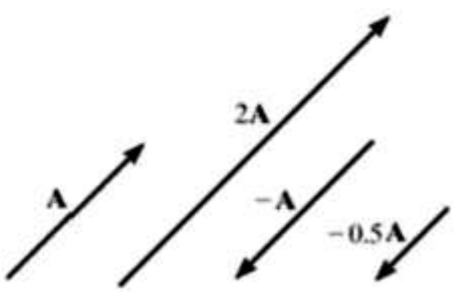
Point O called tail and point P called tip (or head)





Vector operations :-

1. Multiplication of a vector by a scalar. For example

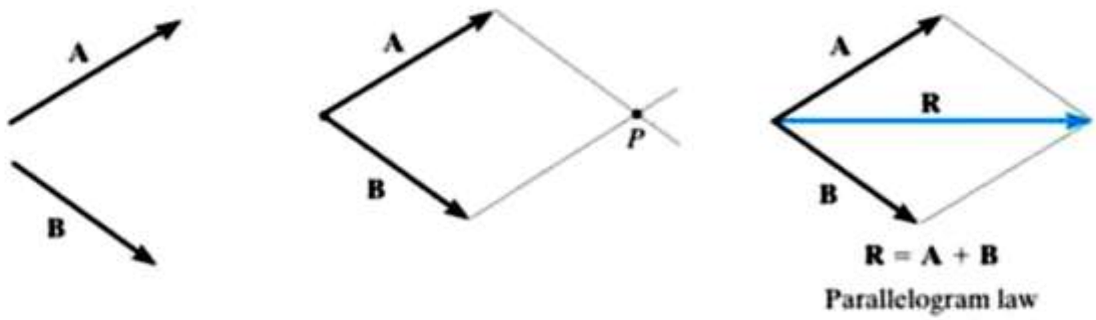


2. Vector addition.

If we have a two vectors A&B. These two vectors can be added to form a resultant vector R = A + B by using the parallelogram law.

To do this A & B are joined together by their tails. Parallel lines drawn from the head of each vector intersect at a common point to form a parallelogram.

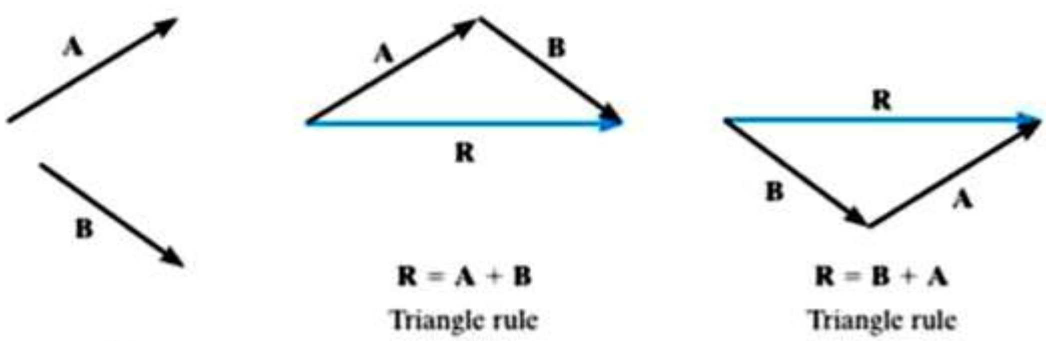
The resultant R is the diagonal of the parallelogram which extends from the tail of A & B to the intersection point.



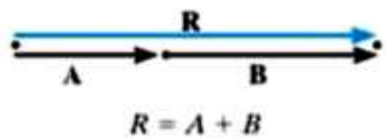
We can also added B to A using the triangle construction which a special case of parallelogram law.

Connect the head of A to the tail of B. The resultant extends from the tail of A to the head of B.

Or, head of A to tail of B.



As a special case, if A & B are collinear (the both have the same line of action), R determined by scalar addition.

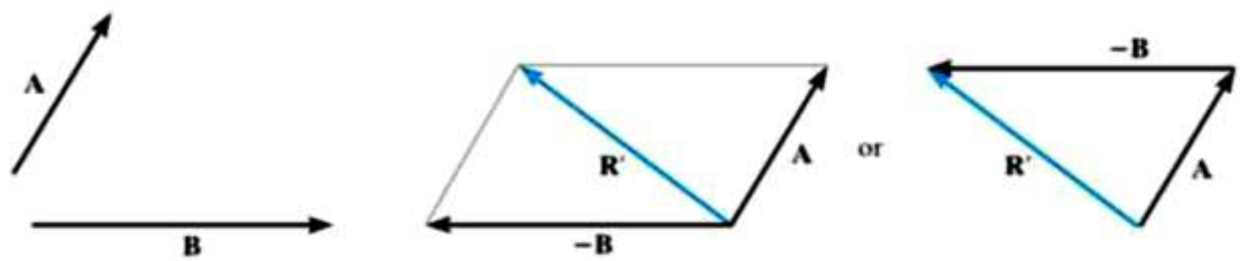


Addition of collinear vectors

3. Vector subtraction :

The resultant difference between A&B may expressed as :

$$\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

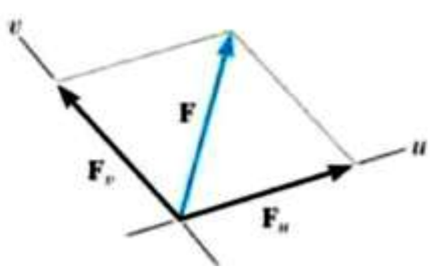


Parallelogram law

Triangle construction

4. Resolution of a vector :

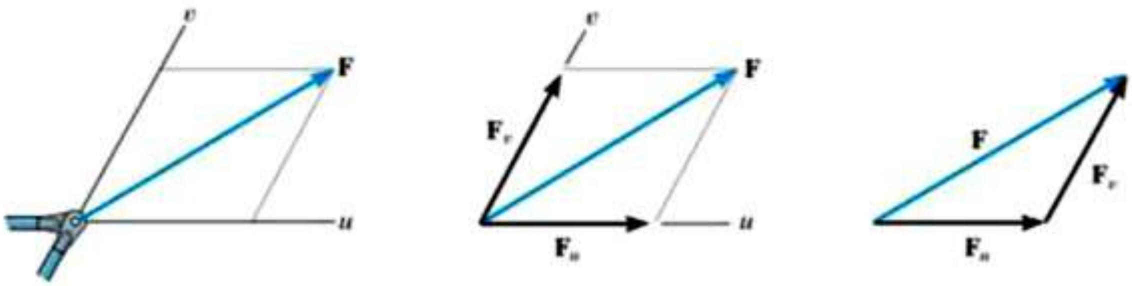
The vector may be resolved into two components having known lines of action by using the parallelogram law. For example: if R is to be resolved into components acting along the lines a & b. Start from the head of R to draw a parallelogram. Then, the components A & B extend from the tail of R to the intersection points.



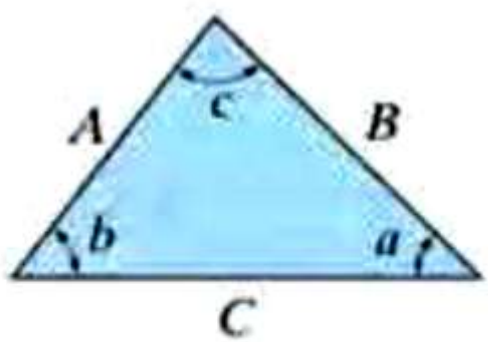
Vector addition of Forces :-

A force is a vector quantity since it has a magnitude and direction.

Therefore, the force addition will be according to parallelogram law.



The sine law and the cosine law may be used.



Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

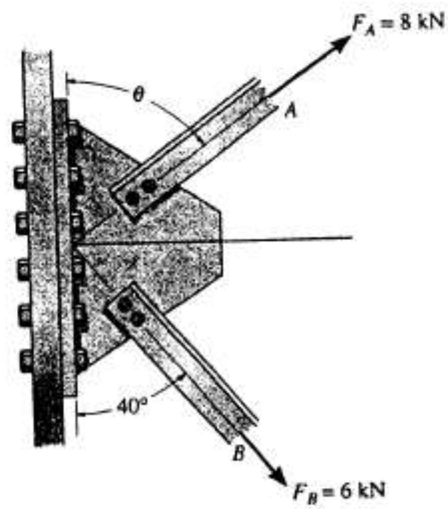
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Examples:-

Ex.1: determine the magnitude of the resultant and the direction measured from the horizontal line.

Sol.:



Parallelogram Law *:* The parallelogram law of addition is shown Fig. (a).

Trigonometry *:* Using law of cosines [Fig. (b)], we have

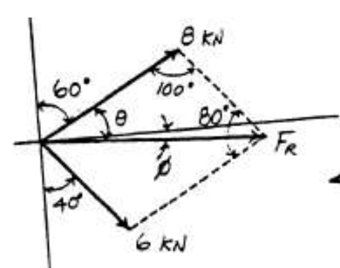
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$
$$= 10.80 \text{ kN} = 10.8 \text{ kN}$$

The angle θ can be determined using law of sines [Fig. (b)].

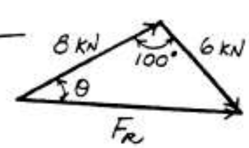
$$\frac{\sin\theta}{6} = \frac{\sin 100^\circ}{10.80}$$
$$\sin\theta = 0.5470$$
$$\theta = 33.16^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ \qquad \textbf{Ans}$$



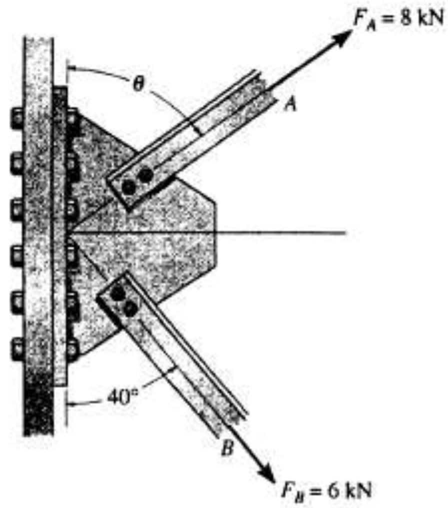
(a)



(b)

Ex.2: determine the angle θ so that the resultant force directed horizontally to the right. Also find magnitude of the resultant.

Sol.:



Parallelogram Law : The parallelogram law of addition is shown in Fig. (a).

Trigonometry : Using law of sines [Fig. (b)], we have

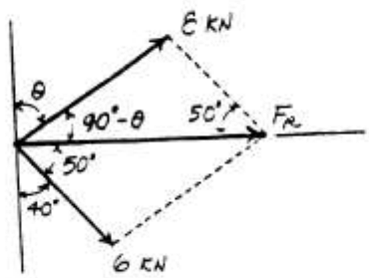
$$\frac{\sin(90° - \theta)}{6} = \frac{\sin 50°}{8}$$

$$\sin(90° - \theta) = 0.5745$$

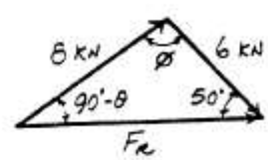
$$\theta = 54.93° = 54.9° \qquad \textbf{Ans}$$

From the triangle, $\phi = 180° - (90° - 54.93°) - 50° = 94.93°$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93°}$$
$$= 10.4 \text{ kN} \qquad \textbf{Ans}$$



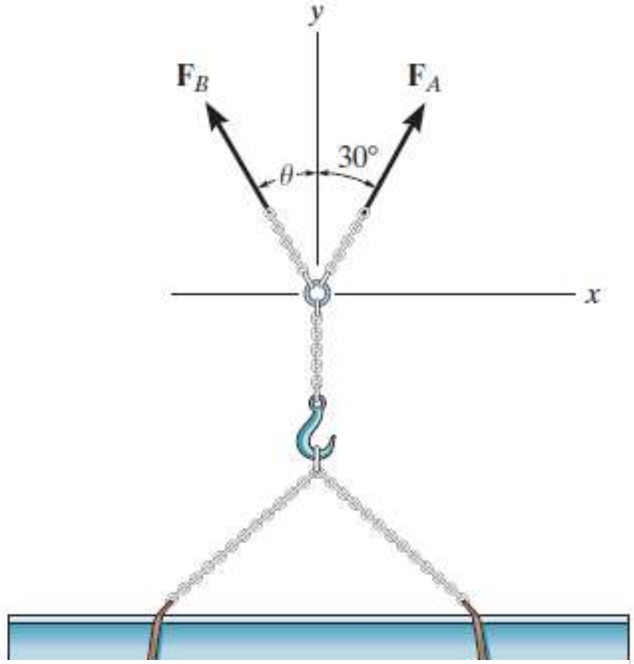
(a)



(b)

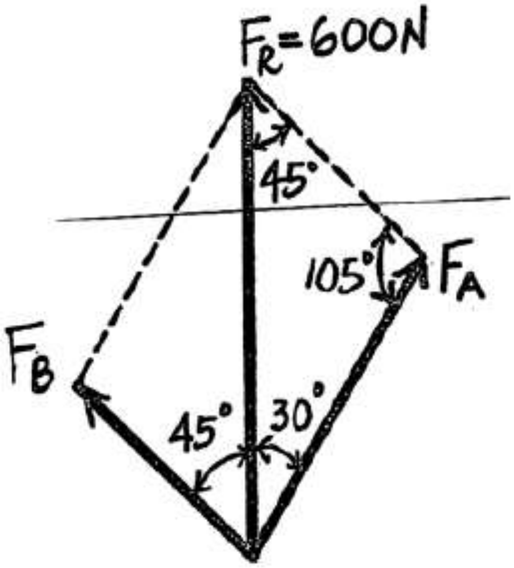
Ex.3: determine the magnitude of forces FA and FB acting on each chain In order to develop a resultant force of 600 N directed along the positive Y-axis.

Sol.:

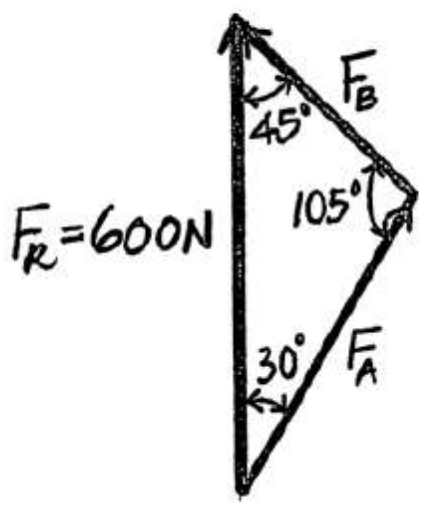


$$\frac{F_A}{\sin 45^\circ} = \frac{600}{\sin 105^\circ}; \qquad F_A = 439 \text{ N} \qquad \text{Ans}$$

$$\frac{F_B}{\sin 30^\circ} = \frac{600}{\sin 105^\circ}; \qquad F_B = 311 \text{ N} \qquad \text{Ans}$$



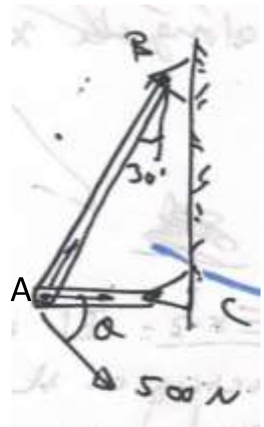
(a)



(b)

Resolution of resultant:

Ex.1: for the frame shown, determine the angle θ so that the horizontal component FAC has a magnitude of 400 N. Also find FAB.



Sol.:

$$\frac{400}{\sin\phi} = \frac{500}{\sin 60}$$

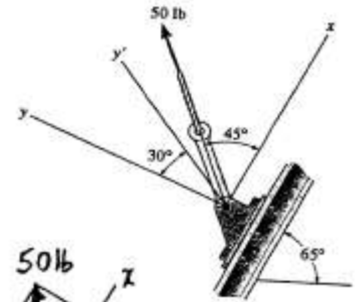
$$\sin\phi = 0.6928$$
$$\Rightarrow \phi = 43.85^\circ$$

$$\Rightarrow \theta = 180 - (60 + 43.85) = 76.15^\circ$$

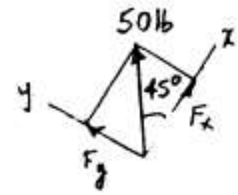
$$\Rightarrow \frac{F_{AB}}{\sin 76.15} = \frac{500}{\sin 60} \Rightarrow F_{AB} = 560.56\ N$$

$$\text{or}\quad F_{AB} = \sqrt{500^2 + 400^2 - 2(500)(400)\cos 76.15} = 56.58\ N$$

Ex.2: Resolve the 50-lb force into components acting along (a) the x and y axes, and (b) the x and y′ axes.



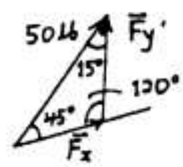
Sol.:



(a) $F_x = 50\cos 45° = 35.4 \text{ lb}$ **Ans**

$F_y = 50\sin 45° = 35.4 \text{ lb}$ **Ans**

(b) $$\frac{F_x}{\sin 15°} = \frac{50}{\sin 120°}$$

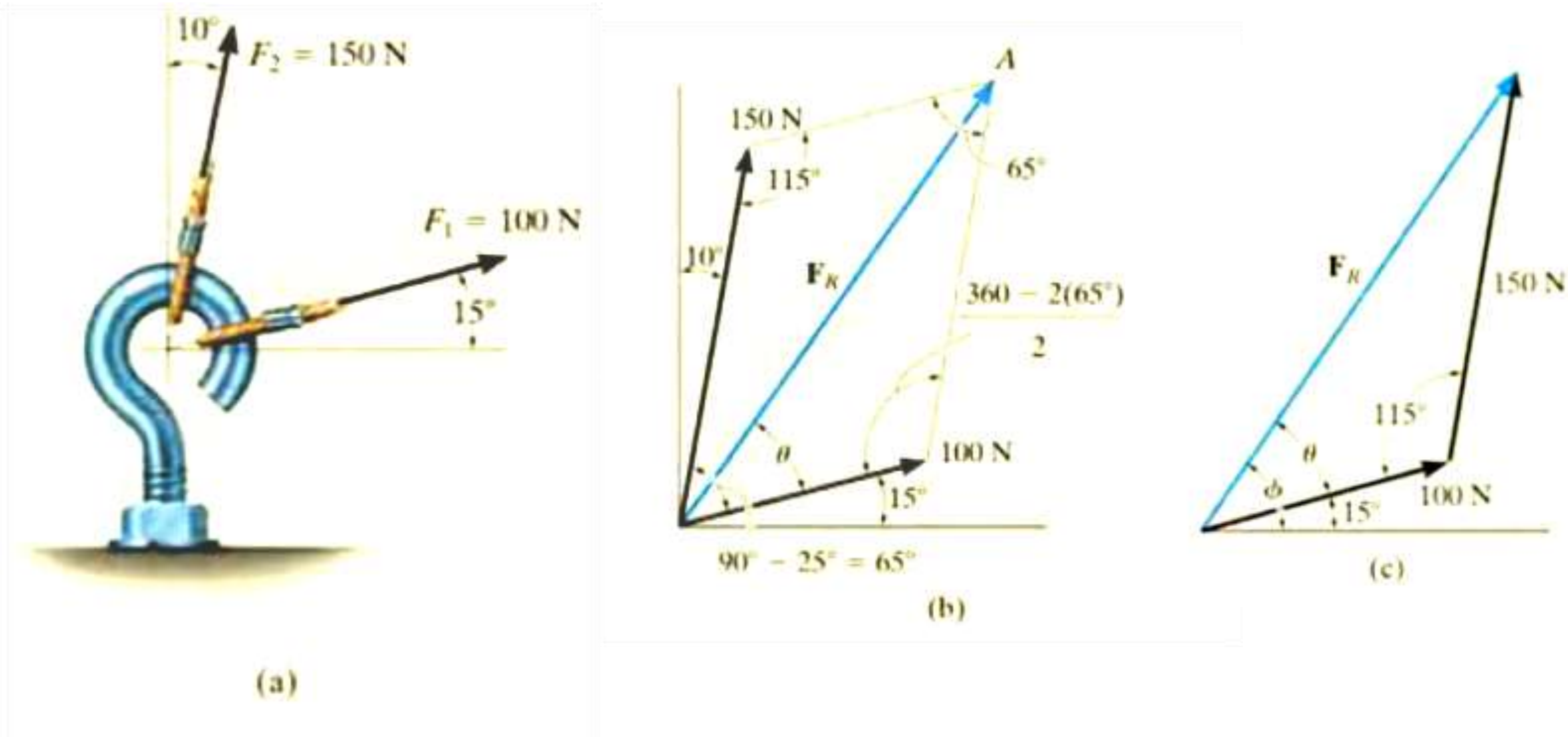


$F_x = 14.9 \text{ lb}$ **Ans**

$$\frac{F_{y'}}{\sin 45°} = \frac{50}{\sin 120°}$$

$F_{y'} = 40.8 \text{ lb}$ **Ans**

<u>Ex.3:-</u> The screw eye shown is subjected to two forces F_1 and F_2. Determine the magnitude and direction of the resultant force .



<u>Sol.:</u>

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$

$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$

$$= 213\text{ N} \qquad \textit{Ans.}$$

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$

$$\theta = 39.8^\circ$$

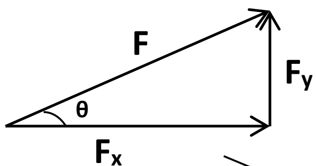
Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

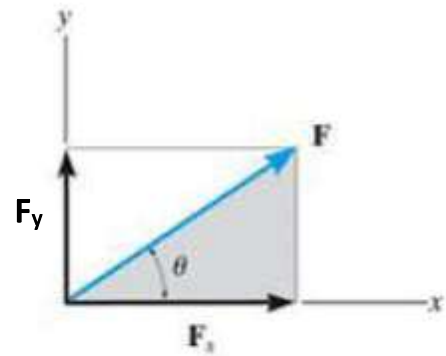
$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ \qquad \textit{Ans.}$$

Resolving a Force into Rectangular Components :-

When a force resolved into perpendicular axes, the resulting components called rectangular components. These components are determined by trigonometry (easier).

For example,

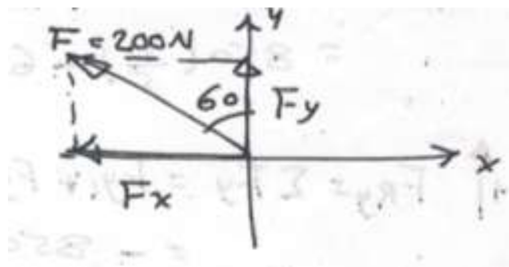




$\cos\theta = F_x / F \implies F_x = F\cos\theta$

$\sin\theta = F_y / F \implies F_y = F\sin\theta$

EX.1: determine the X, Y components of the force F.

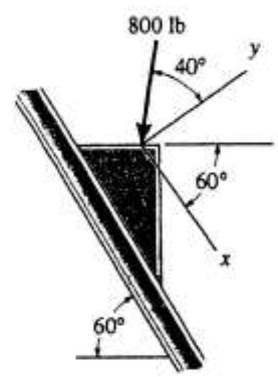


Sol.:

$F_x = 200\sin 60 = 173.2\ N \leftarrow$

$F_y = 200\cos 60 = 100\ N \uparrow$

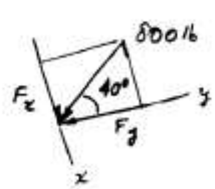
EX.2: Determine the X and Y components of the 800-lb force.



Sol.:

$F_x = 800\sin 40° = 514\ \text{lb}$ **Ans**

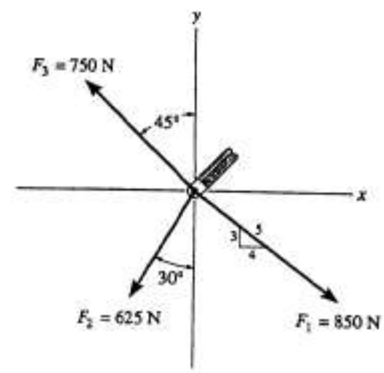
$F_y = -800\cos 40° = -613\ \text{lb}$ **Ans**



Addition of System of Coplanar Forces:-

When a resultant of more than two forces has to be obtained, it is easier to find the components of each force along specified axes, add these components algebraically, and then form the resultant.

EX.1: Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive X axis.



Sol.:

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = \frac{4}{5}(850) - 625\sin 30^\circ - 750\sin 45^\circ = -162.8 \text{ N}$$

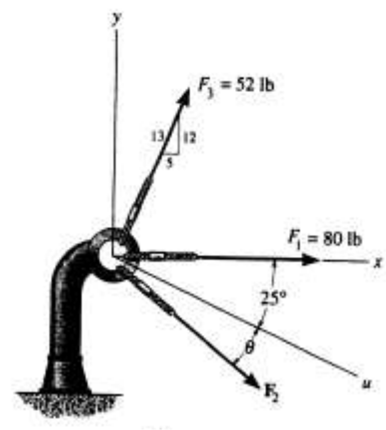
$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -\frac{3}{5}(850) - 625\cos 30^\circ + 750\cos 45^\circ = -520.9 \text{ N}$$

$$F_R = \sqrt{(-162.8)^2 + (-520.9)^2} = 546 \text{ N} \qquad \textbf{Ans}$$

$$\phi = \tan^{-1}\left[\frac{-520.9}{-162.8}\right] = 72.64^\circ$$

$$\theta = 180^\circ + 72.64^\circ = 253^\circ \qquad \textbf{Ans}$$

EX.2: If F_2 =150lb and θ=55°, Determine the magnitude and orientation, measured clockwise from the positive x-axis, of the resultant force of the three forces acting on the bracket.



Sol.:

Scalar Notation : Suming the force components **algebraically**, **we have**

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 80 + 52\left(\frac{5}{13}\right) + 150\cos 80^\circ = 126.05 \text{ lb} \rightarrow$$

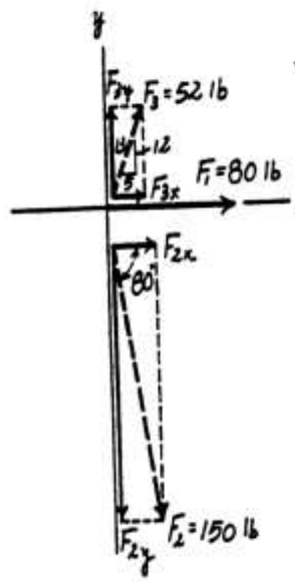
$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 52\left(\frac{12}{13}\right) - 150\sin 80^\circ = -99.72 \text{ lb} = 99.72 \text{ lb} \downarrow$$

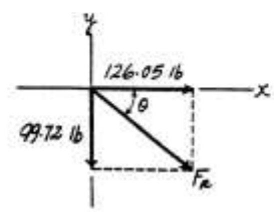
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{126.05^2 + 99.72^2} = 161 \text{ lb} \qquad \textbf{Ans}$$

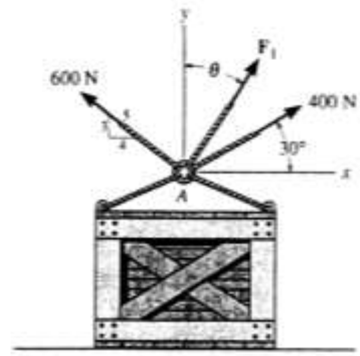
The directional angle θ measured clockwise from positive x axis is

$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{99.72}{126.05}\right) = 38.3^\circ \qquad \textbf{Ans}$$





EX.3: Determine the magnitude and direction θ of F_1 so that the resultant force is directed vertically upward and has a magnitude of 800 N.



Sol.:

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 0 = F_1 \sin\theta + 400\cos 30° - 600\left(\frac{4}{5}\right)$$

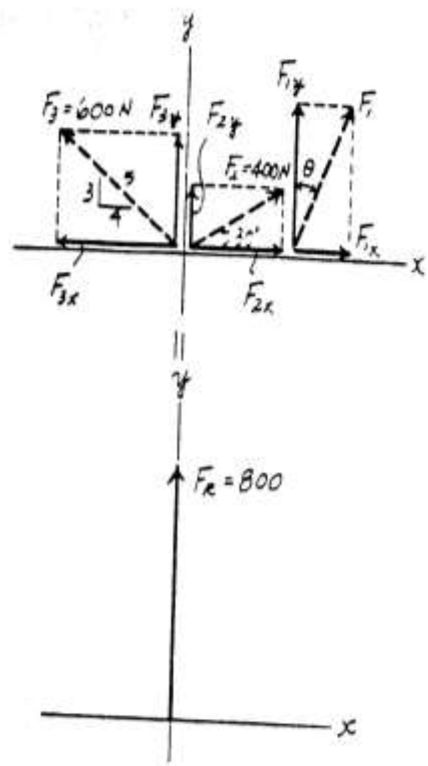
$$F_1 \sin\theta = 133.6 \qquad [1]$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 800 = F_1 \cos\theta + 400\sin 30° + 600\left(\frac{3}{5}\right)$$

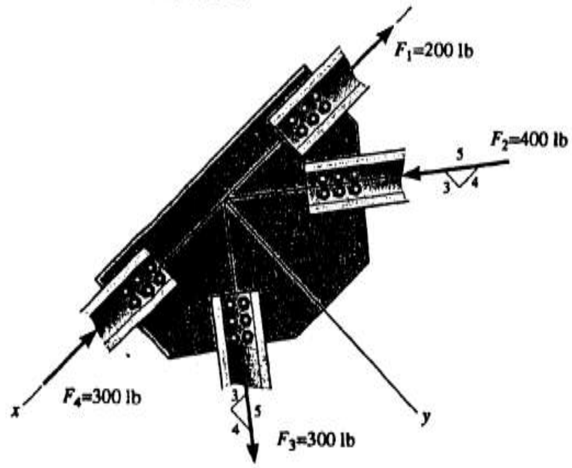
$$F_1 \cos\theta = 240 \qquad [2]$$

Solving Eq. [1] and [2] yields

$\theta = 29.1°$ $\qquad F_1 = 275$ N $\qquad$ **Ans**



Ex.4: Determine the X and Y components of force acting on the qusset plate of the bridge truss. Show that the resultant force is zero.



Sol.:

$$F_{1x} = -200 \text{ lb} \qquad \textbf{Ans}$$

$$F_{1y} = 0 \qquad \textbf{Ans}$$

$$F_{2x} = 400\left(\frac{4}{5}\right) = 320 \text{ lb} \qquad \textbf{Ans}$$

$$F_{2y} = -400\left(\frac{3}{5}\right) = -240 \text{ lb} \qquad \textbf{Ans}$$

$$F_{3x} = 300\left(\frac{3}{5}\right) = 180 \text{ lb} \qquad \textbf{Ans}$$

$$F_{3y} = 300\left(\frac{4}{5}\right) = 240 \text{ lb} \qquad \textbf{Ans}$$

$$F_{4x} = -300 \text{ lb} \qquad \textbf{Ans}$$

$$F_{4y} = 0 \qquad \textbf{Ans}$$

$$F_{Rx} = F_{1x} + F_{2x} + F_{3x} + F_{4x}$$

$$F_{Rx} = -200 + 320 + 180 - 300 = 0$$

$$F_{Ry} = F_{1y} + F_{2y} + F_{3y} + F_{4y}$$

$$F_{Ry} = 0 - 240 + 240 + 0 = 0$$

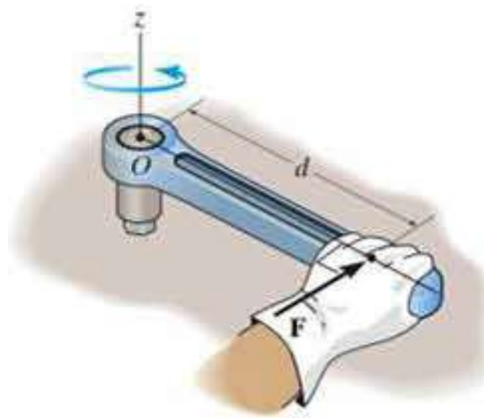
$$\textbf{Thus,} \quad F_R = 0$$

Moment and Couples

(1) Moment of a force:

The moment of a force about a point or axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis.

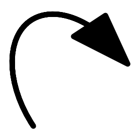
The force F and the point O lie in a plane. The moment about the point O, or about an axis passing through O and perpendicular to the plane is a vector quantity.



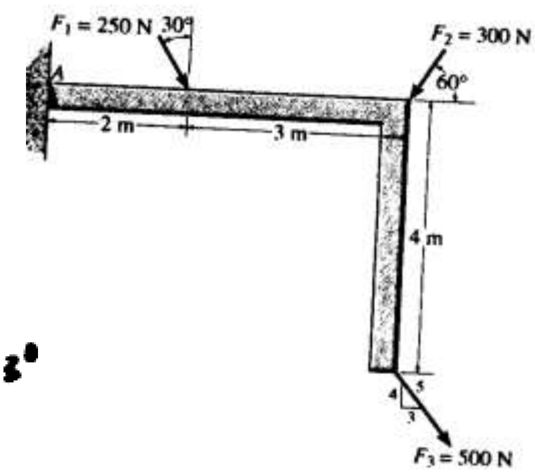
The magnitude of M is : $M = F \cdot d$

Where d is the moment arm or perpendicular distance from point O to the line of action of the force F. Units of moment consist of force times distance, e.g., N.m or lb.ft.

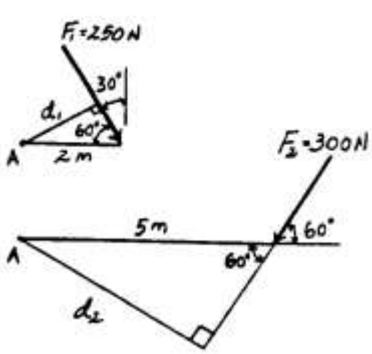
The direction of M is specified by using the right-hand role. The fingers of the right hand is followed the rotation. The thumb then points along the moment axis to give the direction of the moment vector.

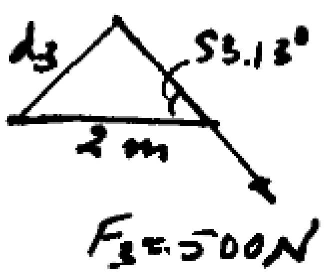
Here the sense of rotation  represents the direction of moment.

<u>Ex.:</u> Determine the moment of each of the three forces about point A.



<u>Sol.:</u>





The moment arm measured perpendicular to each force from point A is

$$d_1 = 2\sin 60° = 1.732 \text{ m}$$
$$d_2 = 5\sin 60° = 4.330 \text{ m}$$
$$d_3 = 2\sin 53.13° = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have

$$\circlearrowleft + \quad (M_{F_1})_A = -250(1.732)$$
$$= -433 \text{ N}\cdot\text{m} = 433 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_2})_A = -300(4.330)$$
$$= -1299 \text{ N}\cdot\text{m} = 1.30 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_3})_A = -500(1.60)$$
$$= -800 \text{ N}\cdot\text{m} = 800 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Note : It is easier to use the principle of moments, that is : the moment of a force about a point is equal to the sum of the moments of the forces components about the point. (Varignon' s Theorem)

$$\circlearrowleft + \quad (M_{F_1})_A = -250\cos 30°(2)$$
$$= -433 \text{ N}\cdot\text{m} = 433 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_2})_A = -300\sin 60°(5)$$
$$= -1299 \text{ N}\cdot\text{m} = 1.30 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_3})_A = 500\left(\frac{3}{5}\right)(4) - 500\left(\frac{4}{5}\right)(5)$$
$$= -800 \text{ N}\cdot\text{m} = 800 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Resultant moment of a System of Coplanar Forces :

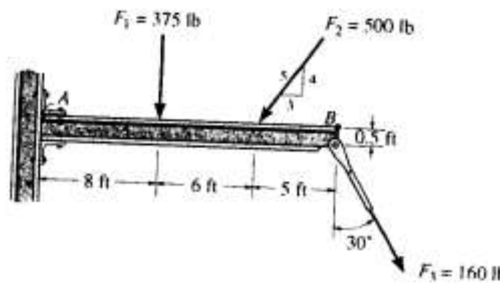
Resultant moment M_R of the system can be determined by adding the moments of all forces algebraically.

Or $\curvearrowright +$ $M_R = \Sigma Fd$

The moment of any force will be positive if is rotate the body clockwise, whereas a negative moment rotate the body counterclockwise.

Examples:

Ex.1 : Determine the moment about point B of each of the three forces acting on the beam.



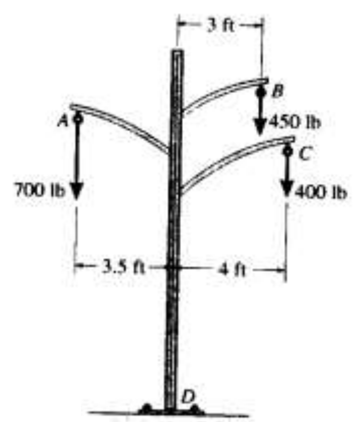
Sol.:

$$\circlearrowleft + \quad (M_{F_1})_B = 375(11)$$
$$= 4125 \text{ lb}\cdot\text{ft} = 4.125 \text{ kip}\cdot\text{ft} \qquad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_2})_B = 500\left(\frac{4}{5}\right)(5)$$
$$= 2000 \text{ lb}\cdot\text{ft} = 2.00 \text{ kip}\cdot\text{ft} \qquad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_3})_B = 160\sin 30^\circ(0.5) - 160\cos 30^\circ(0)$$
$$= 40.0 \text{ lb}\cdot\text{ft} \qquad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

Ex.2 : For the power pole shown, determine the resultant moment about base D. Then determine the resultant moment if line A removed.



Sol.:

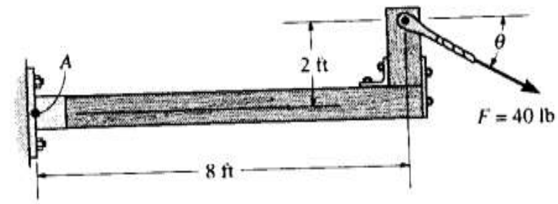
$$\circlearrowleft + \ M_{R_D} = \Sigma Fd; \qquad M_{R_D} = 700(3.5) - 450(3) - 400(4)$$
$$= -500 \text{ lb}\cdot\text{ft} = 500 \text{ lb}\cdot\text{ft} \qquad (\textit{Clockwise}) \qquad \textbf{Ans}$$

When the cable at *A* is removed it will create the greatest moment at point *D*. **Ans**

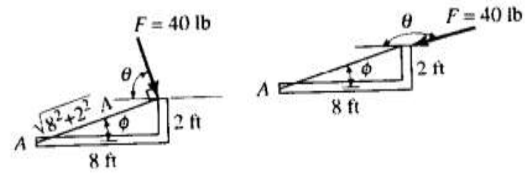
$$\circlearrowleft + \ (M_{R_D})_{max} = \Sigma Fd;$$

$$(M_{R_D})_{max} = -450(3) - 400(4)$$
$$= -2950 \text{ lb}\cdot\text{ft} = 2.95 \text{ kip}\cdot\text{ft} \qquad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Ex.3 : Determine the direction θ ($0^\circ \le \theta \ge 180^\circ$) of the force F = 40 lb so that it produces (a) moment about point A and (b) the minimum moment about point A. Compute the moment in each case.



Sol.:



(a) $\curvearrowleft + (M_A)_{max} = 40(\sqrt{8^2 + 2^2}) = 330 \text{ lb} \cdot \text{ft}$ **Ans**

$$\phi = \tan^{-1}\left(\frac{2}{8}\right) = 14.04^\circ$$

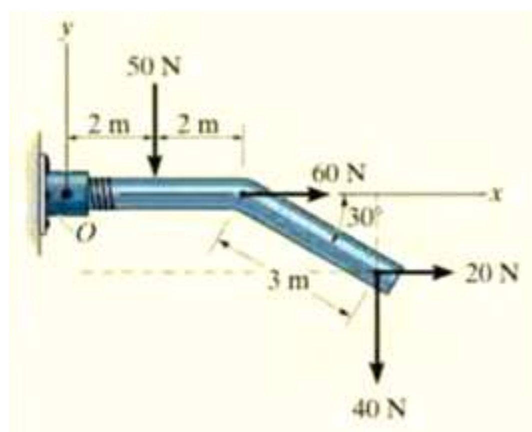
$$\theta = 90^\circ - 14.04^\circ = 76.0^\circ$$ **Ans**

(b) $\curvearrowleft + (M_A)_{min} = 0$ **An**

$$\phi = \tan^{-1}\left(\frac{2}{8}\right) = 14.04^\circ$$

$$\theta = 180^\circ - 14.04^\circ = 166^\circ$$ **An**

Ex.4 : Determine the resultant moment of the forces shown about point O.



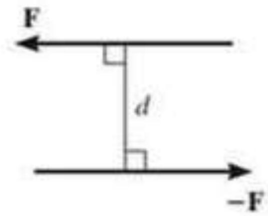
Sol.:

$\zeta + M_{R_O} = \Sigma Fd;$

$$M_{R_O} = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3 \sin 30^\circ\text{ m})$$
$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$
$$M_{R_O} = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m} \circlearrowright$$ *Ans.*

(2) Couples :

A couple is defined as two parallel forces that may have the same magnitude, opposite directions, and are separated by a perpendicular distance.



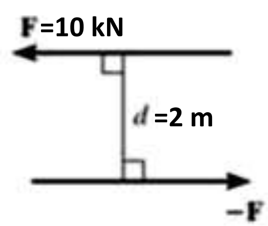
Since the resultant force of the two forces is zero, the only effect of a couple is to produce a rotation.

The moment produced by a couple called a couple moment and is

$$M_C = F \cdot d$$

Where F is a magnitude of one of the forces and d is the perpendicular distance between the two forces.

Ex.1: Determine the magnitude of the couple.

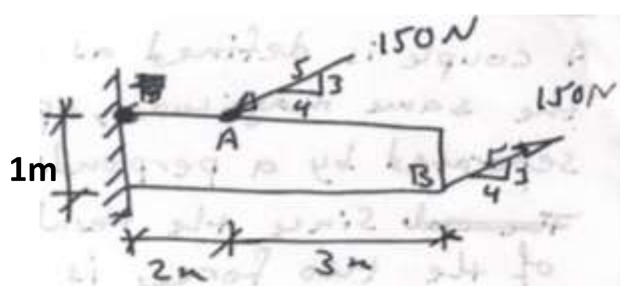


Sol.:

(+↻) $M_C = F \cdot d$

$= 10 * 2 = 20$ kN.m ↻

Ex.2: Determine the magnitude of the couple.



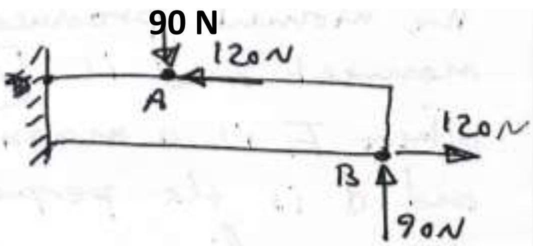
Sol.:

It is difficult to find the perpendicular distance between the forces.

Instead we can resolve each force into components and then use varginon's theorem.

$F_x = 150\left(\frac{4}{5}\right) = 120\,N$

$F_y = 150\left(\frac{3}{5}\right) = 90\,N$



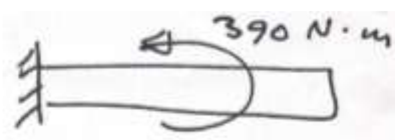
There are two couples:

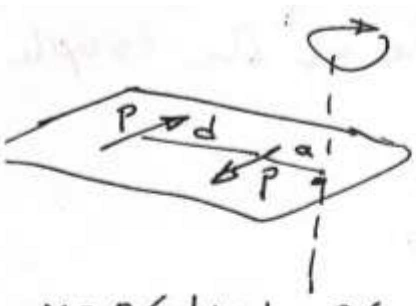
$M_1 = 120(1) = 120\ N\cdot m$ ↺

$M_2 = 90(3) = 270\ N\cdot m$ ↺

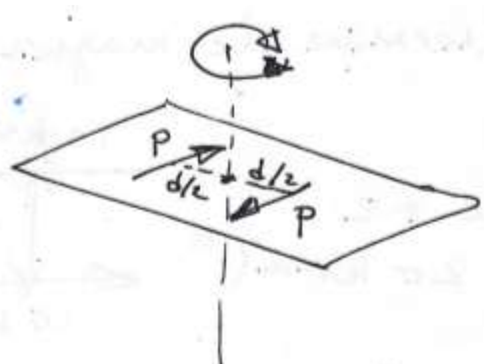
$\Rightarrow M_c = 120 + 270 = 390\ N\cdot m$ ↺

Notice that the couple moment can be act at any point of the member since the M_C is free vector.





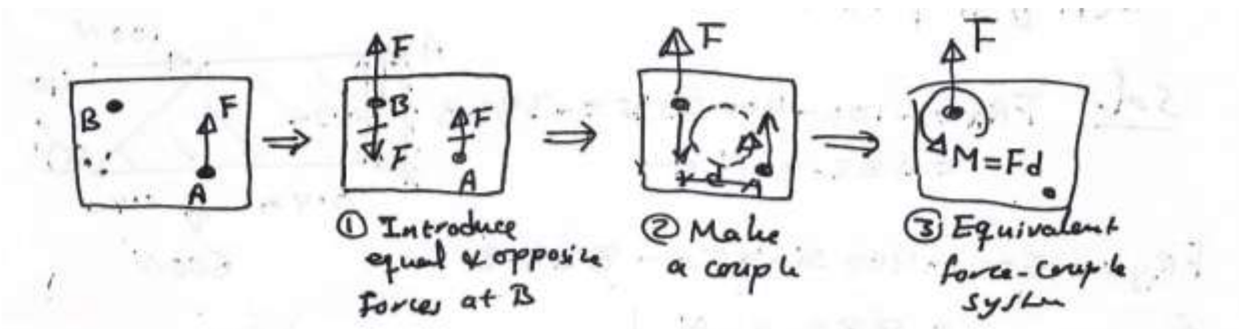
$M = P(d+a) - P(a)$
$= Pd + Pa - Pa$
$= Pd$



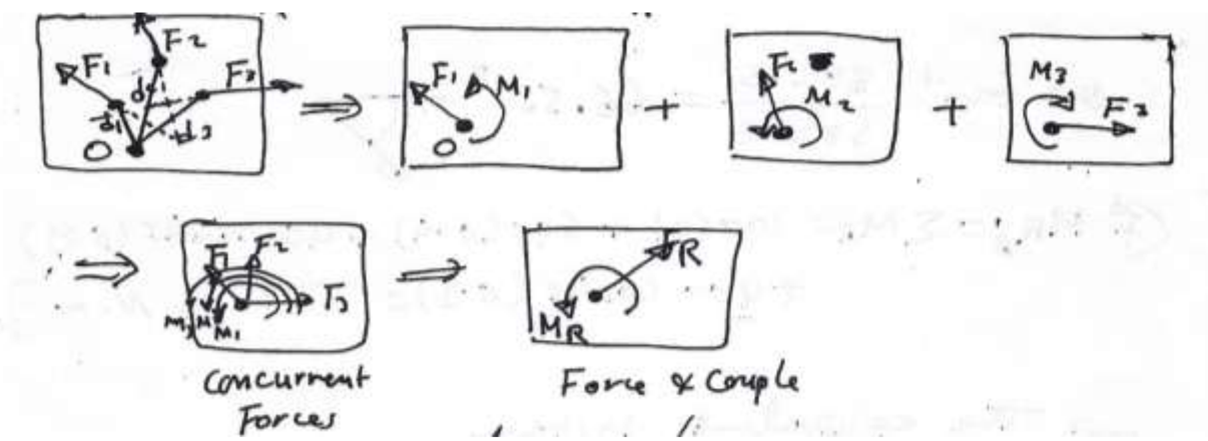
$M = P\left(\frac{d}{2}\right) + P\left(\frac{d}{2}\right) = Pd$

Using of couples in statics:

(1) Changing the line of action of aforce :
We want to move the force F from the point A to B.

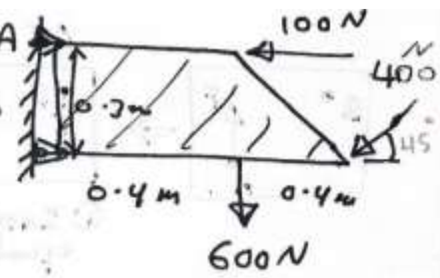


(2) Reduction of a force system to a force and a couple;
We want to reduce the forces F_1, F_2, and F_3 to a force and couple M_R at O.



Examples:

Ex.1: Replace the forces acting on the brace by an equivalent resultant force and couple moment acting at point A.



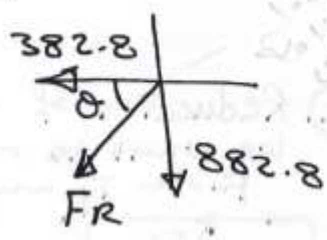
Sol.

$F_{R_x} = -100 - 400\cos 45 = -382.8$

$= 382.8 \text{ N} \leftarrow$

$F_{R_y} = -600 - 400\sin 45 = -882.8$

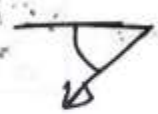
$= 882.8 \text{ N} \downarrow$



$\Rightarrow F_R = \sqrt{(382.8)^2 + (882.8)^2}$

$= 962.2 \text{ N}$

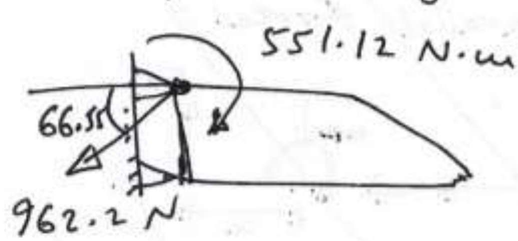
$\theta = \tan^{-1}\dfrac{882.8}{382.8} = 66.55°$



$\circlearrowleft + \; M_{R_A} = \Sigma M_A = 100(0) + 600(0.4) + 400\sin 45(0.8)$

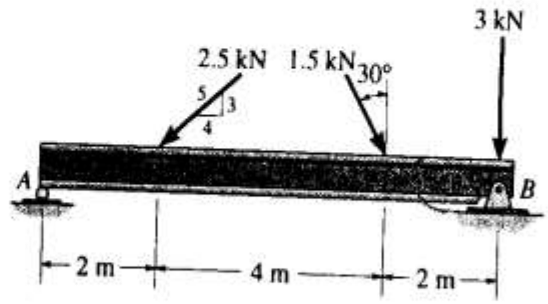
$+ 400\cos 45(0.3) = 551.12 \text{ N.m} \circlearrowright$

$\Rightarrow$ The equivalent system:



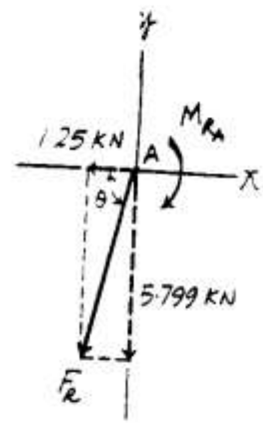
Ex.2: Replace the force system acting on the eam by an equivalent resultant force and couple moment acting at point A.

Sol.:



$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 1.5\sin 30° - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -1.5\cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans}$$

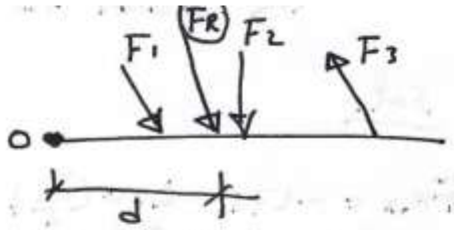
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5\cos 30°(6) - 3(8)$$
$$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Using the moment Concept to find the resultant of Nonconcurrent Forces:

If we have the force system shown:

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$$

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}}$$

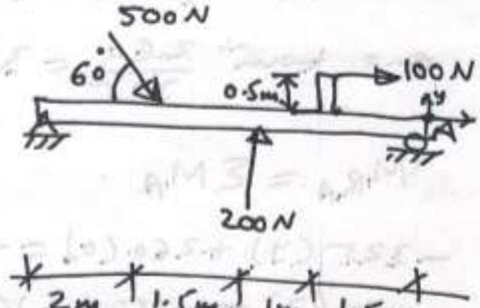


$$F_R \cdot d = F_1 \cdot d_1 + F_2 \cdot d_2 + F_3 \cdot d_3$$ [To find d]

or $\sum M_o = M_{Ro}$

Ex.1: Determine the magnitude, direction, and location of the resultant.

Sol. The origin of the coordinates arbitrary located at point A.



$\rightarrow F_{Rx} = 500 \cos 60 + 100 = 350 N \rightarrow$

$\uparrow F_{Ry} = -500 \sin 60 + 200 = -233 N$
$= 233 N \downarrow$

$$F_R = \sqrt{(350)^2 + (233)^2} = 420.5 \ N$$

$$\theta = \tan^{-1} \frac{233}{350} = 33.7^\circ$$

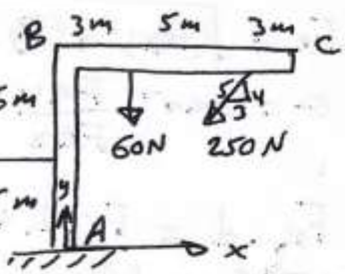
$\circlearrowleft +$ $F_R \cdot d = F_1 \cdot d_1 + F_2 \cdot d_2 + F_3 \cdot d_3$

$$-233(d) + 350(0) = -500 \sin 60 (4) + 500 \cos 60 (0) + 100 (0.5) + 200 (2.5)$$

$$\Rightarrow d = \frac{1182.1}{233} = 5.07 \ m$$

Ex.2: Determine the magnitude of the resultant and specify where its line of action intersects the column AB and the beam BC.

Sol.



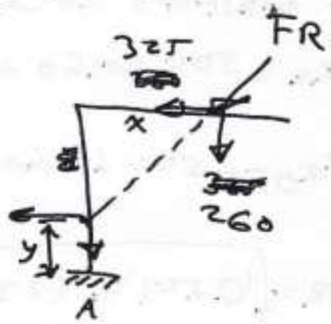
$F_{Rx} = -175 - 250\left(\frac{3}{5}\right)$

$= -325 = 325\ N \leftarrow$

$F_{Ry} = -60 - 250\left(\frac{4}{5}\right) = -260 = 260\ N \downarrow$

$F_R = \sqrt{(325)^2 + (260)^2} = 416\ N$

$\theta = \tan^{-1}\frac{260}{325} = 38.7^\circ$



$M_{R_A} = \Sigma M_A$

$-325(y) + 260(0) = -175(5)$

$+60(3) - 250\cdot\left(\frac{3}{5}\right)(11) + 250\left(\frac{4}{5}\right)(8)$

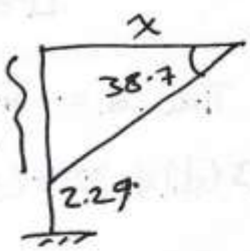
$\Rightarrow y = 2.29\ m$

~~$M_{RA} = \Sigma M_A$~~

$\tan\theta = \frac{8.71}{x}$

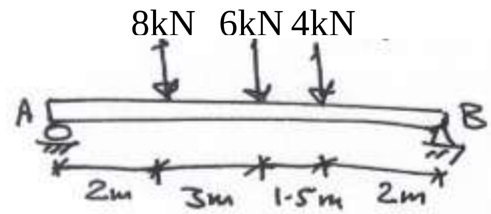
$\Rightarrow x = \frac{8.71}{\tan 38.7} = 10.87\ m$

$11 - 2.29 = 8.71$



Ex.3: Determine the magnitude, direction, and location of the resultant.

Sol.:

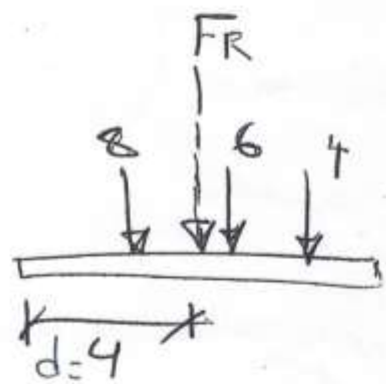


$F_R = \Sigma F_y = 8 + 6 + 4 = 18 \text{ kN} \downarrow$

$\circlearrowleft + \; (M_R)_A = \Sigma M_A$

$18 * d = 8(2) + 6(5) + 4(6.5)$

$\Longrightarrow d = 4 \text{ m}$



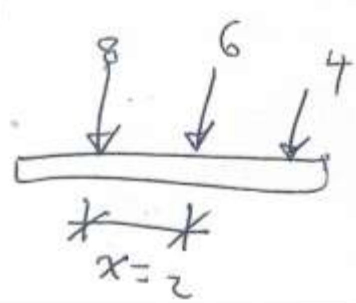
Or

We can find the distance d by taking the moments about any point.

For example: about the force 8 kN.

$18\,x = 6(3) + 4(4.5)$

$\Longrightarrow x = 2$



EQUILIBRIUM

The Concept:

When a system of forces acting on a body has no resultant, the body is in equilibrium.

Newton's first law of motion states that if the resultant force acting on a particle is zero, the particle will remain at rest or move with a constant velocity. This law provides the basis for the equations of equilibrium.

The equations of equilibrium of a rigid body are:

$\xrightarrow{+} \Sigma F_x = 0 \qquad + \uparrow \Sigma F_y = 0 \qquad \Sigma M_O = 0$

Free Body Diagram: (F.B.D):

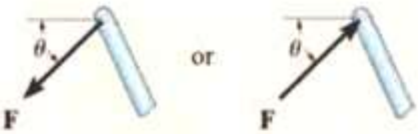
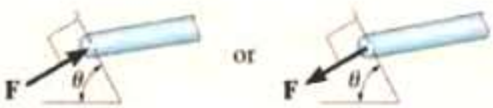
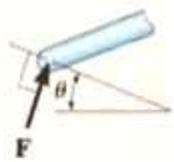
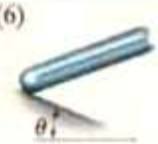
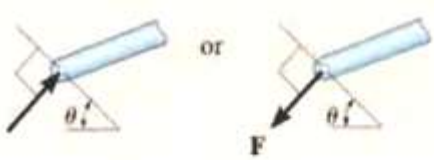
The free body diagram is a sketch of a body or a portion of a body completely issolated (or free) from its surrondings.

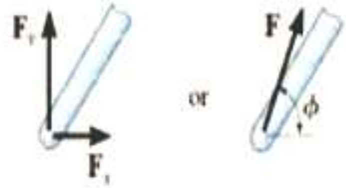
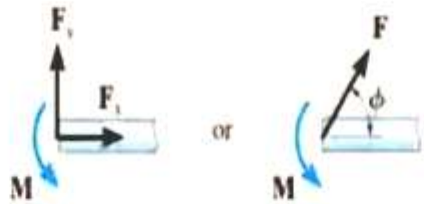
In this sketch, it is necessary to show all the forces and moments that the surroundings exert on the body. By using this diagram, the effect of all applied forces and moments acting on the body can be accounted by the equations of equilibrium.

We need to know the following things before knowing how to draw the F.B.D.

(1) Support Reactions:

There are various types of reactions that occur at supports and points of support between bodies subjected to forces.

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

(2) External and Internal forces:

Since a rigid body is a composition of particles, both external and internal loadings may act on it. Only the external loadinge are represented on the free body diagram because the net effect of the internal forces on the body is zero.

(3) Weight of a Body :

When a body is subjected to a gravitational field, then it has a specified weight. The weight of the body is represented by a resultant force located at the center of gravity of the body.

$$W = m*g \quad \text{where } g = 9.81\ m/s^2$$

Procedure for Analysis

To construct a free-body diagram for a rigid body or any group of bodies considered as a single system, the following steps should be performed:

Draw Outlined Shape.

Imagine the body to be *isolated* or cut "free" from its constraints and connections and draw (sketch) its outlined shape.

Show All Forces and Couple Moments.

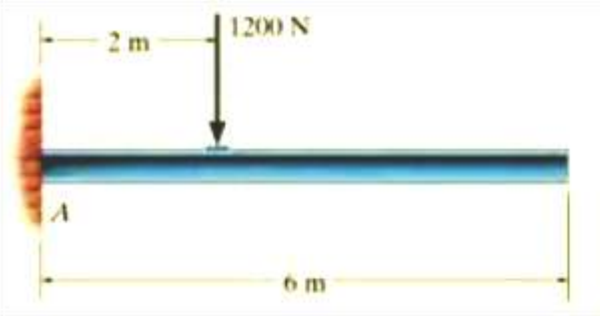
Identify all the known and unknown *external forces* and couple moments that *act on the body*. Those generally encountered are due to (1) applied loadings, (2) reactions occurring at the supports or at points of contact with other bodies (see Table 5-1), and (3) the weight of the body. To account for all these effects, it may help to trace over the boundary, carefully noting each force or couple moment acting on it.

Identify Each Loading and Give Dimensions.

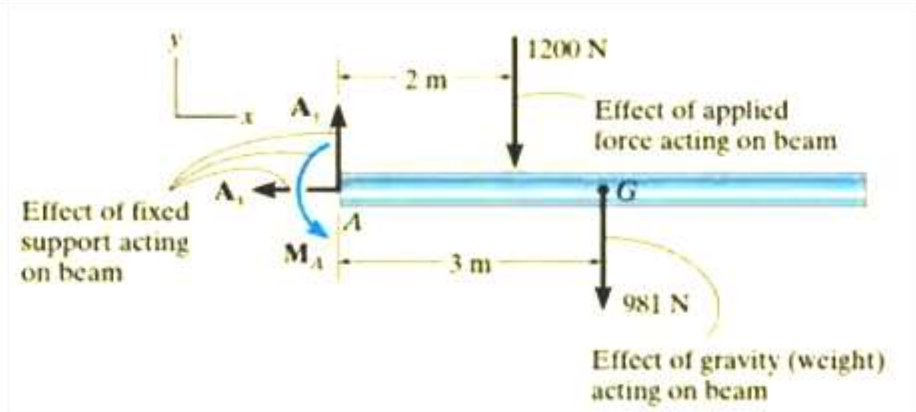
The forces and couple moments that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couple moments that are unknown. Establish an x, y coordinate system so that these unknowns, A_x, A_y, etc., can be identified. Finally, indicate the dimensions of the body necessary for calculating the moments of forces.

Examples:

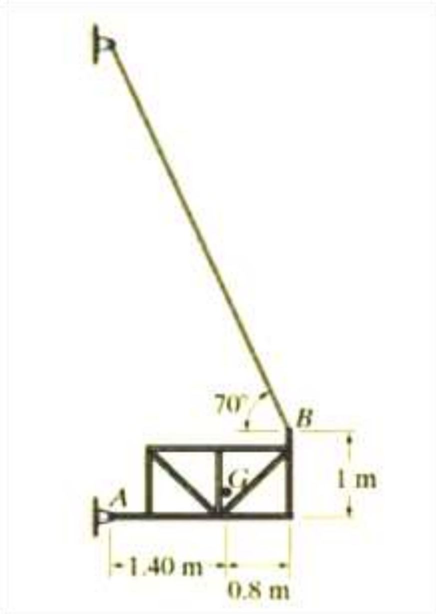
Ex.1: Draw the free-body diagram of the uniform beam shown below. The beam has a mass of 100kg.



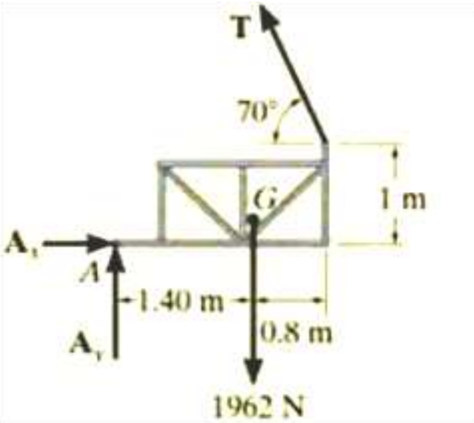
Sol.:



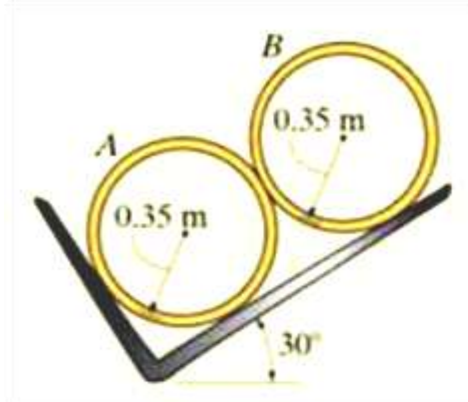
Ex.2: Draw the free-body diagram of the unloaded platform that is suspended off the edge of the oil rig shown. The platform has a mass of 200kg.



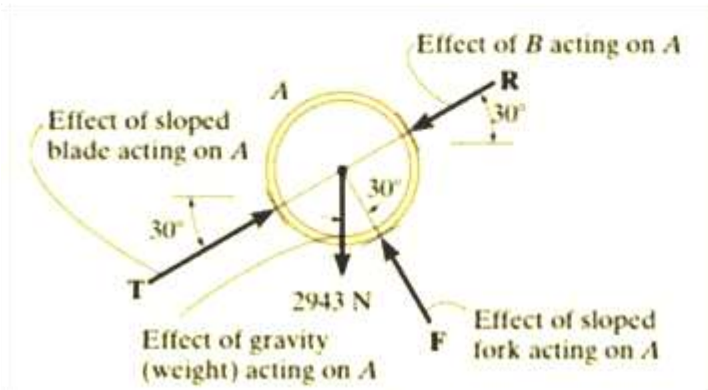
Sol.:

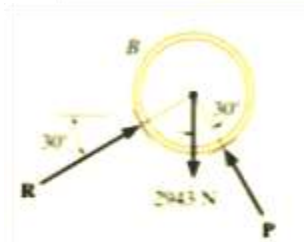


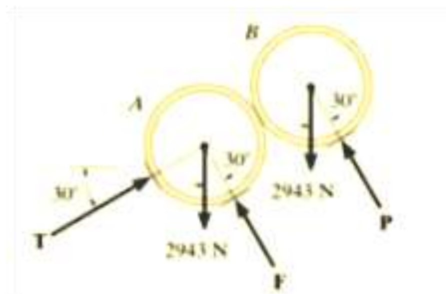
Ex.3: Tow smooth pipes, each having a mass of 300kg, are supported by the forked tines of the tractor as shown. Draw the free-body diagrams for each pipe and both pipes together.



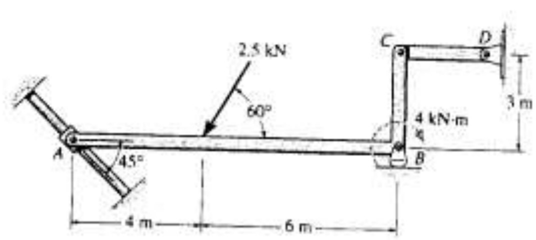
Sol.:







Ex.4: Draw the free-body diagram of the member ABC shown.



Sol.:

Significance of Each Force :

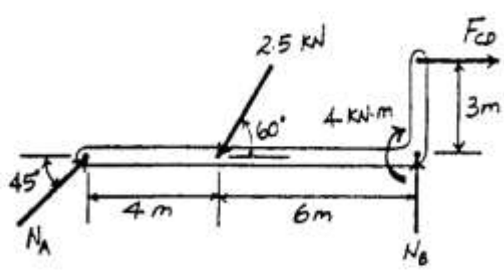
N_A is the smooth collar reaction on member *ABC*.

N_B is the roller support *B* reaction on member *ABC*.

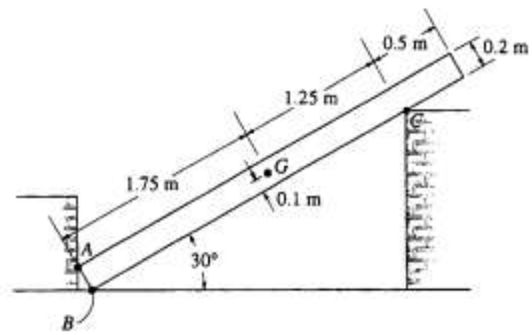
F_{CD} is the short link reaction on member *ABC*.

2.5 kN is the effect of external applied force on member *ABC*.

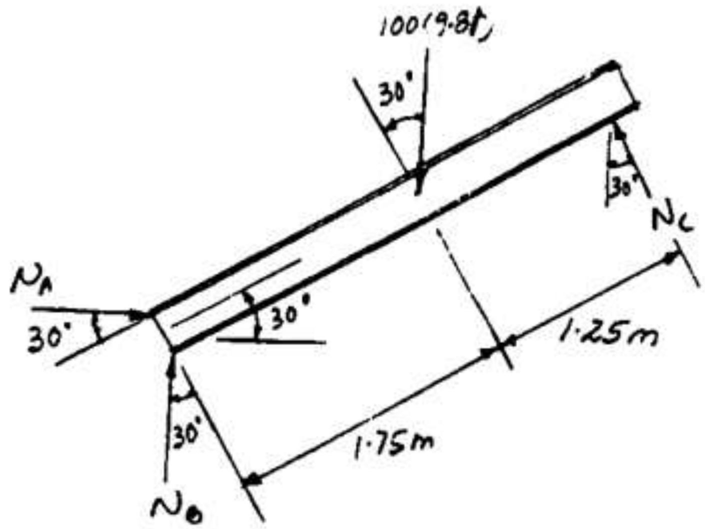
4 kN · m is the effect of external applied couple moment on member *ABC*.



Ex.5: Draw the free-body diagram of the uniform bar which has a mass of 100kg and a center of mass at G.



Sol.:

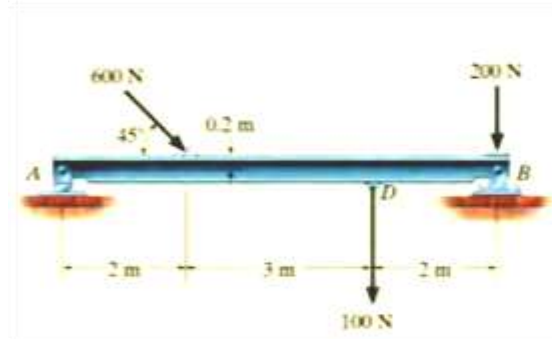


Solving the Equilibrium Problems :

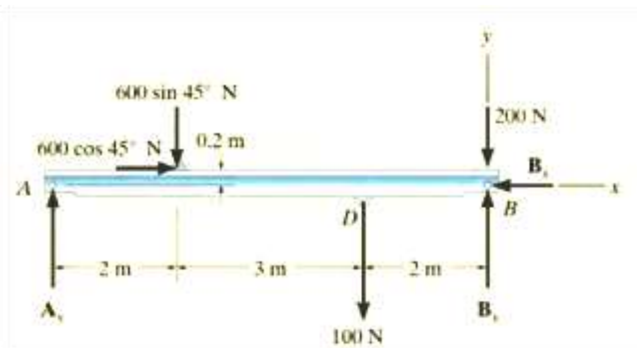
To find the unknowns in the equilibrium equations, the free-body diagram and the equations of equilibrium are used.

Examples:

Ex.1: Determine the horizontal and vertical components on the beam at A and B. Neglect the weight of the beam.



Sol.:



Equations of Equilibrium. Summing forces in the x direction yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

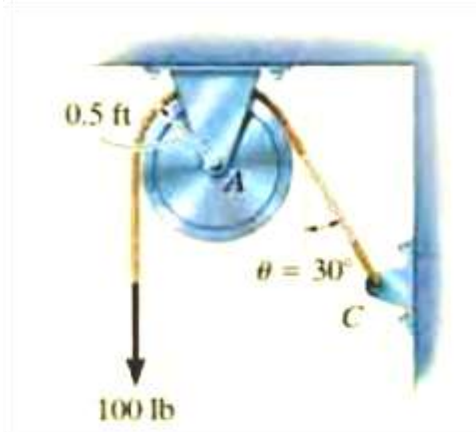
NOTE: We can check this result by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad -(600 \sin 45^\circ \text{ N})(2 \text{ m}) - (600 \cos 45^\circ \text{ N})(0.2 \text{ m})$$

$$-(100 \text{ N})(5 \text{ m}) - (200 \text{ N})(7 \text{ m}) + B_y(7 \text{ m}) = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

Ex.2: The cord shown supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



Sol.:

Equations of Equilibrium. Summing moments about point A to eliminate $\mathbf{A}_x$ and $\mathbf{A}_y$, Fig. 5-13c, we have

$$\circlearrowleft + \Sigma M_A = 0; \qquad 100\text{ lb}\,(0.5\text{ ft}) - T(0.5\text{ ft}) = 0$$

$$T = 100\text{ lb} \qquad \textit{Ans.}$$

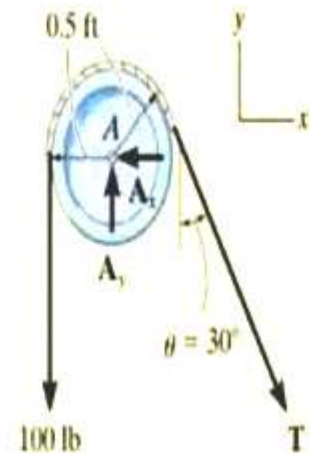
Using the result,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 100 \sin 30^\circ\text{ lb} = 0$$

$$A_x = 50.0\text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 100\text{ lb} - 100 \cos 30^\circ\text{ lb} = 0$$

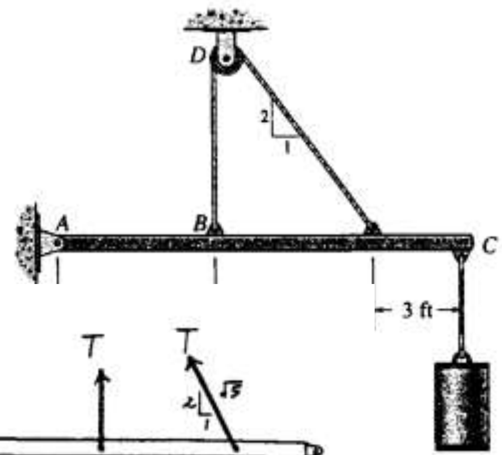
$$A_y = 187\text{ lb} \qquad \textit{Ans.}$$



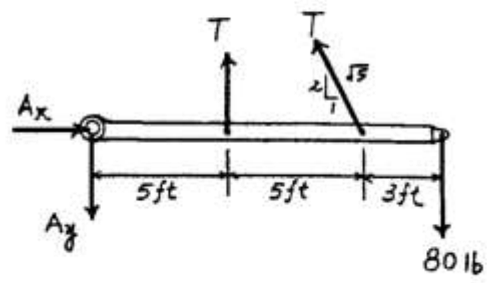
(c)

NOTE: It is seen that the tension remains *constant* as the cord passes over the pulley. (This of course is true for *any angle θ* at which the cord is directed and for *any radius r* of the pulley.)

Ex.3: Determine the tension in the cable and the horizontal and vertical components of reaction at pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



Sol.:



$$\circlearrowleft + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

$$T = 74.583 \text{ lb} = 74.6 \text{ lb} \qquad \textbf{Ans}$$

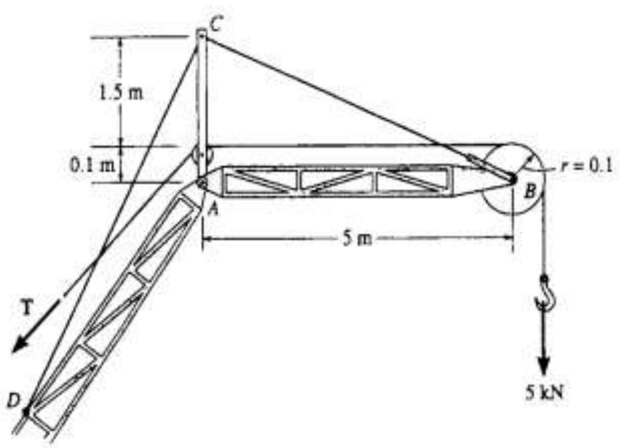
$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$$

$$A_x = 33.4 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$$

$$A_y = 61.3 \text{ lb} \qquad \textbf{Ans}$$

Ex.4: For the upper portion of the crane boom, determine the tension in the cable T, the tension in the cable BC, and the forces at pin A.



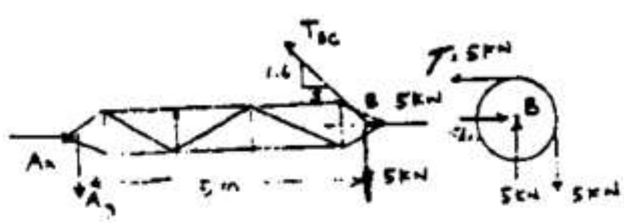
Sol.:

From pulley, tension in the hoist line is

$$\circlearrowleft + \Sigma M_B = 0; \quad T(0.1) - 5(0.1) = 0;$$

$$T = 5 \text{ kN} \qquad \textbf{Ans}$$

From the jib,



$$\circlearrowleft + \Sigma M_A = 0; \quad -5(5) + T_{BC}\left(\frac{1.6}{\sqrt{27.56}}\right)(5) = 0$$

$$T_{BC} = 16.4055 = 16.4 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad -A_y + (16.4055)\left(\frac{1.6}{\sqrt{27.56}}\right) - 5 = 0$$

$$A_y = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 16.4055\left(\frac{5}{\sqrt{27.56}}\right) - 5 = 0$$

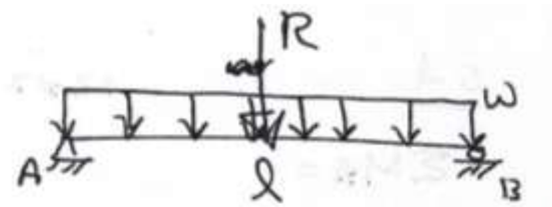
$$F_A = A_x = 20.6 \text{ kN} \qquad \textbf{Ans}$$

The Resultant 0f the Distributed Loadings:

The body may be subjected to distributed loadings such as those caused by wind, fluids, or weight of material over the body's surface.

(1) Uniform Loadings:

The magnitude of the resultant is R = W*L (area under load). The location of R passes through the centroid of the rectangle (middle)



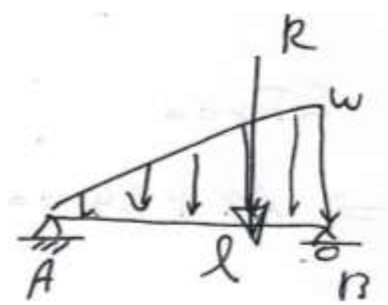
For example : W=5kN/m

L = 6m

R = 5*6= 30 kN at 3m from A

(2) Triangl loading:

R = W*L/2 at a distance of one third the length of triangle measured from the right side.



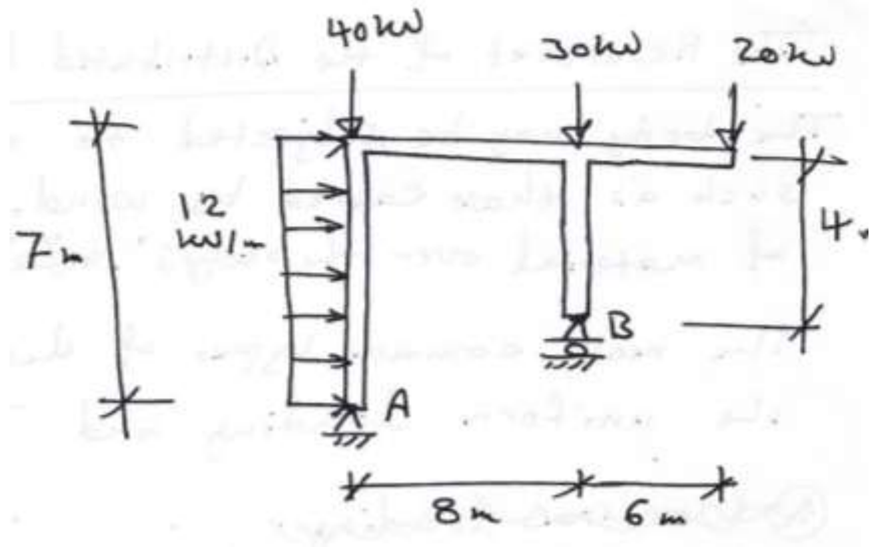
For example : W=5kN/m

L = 6m

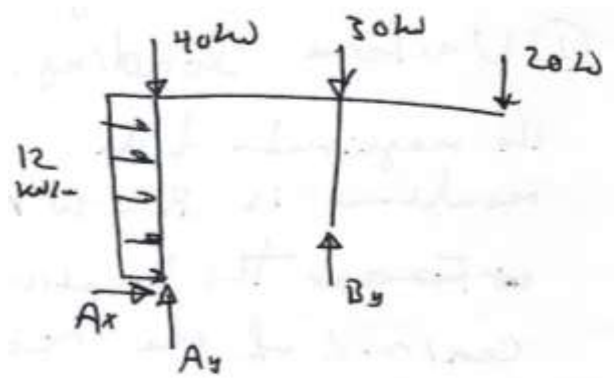
R = 5*6/2= 15 kN at 2m from B

Examples:

Ex.1: Determine the reaction at A and B.



Sol.:



$$\Sigma M_A = 0$$

$$20(14) + 30(8) + 84(3.5) - B_y(8) = 0$$

$$\Rightarrow B_y = 101.75 \text{ kN} \uparrow$$

$$\Sigma F_y = 0$$

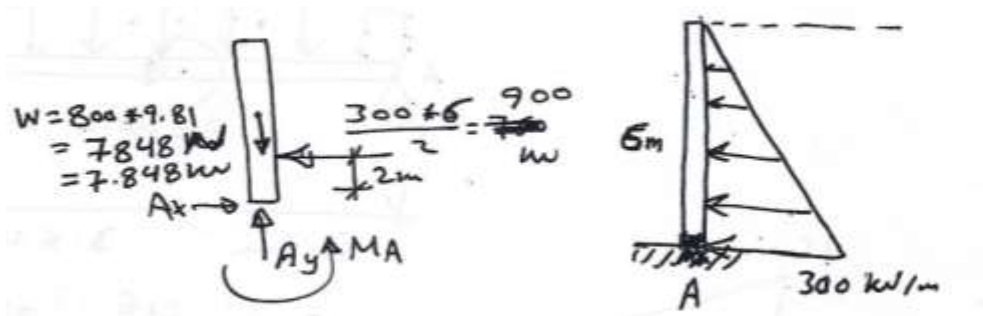
$$A_y + 101.75 - 40 - 30 - 20 = 0 \Rightarrow A_y = -11.75 \text{ kN}$$

$$\Rightarrow A_y = 11.75 \text{ kN} \downarrow$$

$$\Sigma F_x = 0$$

$$\Rightarrow A_x + 84 = 0 \Rightarrow A_x = -84 = 84 \text{ kN} \leftarrow$$

Ex.2: Determine the reactions at the base A for the column shown in the figure below. The mass of the col. Is 800kg.



Sol.:

$\sum F_x = 0$

$A_x - 900 = 0 \Rightarrow A_x = 900 \text{ kN} \rightarrow$

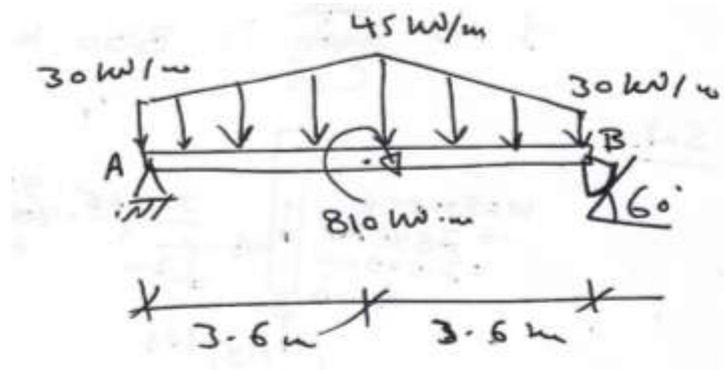
$\sum F_y = 0$

$A_y - 7.848 = 0 \Rightarrow A_y = 7.848 \text{ kN} \uparrow$

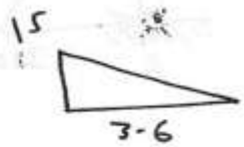
$\sum M_A = 0$

$-M_A - 900(2) = 0 \Rightarrow M_A = -1800 = 1800 \text{ kN.m}$

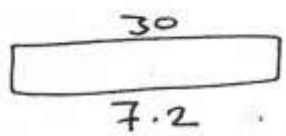
Ex.3: Determine the reactions at the beam.



Sol.:



$\frac{15 * 3.6}{2} = 27 \text{ kN}$



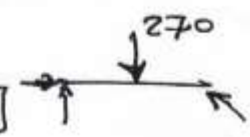
$7.2 * 30 = 216 \text{ kN}$

$\Sigma M_A = 0$

$27(4.8) + 216(3.6) + 810 + 27(6) - F_B \cos 60 = 0$

$\Rightarrow F_B = 495 \text{ kN}$

$[\text{or}: 270(3.6) + 810 - F_B \cos 60 (7.2) = 0]$



$\Sigma F_x = 0$

$A_x - 495 \sin 60 = 0 \Rightarrow A_x = 428.68 \text{ kN}$ →

$\Sigma F_y = 0$

$A_y - 27 - 216 - 27 + 495 \cos 60 = 0$

$\Rightarrow A_y = 22.5 \text{ kN}$ ↑

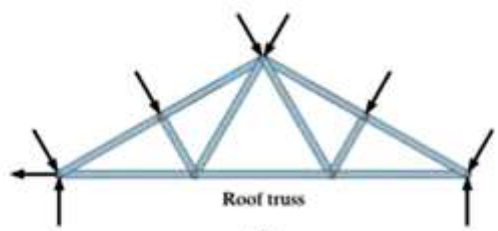
SIMPLE TRUSSES

A truss is a structure composed of slender members joined together at their end points. The members commonly used consist of wooden struts or metal bars. The joint connections are usually formed by bolting or welding the ends of members to a common plate called a gusset plate, or by simply passing a large bolt or pin through each of the members.

Planar Trusses:

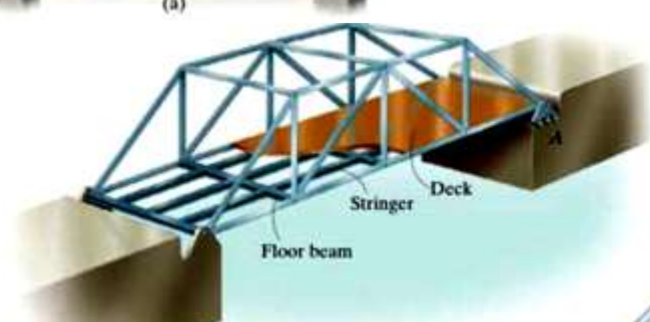
Planar trusses lie in a single plane and are used to support roofs and bridges.

(a)

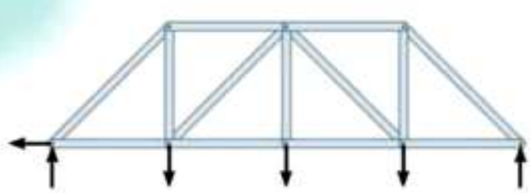


Roof truss

(b)



(a)



Bridge truss

(b)

Assumptions for design:

1. All loadings are applied at the joints.
2. The members are joined together by smooth pins.

Because of these two assumptions, the forces at the ends of the members must be directed along the axis of the member, it is a tensile force (T), whereas if it tends to shorten the member, it is a compressive force (C).

Tension

Compression

Analysis of a truss:

The analysis means finding the reactions and the forces in the truss members.

There are two methods to find the forces in the members of the truss:

(1) The Joints method. (2) The Sections method.

(1) The Method of Joints:

In this method, the F.B.D. of each joint is drawn. Then the equilibrium equations $\sum Fx=0$ and $\sum Fy=0$ are applied (The equilibrium equation $\sum M=0$ is satisfied since the forces in each joint are concurrent). If the sense of direction of force in the member is unknown, assume the force is tension (T). Notice that the tension force pulling on the joint while the compression force pushing 0n the joint.

Procedure for Analysis:

1. Neglect the weight of all members.

2. Draw free-body diagram of a joint have at least one unknown force and at most two unknown member forces.

3. If the sense of direction of force in the member is unknown, assume the force is tension (T).

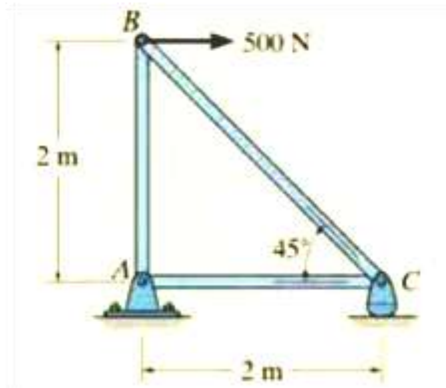
4. Apply $\Sigma F_x=0$ and $\Sigma F_y=0$ to find the tow unknown member forces.

5. Repeat steps (2) to (4) for the other joints.

Note:

Orient x & y axes such that the forces in F.B.D. can be easily resolved.

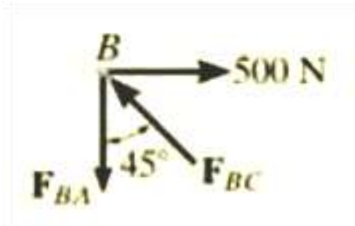
Examples:(Joint Method)



Ex.1: Determine the force in each member of the truss shown and indicate whether the members are in tension or compression.

Sol.:

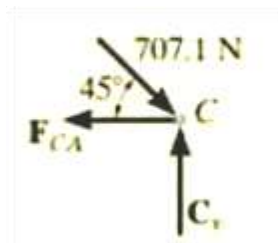
Joint B.



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500\text{ N} - F_{BC}\sin 45^\circ = 0 \quad F_{BC} = 707.1\text{ N (C)} \quad Ans.$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\cos 45^\circ - F_{BA} = 0 \quad F_{BA} = 500\text{ N (T)} \quad Ans.$$

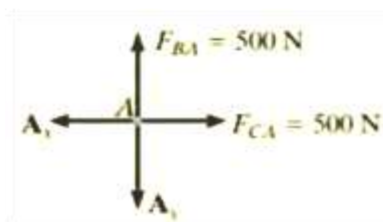
Joint C.



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{CA} + 707.1\cos 45^\circ\text{ N} = 0 \quad F_{CA} = 500\text{ N (T)} \quad Ans.$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 707.1\sin 45^\circ\text{ N} = 0 \quad C_y = 500\text{ N} \quad Ans.$$

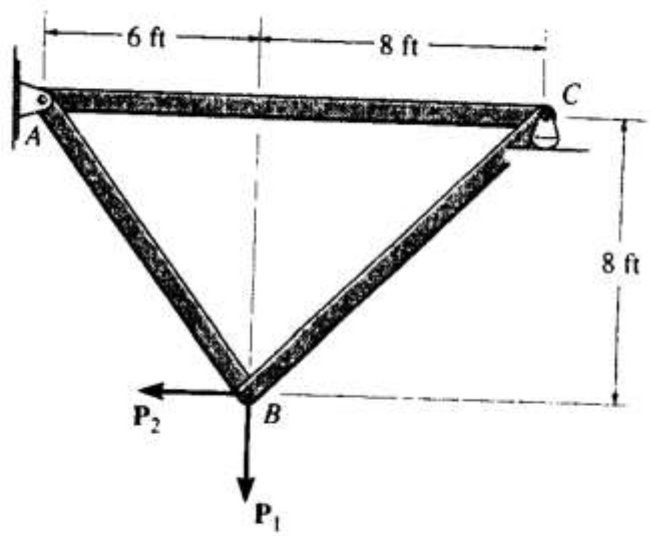
Joint A.



$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 500\text{ N} - A_x = 0 \quad A_x = 500\text{ N}$$

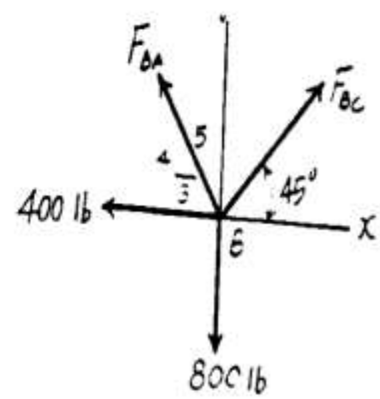
$$+\uparrow \Sigma F_y = 0; \quad 500\text{ N} - A_y = 0 \quad A_y = 500\text{ N}$$

Ex.2: Determine the force in each member of the truss and state if the members are in tension or compression. Set P_1 = 800lb and P_2 = 400lb.



Sol.:

Joint *B*



$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{BC}\cos 45° - F_{BA}\left(\frac{3}{5}\right) - 400 = 0 \qquad [1]$$

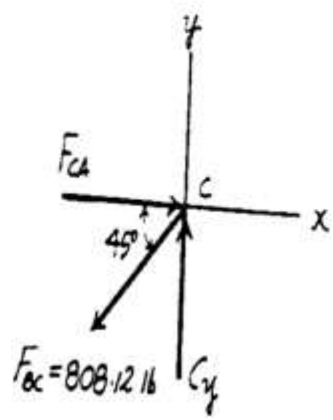
$$+\uparrow \Sigma F_y = 0;\quad F_{BC}\sin 45° + F_{BA}\left(\frac{4}{5}\right) - 800 = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$F_{BA} = 285.71 \text{ lb (T)} = 286 \text{ lb (T)} \qquad \textbf{Ans}$$

$$F_{BC} = 808.12 \text{ lb (T)} = 808 \text{ lb (T)} \qquad \textbf{Ans}$$

Joint *C*



$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CA} - 808.12\cos 45° = 0$$

$$F_{CA} = 571 \text{ lb (C)} \qquad \textbf{Ans}$$

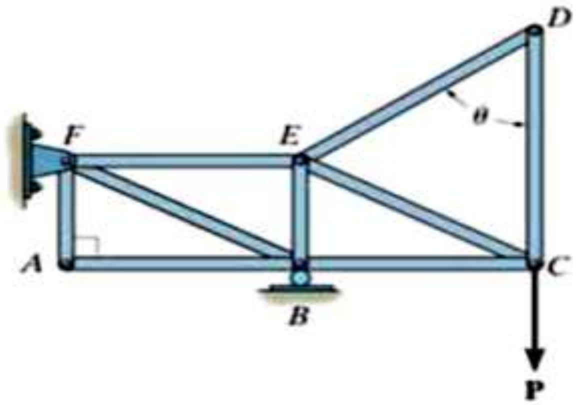
$$+\uparrow \Sigma F_y = 0;\quad C_y - 808.12\sin 45° = 0$$

$$C_y = 571 \text{ lb}$$

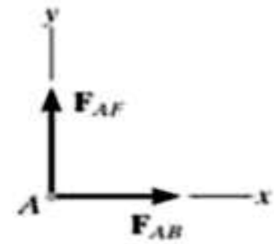
Zero-Force Members:

The zero force members are used to increase the stability of the truss during construction and to provide support if the applied loading is changed. The zero-force member can be determined by inspection. In general, there are two cases:

1. If only two members form a truss joint and no external load or support reaction is applied to the joint, these two members are zero-force members.



(a)

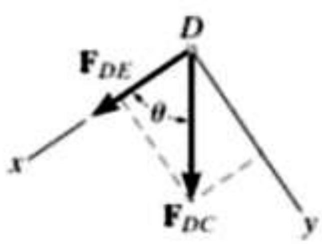


$\overset{+}{\rightarrow} \Sigma F_x = 0;\ F_{AB} = 0$

$+\uparrow \Sigma F_y = 0;\ F_{AF} = 0$

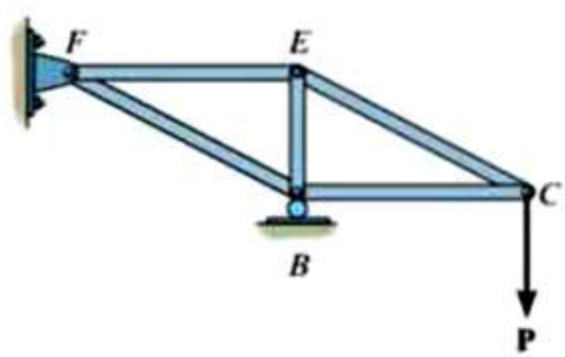
(b)

2. If three members form a truss joint for which two of the members are collinear and no external load or support reaction is applied to the joint, the third member is zero-force member.

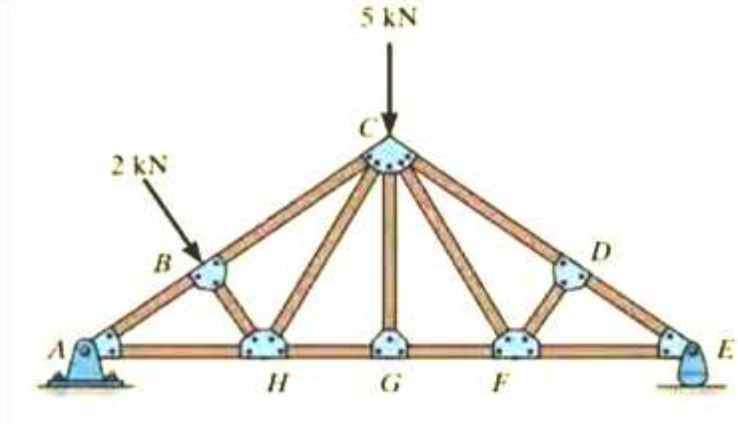


$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\quad F_{DC} = 0 \text{ since } \sin\theta \neq 0$

$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\quad F_{DE} = 0$



Ex.3: Using the method of joints, determine all the zero-force members of the truss shown..



Sol.:

Joint G. (Fig. 6–13*b*).

$+\uparrow \Sigma F_y = 0;$ $F_{GC} = 0$ *Ans.*

Realize that we could not conclude that *GC* is a zero-force member by considering joint *C*, where there are five unknowns. The fact that *GC* is a zero-force member means that the 5-kN load at *C* must be supported by members *CB*, *CH*, *CF*, and *CD*.

Joint D. (Fig. 6–13*c*).

$+\swarrow \Sigma F_x = 0;$ $F_{DF} = 0$ *Ans.*

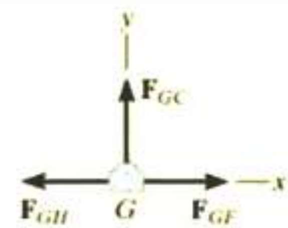
Joint F. (Fig. 6–13*d*).

$+\uparrow \Sigma F_y = 0;$ $F_{FC} \cos \theta = 0$ Since $\theta \neq 90^\circ$, $F_{FC} = 0$ *Ans.*

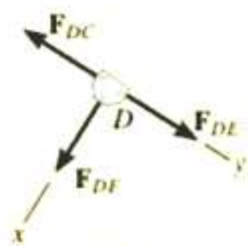
NOTE: If joint *B* is analyzed, Fig. 6–13*e*,

$+\searrow \Sigma F_x = 0;$ $2 \text{ kN} - F_{BH} = 0$ $F_{BH} = 2 \text{ kN}$ (C)

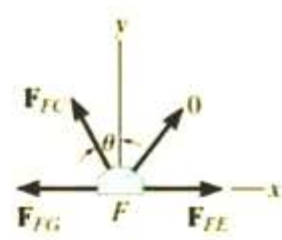
Also, F_{HC} must satisfy $\Sigma F_y = 0$, Fig. 6–13*f*, and therefore *HC* is *not* a zero-force member.



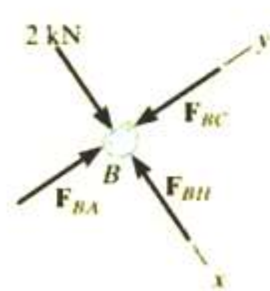
(b)



(c)



(d)



(e)

(2) The Method of Sections:

This method is based on the principle that if a body is in equilibrium, then any part of the body is also in equilibrium.

The method of sections involves cutting the truss into two portions by passing an imaginary section through the members whose forces are desired.

Then the equilibrium equations ($\sum F_x=0$, $\sum F_y=0$ and $\sum m=0$) are applied to the isolated part of the truss.

Procedure for analysis:

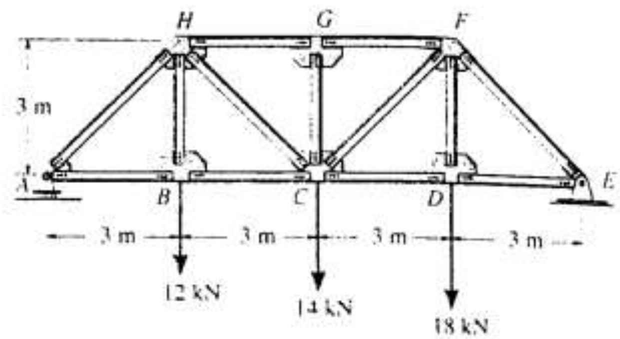
1. Select a section that passes through members whose forces are desired, but generally not more than three members with unknown forces.

2. Draw free body diagram of the part of truss which has the least member of forces acting on it.

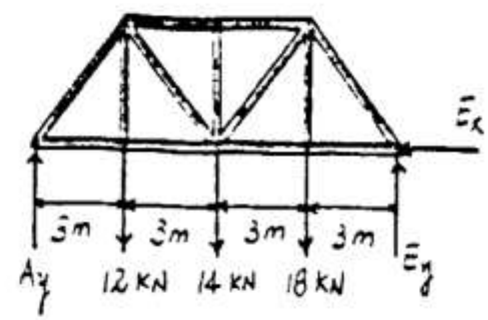
3. Apply the equilibrium equations to find the unknowns.

Examples:

Ex.1: Determine the force in members BC, HC, and HG of the bridge truss shown, and indicate whether the members are in tension or compression.



Sol.:



Support Reactions :

$\circlearrowleft + \Sigma M_E = 0;\quad 18(3) + 14(6) + 12(9) - A_y(12) = 0\quad A_y = 20.5\text{ kN}$

Method of Sections :

$\circlearrowleft + \Sigma M_C = 0;\quad F_{HG}(3) + 12(3) - 20.5(6) = 0$

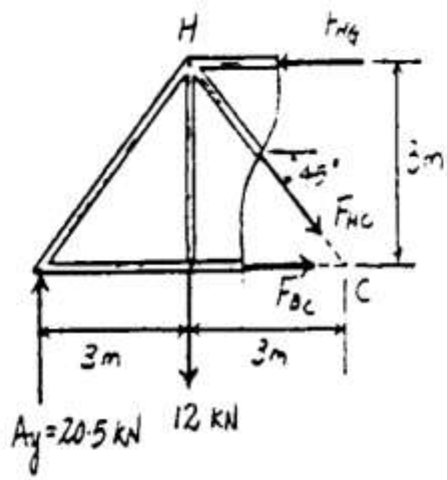
$F_{HG} = 29.0\text{ kN (C)}$ **Ans**

$\circlearrowleft + \Sigma M_H = 0;\quad F_{BC}(3) - 20.5(3) = 0$

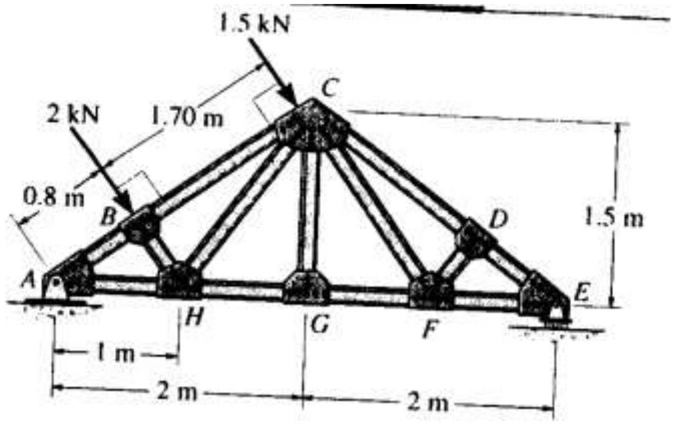
$F_{BC} = 20.5\text{ kN (T)}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 20.5 - 12 - F_{HC}\sin 45° = 0$

$F_{HC} = 12.0\text{ kN (T)}$ **Ans**



Ex.2: Determine the force in members GF, CF, and CD of the roof truss shown, and indicate whether the members are in tension or compression.



Sol.:

$\curvearrowleft + \Sigma M_A = 0;\quad E_y(4) - 2(0.8) - 1.5(2.50) = 0 \quad E_y = 1.3375\text{ kN}$

Method of Sections :

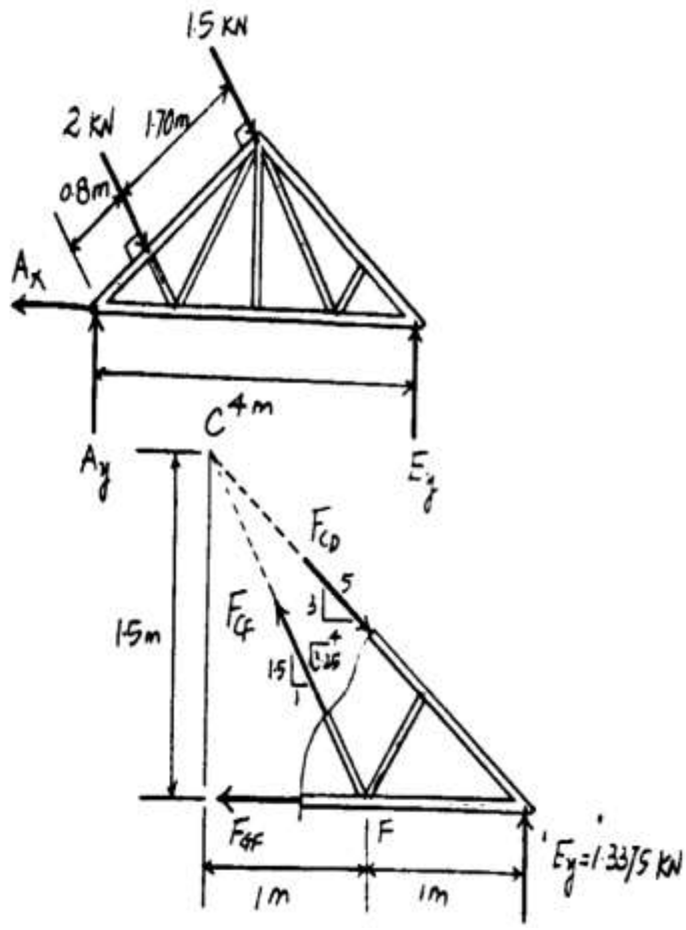
$\curvearrowleft + \Sigma M_C = 0;\quad 1.3375(2) - F_{GF}(1.5) = 0$

$F_{GF} = 1.78\text{ kN (T)}$ **Ans**

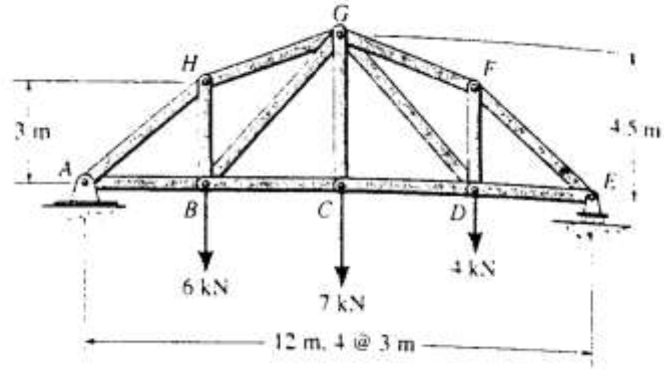
$\curvearrowleft + \Sigma M_F = 0;\quad 1.3375(1) - F_{CD}\left(\frac{3}{5}\right)(1) = 0$

$F_{CD} = 2.23\text{ kN (C)}$ **Ans**

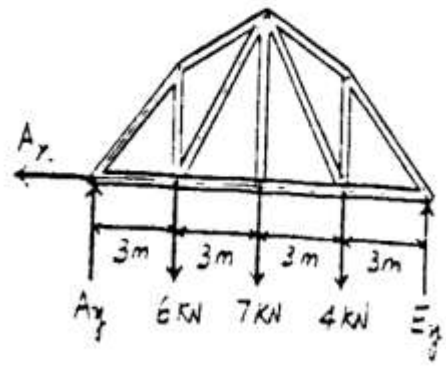
$\curvearrowleft + \Sigma M_E = 0;\quad F_{CF}\left(\frac{1.5}{\sqrt{3.25}}\right)(1) = 0 \quad F_{CF} = 0$ **Ans**



Ex.3: Determine the force in members BG, HG, and BC of the truss shown, and indicate whether the members are in tension or compression.



Sol.:



$\circlearrowleft + \Sigma M_E = 0; \quad 6(9) + 7(6) + 4(3) - A_y(12) = 0 \quad A_y = 9.00 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

Method of Sections :

$\circlearrowleft + \Sigma M_G = 0; \quad F_{BC}(4.5) + 6(3) - 9(6) = 0$

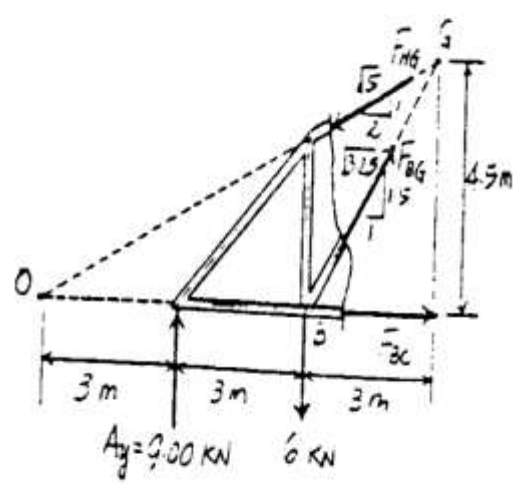
$F_{BC} = 8.00 \text{ kN (T)}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0; \quad F_{HG}\left(\frac{1}{\sqrt{5}}\right)(6) - 9(3) = 0$

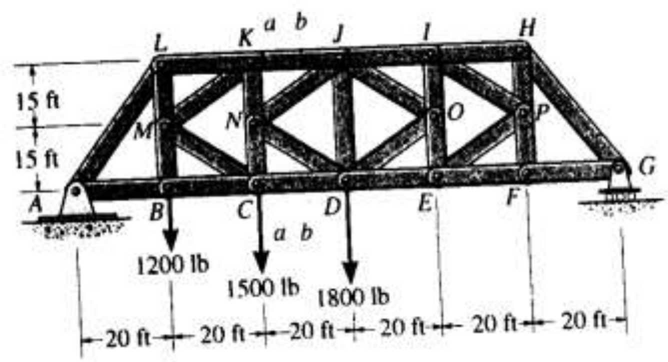
$F_{HG} = 10.1 \text{ kN (C)}$ **Ans**

$\circlearrowleft + \Sigma M_O = 0; \quad F_{BG}\left(\frac{1.5}{\sqrt{3.25}}\right)(6) + 9(3) - 6(6) = 0$

$F_{BG} = 1.80 \text{ kN (T)}$ **Ans**



Ex.4: Determine the force in members KJ, NJ, and CD of the K truss shown, and indicate whether the members are in tension or compression.



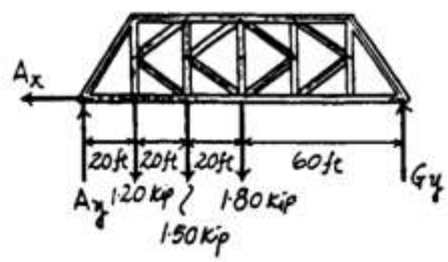
Sol.:

Support Reactions :

$\curvearrowleft + \Sigma M_G = 0;\quad 1.20(100) + 1.50(80) + 1.80(60) - A_y(120) = 0$

$A_y = 2.90 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$



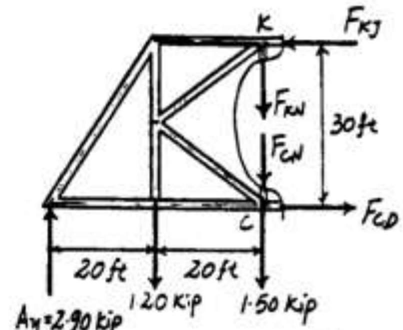
Method of Sections : From section $a-a$, F_{KJ} and F_{CD} can be obtained directly by summing moment about points C and K respectively.

$\curvearrowleft + \Sigma M_C = 0;\quad F_{KJ}(30) + 1.20(20) - 2.90(40) = 0$

$F_{KJ} = 3.067 \text{ kip (C)} = 3.07 \text{ kip (C)}$ **Ans**

$\curvearrowleft + \Sigma M_K = 0;\quad F_{CD}(30) + 1.20(20) - 2.90(40) = 0$

$F_{CD} = 3.067 \text{ kip (T)} = 3.07 \text{ kip (T)}$ **Ans**



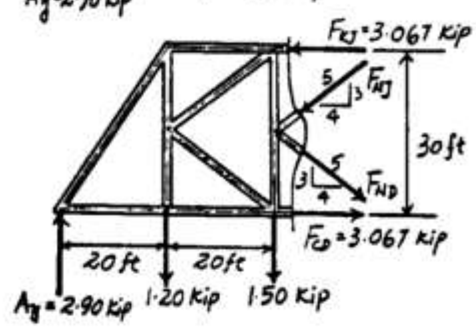
From sec $b-b$, summing forces along x and y axes yields

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{ND}\left(\frac{4}{5}\right) - F_{NJ}\left(\frac{4}{5}\right) + 3.067 - 3.067 = 0$

$F_{ND} = F_{NJ}$ **[1]**

$+\uparrow \Sigma F_y = 0;\quad 2.90 - 1.20 - 1.50 - F_{ND}\left(\frac{3}{5}\right) - F_{NJ}\left(\frac{3}{5}\right) = 0$

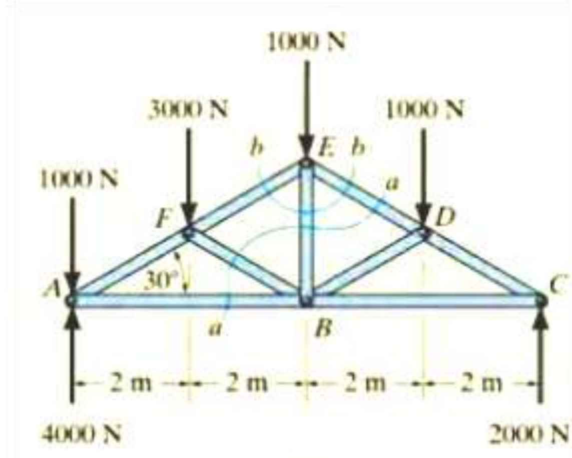
$F_{ND} + F_{NJ} = 0.3333$ **[2]**



Solving Eqs.[1] and [2] yields

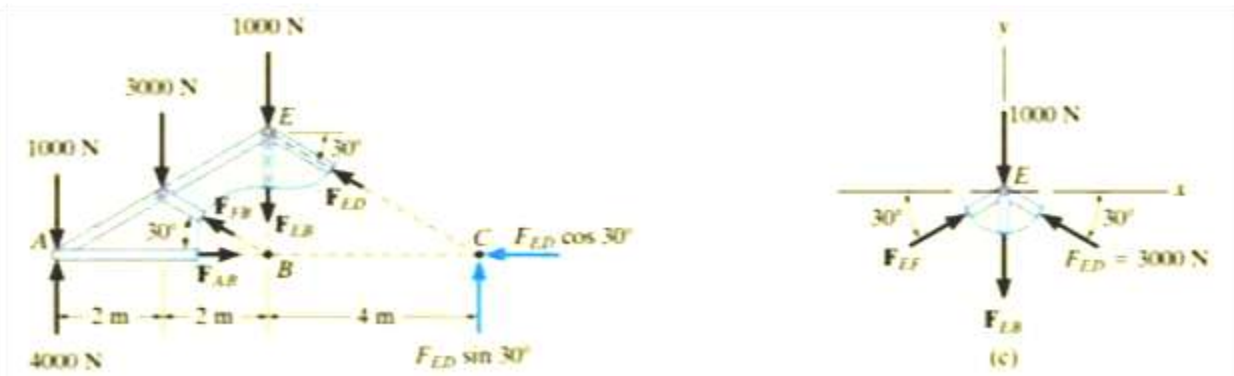
$F_{ND} = 0.167 \text{ kip (T)}\qquad F_{NJ} = 0.167 \text{ kip (C)}$ **Ans**

Ex.5: Determine the force in members EB of the roof truss shown, and indicate whether the members are in tension or compression.



(a)

Sol.:



(b) (c)

$\circlearrowleft + \Sigma M_B = 0;$ $1000\ \text{N}(4\ \text{m}) + 3000\ \text{N}(2\ \text{m}) - 4000\ \text{N}(4\ \text{m}) + F_{ED}\sin 30^\circ(4\ \text{m}) = 0$

$$F_{ED} = 3000\ \text{N} \quad (\text{C})$$

Considering now the free-body diagram of section *bb*, Fig. 6–18*c*, we have

$\xrightarrow{+} \Sigma F_x = 0;$ $F_{EF}\cos 30^\circ - 3000\cos 30^\circ\ \text{N} = 0$

$$F_{EF} = 3000\ \text{N} \quad (\text{C})$$

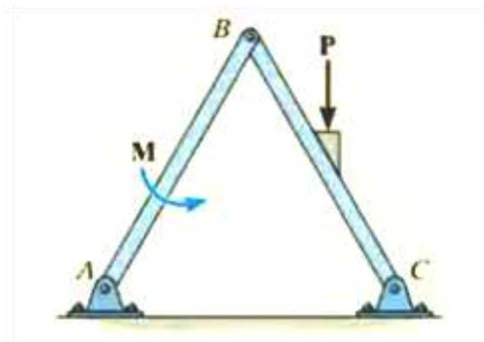
$+\uparrow \Sigma F_y = 0;$ $2(3000\sin 30^\circ\ \text{N}) - 1000\ \text{N} - F_{EB} = 0$

$$F_{EB} = 2000\ \text{N} \quad (\text{T}) \qquad \textit{Ans.}$$

Frames and Machines

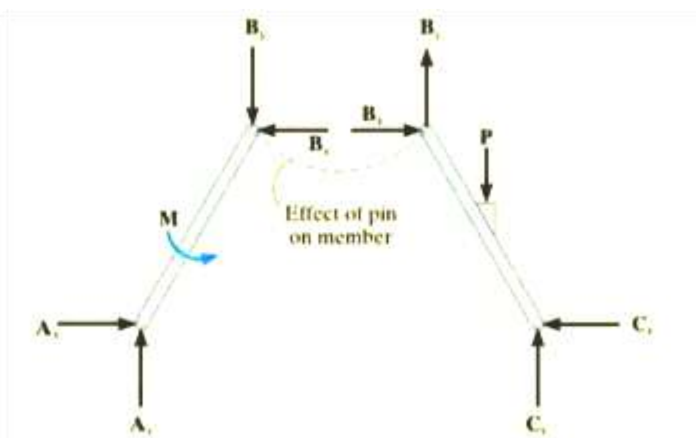
Frames and machines are two common types of structure which are composed of pin-connected multiforce members. Frames are used to support loads, whereas machines contain moving parts and are designed to transmit and alter the forces. To determine the forces acting at the joints and supports of a frame or machine, the structure must be disassembled and free body diagram of its parts must be drawn. When free body diagram is drawn for each of connected members, the forces at pin-connection must be equal and opposite for each member.

Ex.1: For the frame shown, draw the free-body diagram of (a) each member, (b) the pin at B, and (c) the two members connected together.

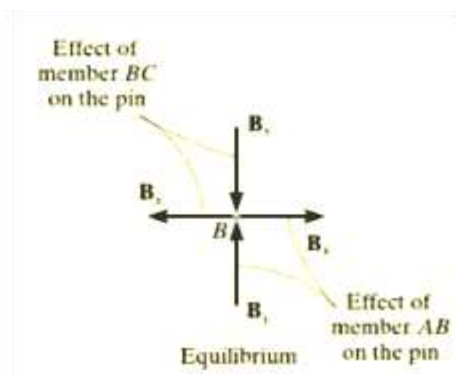


Sol.:

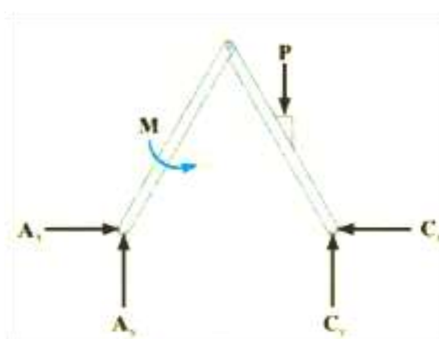
(a)



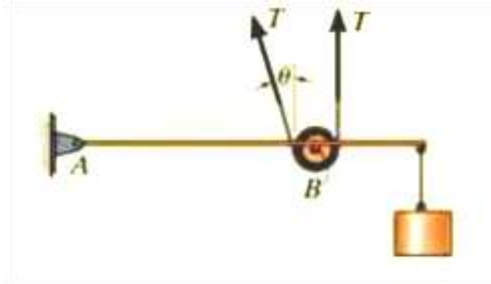
(b)



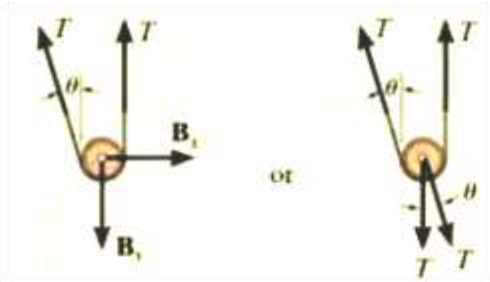
(c)



Ex.2: Draw the free-body diagrams of the frame and the cylinder. The suspended block has a weight of W.



Sol.:

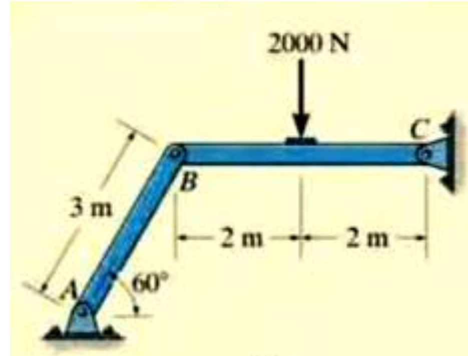


Procedure for Analysis:

1. Draw free-body diagram of the entire structure, a portion of the structure, or each of the members. The choice should be made so that it leads to the most direct solution.

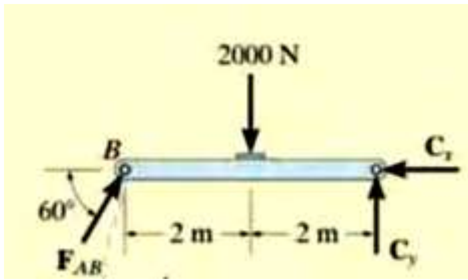
2. Apply the equations of equilibrium for the entire structure or a portion of structure to find unknowns.

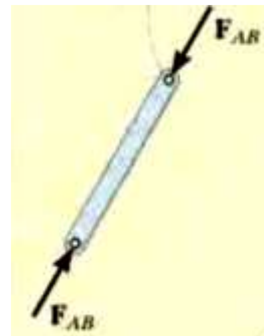
Ex.1: Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame shown.



Sol.:

Method (1):





$$\circlearrowleft + \Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - (F_{AB}\sin 60^\circ)(4\text{ m}) = 0 \quad F_{AB} = 1154.7\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0;\quad 1154.7\cos 60^\circ\text{ N} - C_x = 0 \qquad C_x = 577\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0;\quad 1154.7\sin 60^\circ\text{ N} - 2000\text{ N} + C_y = 0 \quad C_y = 1000\text{ N} \quad \textit{Ans.}$$

Method (2):

Member AB

$$\circlearrowleft + \Sigma M_A = 0;\quad B_x(3\sin 60^\circ\text{ m}) - B_y(3\cos 60^\circ\text{ m}) = 0 \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - B_x = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0;\quad A_y - B_y = 0 \qquad (3)$$

Member BC

$$\circlearrowleft + \Sigma M_C = 0;\quad 2000\text{ N}(2\text{ m}) - B_y(4\text{ m}) = 0 \qquad (4)$$

$$\xrightarrow{+} \Sigma F_x = 0;\quad B_x - C_x = 0 \qquad (5)$$

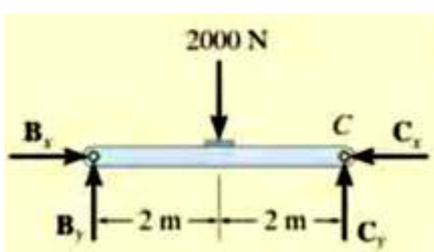
$$+\uparrow \Sigma F_y = 0;\quad B_y - 2000\text{ N} + C_y = 0 \qquad (6)$$

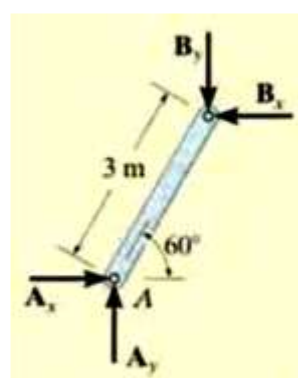
$B_y = 1000$ N

$B_x = 577$ N

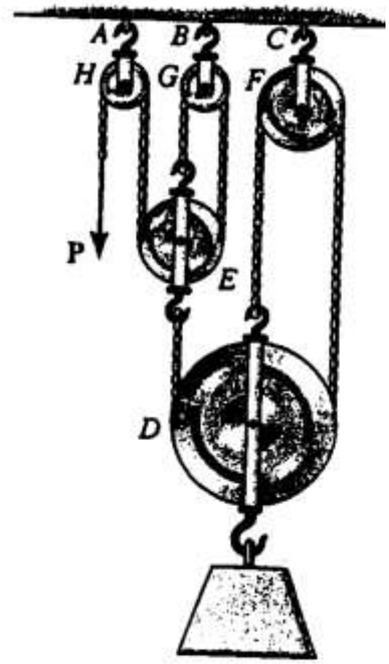
$C_x = 577$ N

$C_y = 1000$ N





Ex.2: Determine the force P needed to support the 20-Kg mass. Also, what are the reactions at the supporting hooks A, B, and C?



Sol.:

For pulley D:

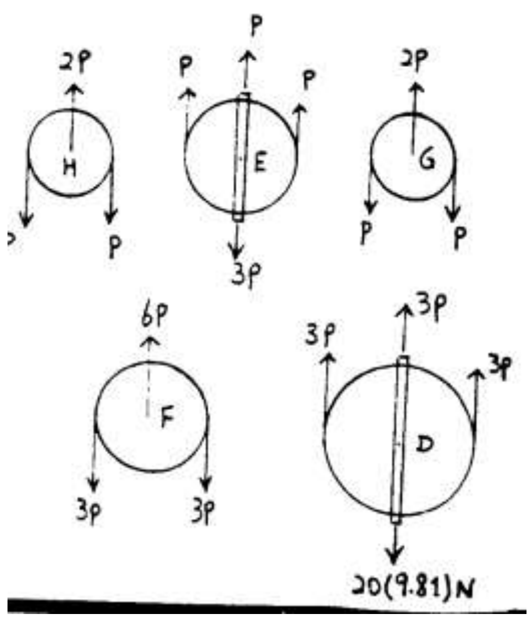
$+\uparrow \Sigma F_y = 0;\qquad 9P - 20(9.81) = 0$

$P = 21.8\ \text{N}$

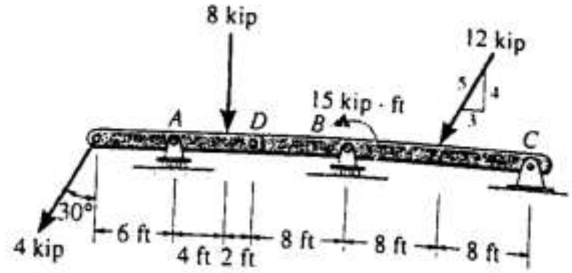
At A, $\qquad R_A = 2P = 43.6\ \text{N}$

At B, $\qquad R_B = 2P = 43.6\ \text{N}$

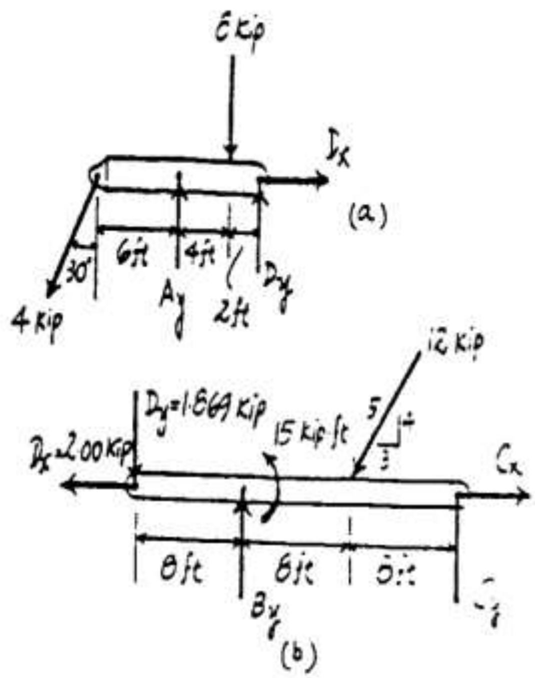
At C, $\qquad R_C = 6P = 131\ \text{N}$



Ex.3: Determine the reactions at supports. Neglect the thickness of the beam.



Sol.:



$\curvearrowleft + \Sigma M_D = 0; \quad 4\cos 30°(12) + 8(2) - A_y(6) = 0$

$A_y = 9.595 \text{ kip} = 9.59 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad D_y + 9.595 - 4\cos 30° - 8 = 0$

$D_y = 1.869 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0; \quad D_x - 4\sin 30° = 0 \quad D_x = 2.00 \text{ kip}$

From FBD(b),

$\curvearrowleft + \Sigma M_C = 0; \quad 1.869(24) + 15 + 12\left(\frac{4}{5}\right)(8) - B_y(16) = 0$

$B_y = 8.541 \text{ kip} = 8.54 \text{ kip}$ **Ans**

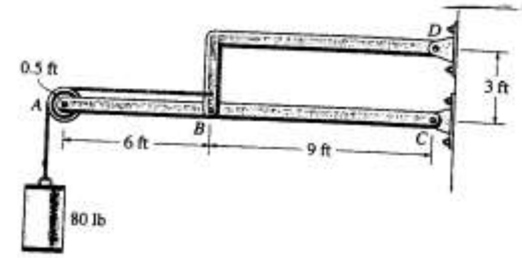
$+\uparrow \Sigma F_y = 0; \quad C_y + 8.541 - 1.869 - 12\left(\frac{4}{5}\right) = 0$

$C_y = 2.93 \text{ kip}$ **Ans**

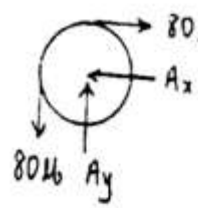
$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 2.00 - 12\left(\frac{3}{5}\right) = 0$

$C_x = 9.20 \text{ kip}$ **Ans**

Ex.4: Determine the horizontal and vertical components of force which the pins exert on member ABC.



Sol.:

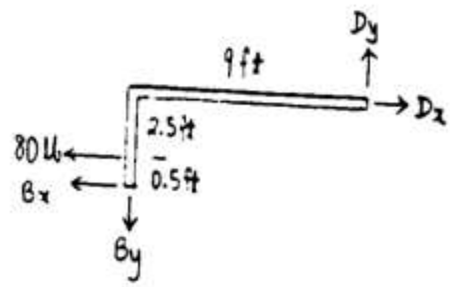


$\xrightarrow{+}\Sigma F_x = 0;\quad A_x = 80 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y = 80 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad 80(15) - B_y(9) = 0$

$B_y = 133.3 = 133 \text{ lb}$ **Ans**

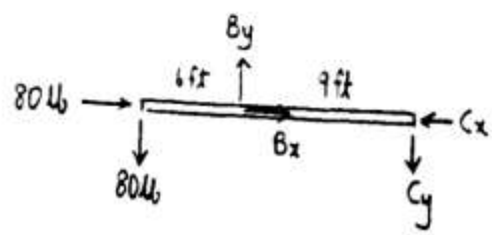


$\circlearrowleft + \Sigma M_D = 0;\quad -80(2.5) + 133.3(9) - B_x(3) = 0$

$B_x = 333 \text{ lb}$ **Ans**

$\xrightarrow{+}\Sigma F_x = 0;\quad 80 + 333 - C_x = 0$

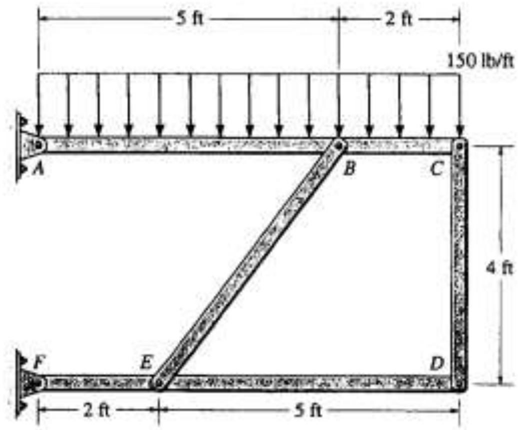
$C_x = 413 \text{ lb}$ **Ans**



$+\uparrow \Sigma F_y = 0;\quad -80 + 133.3 - C_y = 0$

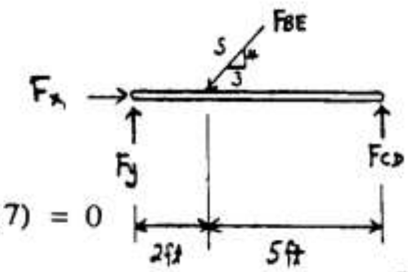
$C_y = 53.3 \text{ lb}$ **Ans**

Ex.5: Determine the resultant force at pins B and C on member ABC of the frame shown.



Sol.:

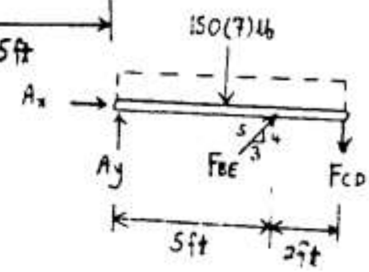
$$\curvearrowleft + \Sigma M_F = 0; \qquad F_{CD}(7) - \frac{4}{5}F_{BE}(2) = 0$$



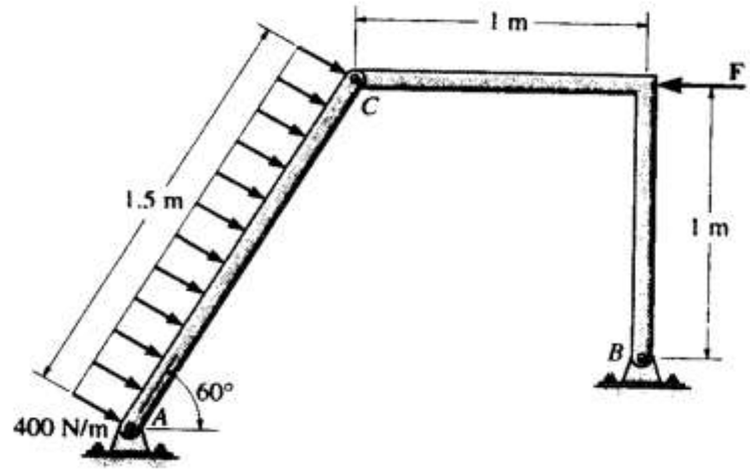
$$\curvearrowleft + \Sigma M_A = 0; \qquad -150(7)(3.5) + \frac{4}{5}F_{BE}(5) - F_{CD}(7) = 0$$

$$F_{BE} = 1531 \text{ lb} = 1.53 \text{ kip} \qquad \textbf{Ans}$$

$$F_{CD} = 350 \text{ lb} \qquad \textbf{Ans}$$



Ex.6: Determine the horizontal and vertical components of force that pins A and B exert on the frame shown. Set F = 500 N.



Sol.:

Member AC:

$\circlearrowleft + \Sigma M_A = 0; \quad -600\,(0.75) - C_y\,(1.5\cos 60^\circ) + C_x\,(1.5\sin 60^\circ) = 0$

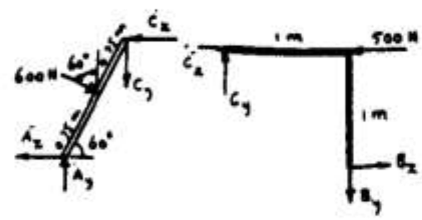
Member CB:

$\circlearrowleft + \Sigma M_B = 0; \quad -C_x\,(1) - C_y\,(1) + 500\,(1) = 0$

Solving,

$C_x = 402.6\text{ N}$

$C_y = 97.4\text{ N}$



Member AC:

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 600\sin 60^\circ - 402.6 = 0$

$A_x = 117\text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y - 600\cos 60^\circ - 97.4 = 0$

$A_y = 397\text{ N}$ **Ans**

Member CB:

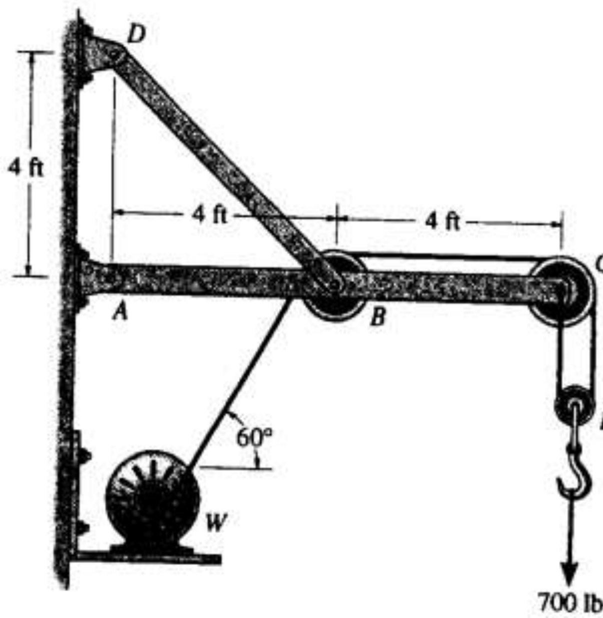
$\xrightarrow{+} \Sigma F_x = 0; \quad 402.6 - 500 + B_x = 0$

$B_x = 97.4\text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -B_y + 97.4 = 0$

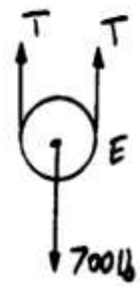
$B_y = 97.4\text{ N}$ **Ans**

Ex.7: Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?



Sol.:

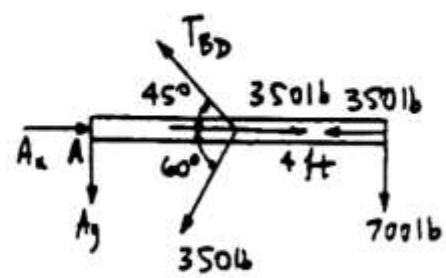
Pulley E :



$+\uparrow \Sigma F_y = 0; \quad 2T - 700 = 0$

$T = 350$ lb **Ans**

Member ABC :



$\circlearrowleft + \Sigma M_A = 0; \quad T_{BD} \sin 45° (4) - 350 \sin 60° (4) - 700 (8) = 0$

$T_{BD} = 2409$ lb

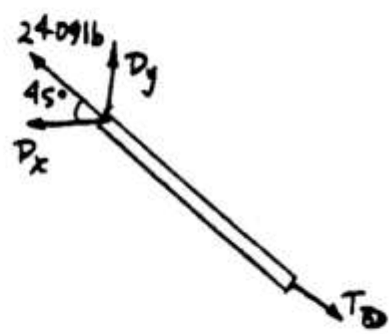
$+\uparrow \Sigma F_y = 0; \quad -A_y + 2409 \sin 45° - 350 \sin 60° - 700 = 0$

$A_y = 700$ lb **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 2409 \cos 45° - 350 \cos 60° + 350 - 350 = 0$

$A_x = 1.88$ kip **Ans**

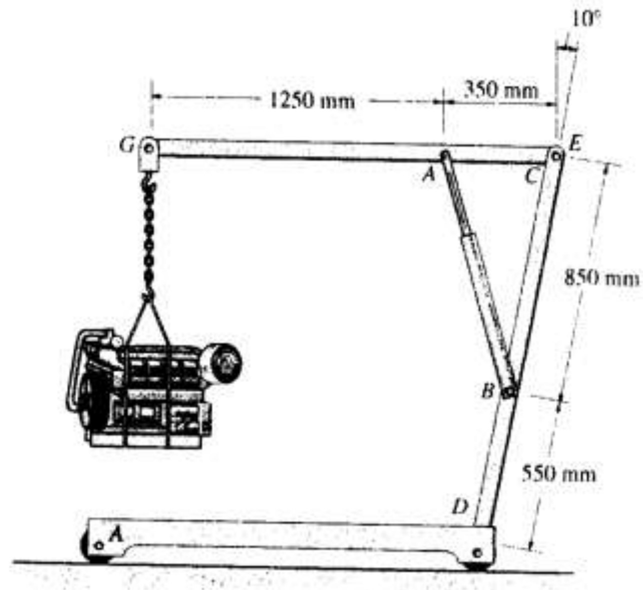
At D :



$D_x = 2409 \cos 45° = 1703.1$ lb $= 1.70$ kip **Ans**

$D_y = 2409 \sin 45° = 1.70$ kip **Ans**

Ex.8: The engine hoist is used to support the 200-kg engine. Determine the force acting in the hydraulic cylinder AB, the horizontal and vertical components of force at the pin C, and the reactions at the fixed support D.

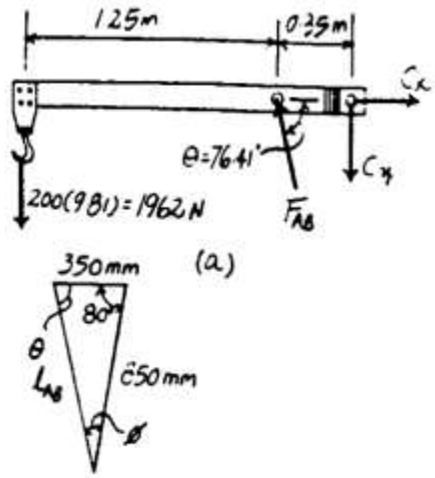


Sol.:

Free Body Diagram : The solution for this problem will be simplified if one realizes that member AB is a two force member. From the geometry,

$$l_{AB} = \sqrt{350^2 + 850^2 - 2(350)(850)\cos 80^\circ} = 861.21 \text{ mm}$$

$$\frac{\sin\theta}{850} = \frac{\sin 80^\circ}{861.21} \qquad \theta = 76.41^\circ$$



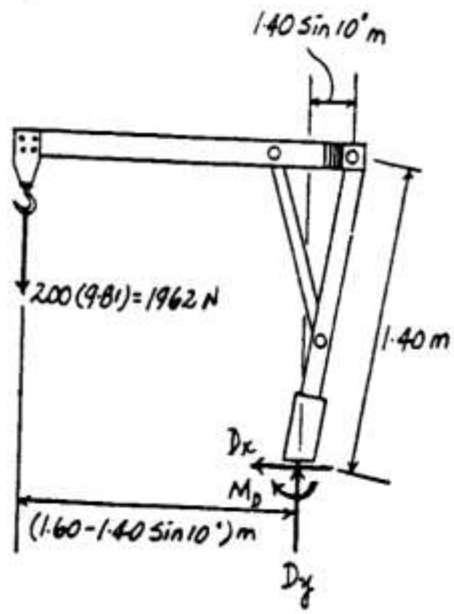
Equations of Equilibrium : From FBD (a).

$$\curvearrowleft + \Sigma M_C = 0; \quad 1962(1.60) - F_{AB}\sin 76.41^\circ(0.35) = 0$$
$$F_{AB} = 9227.60 \text{ N} = 9.23 \text{ kN} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad C_x - 9227.60\cos 76.41^\circ = 0$$
$$C_x = 2168.65 \text{ N} = 2.17 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 9227.60\sin 76.41^\circ - 1962 - C_y = 0$$
$$C_y = 7007.14 \text{ N} = 7.01 \text{ kN} \qquad \textbf{Ans}$$

From FBD (b).



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad D_x = 0 \qquad \textbf{Ans}$$

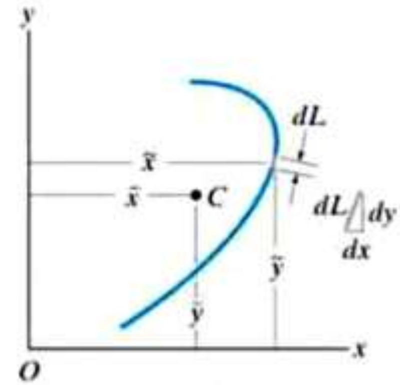
$$+\uparrow \Sigma F_y = 0; \quad D_y - 1962 = 0$$
$$D_y = 1962 \text{ N} = 1.96 \text{ kN} \qquad \textbf{Ans}$$

$$+\Sigma M_D = 0; \quad 1962(1.60 - 1.40\sin 10^\circ) - M_D = 0$$
$$M_D = 2662.22 \text{ N}\cdot\text{m} = 2.66 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

CENTROID

The centroid is a point which defines the geometric center of an object. The lines, areas, and volumes all have centroids. We will study the centroids of areas and lines in plane.

Centroid of a line in a plane:



The centroid C represents the center of a homogenous wire of length L and is specified by the distances $\overline{x}$ &$\bar{y}$, where:

$\overline{x}$: horizontal distance from the centroid to the y-axis,

$\bar{y}$: vertical distance from the centroid to the x-axis.

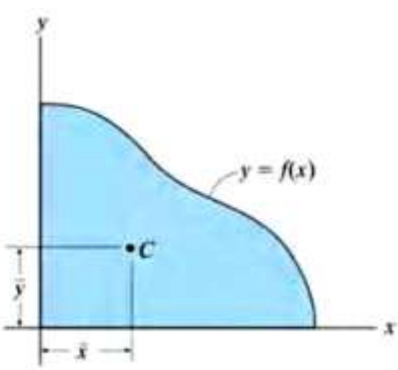
If the length L is subdivided into differential elements dl, then the moments of these elements about an axis is equal to the moment of total length about the same axis.

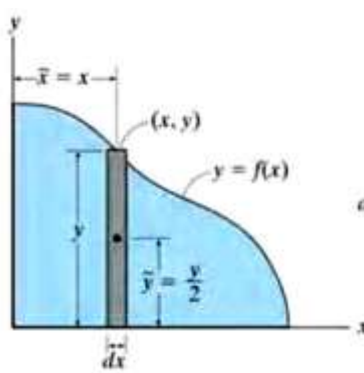
$$L.\,\overline{x} = \Sigma\, \check{x}.dl \quad \Rightarrow \quad \overline{x} = \frac{\Sigma\, \check{x}.dl}{L}$$

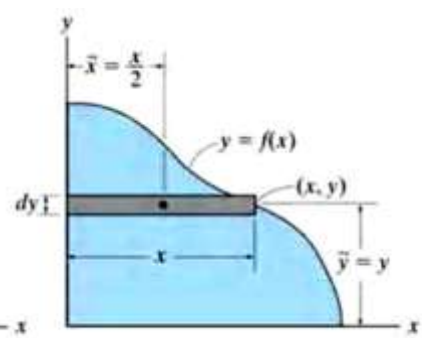
$$L.\,\bar{y} = \Sigma\, \tilde{y}.dl \Rightarrow \quad \bar{y} = \frac{\Sigma\, \tilde{y}.dl}{L}$$

In integral form : $\overline{x} = \dfrac{\int \check{x}.dl}{L}$, $\bar{y} = \dfrac{\int \tilde{y}.dl}{L}$

Centroid of Area:



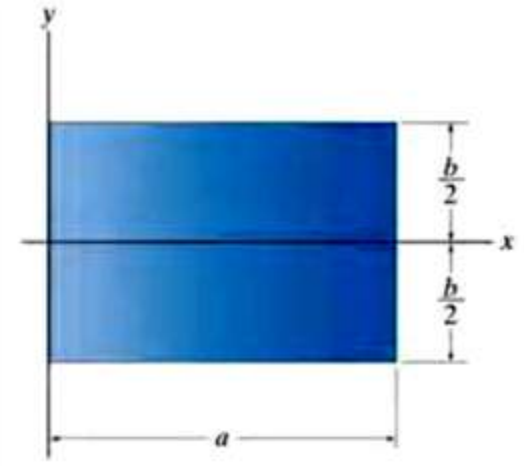


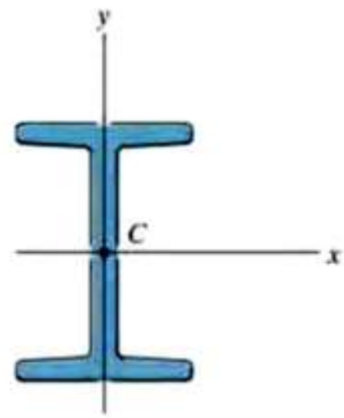


$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

Symmetry :

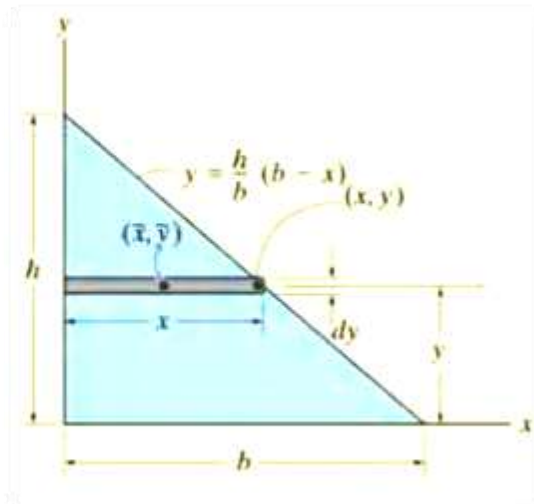
When the shape has an axis of symmetry, the centroid of the shape will lie along that axis. When the shape has two or more than two axes, the centroid lies at the intersection of the axes.





Examples:

Ex.1: Determine the distance $\overline{y}$ measured from the x-axis to the centroid of the area of the triangle shown.



Sol.:

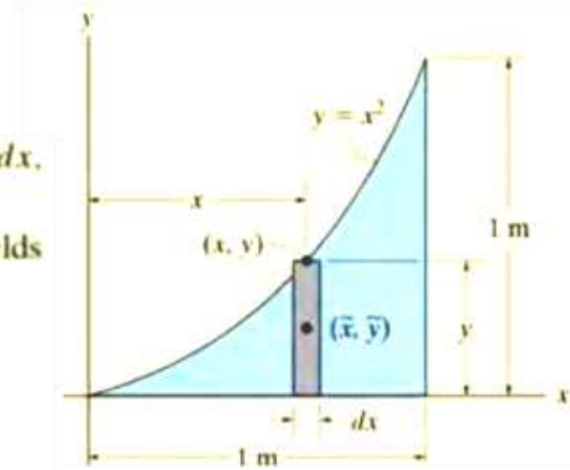
Area and Moment Arms. The area of the element is $dA = x\,dy$ $= \dfrac{b}{h}(h - y)\,dy$, and its centroid is located a distance $\tilde{y} = y$ from the x axis.

Integration. Applying the second of Eqs. 9-4 and integrating with respect to y yields

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left[\frac{b}{h}(h - y)\,dy\right]}{\int_0^h \frac{b}{h}(h - y)\,dy} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh}$$

$$= \frac{h}{3} \qquad \textit{Ans.}$$

<u>Ex.2:</u> Locate the centroid of the area shown.



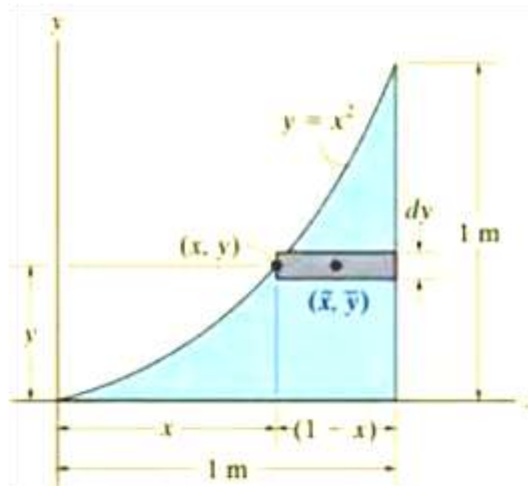
<u>Sol.:</u>

Area and Moment Arms. The area of the element is $dA = y\,dx$, and its centroid is located at $\tilde{x} = x$, $\tilde{y} = y/2$.

Integrations. Applying Eqs. 9–4 and integrating with respect to x yields

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} xy\,dx}{\int_0^{1\text{m}} y\,dx} = \frac{\int_0^{1\text{m}} x^3\,dx}{\int_0^{1\text{m}} x^2\,dx} = \frac{0.250}{0.333} = 0.75\text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} (y/2)y\,dx}{\int_0^{1\text{m}} y\,dx} = \frac{\int_0^{1\text{m}} (x^2/2)x^2\,dx}{\int_0^{1\text{m}} x^2\,dx} = \frac{0.100}{0.333} = 0.3\text{ m}$$



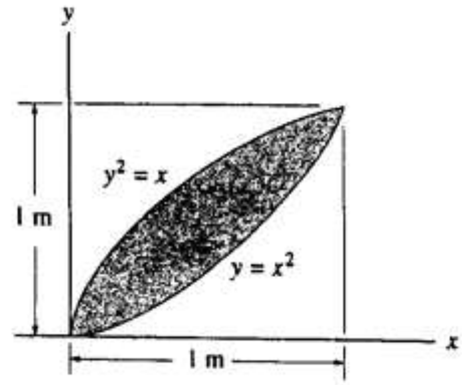
Area and Moment Arms. The area of the element is $dA = (1 - x)\,dy$, and its centroid is located at

$$\tilde{x} = x + \left(\frac{1 - x}{2}\right) = \frac{1 + x}{2},\ \tilde{y} = y$$

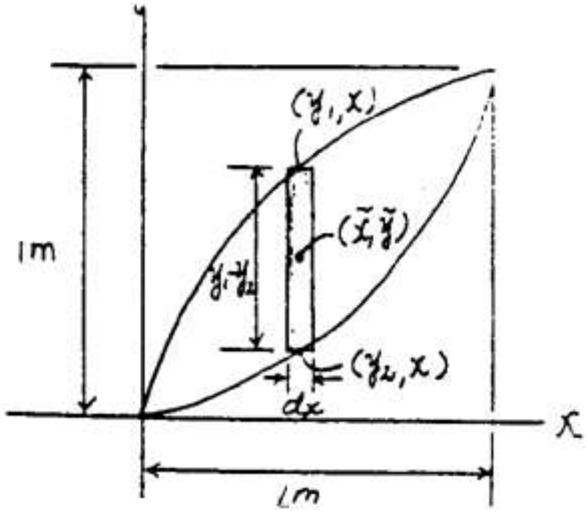
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} [(1 + x)/2](1 - x)\,dy}{\int_0^{1\text{m}} (1 - x)\,dy} = \frac{\frac{1}{2}\int_0^{1\text{m}} (1 - y)\,dy}{\int_0^{1\text{m}} (1 - \sqrt{y})\,dy} = \frac{0.250}{0.333} = 0.75\text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{m}} y(1 - x)\,dy}{\int_0^{1\text{m}} (1 - x)\,dy} = \frac{\int_0^{1\text{m}} (y - y^{3/2})\,dy}{\int_0^{1\text{m}} (1 - \sqrt{y})\,dy} = \frac{0.100}{0.333} = 0.3\text{ m}$$

Ex.3: Locate the centroid $\overline{x}$ of the shaded area shown.



Sol.:



Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\,\mathrm{m}} x\left[\left(x^{\frac{1}{2}} - x^2\right) dx\right]}{\int_0^{1\,\mathrm{m}} \left(x^{\frac{1}{2}} - x^2\right) dx}$$

$$= \frac{\left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^{1\,\mathrm{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\,\mathrm{m}}} = \frac{9}{20}\ \mathrm{m} = 0.45\ \mathrm{m} \qquad \textbf{Ans}$$

Centroid of composite areas:

The centroid of composite areas can be found using the relations:

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \qquad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

Where:

x,y: centroids of each composite part of the area.

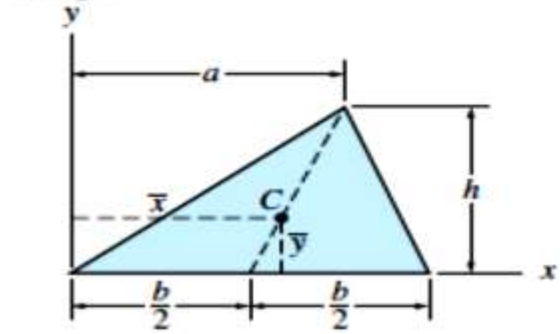
ƸA: sum of the areas of all parts (total areas).

$\bar{x}, \bar{y}$: centroids of the total area.

Centroids for common shapes of areas are given in the table below:

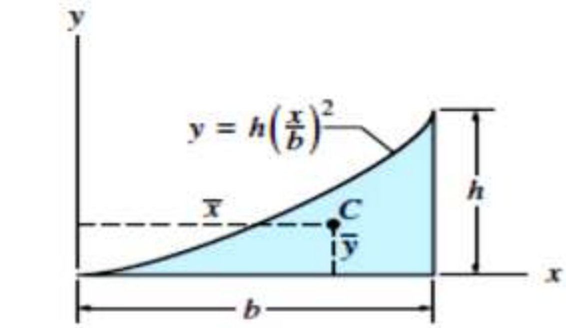
Centroids of Plane Areas

Triangle



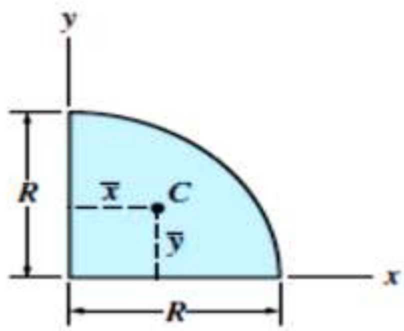
$\bar{x} = \frac{1}{3}(a+b) \quad \bar{y} = \frac{1}{3}h \quad A = \frac{1}{2}bh$

Half parabolic complement



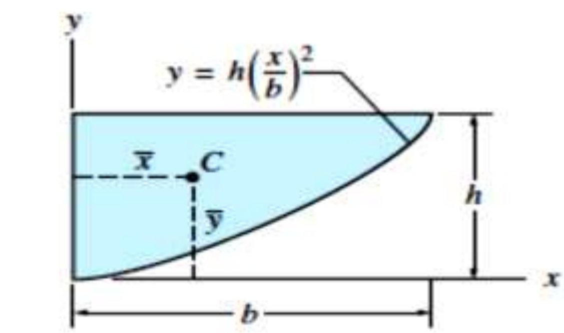
$\bar{x} = \frac{3}{4}b \quad \bar{y} = \frac{3}{10}h \quad A = \frac{1}{3}bh$

Quarter circle



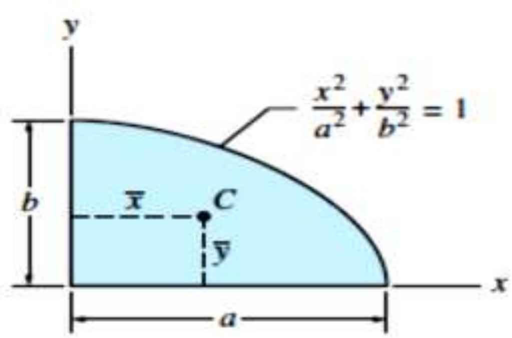
$\bar{x} = \frac{4}{3\pi}R \quad \bar{y} = \frac{4}{3\pi}R \quad A = \frac{\pi}{4}R^2$

Half parabola



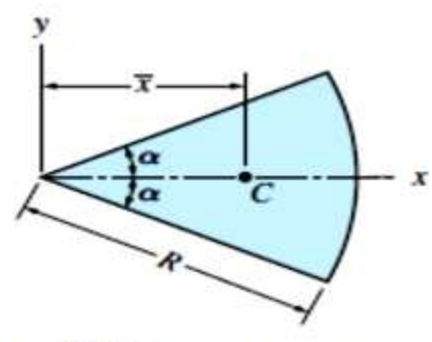
$\bar{x} = \frac{3}{8}b \quad \bar{y} = \frac{3}{5}h \quad A = \frac{2}{3}bh$

Quarter ellipse



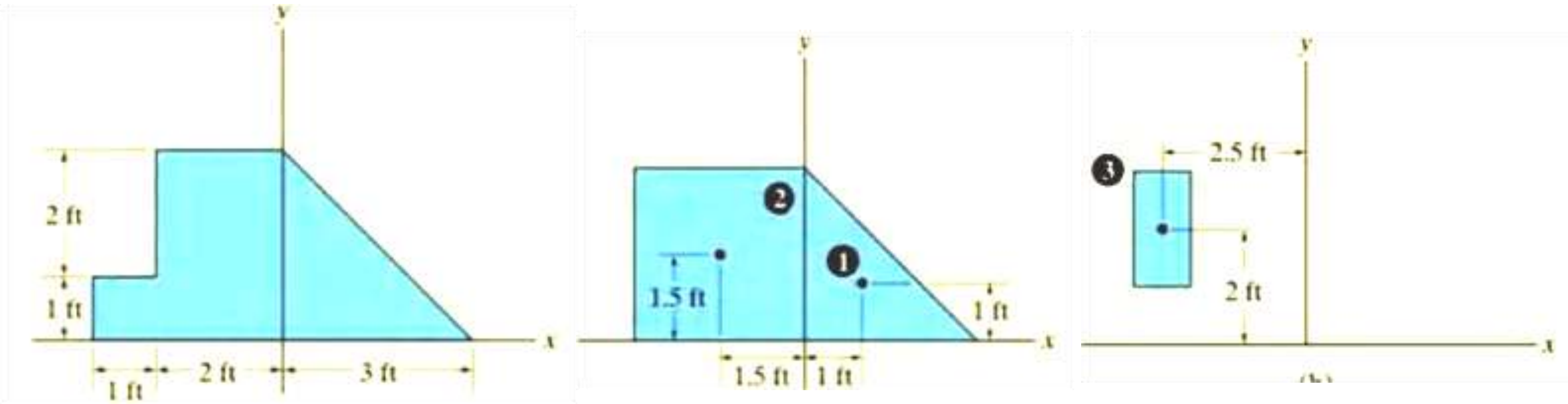
$\bar{x} = \frac{4}{3\pi}a \quad \bar{y} = \frac{4}{3\pi}b \quad A = \frac{\pi}{4}ab$

Circular sector



$\bar{x} = \frac{2R\sin\alpha}{3\alpha} \quad A = \alpha R^2$

<u>Ex.1:</u> Locate the centroid of the area shown.



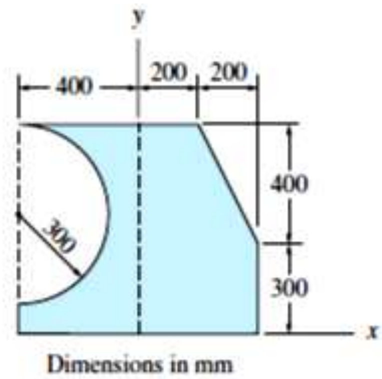
Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$

Thus,

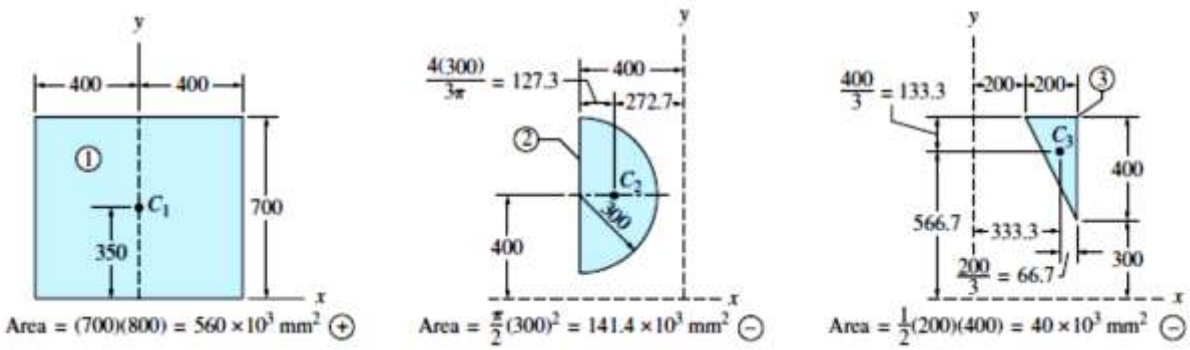
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

Ex.2: Using the method of composite areas, determine the location of the centroid of the shaded area shown in figure below.



Sol.:

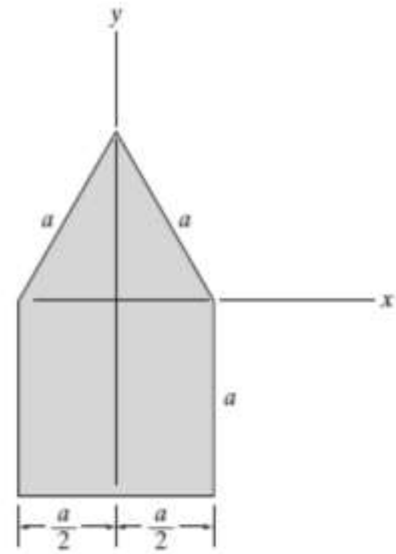


Shape	Area A (mm²)	$\bar{x}$ (mm)	$A\bar{x}$ (mm³)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)
1 (Rectangle)	$+560.0 \times 10^3$	0	0	+350	196.0×10^6
2 (Semicircle)	-141.4×10^3	−272.7	$+38.56 \times 10^6$	+400	-56.56×10^6
3 (Triangle)	-40.0×10^3	+333.3	-13.33×10^6	+566.7	-22.67×10^6
Σ	$+378.6 \times 10^3$	...	$+25.23 \times 10^6$	...	$+116.77 \times 10^6$

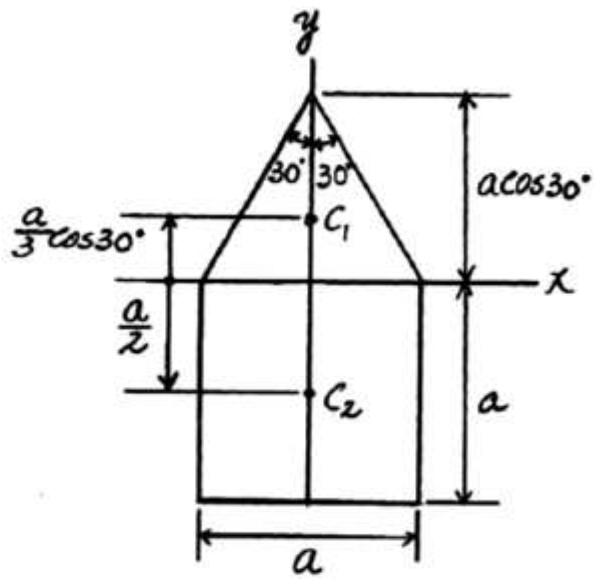
$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{+25.23 \times 10^6}{+378.6 \times 10^3} = 66.6 \text{ mm}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{+116.77 \times 10^6}{+378.6 \times 10^3} = 308 \text{ mm}$$

Ex.3: Using the method of composite areas, determine the location of the centroid $\bar{y}$ of the shaded area shown in figure below.

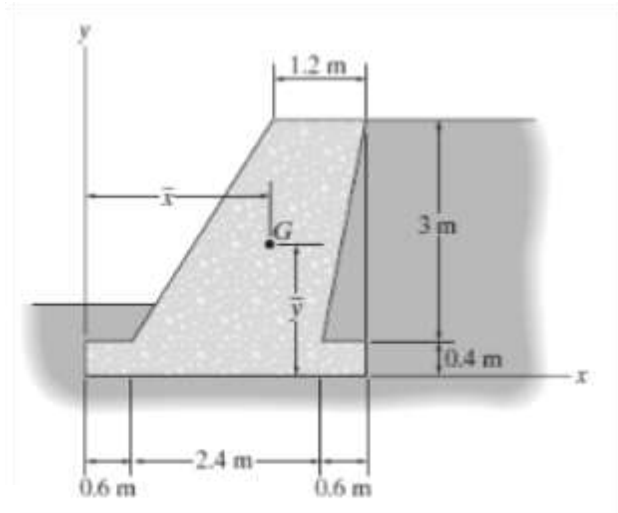


Sol.:

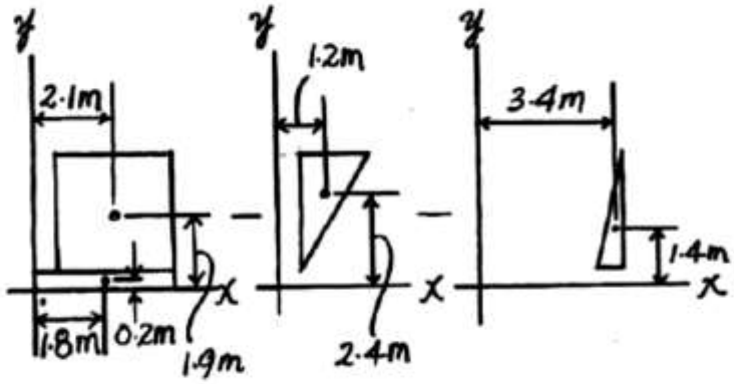


$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{\frac{a}{3}\cos 30^\circ\left[\frac{1}{2}(a)(a\cos 30^\circ)\right] - \frac{a}{2}[a(a)]}{\frac{1}{2}(a)(a\cos 30^\circ) + [a(a)]} = -0.262a$$

Ex.4: The gravity wall is made of concrete. Determine the location (x, y) of the center of mass *G* for the wall.



Sol.:



$$\Sigma\tilde{x}A = 1.8(3.6)(0.4) + 2.1(3)(3) - 3.4\left(\frac{1}{2}\right)(3)(0.6) - 1.2\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 15.192 \text{ m}^3$$

$$\Sigma\tilde{y}A = 0.2(3.6)(0.4) + 1.9(3)(3) - 1.4\left(\frac{1}{2}\right)(3)(0.6) - 2.4\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 9.648 \text{ m}^3$$

$$\Sigma A = 3.6(0.4) + 3(3) - \frac{1}{2}(3)(0.6) - \frac{1}{2}(1.8)(3)$$

$$= 6.84 \text{ m}^2$$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{15.192}{6.84} = 2.22 \text{ m} \qquad \text{Ans}$$

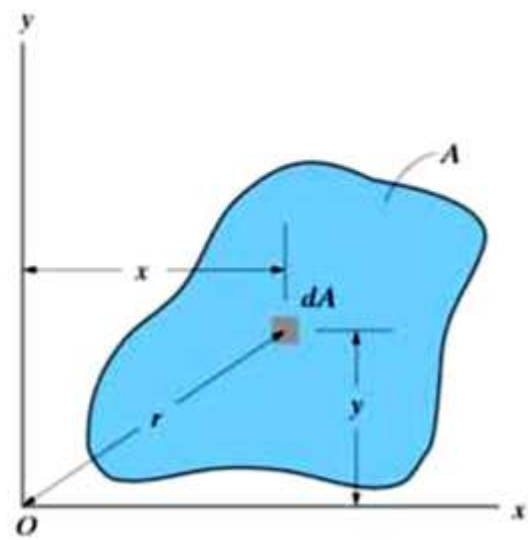
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{9.648}{6.84} = 1.41 \text{ m} \qquad \text{Ans}$$

THE MOMENT OF INERTIAFOR AREA

The inertia is the resistance of any object to a change.
The moment of inertia is a measure of an object's resistance to changes its rotation.
The moment of inertia for an area is important property in analysis and design of structural members.
The centroid represents the moment of area ($\int xdA$), while the moment

of area represents the second moment of area ($\int_A x^2 \, dA$).
Consider the figure:
The moment of inertia of the area about x & y axes are:

$$I_x = \int_A y^2 \, dA$$
$$I_y = \int_A x^2 \, dA$$



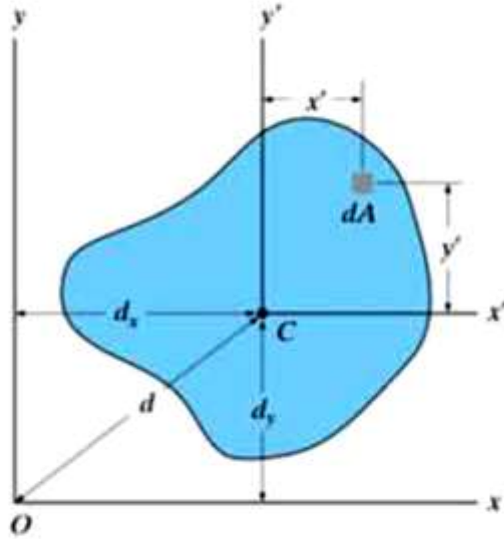
The polar moment of inertia J_o is:

$$J_O = \int_A r^2 \, dA = I_x + I_y$$

The moment of inertia is alawys positive (product of distance squered and area), and the units are length raised to the fourth power e.g. m^4, mm^4,….

Parallel Axis Theorem for an Area:

This theorem is used to find the moment of inertia about an axis parallel to the axis passing through the centroid.
This theorem says:



$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$I_y = \bar{I}_{y'} + Ad_x^2$$

$$J_O = \bar{J}_C + Ad^2$$

Radius of Gyration of an Area:

The redius of gyration is often used in design of columns. The formulas are:

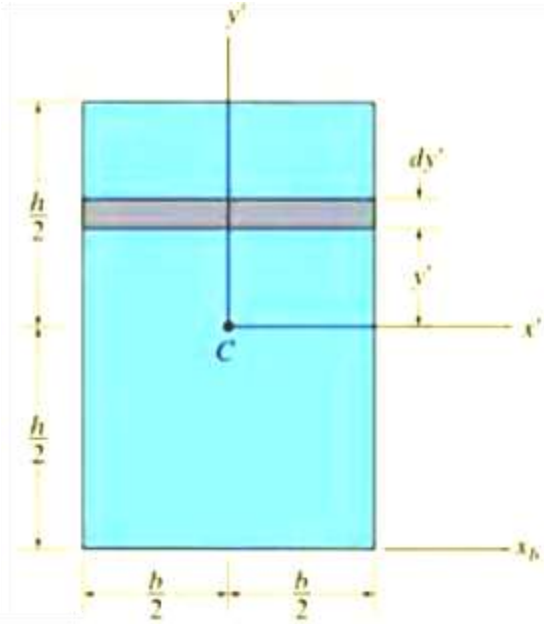
$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_O = \sqrt{\frac{J_O}{A}}$$

Moment of Inertia By Integration:

EX.1: Determine the moment of inertia for the rectangular shown with respect to (a) the centroidal x' axis, (b) the axis x_b passing through the base of the rectangle, and (c) the z' axis perpendicular to the x'-y' plane and passing through the centroid C.



Sol.:

(a)

$$\bar{I}_{x'} = \int_A y'^2\, dA = \int_{-h/2}^{h/2} y'^2 (b\, dy') = b \int_{-h/2}^{h/2} y'^2\, dy'$$

$$\bar{I}_{x'} = \frac{1}{12} b h^3 \qquad \textit{Ans.}$$

(b)

$$I_{x_b} = \bar{I}_{x'} + A d_y^2$$

$$= \frac{1}{12} b h^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3} b h^3 \qquad \textit{Ans.}$$

(c)

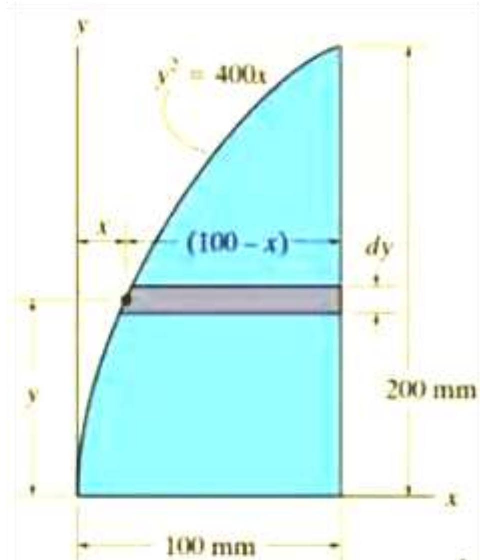
$$\bar{I}_{y'} = \frac{1}{12} h b^3$$

$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'} = \frac{1}{12} bh(h^2 + b^2) \qquad \textit{Ans.}$$

EX.2: Determine the moment of inertia of the shaded area shown about the x axis.

Sol.:

Case (1):

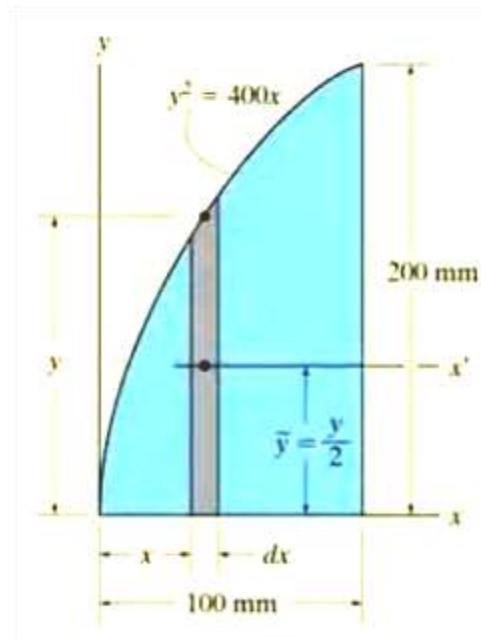


$$I_x = \int_A y^2\,dA = \int_0^{200\text{ mm}} y^2(100 - x)\,dy$$

$$= \int_0^{200\text{ mm}} y^2\left(100 - \frac{y^2}{400}\right)dy = \int_0^{200\text{ mm}} \left(100y^2 - \frac{y^4}{400}\right)dy$$

$$= 107(10^6)\text{ mm}^4$$

Case (2):



$$I_x = \int dI_x = \int_0^{100\text{ mm}} \frac{1}{3}y^3\,dx = \int_0^{100\text{ mm}} \frac{1}{3}(400x)^{3/2}\,dx$$

$$= 107(10^6)\text{ mm}^4$$

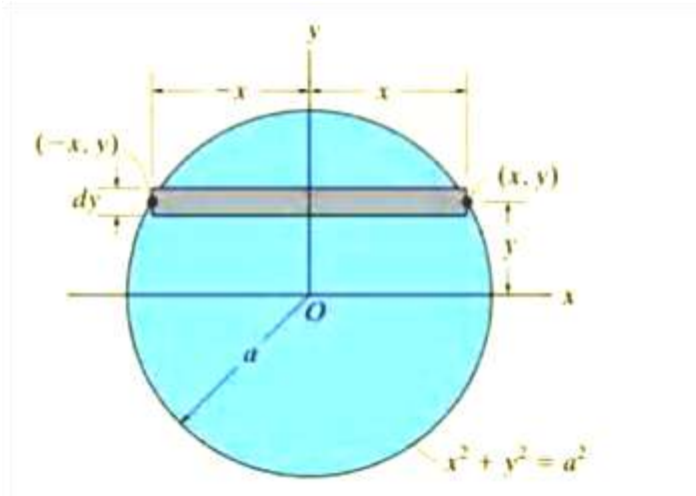
EX.3: Determine the moment of inertia with respect to the x axis for the circular area shown.

Sol.:

Case (1):

$$I_x = \int_A y^2\,dA = \int_A y^2(2x)\,dy$$

$$= \int_{-a}^{a} y^2\left(2\sqrt{a^2 - y^2}\right) dy = \frac{\pi a^4}{4}$$



Case (2):

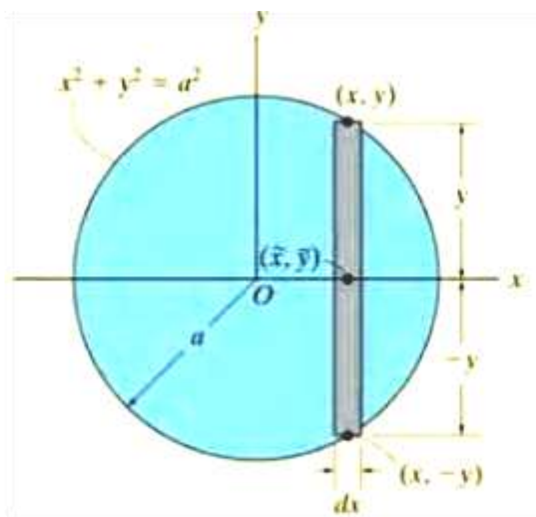
$\bar{I}_{x'} = \frac{1}{12}bh^3$ for a rectangle, we have

$$dI_x = \frac{1}{12}dx(2y)^3$$

$$= \frac{2}{3}y^3\,dx$$

Integrating with respect to x yields

$$I_x = \int_{-a}^{a} \frac{2}{3}(a^2 - x^2)^{3/2}\,dx = \frac{\pi a^4}{4}$$



Moment of Inertia for Composite Areas:

The following procedures provides a method for determing the moment of inertia of a composite areas about a reference axis.

1. Divide the area into its composite parts and indicate the perpendicular distance from the centroid of each part to the reference axis.

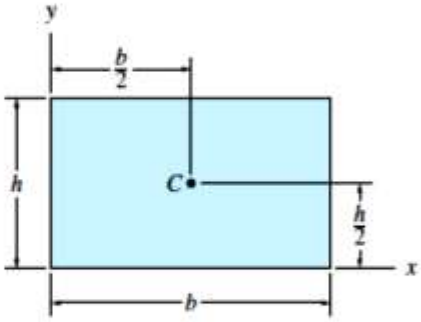
2. Find the moment of inertia of each part about its centroidal axis (use the table of moment of inertia). If the centroidal axis does not coincide with the reference axis; use the parallel axis theorem $I = \bar{I} + Ad^2$.

3. The moment of inertia of the total area about the reference axis is determing by summing the results of parts.

The table below shows the moment of inertia for more common shapes :

Moments of Inertia of Areas and Polar Moments of Inertia

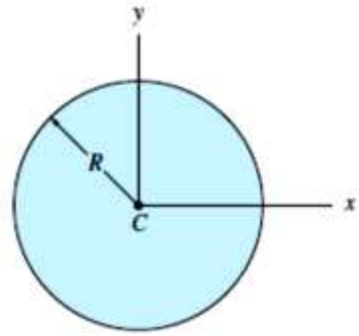
Rectangle



$\bar{I}_x = \frac{bh^3}{12}$ $\quad \bar{I}_y = \frac{b^3h}{12}$ $\quad \bar{I}_{xy} = 0$

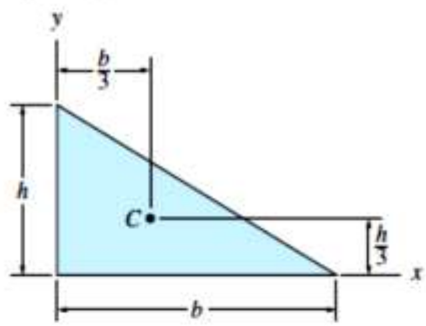
$I_x = \frac{bh^3}{3}$ $\quad I_y = \frac{b^3h}{3}$ $\quad I_{xy} = \frac{b^2h^2}{4}$

Circle



$\bar{I}_x = \bar{I}_y = \frac{\pi R^4}{4}$ $\quad \bar{I}_{xy} = 0$

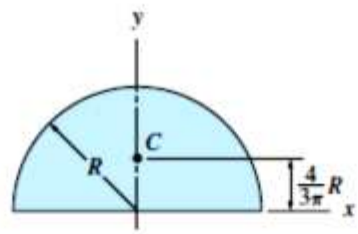
Right triangle



$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{36}$ $\quad \bar{I}_{xy} = -\frac{b^2h^2}{72}$

$I_x = \frac{bh^3}{12}$ $\quad I_y = \frac{b^3h}{12}$ $\quad I_{xy} = \frac{b^2h^2}{24}$

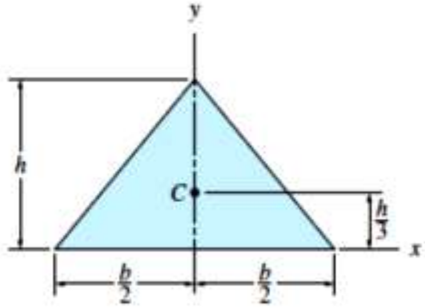
Semicircle



$\bar{I}_x = 0.1098R^4$ $\quad \bar{I}_{xy} = 0$

$I_x = I_y = \frac{\pi R^4}{8}$ $\quad I_{xy} = 0$

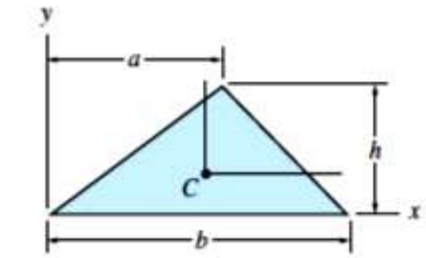
Isosceles triangle



$\bar{I}_x = \frac{bh^3}{36}$ $\quad \bar{I}_y = \frac{b^3h}{48}$ $\quad \bar{I}_{xy} = 0$

$I_x = \frac{bh^3}{12}$ $\quad I_{xy} = 0$

Triangle

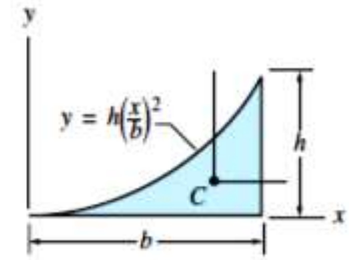


$\bar{I}_x = \frac{bh^3}{36}$ $\quad I_x = \frac{bh^3}{12}$

$\bar{I}_y = \frac{bh}{36}(a^2 - ab + b^2)$ $\quad I_y = \frac{bh}{12}(a^2 + ab + b^2)$

$\bar{I}_{xy} = \frac{bh^2}{72}(2a - b)$ $\quad I_{xy} = \frac{bh^2}{24}(2a + b)$

Half parabolic complement

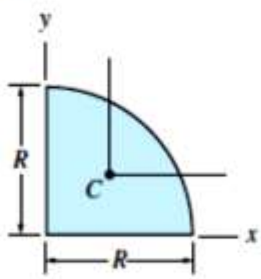


$\bar{I}_x = \frac{37bh^3}{2100}$ $\quad I_x = \frac{bh^3}{21}$

$\bar{I}_y = \frac{b^3h}{80}$ $\quad I_y = \frac{b^3h}{5}$

$\bar{I}_{xy} = \frac{b^2h^2}{120}$ $\quad I_{xy} = \frac{b^2h^2}{12}$

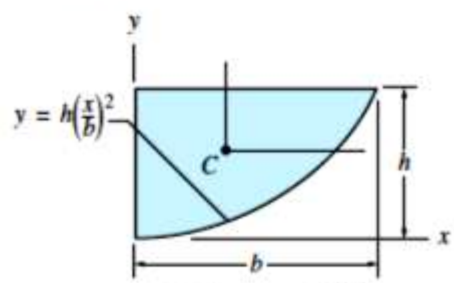
Quarter circle



$\bar{I}_x = \bar{I}_y = 0.05488R^4$ $\quad I_x = I_y = \frac{\pi R^4}{16}$

$\bar{I}_{xy} = -\ 0.01647R^4$ $\quad I_{xy} = \frac{R^4}{8}$

Half parabola

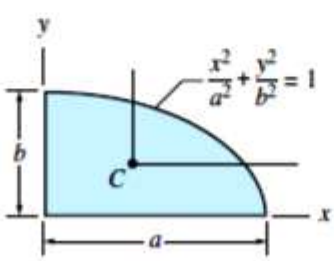


$\bar{I}_x = \frac{8bh^3}{175}$ $\quad I_x = \frac{2bh^3}{7}$

$\bar{I}_y = \frac{19b^3h}{480}$ $\quad I_y = \frac{2b^3h}{15}$

$\bar{I}_{xy} = \frac{b^2h^2}{60}$ $\quad I_{xy} = \frac{b^2h^2}{6}$

Quarter ellipse

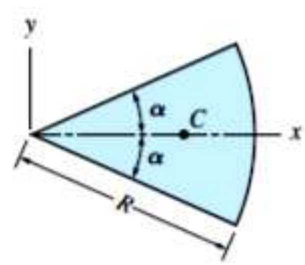


$\bar{I}_x = 0.05488ab^3$ $\quad I_x = \frac{\pi ab^3}{16}$

$\bar{I}_y = 0.05488a^3b$ $\quad I_y = \frac{\pi a^3b}{16}$

$\bar{I}_{xy} = -\ 0.01647a^2b^2$ $\quad I_{xy} = \frac{a^2b^2}{8}$

Circular sector

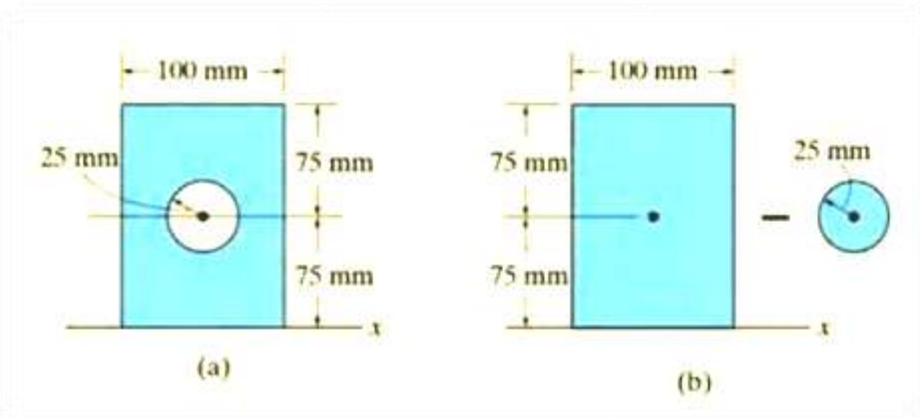


$I_x = \frac{R^4}{8}(2\alpha - \sin 2\alpha)$

$I_y = \frac{R^4}{8}(2\alpha + \sin 2\alpha)$

$I_{xy} = 0$

Ex.1: Determine the moment of inertia of the area shown about the x axis.



Sol.:

Circle

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{4}\pi(25)^4 + \pi(25)^2(75)^2 = 11.4(10^6)\ \text{mm}^4$$

Rectangle

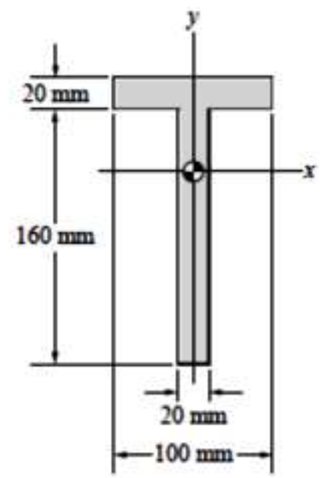
$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{12}(100)(150)^3 + (100)(150)(75)^2 = 112.5(10^6)\ \text{mm}^4$$

Summation. The moment of inertia for the area is therefore

$$I_x = -11.4(10^6) + 112.5(10^6)$$
$$= 101(10^6)\ \text{mm}^4 \qquad \textit{Ans.}$$

Ex.2: Determine the moment of inertia of the beam cross section about the x axis.

Sol.:



For the composite

$$X_c = \frac{X_{c1}A_1 + X_{c2}A_2}{A_1 + A_2} = 0$$

$$Y_c = \frac{Y_{c1}A_1 + Y_{c2}A_2}{A_1 + A_2}$$

Substituting, we get

$X_c = 0$ mm

$Y_c = 114.6$ mm

We now find I_x for each part about its center and use the parallel axis theorem to find I_x about C.

Part (1): $b_1 = 100$ mm, $h_1 = 20$ mm

$$I_{x'1} = \frac{1}{12}b_1h_1^3 = \frac{1}{12}(100)(20)^3 \text{ mm}^4$$

$I_{x'1} = 6.667 \times 10^4$ mm^4

$dy_1 = Y_{c1} - Y_c = 55.38$ mm

$I_{x1} = I_{x'1} + (dy_1)^2(A_1)$

$I_{x1} = 6.20 \times 10^6$ mm^4

Part (2) $b_2 = 20$ mm, $h_2 = 160$ mm

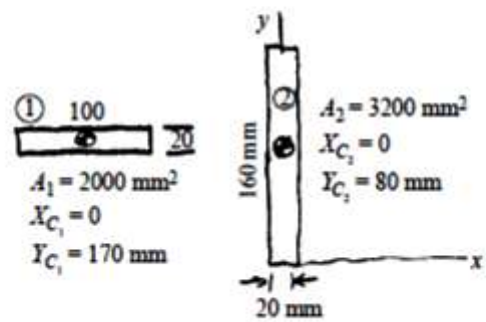
$$I_{x'2} = \frac{1}{12}(b_2)(h_2)^3 = \frac{1}{12}(20)(160)^3 \text{ mm}^4$$

$I_{x'2} = 6.827 \times 10^6$ mm^4

$dy_2 = Y_{c2} - Y_c = -34.61$ mm

$I_{x2} = I_{x'2} + (dy)^2A_2$

$I_{x2} = 1.066 \times 10^7$ mm^4



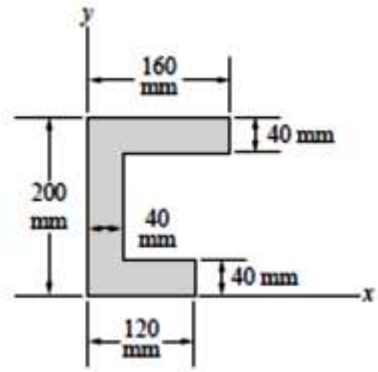
Finally, $I_x = I_{x1} + I_{x2}$

$I_x = 1.686 \times 10^7$ mm^4

for our composite shape.

Ex.3: Determine I_y and K_y of the shaded area shown.

Sol.:



Part (1): The top rectangle.

$$A_1 = 160(40) = 6.4 \times 10^3 \text{ mm}^2,$$

$$d_{x1} = \frac{160}{2} = 80 \text{ mm},$$

$$I_{yy1} = \left(\frac{1}{12}\right)(40)(160^3) = 1.3653 \times 10^7 \text{ mm}^4.$$

From which

$$I_{y1} = d_{x1}^2 A_1 + I_{yy1} = 5.4613 \times 10^7 \text{ mm}^4.$$

Part (2): The middle rectangle:

$$A_2 = (200 - 80)(40) = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{x2} = 20 \text{ mm},$$

$$I_{yy2} = \left(\frac{1}{12}\right)(120)(40^3) = 6.4 \times 10^5 \text{ mm}^4.$$

From which,

$$I_{y2} = d_{x2}^2 A_2 + I_{yy2} = 2.56 \times 10^6 \text{ mm}^4.$$

Part (3) The bottom rectangle:

$$A_3 = 120(40) = 4.8 \times 10^3 \text{ mm}^2,$$

$$d_{x3} = \frac{120}{2} = 60 \text{ mm},$$

$$I_{yy3} = \left(\frac{1}{12}\right) 40(120^3) = 5.76 \times 10^6 \text{ mm}^4$$

From which

$$I_{y3} = d_{x3}^2 A_3 + I_{yy3} = 2.304 \times 10^7 \text{ mm}^4$$

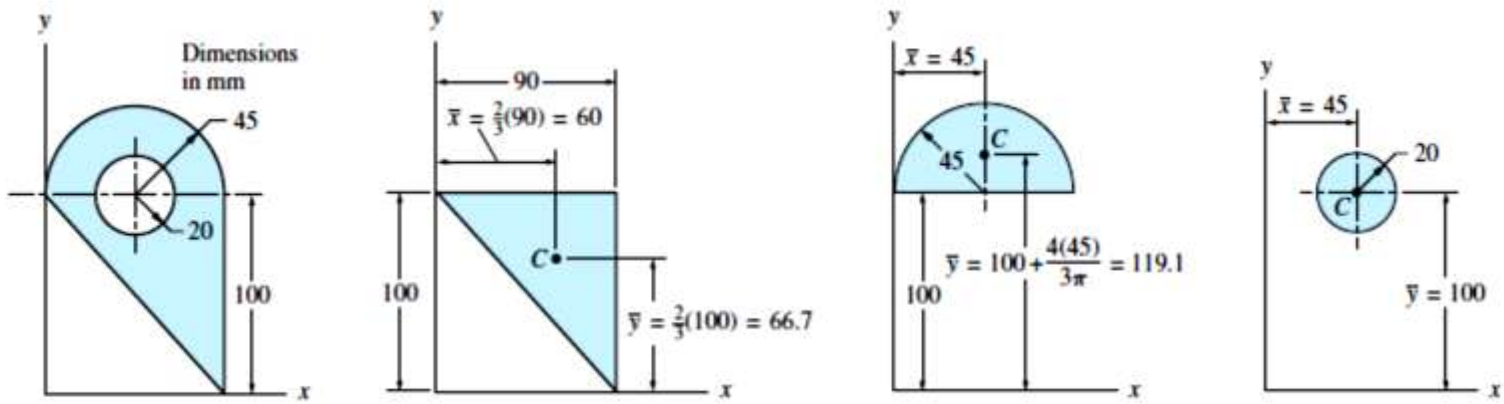
The composite:

$$I_y = I_{y1} + I_{y2} + I_{y3} = 8.0213 \times 10^7 \text{ mm}^4$$

$$k_y = \sqrt{\frac{I_y}{(A_1 + A_2 + A_3)}} = 70.8 \text{ mm}$$

Ex.4: For the area shown in Fig. (a), calculate the radii of gyration about the *x*- and y-axes.

Sol.:



Triangle

$$A = \frac{bh}{2} = \frac{90(100)}{2} = 4500 \text{ mm}^2$$

$$\bar{I}_x = \frac{bh^3}{36} = \frac{90(100)^3}{36} = 2.50 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (2.50 \times 10^6) + (4500)(66.7)^2 = 22.52 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{hb^3}{36} = \frac{100(90)^3}{36} = 2.025 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (2.025 \times 10^6) + (4500)(60)^2 = 18.23 \times 10^6 \text{ mm}^4$$

Semicircle

$$A = \frac{\pi R^2}{2} = \frac{\pi(45)^2}{2} = 3181 \text{ mm}^2$$

$$\bar{I}_x = 0.1098R^4 = 0.1098(45)^4 = 0.450 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (0.450 \times 10^6) + (3181)(119.1)^2 = 45.57 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{\pi R^4}{8} = \frac{\pi(45)^4}{8} = 1.61 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (1.61 \times 10^6) + (3181)(45)^2 = 8.05 \times 10^6 \text{ mm}^4$$

Circle

$$A = \pi R^2 = \pi(20)^2 = 1257 \text{ mm}^2$$

$$\bar{I}_x = \frac{\pi R^4}{4} = \frac{\pi(20)^4}{4} = 0.1257 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (0.1257 \times 10^6) + (1257)(100)^2 = 12.70 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{\pi R^4}{4} = \frac{\pi(20)^4}{4} = 0.1257 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (0.1257 \times 10^6) + (1257)(45)^2 = 2.67 \times 10^6 \text{ mm}^4$$

Composite Area

To determine the properties for the composite area, we superimpose the foregoing results (taking care to subtract the quantities for the circle) and obtain

$$A = \Sigma A = 4500 + 3181 - 1257 = 6424 \text{ mm}^2$$

$$I_x = \Sigma I_x = (22.52 + 45.57 - 12.70) \times 10^6 = 55.39 \times 10^6 \text{ mm}^4$$

$$I_y = \Sigma I_y = (18.23 + 8.05 - 2.67) \times 10^6 = 23.61 \times 10^6 \text{ mm}^4$$

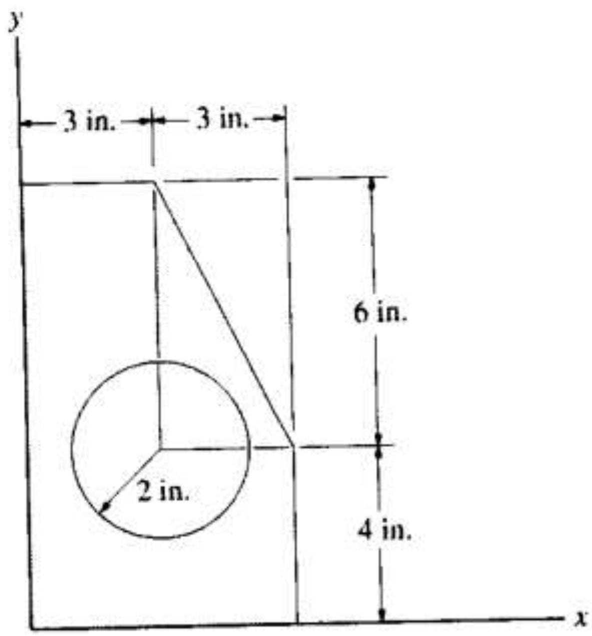
Therefore, for the radii of gyration we have

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{55.39 \times 10^6}{6424}} = 92.9 \text{ mm} \qquad \textit{Answer}$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{23.61 \times 10^6}{6424}} = 60.6 \text{ mm} \qquad \textit{Answer}$$

<u>Ex.5:</u> Determine the moments of inertia of the shaded area about the x and y axes.

<u>Sol.:</u>



$$I_x = \left[\frac{1}{12}(6)(10)^3 + 6(10)(5)^2\right] - \left[\frac{1}{36}(3)(6)^3 + \left(\frac{1}{2}\right)(3)(6)(8)^2\right]$$

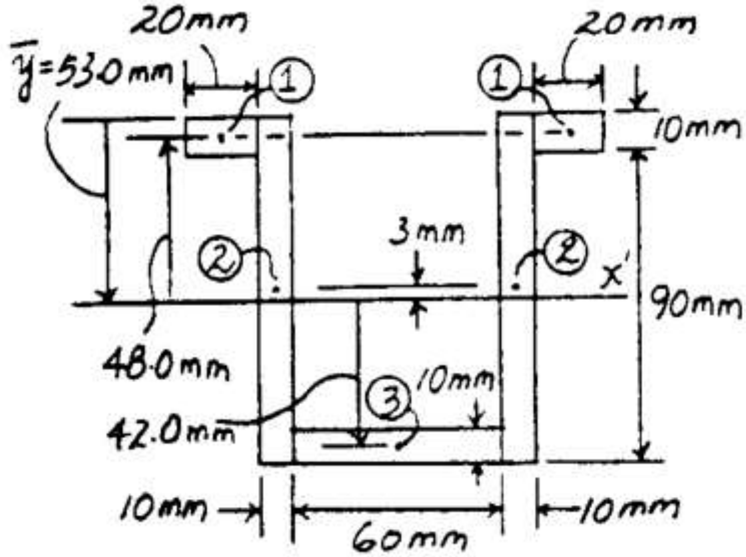
$$- \left[\frac{1}{4}\pi(2)^4 + \pi(2)^2(4)^2\right] = 1.192(10^3) \qquad \textbf{Ans}$$

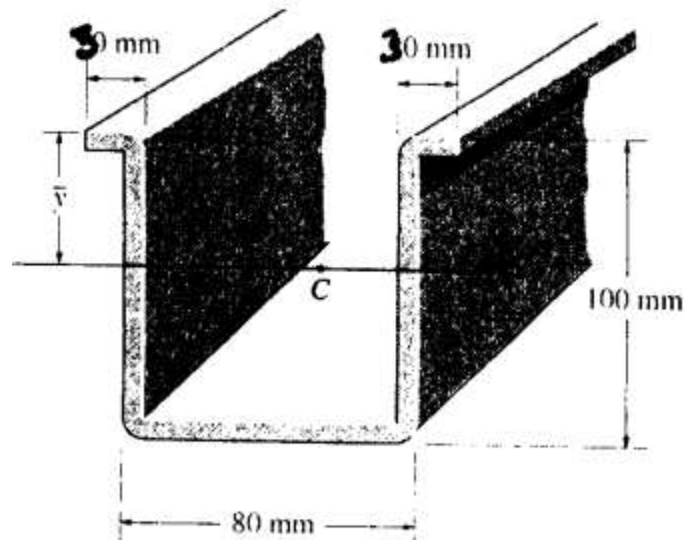
$$I_y = \left[\frac{1}{12}(10)(6)^3 + 6(10)(3)^2\right] - \left[\frac{1}{36}(6)(3)^3 + \left(\frac{1}{2}\right)(6)(3)(5)^2\right]$$

$$- \left[\frac{1}{4}\pi(2)^4 + \pi(2)^2(3)^2\right] = 364.8 \text{ in}^4 \qquad \textbf{Ans}$$

Ex.6: Determine the moments of inertia of the shaded area about the $\overline{x}$ axis. Each segment has a thickness of 10mm,$\overline{y}$=53mm from top.

Sol.:



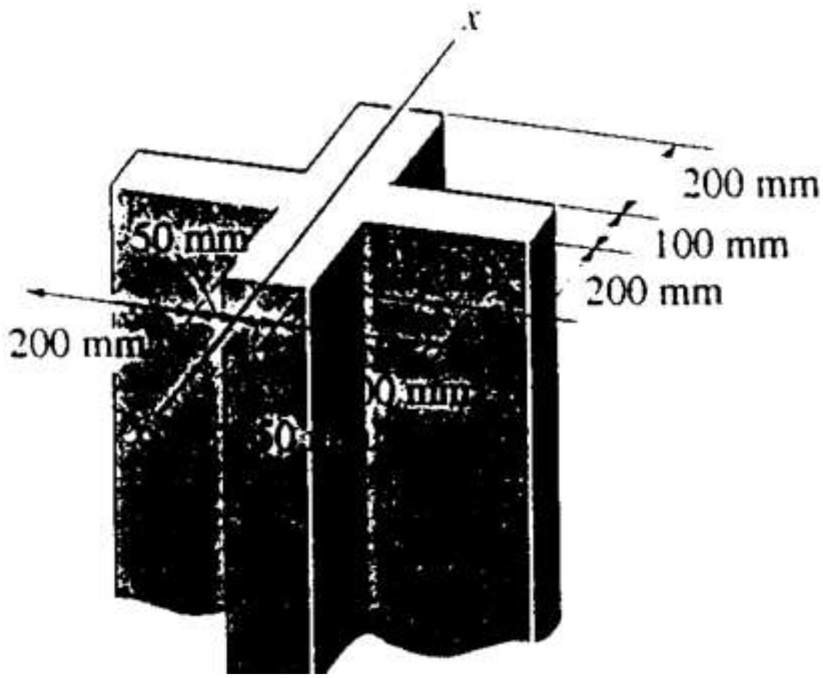


Segment	A_i (mm²)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm⁴)	$(Ad_y^2)_i$ (mm⁴)	$(I_{x'})_i$ (mm⁴)
1	40(10)	48.0	$\frac{1}{12}(40)(10^3)$	$0.9216(10^6)$	$0.9249(10^6)$
2	20(100)	3.00	$\frac{1}{12}(20)(100^3)$	$0.018(10^6)$	$1.6847(10^6)$
3	60(10)	42.0	$\frac{1}{12}(60)(10^3)$	$1.0584(10^6)$	$1.0634(10^6)$

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 3.673\left(10^6\right)\ \text{mm}^4 = 3.67\left(10^6\right)\ \text{mm}^4 \quad \textbf{Ans}$$

Ex.7: Determine the radius of gyration K_x for the column's cross-sectional area.



Sol.:

$$I_x = \frac{1}{12}(500)(100)^3 + 2\left[\frac{1}{12}(100)(200)^3 + (100)(200)(150)^2\right]$$

$$= 1.075(10^9)\ \text{mm}^4$$

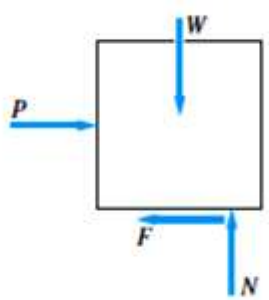
$$k_x = \sqrt{\frac{1.075(10^9)}{90(10^3)}} = 109\ \text{mm} \qquad \textbf{Ans}$$

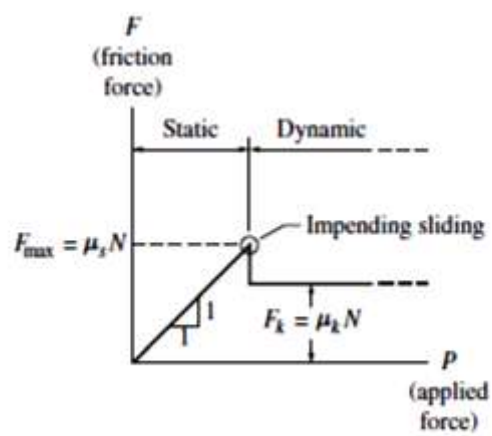
FRICTION

When a body slides or tend to slide on another body the force tangent to the contact surface which resists the motion, or the tendency toward motion, of one body relative to the other is defined as friction.

When two bodies are in contact and assumed to be smooth, the reaction of one body on the other is a force normal to the contact surface. In actual practice the contact surface is not smooth, and the reaction is resolved into two components, one perpendicular and the other tangent to the contact surface. The component tangent to the surface is called frictional force or the friction. When one body moves relative to another body, the resistance force between the bodies tangent to the contact surface is called kinetic friction.

The static frictional force is the minimum force required to maintain equilibrium or prevent relative motion between the bodies. The kinetic friction varies somewhat with the velocity. The variation of the frictional force versus the applied load on the body is shown in the figure below.





Here, we will study the static friction.

Coefficient of Friction:

The coefficient of static friction μ is the ratio of the maximum static frictional force F_s to the normal force N, or

$$\mu = \frac{Fs}{N}$$

The coefficient of static friction is experimentally determined, and depends on the materials from which the contact bodies are made. The table below show the values of coefficient of static friction obtained by experiments on dry surface:

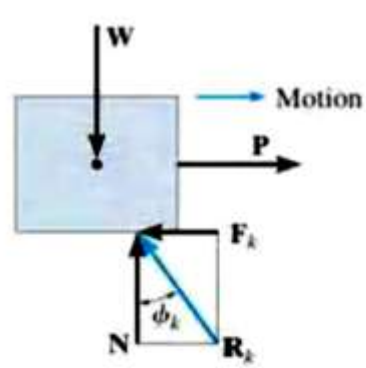
Typical Values for μ_s

Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Aluminum on aluminum	1.10–1.70

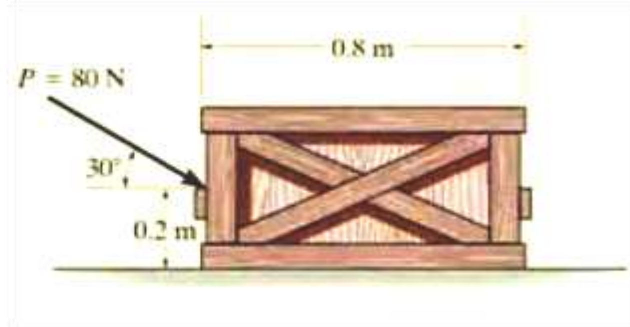
Angle of Friction:

The angle ø which R makes with n is defined as the angle of friction.

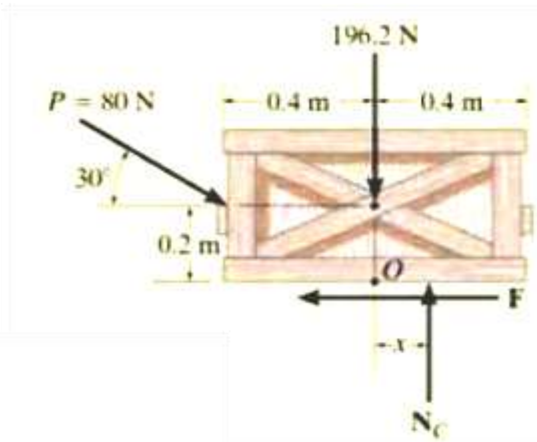
$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$



EX.1: The uniform crate shown has a mass of 20kg. if a force P = 80N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.



Sol.:



$$\xrightarrow{+} \Sigma F_x = 0: \qquad 80 \cos 30^\circ \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} = 0$$

$$\circlearrowleft + \Sigma M_O = 0: \qquad 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

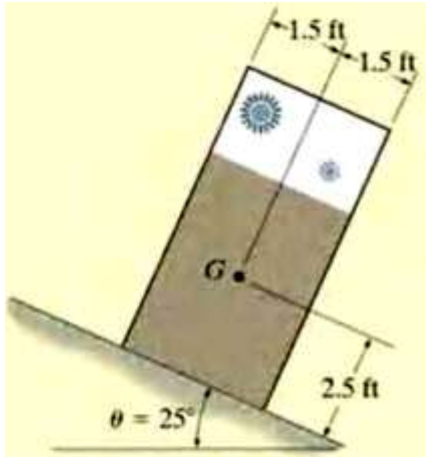
$$F = 69.3 \text{ N}$$

$$N_C = 236 \text{ N}$$

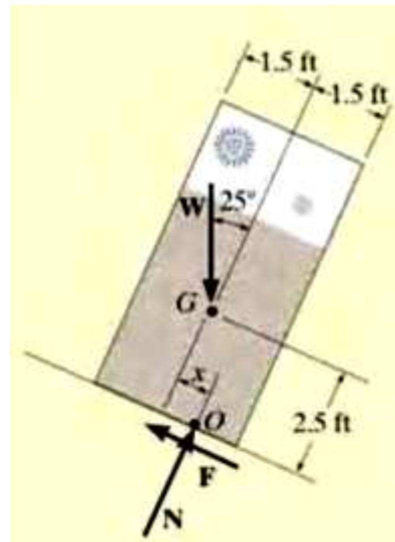
$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236 \text{ N}) = 70.8$ N. Since $F = 69.3 \text{ N} < 70.8$ N, the crate will *not slip*, although it is very close to doing so.

Ex.2: Determine the static coefficient of friction between a block shown and the surface.



Sol.:



$+\searrow\Sigma F_x = 0;\qquad W \sin 25^\circ - F = 0 \qquad (1)$

$+\nearrow\Sigma F_y = 0;\qquad N - W \cos 25^\circ = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_O = 0;\ -W \sin 25^\circ(2.5\text{ ft}) + W \cos 25^\circ(x) = 0 \qquad (3)$

Since slipping impends at $\theta = 25^\circ$, using Eqs. 1 and 2, we have

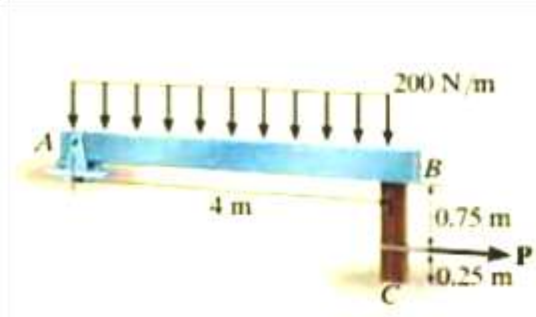
$F_s = \mu_s N;\qquad W \sin 25^\circ = \mu_s(W \cos 25^\circ)$

$\mu_s = \tan 25^\circ = 0.466$ *Ans.*

NOTE: From Eq. 3, we find $x = 1.17$ ft. Since 1.17 ft $<$ 1.5 ft,

The block will slip before it can tip.

Ex.3: Beam AB is subjected to a uniform load of 200N/m and is supported at B by post BC as shown. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force P needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.

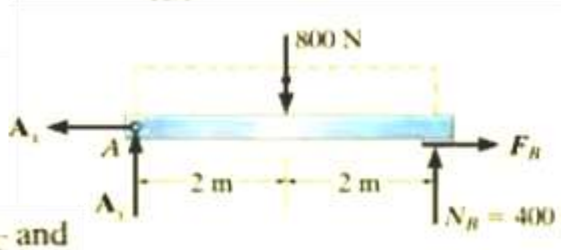


Sol.:

$\xrightarrow{+} \Sigma F_x = 0;\qquad P - F_B - F_C = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0;\qquad N_C - 400\text{ N} = 0 \qquad (2)$

$\circlearrowleft + \Sigma M_C = 0;\qquad -P(0.25\text{ m}) + F_B(1\text{ m}) = 0 \qquad (3)$



(Post Slips at B and Rotates about C.) This requires $F_C \le \mu_C N_C$ and

$F_B = \mu_B N_B;\qquad F_B = 0.2(400\text{ N}) = 80\text{ N}$

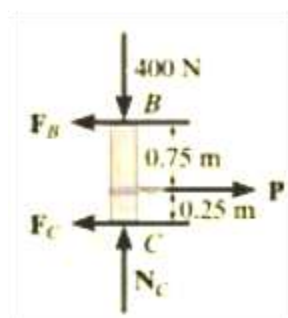
Using this result and solving Eqs. 1 through 3, we obtain

$$P = 320\text{ N}$$
$$F_C = 240\text{ N}$$
$$N_C = 400\text{ N}$$

Since $F_C = 240\text{ N} > \mu_C N_C = 0.5(400\text{ N}) = 200\text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at C and Rotates about B.) Here $F_B \le \mu_B N_B$ and

$F_C = \mu_C N_C;\qquad F_C = 0.5 N_C \qquad (4)$

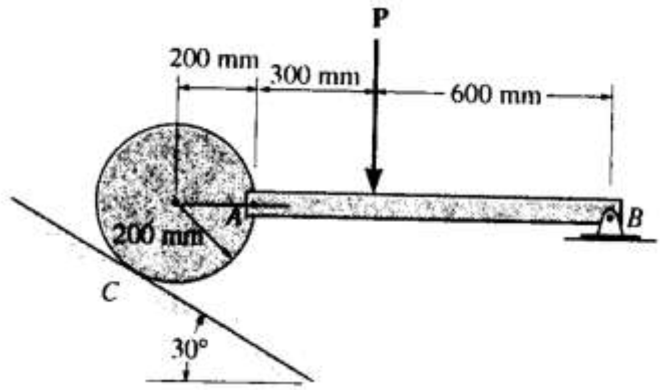


Solving Eqs. 1 through 4 yields

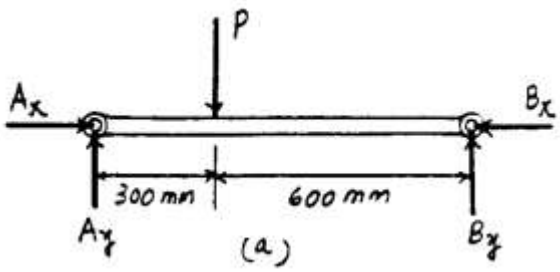
$$P = 267\text{ N} \qquad \textit{Ans.}$$
$$N_C = 400\text{ N}$$
$$F_C = 200\text{ N}$$
$$F_B = 66.7\text{ N}$$

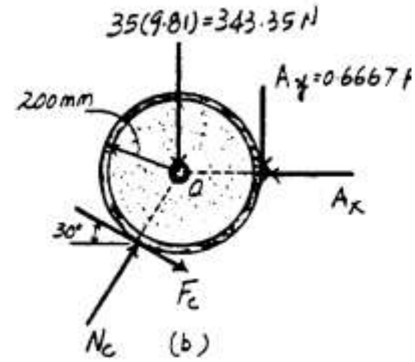
Obviously, this case occurs first since it requires a *smaller* value for P.

Ex.4: A 35-kg disk rests on an inclined surface for which $\mu_s = 0.2$. Determine the maximum vertical force P that may be applied to link AB without causing the disk to slip at C.



Sol.:





Equations of Equilibrium : From FBD (a),

$$\curvearrowleft + \Sigma M_B = 0; \quad P(600) - A_y(900) = 0 \quad A_y = 0.6667P$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0 \quad N_C \sin 60^\circ - F_C \sin 30^\circ - 0.6667P - 343.35 = 0 \quad [1]$$

$$\curvearrowleft + \Sigma M_O = 0; \quad F_C(200) - 0.6667P(200) = 0 \quad [2]$$

Friction : If the disk is on the verge of moving, slipping would have to occur at point C. Hence, $F_C = \mu_s N_C = 0.2N_C$. Substituting this value into Eqs. [1] and [2] and solving, we have

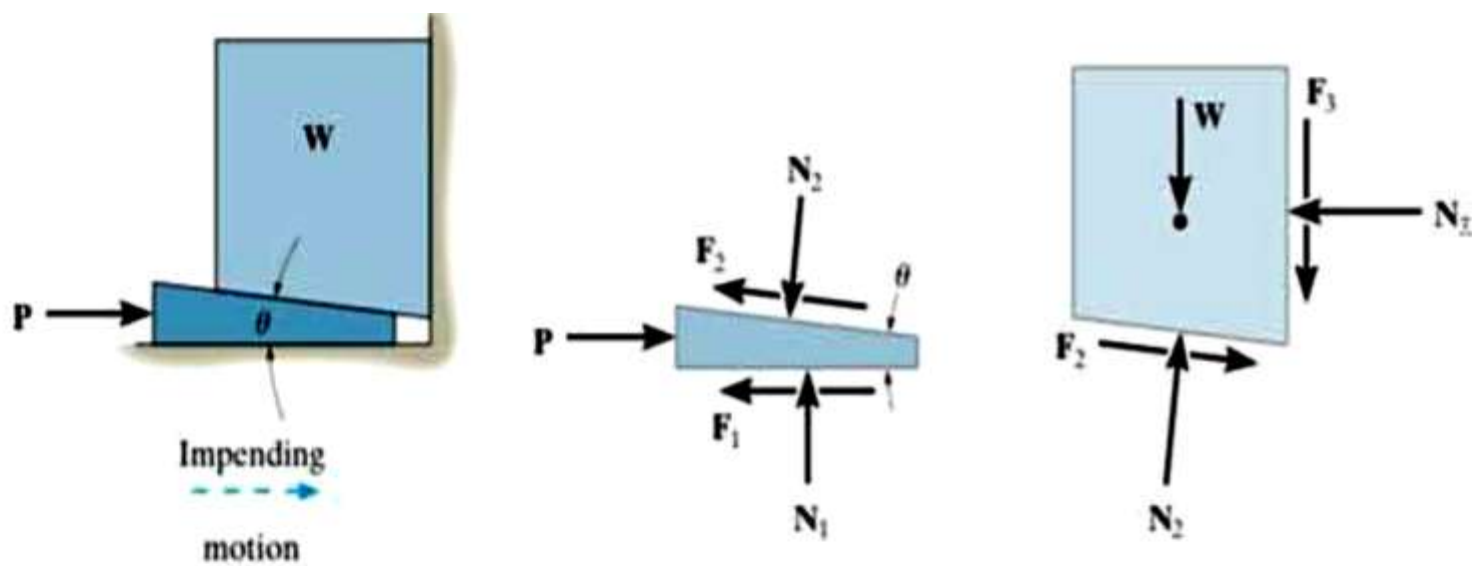
$$P = 182 \text{ N} \qquad \textbf{Ans}$$
$$N_C = 606.60 \text{ N}$$

Wedges:

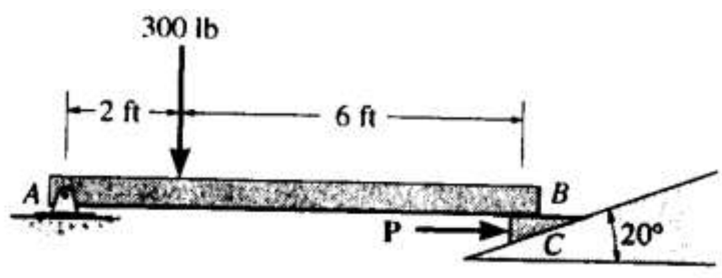
A wedge is a simple machine which is often used to transform an applied force into much larger forces, directed approximately at right angles to the applied force. Also wedges can be used to give small displacement or adjustments to heavy loads.

Notes:

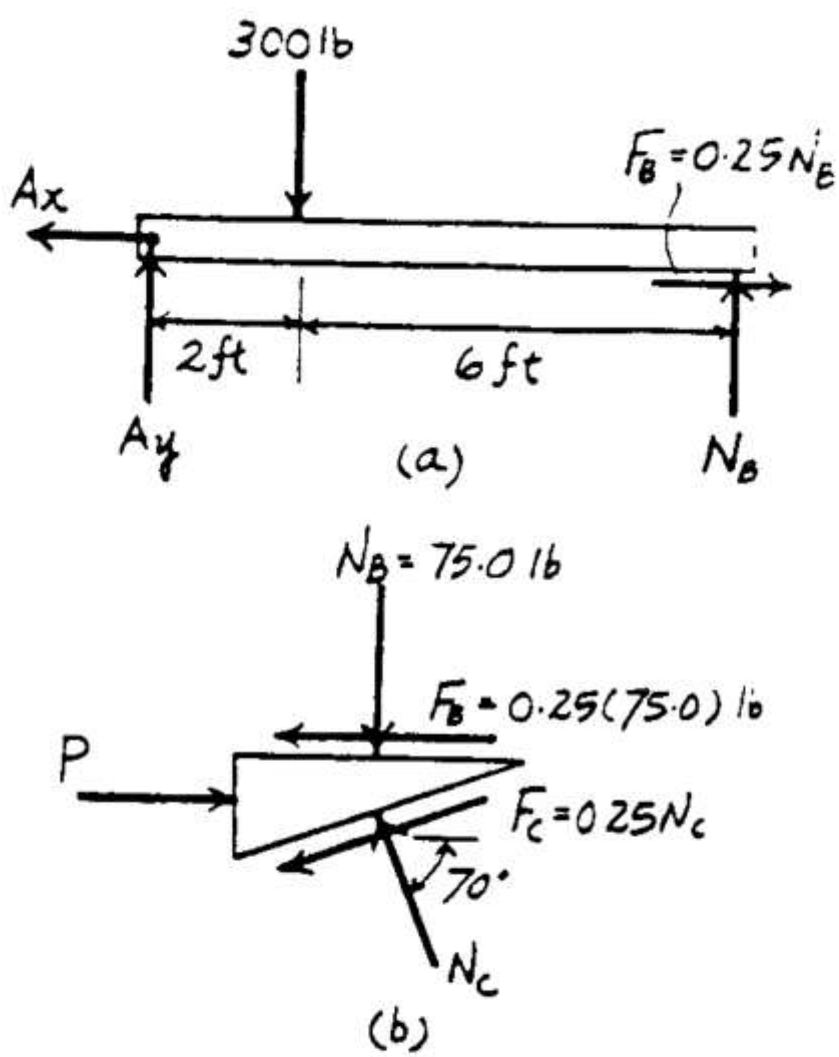
- Weight of wedge is neglected.
- The location of N is not important since neither block or wedge will tip.
-



EX.1: The beam is adjusted to the horizontal position by means of a wedge located at its right support. If the coefficient of static friction between the wedge and the two surfaces of contact is $\mu_s = 0.25$, determine the horizontal force P required to push the wedge forward. Neglect the weight and size of the wedge and the thickness of the beam.



Sol.:



Equations of Equilibrium and Friction : If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_B = \mu_s N_B = 0.25N_A$ and $F_C = \mu_s N_C = 0.25N_C$. From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(8) - 300(2) = 0 \quad N_B = 75.0 \text{ lb}$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_C \sin 70° - 0.25N_C \sin 20° - 75.0 = 0$$

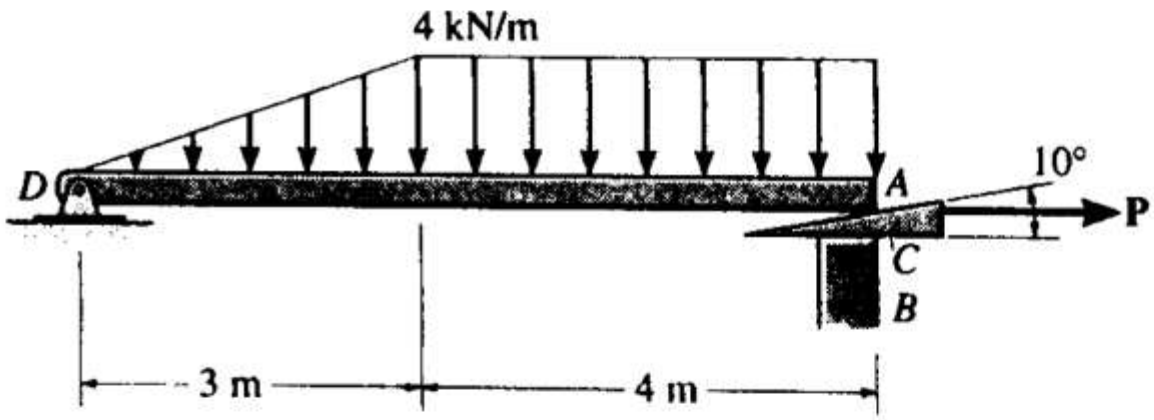
$$N_C = 87.80 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.25(75.0) - 0.25(87.80)\cos 20° - 87.80\cos 70° = 0$$

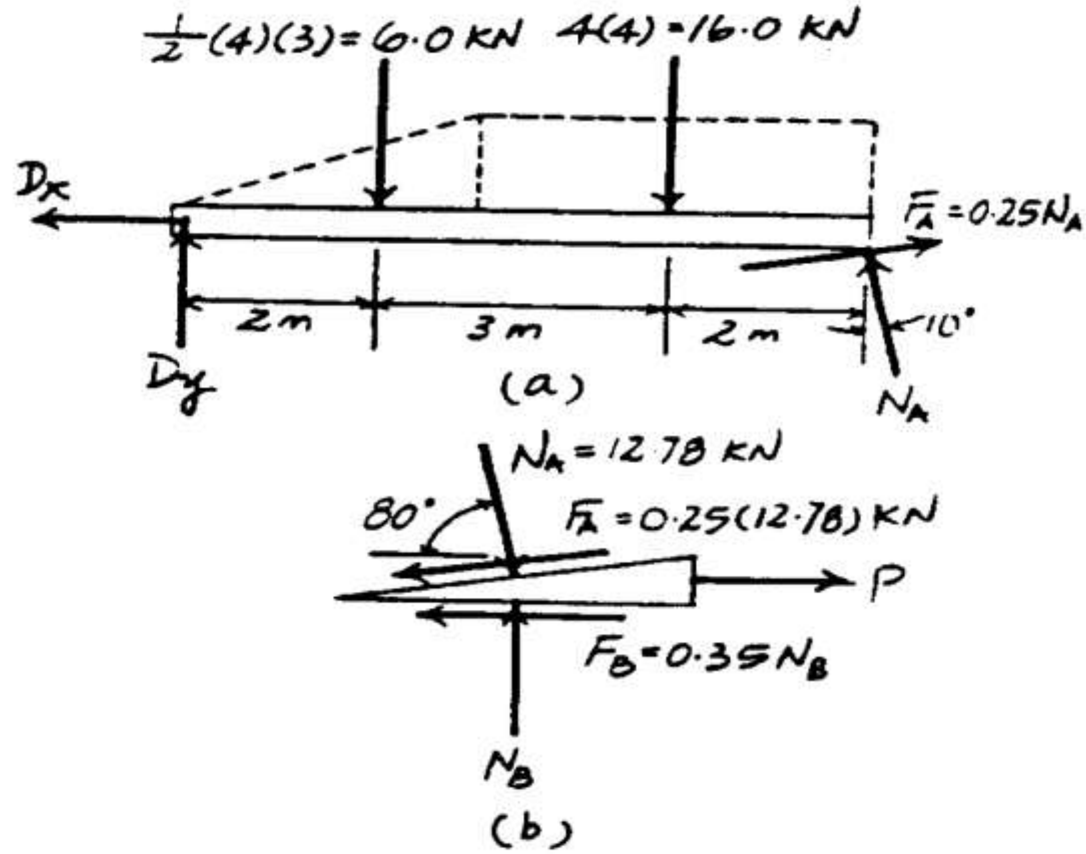
$$P = 69.4 \text{ lb}$$

Ans

EX.2: If the beam AD is loaded as shown, determine the horizontal force P which must be applied to the wedge in order to remove it from under the beam. The coefficient of static friction at the wedge's top and bottom surfaces are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. Neglect the weight and size of the wedge and the thickness of the beam.



Sol.:



Equations of Equilibrium and Friction : If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_{s\,A} N_A = 0.25N_A$ and $F_B = \mu_{s\,B} N_B = 0.35N_B$. From FBD (a),

$$\circlearrowleft + \Sigma M_D = 0; \quad N_A \cos 10^\circ (7) + 0.25N_A \sin 10^\circ (7) - 6.00(2) - 16.0(5) = 0$$

$$N_A = 12.78 \text{ kN}$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_B - 12.78\sin 80^\circ - 0.25(12.78)\sin 10^\circ = 0$$

$$N_B = 13.14 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P + 12.78\cos 80^\circ - 0.25(12.78)\cos 10^\circ - 0.35(13.14) = 0$$

$$P = 5.53 \text{ kN}$$

Ans

Since a force $P(> 0)$ is required to pull out the wedge, **the wedge will be self-locking when** $P = 0$.

Ans