Heat transfer

Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference.

The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange will take place under certain specified conditions. Supplements the first and second principles of thermodynamics by providing additional experimental rules that may be used to establish energy-transfer rates. As in the science of thermodynamics, the experimental rules used as basis of the subject of heat transfer are rather simple and easily expanded to encompass a variety of practical situations.

Heat transfer modes:

1. Conduction
2. Convection
3. Radiation

Conduction in Heat Transfer

Whenever a temperature gradient exists in a solid medium heat will flow from the higher-temperature to the lower-temperature region. The rate at which heat is transferred by conduction.

Examples of Conduction Problems

Conduction heat transfer problems are encountered in many engineering applications. The following examples illustrate the broad range of conduction problems.

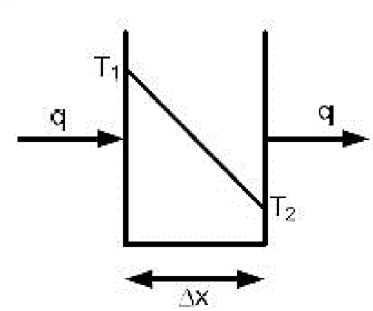
1) Design. A small electronic package is to be cooled by free convection. A heat sink consisting of fins is recommended to maintain the electronic components below a specified temperature. Determine the required number of fins, configuration, size and material.
2) Nuclear Reactor Core. In the event of coolant pump failure, the temperature of a nuclear element begins to rise. How long does it take before meltdown occurs?
3) Re-entry Shield. A heat shield is used to protect a space vehicle during re-entry. The shield ablates as it passes through the atmosphere. Specify the required shield thickness and material to protect a space vehicle during re-entry.
4) Casting. Heat conduction in casting is accompanied by phase change. Determine the transient temperature distribution and the interface motion of a solid-liquid front for use in thermal stress analysis.
5) Food Processing. In certain food processing operations conveyor belts are used to move food products through a refrigerated room. Use transient conduction analysis to determine the required conveyor speed.

Fourier's law

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region.

$$\frac{q}{A} \alpha \frac{\Delta T}{\Delta x}$$

For small difference in temperature in the thin wall



$$\frac{q}{A} \alpha \frac{dT}{dx}$$

The constant of proportionality is inserted is the thermal conductivity of solid material (k)

$$\frac{q}{A} = -k\frac{dT}{dx} \quad or \quad q = -kA\frac{dT}{dx}$$

This is called Fourier's law of heat conduction.

q is the heat-transfer rate

dT/dx is the temperature gradient in the direction of the heat flow. The minus sign is inserted to make clear that the heat must flow in a direction of temperature decrease.

A area

K thermal conductivity w/m.C

Thermal Conductivity

Thermal conductivity, (k), is the property of a material's ability to conduct heat. It appears primarily in Fourier's Law for heat conduction.

- ❖ Experimental measurements may be made to determine the thermal conductivity of different materials.
- ❖ In general, the thermal conductivity is strongly temperature-dependent.
- ❖ The numerical value of the thermal conductivity indicates how fast heat will flow in a given material.

Some values of thermal conductivity of various materials are shown below:

Gases	Liquids	Solids
H_2 = 0.175 W/m.°C He = 0.141 W/m.°C Air = 0.024 W/m.°C CO_2 = 0.0146 W/m.°C	H_2O = 0.556 W/m.°C Hg = 8.21 W/m.°C NH_3 = 0.540 W/m.°C Freon = 0.073 W/m.°C	Ag = 410 W/m.°C Cu = 385 W/m.°C AL = 202 W/m.°C Ni = 93 W/m.°C

Example (1)

One face of a copper plate 3 cm thick is maintained at 400◦C, and the other face is maintained at100◦C. How much heat is transferred through the plate? (K=370W/m· ◦C)

Solution

$$\frac{q}{A} = -k\frac{dT}{dx}$$

$$\frac{q}{A} = -k\frac{dT}{dx} = -\frac{(370)(100-400)}{0.03} = 3700000 w/m^2$$

Example (2)

A plane wall is 150mm thick and its wall area is 4.5m2. Its thermal conductivity is 9. 35 W/m.^{0}C and surface temperatures are steady at 150^{0}C and 45^{0}C. Determine:

A. Heat flow across the plane wall
B. Temperature gradient in the flow direction

Solution:

Wall thickness, dx = 150mm = 0.15m   Area, A = 4.5m^2

Temperature difference, dt = 45 – 150 = -105^{0}c

Thermal conductivity, k = 9.35 W/m.^{0}c

A. Applying the Fourier's Law of heat conduction

$$q = -kA\frac{dT}{dx} - kA\frac{T2 - T1}{L}$$

$$= -9.35 \times 4.5 \times \frac{-105}{0.15} = 29452.5W$$

B. Temperature gradient

$$\frac{dT}{dx} = -\frac{q}{kA} = \frac{29452.5}{9.35 * 4.5} = -700\ C/m$$

Homework

The following data relate to an oven:

Thickness of the inside wall of the oven=82.5mm, thermal conductivity of Wall insulation=0.044W/m°C , temperature on side and outside of wall =(175°C , 75°C) , energy dissipated by the electric coil with in the oven =40.5W , determine the area of surface

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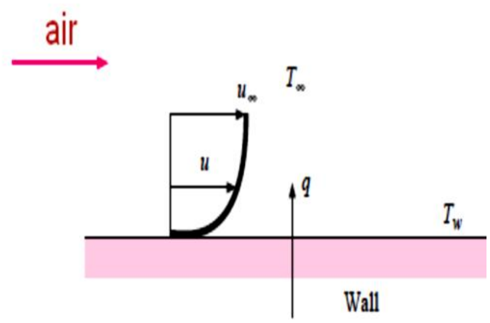
Convection in Heat Transfer

In general, convection heat transfer deals with thermal interaction between a surface and an adjacent moving fluid. Examples include the flow of fluid over a cylinder, inside a tube and between parallel plates. Convection also includes the study of thermal interaction between fluids. An example is a jet issuing into a medium of the same or a different fluid.

- ❖ Convection was considered as it related to the boundary conditions of a conduction problem.

- ❖ The hot metal will cool faster when placed in front of a fan (forced convection) than when exposed to still air (natural convection).
- ❖ If a heated plate were exposed to ambient room air without an external source of motion, a movement of the air would be experienced as a result of the density gradients near the plate. We call this natural, or free, convection as opposed to forced convection, which is experienced in the case of the fan blowing air over a plate.

$$q = hA\,(T_w - T_\infty)$$



h = convection heat transfer coeff.(W/m2.°C)

Tw= The temperature of the plate

T∞= the temperature of the fluid

A= surface area

Example (3)

Air at 20°C blows over a hot plate 50 by 75 cm maintained at 250◦C. The convection heat-transfer coefficient is 25 W/m2 · °C. Calculate the heat transfer.

Solution:

$q = hA\,(T_w - T_\infty)$

$= (25)(0.50)(0.75)(250-20)$

$= 2156$ W

Radiation in Heat Transfer

In contrast to the mechanisms of conduction and convection, where energy transfer through a material medium is involved, heat may also be transferred through regions where a perfect vacuum exists. The mechanism in this case is electromagnetic radiation. We shall limit our discussion to electromagnetic radiation that is propagated as a result of a temperature difference; this is called thermal radiation.

- ❖ In conduction and convection, the energy transfer through a material medium.
- ❖ Radiation: the energy can be transferred through vacuum by propagation of electromagnetic radiation.
- ❖ Black boby (ideal radiation): it's a body emit energy at a rate proportional to the fourth power of the absolute temperature (in Kelvin) of the body and directly proportional to its surface area. Thus

Stefan-Boltzmann law of thermal radiation is

$q = \sigma \in (T1^4 - T2^4)$

T1 is the temperature of radiative body (K)

T2 is the temperature of receiving body (K)

σ planks constant $5.667*10^{-8}$

$\in$ emissivity for black body =1

Example (4)

Two infinite black plates at 800°C and 300°C exchange heat by radiation. Calculate the heat transfer per unit area.

Solution:

$q/A = \sigma \in (T^4_1 - T^4_2)$

$= (5.669\times10^{-8})(1073^4 - 573^4)$

$= 69034.73 W/m^2$

Exercise

A carbon steel plate (thermal conductivity=45W/m.C) 600mm*900mm*25mm is maintained at 310C . air at 16 C blows over the hot plate . if convection heat transfer coefficient is $22W/m^2.C$ and 250W is lost from the plate surface by radiation calculate the inside plate temperature

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The general equation for heat transfer by conduction

The general equation for heat transfer by conduction can be derived by making energy balance on a solid system. For the element of thickness dx, the following energy balance may be made:

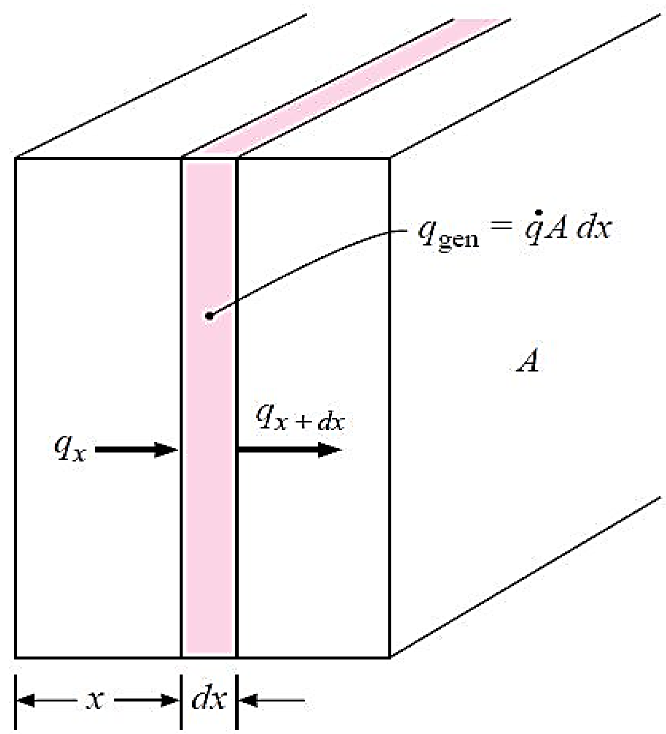
Energy conducted in left face + heat generated within element

=change in internal energy + energy conducted out right face

These energy quantities are given as follows:

$$\text{Energy in left face} = q_x = -kA\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$



Change in internal energy $= \rho c A \dfrac{\partial T}{\partial \tau} dx$

Energy out right face $= q_{x+dx} = -kA \left.\dfrac{\partial T}{\partial x}\right]_{x+dx}$

$$= -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

where

$\dot{q} =$ energy generated per unit volume, W/m^3

$c =$ specific heat of material, $J/kg \cdot {}^{\circ}C$

$\rho =$ density, kg/m^3

Combining the relations above gives

$$-kA\frac{\partial T}{\partial x} + \dot{q}A\,dx = \rho c A\frac{\partial T}{\partial \tau}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

or

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

This is the one-dimensional heat-conduction equation.

General three dimension heat transfer by conduction

(A) Cartesian coordinates

so that the general three-dimensional heat-conduction equation is

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

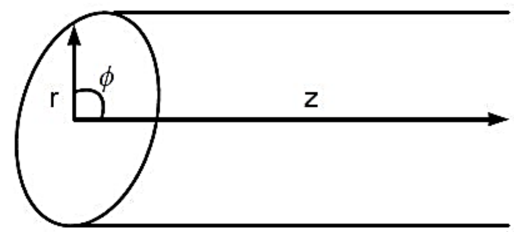
For constant thermal conductivity,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

where the quantity $\alpha = k/\rho c$ is called the *thermal diffusivity* of the material (m^2/s).

B) Cylindrical Coordinate

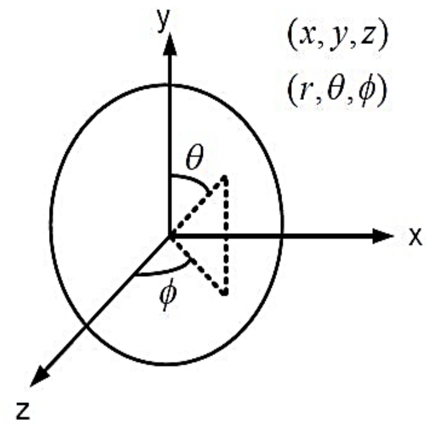
$x = r\,Cos\phi$
$y = r\,Sin\phi$
$z = z$



$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

C) Spherical Coordinate

$x = r\,Cos\theta + Sin\phi$
$y = r\,Sin\theta + Cos\phi$
$z = r\,Cos\theta$



The general equation is written:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

- **Steady-state one-dimensional heat flow (no heat generation):**

$$\frac{d^2T}{dx^2}=0$$

Note that this equation is the same as Equation (1-1) when $q=$ constant.

- **Steady-state one-dimensional heat flow in cylindrical coordinates (no heat generation):**

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}=0$$

- **Steady-state one-dimensional heat flow with heat sources:**

$$\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0$$

- **Two-dimensional steady-state conduction without heat sources:**

$$\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}=0$$

Example

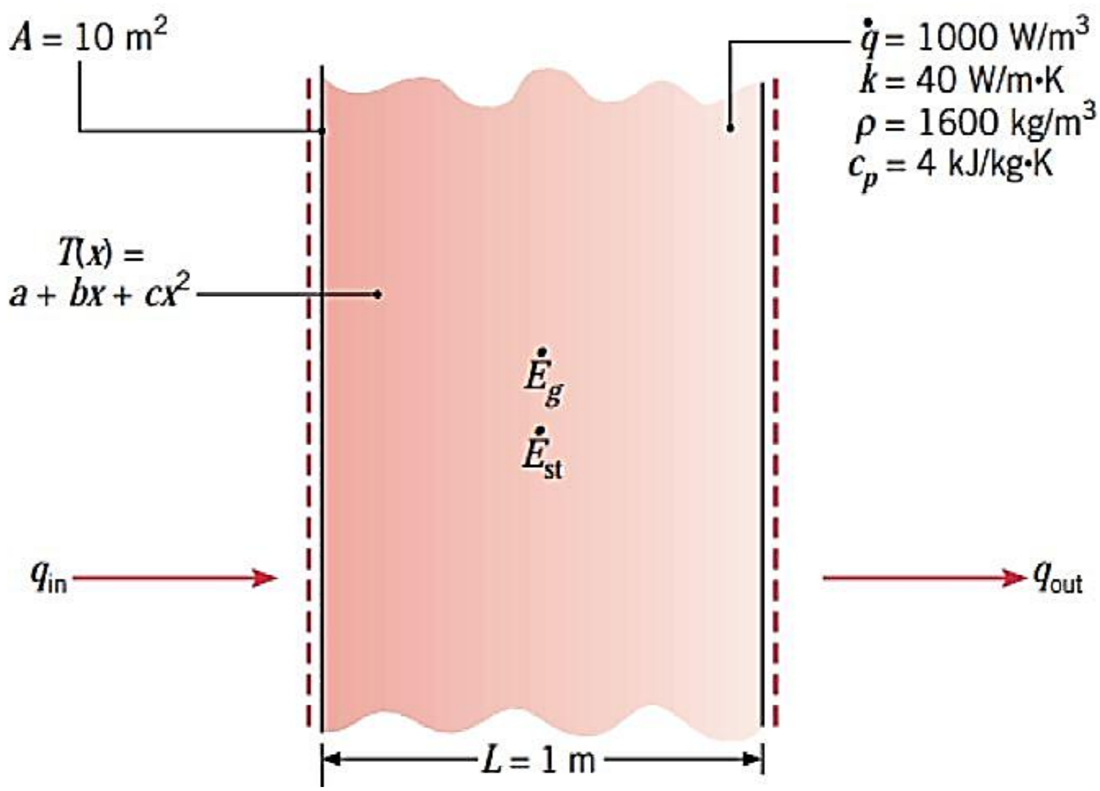
The temperature distribution across a wall (1m) thick at a certain instant of time is given as $T(x)=a+bx+cx2$ where T is in degrees Celsius and x is in meters, while a =900 $°C$, $b=-300\ °C/m\ and\ c=-50\ °C/m2$. A uniform heat generation $\dot{q}=1000\ W/m3$ is present in the wall of area 10m^2 having the properties $\rho=1600\ kg/m^3$, k =40 $W/m.K$ and $Cp=4kJ/kg.K$.

1. Determine the rate of heat transfer entering the wall (x=0) and leaving the wall (x=1 m).

2. Determine the rate of change of energy storage in the wall.

3. Determine the time rate of temperature change at x= 0.25, 0.5 m.

Solution :



Assumptions:

1- One-dimensional conduction in the x direction.

2- Isotropic medium with constant properties.

3- Uniform internal heat generation, ($\dot{q}$=1000 $W/m3$).

1- Heat rates entering $q_{in}\ (x = 0)$ and leaving $q_{out}\ (x = 1\ m)$ the wall.

$$q_{in} = -kA\frac{dT}{dx}\bigg|_{x=0} = -kA(b + 2cx)_{x=0}$$

$$q_{in} = -kAb$$

$$q_{in} = -40 * 10 * -300 = 120000\ W = 120KW$$

$$q_{out} = -kA\frac{dT}{dx}\bigg|_{x=L} = -kA(b + 2cx)_{x=L}$$

$$q_{out} = -kA(b + 2cL)$$

$$q_{out} = 40 * 10 * (-300 + 2 * -50 * 1)$$

$$q_{out} = 160000\ W = 160\ KW$$

2- Rate of change of energy storage in the wall $\dot{E}_{st}$.

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

Where $\dot{E}_g = \dot{q}V = \dot{E}_g = \dot{q}AL$

$$\dot{E}_{st} = \dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{in} + \dot{q}AL - \dot{E}_{out}$$

$$\dot{E}_{st} = 120000 + 1000 * 10 * 1 - 160000$$

$$\dot{E}_{st} = -30000\,W = -30\,KW$$

3- Time rate of temperature change at $x = 0, 0.25, and\ 0.5\ m$.

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho C_p}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}(b + 2cx) = 2c = 2 * -50 = -100\ °C/m^2$$

$$\frac{\partial T}{\partial t} = \frac{40}{1600 * 4} * (-100) + \frac{1000}{1600 * 4}$$

$$\frac{\partial T}{\partial t} = -0.625 + 0.156 = -0.468\ °C/s \quad for\ x = 0, 0.25\ and\ 0.5$$

Steady-State Conduction One Dimension

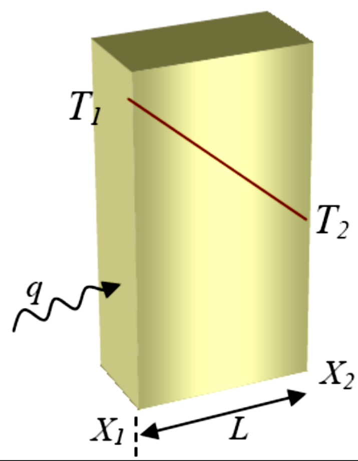
To examine the applications of Fourier's law of heat conduction to calculation of heat flow in some simple one-dimensional systems, we may take the following different cases:

1- The plane wall

A) One material

Using Fourier's law

$$\frac{dT}{dx} = \frac{\Delta T}{\Delta \chi} = \frac{T_2 - T_1}{\chi_2 - \chi_1}$$



$T_1 > T_2$

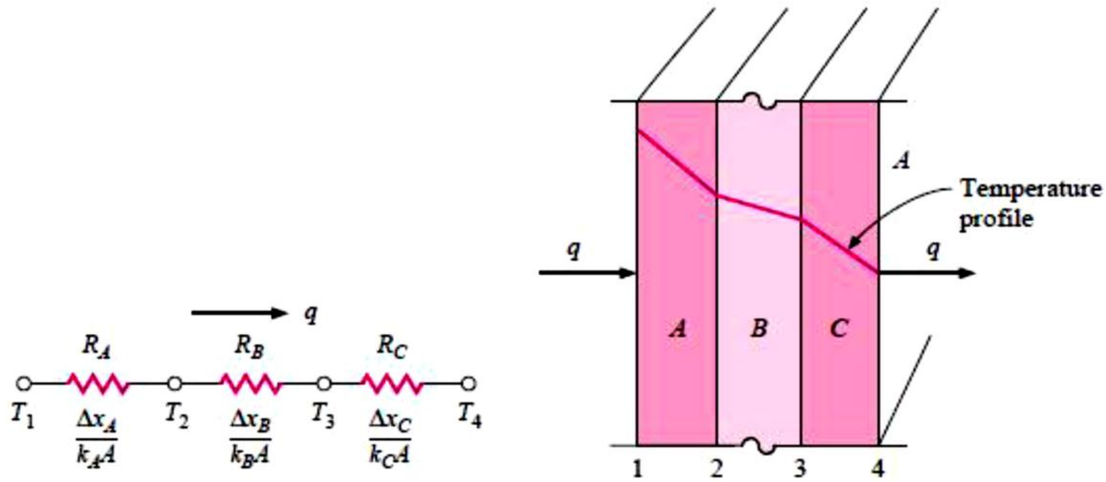
When the thermal conductivity is considered constant

$$q = -\frac{kA}{\Delta x}(T_2 - T_1)$$

B) More than one material (Composite wall)

If more than one material is present, as in the multilayer wall shown in Figure the analysis would proceed as follows: The temperature gradients in the three materials are shown, and the heat flow may be written

$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}$$



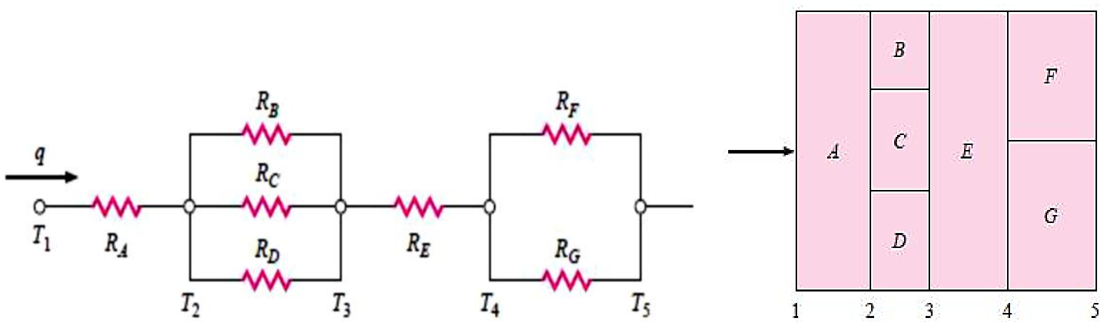
The heat flow must be the same through all sections, therefore

Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A}$$

The one-dimensional heat-flow equation for this type of problem may be written

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}}$$



$$\frac{1}{R_1}=\frac{1}{R_A};\quad \frac{1}{R_2}=\frac{1}{R_B}+\frac{1}{R_C}+\frac{1}{R_D};\quad \frac{1}{R_3}=\frac{1}{R_E};\quad \frac{1}{R_4}=\frac{1}{R_F}+\frac{1}{R_G}$$

$$\therefore\ \frac{1}{R_{th}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}$$

(R_{th} is the thermal resistances)

Example (5)

An outside wall of a building consists of 0.1m layer of common brick [k=0.69W/m.K] and 25mm layer of fiber glass [k=o.o5W/m.K]. Calculate the heat flow with through the wall for a 45°C temperature differences.

Solution:

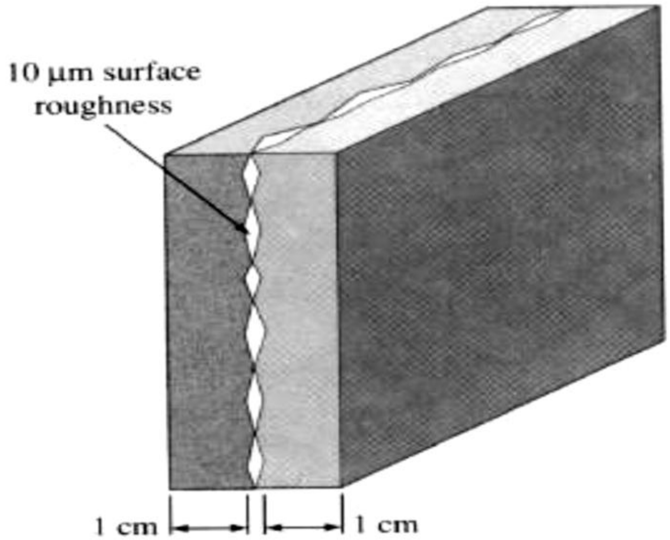
$$q=\frac{\Delta T}{\sum R_{th}}=\frac{\Delta T_{overall}}{\dfrac{\Delta x_b}{k_b A}+\dfrac{\Delta x_f}{k_f A}}$$

$$\Rightarrow\quad q=\frac{45}{\dfrac{0.1}{0.69}+\dfrac{0.025}{0.05}}=69.78\,W/m^2$$

Example

Two large aluminum plates (k = 240 W/m K), each 1 cm thick, with 10 μm surface roughness the contact resistance

Ri = 2.75 x 10-^{4} m^{2} K/W. The temperatures at the outside surfaces are 395°C and 405°C. Calculate the heat flux



Solution

The rate of heat flow per unit area, q" through the sandwich wall is

$$q'' = \frac{T_{s1} - T_{s3}}{R_1 + R_2 + R_3} = \frac{\Delta T}{(L/k)_1 + R_i + (L/k)_2}$$

The two resistances is equal to

(L/k) = (0.01 m)/(240 W/m.K) = 4.17 x 10^{-5} m^{2} K/W

Hence, the heat flux is

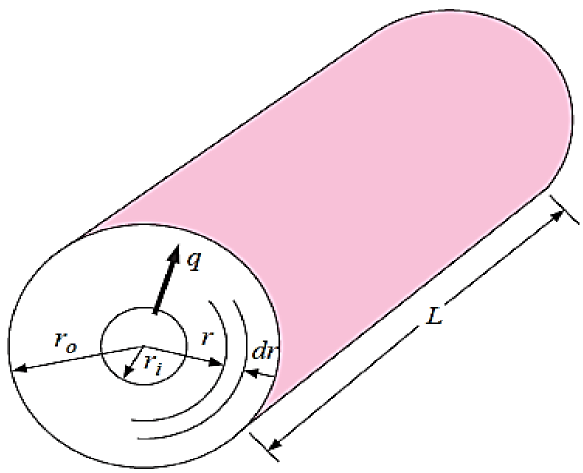
$$q'' = \frac{(405 - 395)^\circ\text{C}}{(4.17 \times 10^{-5} + 2.75 \times 10^{-4} + 4.17 \times 10^{-5})\ \text{m}^2\ \text{K/W}}$$
$$= 2.79 \times 10^{4}\ \text{W/m}^2\ \text{K}$$

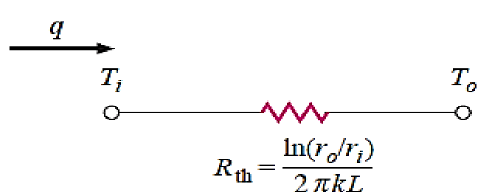
2- Radial systems

A) Cylindrical

i- One material

Consider a long cylinder of inside radius ri, outside radius ro, and length L. The inner side temperature is Ti, The outer side is T0, when the heat flows only in a radial direction. The area for heat flow in the cylindrical system is





$$A_r = 2\pi rL$$

So that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi\, krL \frac{dT}{dr}$$

$$\frac{q}{2\pi kL}\int_{r_i}^{r_0}\frac{dr}{r} = -\int_{T_i}^{T_0} dT$$

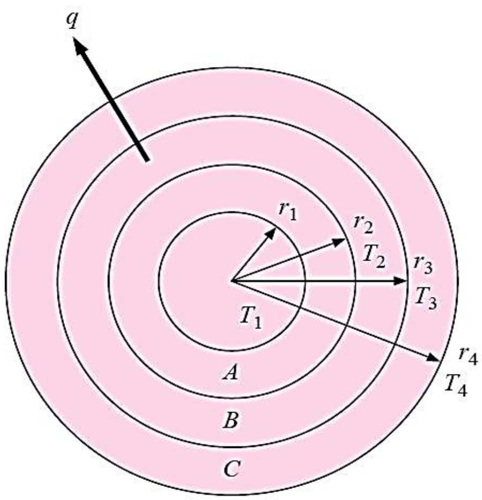
The solution is

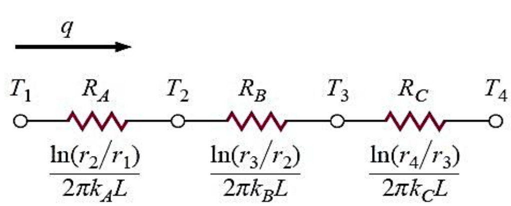
$$q = \frac{2\pi kL\,(T_i - T_o)}{\ln(r_o/r_i)}$$

and the thermal resistance in this case is

$$R_{th} = \frac{\ln(r_o/r_i)}{2\pi kL}$$

ii- Multi-Layer cylindrical wall





T =Ti at r = ri

T =To at r = ro

The solution to Equation

$$q = \frac{2\pi kL\,(T_i - T_o)}{\ln\,(r_o/r_i)}$$

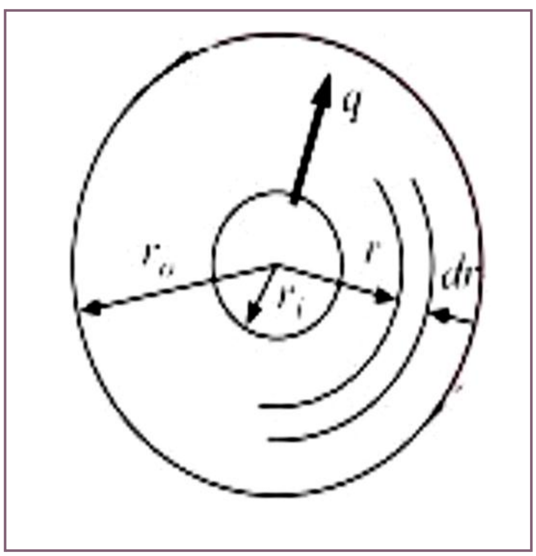
and the thermal resistance in this case is

$$R_{\text{th}} = \frac{\ln\,(r_o/r_i)}{2\pi kL}$$

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln\,(r_2/r_1)/k_A + \ln\,(r_3/r_2)/k_B + \ln\,(r_4/r_3)/k_C}$$

B) Spherical

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then



$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -4k\pi\, r^2 \frac{dT}{dr}$$

$$\frac{q}{4\pi\, k} \cdot \int_{r_i}^{r_0} \frac{dr}{r^2} = -\int_{T_i}^{T_0} dT$$

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o}$$

The thermal resistance in spherical system is:

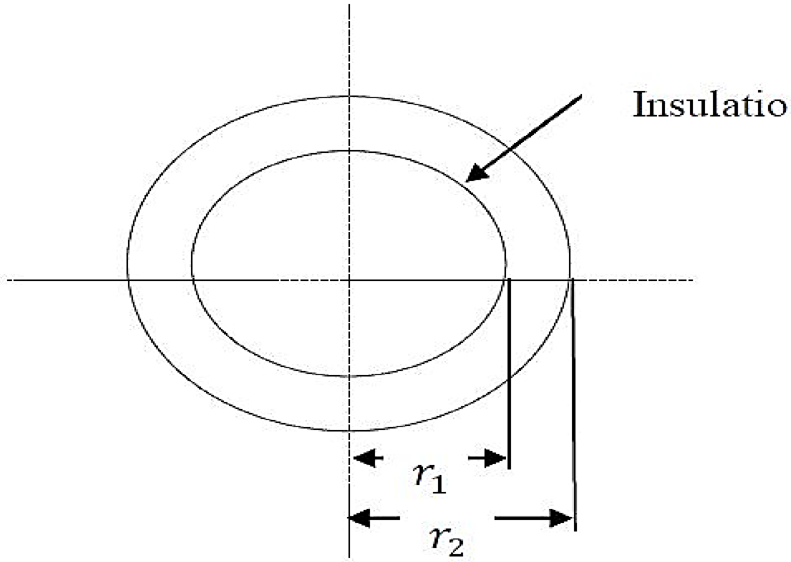
$$R_{th} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right)$$

The solution of heat equation is:

$$q_r = \frac{4\pi k(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Example

A spherical container having outer diameter (500 mm) is insulated by (100 mm) thick layer of material with thermal conductivity (k=0.03(1+0.006T)) W/m. °C, where T in °C. If the surface temperature of sphere is (-200 °C) and temperature of outer surface is (30 °C) determine the heat flow.



$$q = \frac{4\pi k(T_1 - T_2)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$r_1 = \frac{D}{2} = \frac{500}{2} = 250\ mm$$

$$r_2 = r_1 + 100 = 350\ mm$$

$$k = 0.3(1 + 0.006T) = 0.3(1 + 0.006\left(\frac{-200 + 30}{2}\right))$$

$$k = 0.147\ W$$

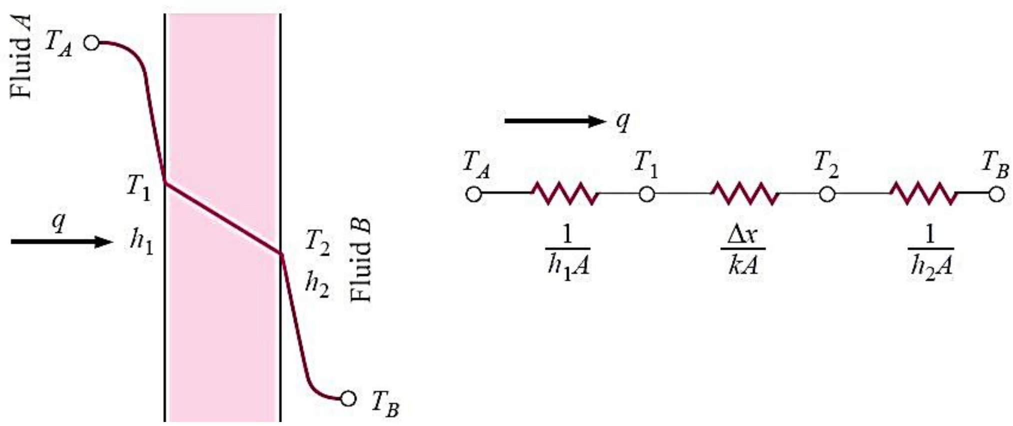
$$q = \frac{4\pi * 0.147(-200 - 30)}{\left(\frac{1}{0.025} - \frac{1}{0.035}\right)} = -37.14\ W$$

THE OVERALL HEAT-TRANSFER COEFFICIENT

We noted previously that a common heat transfer problem is to determine the rate of heat flow between two fluids, gaseous or liquid, separated by a wall. If the wall is plane and heat is transferred only by convection on both sides, the rate of heat transfer in terms of the two fluid temperatures is given by

$$q = h_1 A (T_A - T_1) = \frac{kA}{\Delta x}(T_1 - T_2) = h_2 A (T_2 - T_B)$$

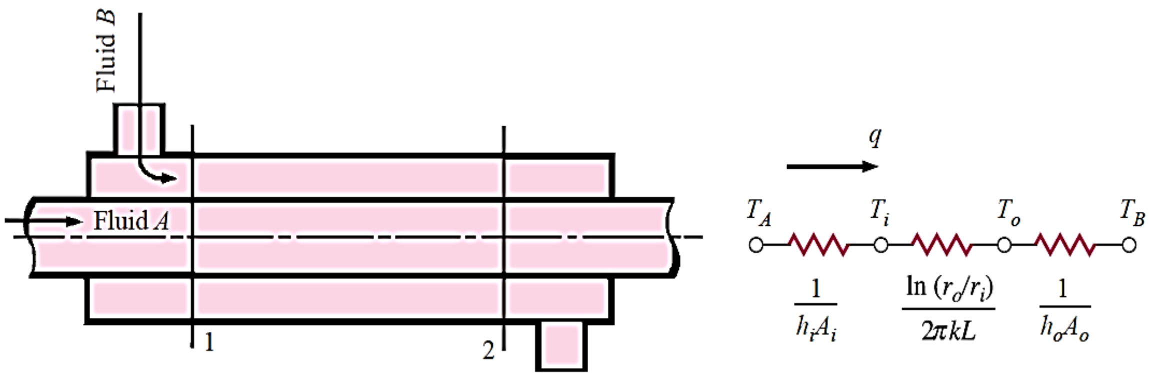
The heat-transfer process may be represented by the resistance network in Figure



and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A}$$

And the figure below show Resistance analogy for hollow cylinder with convection boundaries



Observe that the value $1/hA$ is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient U, defined by the relation

$$q = UA\Delta T_{\text{overall}}$$

where A is some suitable area for the heat flow. In accordance with Equation the overall heat-transfer coefficient would be

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

The overall heat-transfer coefficient is also related to the R value of Equation through

$$U = \frac{1}{R \text{ value}}$$

For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electric-resistance analogy would appear as in Figure where, again, TA and TB are the two fluid temperatures. Note that the area for convection is not the same for both fluids in this case, these areas depending on the inside tube diameter and wall thickness. The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}}$$

in accordance with the thermal network shown in Figure. The terms *Ai* and *Ao* represent the inside and outside surface areas of the inner tube. The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i}\dfrac{1}{h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o}}$$

The general notion, for either the plane wall or cylindrical coordinate system, is that

$$UA = 1/\sum R_{th} = 1/R_{th,overall}$$

Example

Water flows at 50◦C inside a 2.5-cm-inside-diameter tube such that *hi* =3500 W/m2 · ◦C. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · ◦C. The outside of the tube loses heat by free convection with *ho* =7.6W/m2 · ◦C. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20◦C.

Solution

There are three resistances in series for this problem L=1.0 m, di =0.025 m, and do =0.025+(2)(0.0008)=0.0266 m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ ^{\circ}\mathrm{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L}$$

$$= \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ ^{\circ}\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ ^{\circ}\mathrm{C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so*. This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad [a]$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)}$$

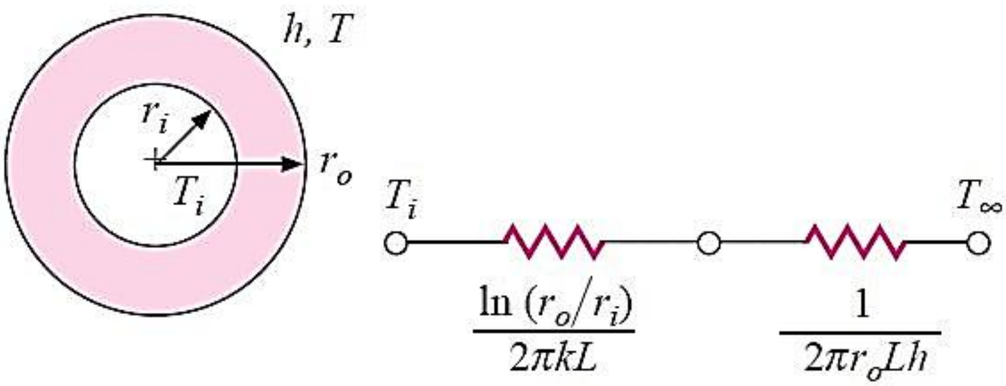
$$= 7.577\ \mathrm{W/m^2 \cdot {}^{\circ}C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (a), with

$$q = U A_o\, \Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19\ \mathrm{W}\ \text{(for 1.0 m length)}$$

CRITICAL THICKNESS OF INSULATION

Let us consider a layer of insulation which might be installed around a circular pipe,. The inner temperature of the insulation is fixed at *Ti*, and the outer



surface is exposed to a convection environment at *T*∞. From the thermal network the heat transfer is

$$q = \frac{2\pi L\,(T_i - T_\infty)}{\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}}$$

Now let us manipulate this expression to determine the outer radius of insulation *ro*, which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dr_o} = 0 = \frac{-2\pi L\,(T_i - T_\infty)\left(\dfrac{1}{kr_o} - \dfrac{1}{hr_o^2}\right)}{\left[\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}\right]^2}$$

which gives the result

$$r_o = \frac{k}{h}$$

expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be *increased* by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of *h* the convection heat loss may actually increase with the addition of insulation because of increased surface area.

Example

Calculate the critical radius of insulation for asbestos [k =0.17 W/^{0}C] surrounding a pipe and exposed to room air at 20◦C with h=3.0 W/m2 · ◦C. Calculate the heat loss from a 200◦C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567\ \text{m} = 5.67\ \text{cm}$$

The inside radius of the insulation is $5.0/2 = 2.5$ cm, so the heat transfer is calculated from Equation

$$\frac{q}{L} = \frac{2\pi\,(200-20)}{\dfrac{\ln(5.67/2.5)}{0.17} + \dfrac{1}{(0.0567)(3.0)}} = 105.7\ \text{W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200-20) = 84.8\ \text{W/m}$$

HEAT-SOURCE SYSTEMS

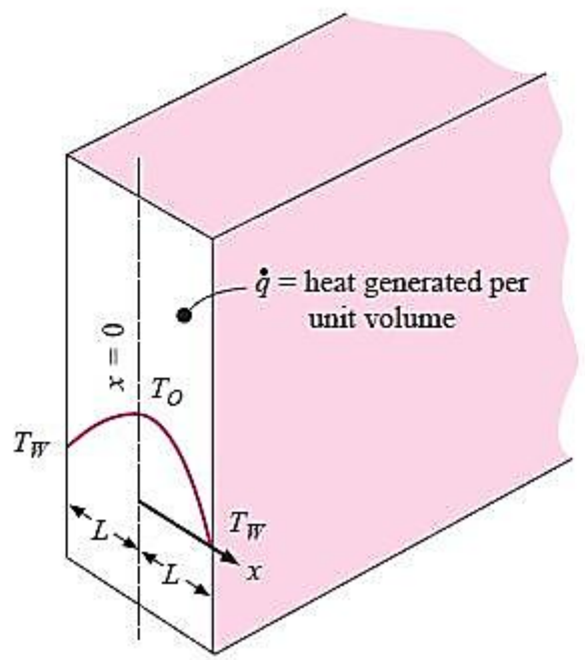
A number of interesting applications of the principles of heat transfer are concerned with systems in which heat may be generated internally.

1. Nuclear reactors are one example
2. electrical conductors
3. chemically reacting systems

At this point we shall confine our discussion to one-dimensional systems, or, more specifically, systems where the temperature is a function of only one space coordinate.

1- Plane Wall with Heat Sources

Consider the plane wall shown with uniformly distributed heat sources as shown in the figure. The heat generated per unit volume is The general equation is ***q·***



$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

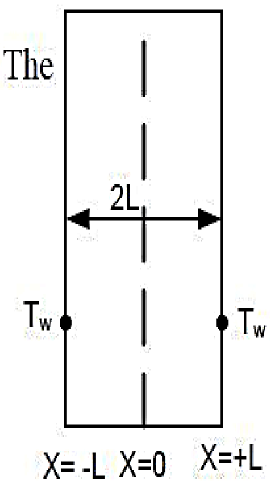
For one-dimensional, steady state with heat generation

$$\frac{d^2T}{dx^2}+\frac{q^\bullet}{k}=0 \Rightarrow \frac{d^2T}{dx^2}=-\frac{q^\bullet}{k} \Rightarrow by\int \Rightarrow \frac{dT}{dx}=-\frac{q^\bullet x}{k}+C_1 \Rightarrow by\int \Rightarrow$$

$$T=-\frac{q^\bullet x^2}{2k}+C_1X+C_2 \quad \textit{(general solution)}$$

The both sides of the plane wall are subjected to a constant temperature Tw. The Boundary conditions will be

$$T=T_w \quad \text{at } x=\pm L$$



By applying the boundary conditions above,

$$C_1=0$$

والثابت الثاني مباشرة عن طريق التعويض

$$T_W=-\frac{q^\bullet L^2}{2k}+C_2 \Rightarrow C_2=T_W+\frac{q^\bullet L^2}{2k}$$

$$\therefore\ T=-\frac{q^\bullet x^2}{2k}+T_W+\frac{q^\bullet L^2}{2k}$$

$$T_{\max} = T_W + \frac{q^{\bullet} L^2}{2k}$$

$$T = -\frac{q^{\bullet} x^2}{2k} + C_1 X + C_2$$

B.C.1 $x = 0, \;\; T = T_1$

B.C. 2 $x = L, \;\; T = T_2$

From B.C.1, $T_1 = C_2$

From B.C.2, $T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + C_2$

By sub. C_2, we get

$$T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + T_1 \Rightarrow$$

$$C_1 = \frac{(T_2 - T_1)}{L} + \frac{q^{\bullet} L}{2k}$$

Application

Consider the plane wall with uniformly distributed heat sources shown in Figure below. The thickness of the wall in the x direction is 2L, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one dimensional. The heat generated per unit volume is $\dot{q}$, and assume that the thermal conductivity does not vary with temperature. Derive an expression of the temperature distribution

Solution:

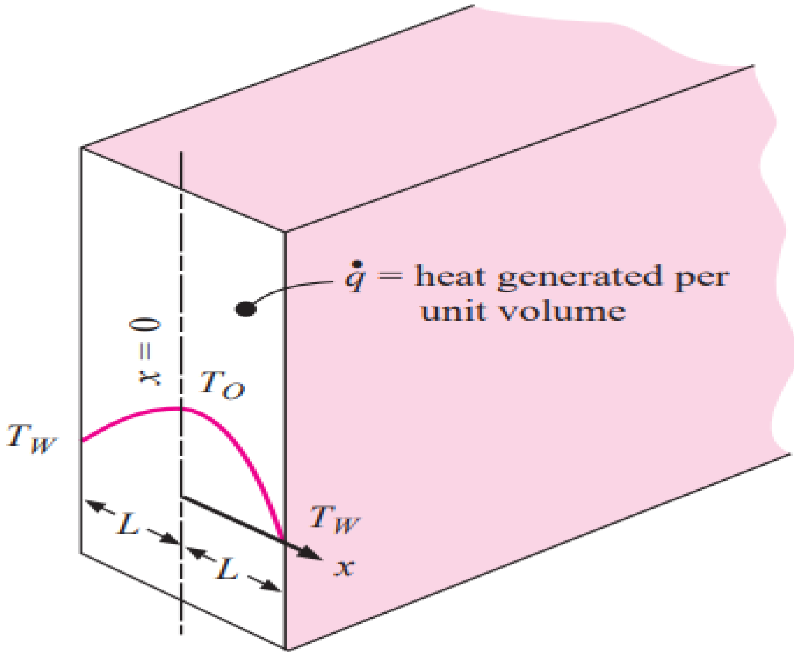
Assumption:

1- One-Dimension ($\partial/\partial y=0, \partial/\partial z=0$).

2- Steady state ($\partial/\partial t$=0).

3- Uniform heat generation ($\dot{q}$).

4- Homogeneous (k=$constant$).



$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0 \qquad \textit{integrate}$$

$$\frac{\partial T}{\partial x} + \frac{\dot{q}}{k}x = C_1 \qquad (1) \quad \textit{integrate again}$$

$$T + \frac{\dot{q}}{k}x^2 = C_1 x + C_2$$

$$T = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \qquad (2)$$

B.C1: $at\ x = 0$ $\quad T = T_0$ $\quad$ Sub. in Eq. (2)

$$T_0 = -\frac{\dot{q}}{2k}(0)^2 + C_1 * 0 + C_2$$

$C_2 = T_0$ Sub. in Eq. (2)

B.C2: $at\ x = \pm L$ $T = T_w$ Sub. in Eq. (2)

$$T_w = -\frac{\dot{q}}{2k}L^2 + C_1 L + T_0 \qquad (3)$$

$$T_w = -\frac{\dot{q}}{2k}L^2 - C_1 L + T_0 \qquad (4)$$

Subtract

$$0 = 0 + 2LC_1 + 0$$

$C_1 = 0$ Sub. in Eq. (2)

$$T = -\frac{\dot{q}}{2k}x^2 + T_0$$

$$T - T_0 = -\frac{\dot{q}}{2k}x^2 \qquad (5)$$

Example

Consider a shielding wall for a nuclear reactor. The wall receives a gamma-ray flux such that heat is generated within the wall according to the relation

$$\dot{q} = \dot{q}_0 e^{-ax}$$

where $\dot{q}0$ is the heat generation at the inner face of the wall exposed to the gamma-ray flux and a is a constant. Using this relation for heat generation, derive an expression for the temperature distribution in a wall of thickness L, where the inside and outside temperatures are maintained at Ti and $T0$, respectively. Also obtain an expression for the maximum temperature in the wall.

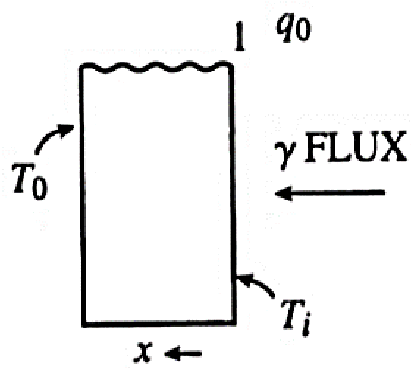
Assumption:

1- One-Dimension ($\partial/\partial y=0, \partial/\partial z=0$).

2- Steady state ($\partial/\partial t=0$).

3- heat generation ($\dot{q}$).

4- Homogeneous ($k=constant$).



$$\dot{q}_x = \dot{q}_0 e^{-ax} \qquad \frac{d^2T}{dx^2} = \frac{-\dot{q}_0 e^{-ax}}{k}$$

$$T = c_1 + c_2 x - \frac{\dot{q}_0}{a^2 k} e^{-ax}$$

Boundary conditions:

(1) $T = T_i$ at $x = 0$

(2) $T = T_0$ at $x = L$

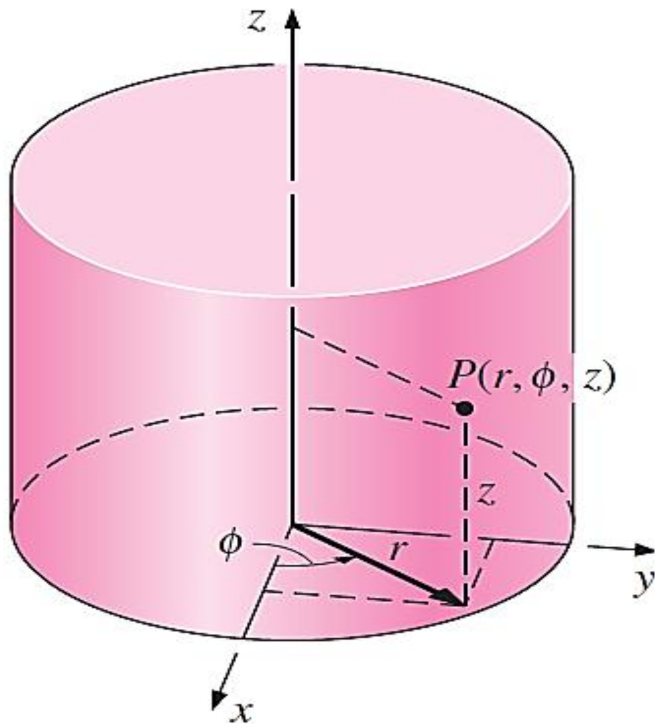
$$c_1 = T_i + \frac{\dot{q}_0}{a^2 k} \qquad c_2 = \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})}{L}$$

$$T = T_i + \frac{\dot{q}_0}{a^2 k} + \frac{T_0 - T_i - \frac{\dot{q}_0}{a^2 k}(1 - e^{-aL})x}{L} + \frac{-\dot{q}_0}{a^2 k} e^{-ax}$$

The Conduction Equation of Cylindrical Coordinate

A common example is the hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures. For a general transient three dimensional in the cylindrical coordinates *T=T(r, φ ,z, t)*, the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



For a general transient three-dimensional in the cylindrical coordinates *T= T(r, φ ,z, t)*,the general form of the conduction equation in cylindrical coordinates becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If the heat flow in a cylindrical shape is only in the radial direction and for steady-state conditions with no heat generation, the conduction equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

Integrating once with respect to radius gives

$$r\frac{\partial T}{\partial r} = C_1 \quad \text{and} \quad \frac{\partial T}{\partial r} = \frac{C_1}{r}$$

A second integration gives $T = C_1 \ln r + C_2$.

To obtain the constants (C_1 and C_2), we introduce the following boundary conditions

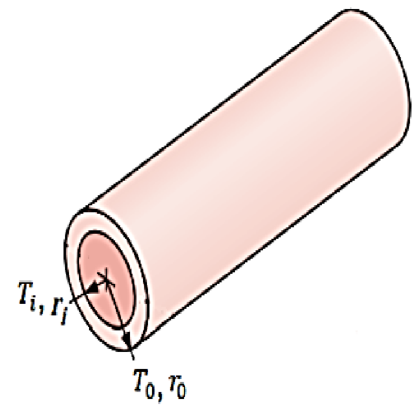
B.C.1 $T=T_i$ *at* $r=r_i$ $T_i = C_1 \ln r_i + C_2$.

B.C.2 $T=T_o$ *at* $r=r_o$ $T_o = C_1 \ln r_o + C_2$.

Example

consider a steam pipe of length (L), inner radius (ri), outer radius ($r0$) and thermal conductivity (k). The inner and outer surface of pipe are maintained at average temperature of (Ti) and ($T0$) respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions and determine the rate of heat loss from the steam through the pipe

Solution:



Assumption:

1- Steady state ($\partial/\partial t = 0$).

2- Homogenous material (isotropic material).

3- With heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad multiply\ by\ r$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad integrate$$

$$r\frac{\partial T}{\partial r} = C_1 \quad \rightarrow \quad \frac{\partial T}{\partial r} = \frac{C_1}{r} \qquad integrate\ again$$

$$T = C_1 \ln r + C_2 \qquad (1)$$

B.C1: $at\ r = r_i \qquad T = T_i \qquad sub.in\ Eq.(1)$

$$T_i = C_1 \ln r_i + C_2 \qquad (3)$$

B.C2: $at\ r = r_0 \qquad T = T_0 \qquad sub.in\ Eq.(1)$

$$T_0 = C_1 \ln r_0 + C_2 \qquad (4)$$

Subtract Eq. (3) and Eq. (4)

$$T_i - T_0 = C_1 \ln\frac{r_i}{r_0}$$

$$C_1 = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \qquad sub.in\ Eq.(3)$$

$$T_i = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i + C_2$$

$$C_2 = T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

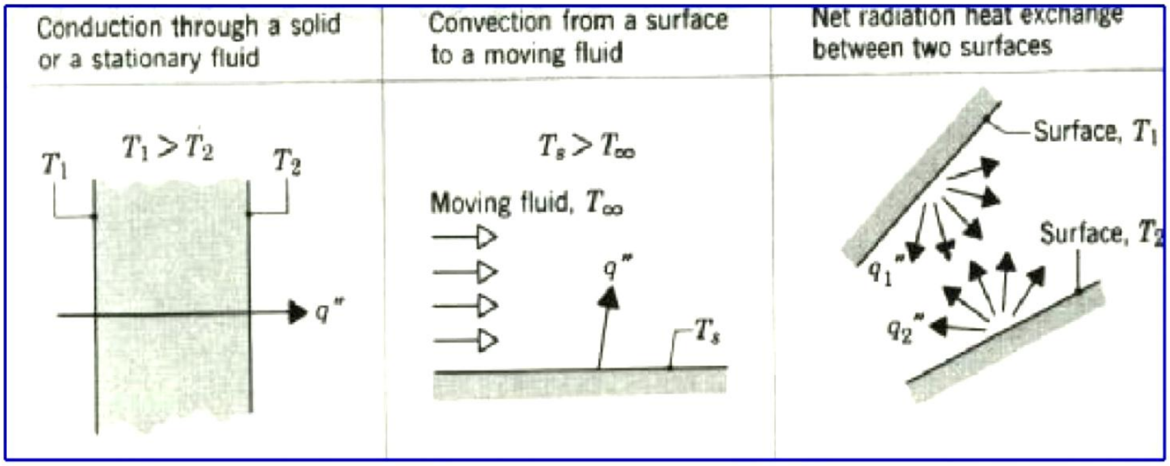
Sub. C_1 and C_2 in Eq. (1)

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r + T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln\frac{r}{r_i} + T_i$$

$$q = -kA\frac{\partial T}{\partial r} = -k(2\pi rL)\frac{C_1}{r} = -\frac{(2\pi rLk)}{r}\frac{T_i - T_0}{\ln\frac{r_i}{r_0}}$$

$$q = -2\pi Lk\frac{(T_i - T_0)}{\ln\frac{r_i}{r_0}}$$



Mode	Transfer Mechanism	Rate of heat transfer(W)	Thermal Resistance (K/W)
Conduction	Diffusion of energy due to random molecular motion	$q = -kA\frac{dT}{dx}$	$R_k = \frac{L}{kA}$
Convection	Diffusion of energy due to random molecular motion plus bulk motion	$q = hA(T_s - T_\infty)$	$R_c = \frac{1}{h\ A}$
Radiation	Energy transfer by electromagnetic Waves	$q = \sigma\varepsilon A(T_s^4 - T_{sur}^4)$	$R_r = \frac{T_s - T_{sur}}{\sigma\varepsilon A(T_s^4 - T_{sur}^4)}$

Consider the plane wall with uniformly distributed heat sources shown in Figure below. The thickness of the wall in the x direction is 2L, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one dimensional. The heat generated per unit volume is $\dot{q}$, and assume that the thermal conductivity does not vary with temperature. Derive an expression of the temperature distribution

[HEAT TRANSFER]

Second term

Assist lecture Esraa Adil

- **Course content:**

1. Basics
2. Fins
3. Principle of convection heat transfer
4. Forced convection
5. Natural convection

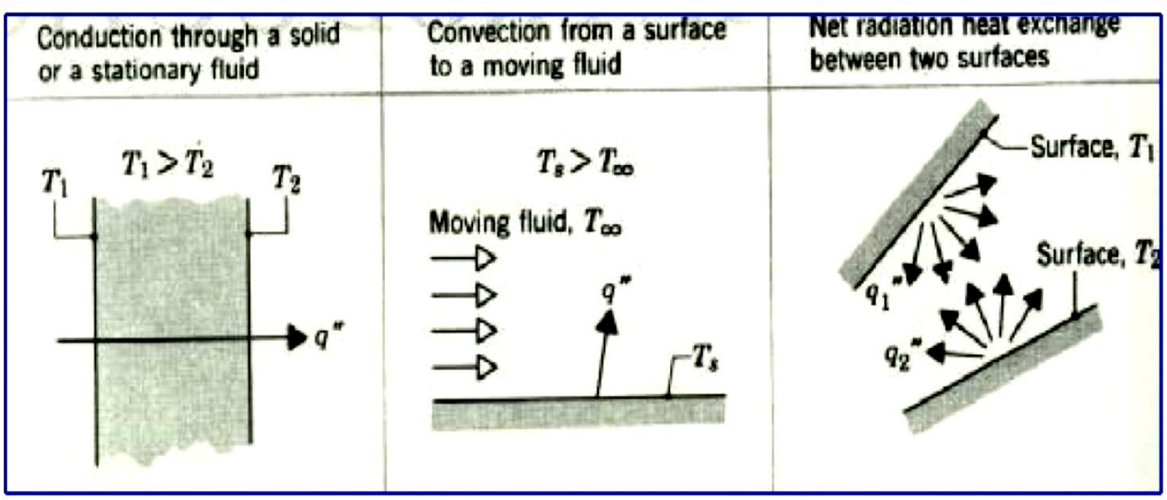
Basics

Heat Transfer Rate Processes

Mode	*Transfer Mechanism*	*Rate of heat transfer (W)*
Conduction	Diffusion of energy due to random molecular motion	$q = -kA\frac{dT}{dx}$
Convection	Diffusion of energy due to random molecular motion plus bulk motion	$q = hA(T_s - T_\infty)$
Radiation	Energy transfer by electromagnetic Waves	$q = \sigma\varepsilon A(T_s^4 - T_{sur}^4)$

Summary of heat transfer rate processes

Mode	Transfer Mechanism	Rate of heat transfer(W)	Thermal Resistance (K/W)
Conduction	Diffusion of energy due to random molecular motion	$q = -kA\frac{dT}{dx}$	$R_k = \frac{L}{kA}$
Convection	Diffusion of energy due to random molecular motion plus bulk motion	$q = hA(T_s - T_\infty)$	$R_c = \frac{1}{h\,A}$
Radiation	Energy transfer by electromagnetic Waves	$q = \sigma\varepsilon A(T_s^4 - T_{sur}^4)$	$R_r = \frac{T_s - T_{sur}}{\sigma\varepsilon A(T_s^4 - T_{sur}^4)}$



HEAT TRANSFER FROM EXTENDED SURFACES (FINS)

Introduction

Extended surfaces have wide industrial application as fins attached to the walls of heat transfer equipment in order to increase the rate of heating or cooling.

Fins are generally used to enhance the heat transfer from a given surface.

Consider a surface losing heat to the surroundings by convection. Then, the heat transfer rate Q, is given by Newton's law of cooling:

$Q = h\,A\,(Tw - T_{\infty})$

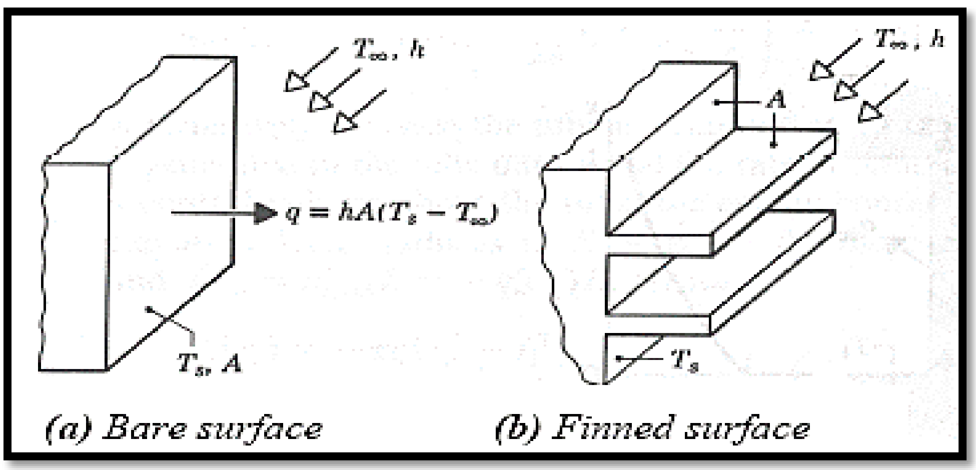
Where,

h = heat transfer coefficient between the surface and the ambient

A = exposed area of the surface

Tw = temperature of the surface, and

T_{∞} = temperature of the surroundings.



So if we need to increase the heat transfer rate from the surface, we can:

1. Increase the temperature potential $(Tw - T_{\infty})$ but, this may not be possible always since both these temperature may not be in our control.

2. increase the heat transfer coefficient h; this also may not be always possible or it may need installing an external fan or pump to increase the fluid velocity and this may involve cost consideration, or
3. Increase the surface area A; in fact, this is the solution generally adopted. Surface area is increased by adding an 'extended surface' (or, fin) to the 'base surface' by extruding, welding or by simply fixing it mechanically.

Adding of fins can increase the heat transfer from the surface by several folds, e.g. an automobile radiator has thin sheets fixed over the tubes to increase the area several folds and thus increase the rate of heat transfer
Generally, fins are fixed on that side of the surface when the heat transfer coefficient is low.

Heat transfer coefficient are lower for gases as compared to liquids. Therefore, one can observe that fins are fixed on the outside of the tubes in a car radiator, where cooling liquid flows inside the tubes and air flows on the outside across fins.

Likewise, in the condenser of a household refrigerator, freon flows inside the tubes and the fins are fixed on the outside of these tubes to enhance the heat transfer rate.

Application areas of fins are:

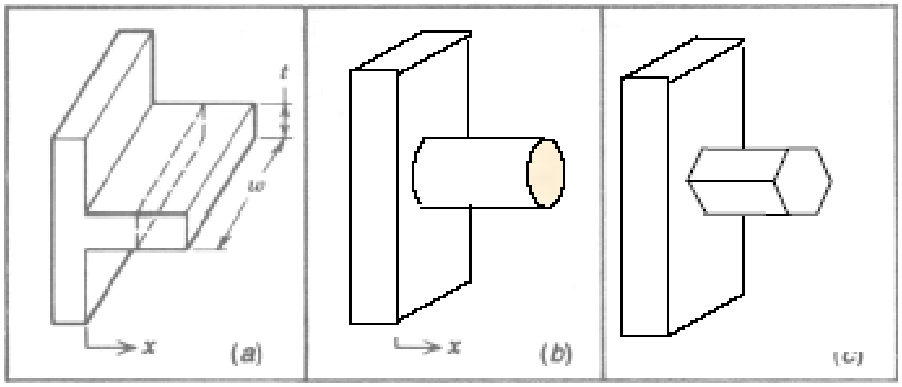
1. Radiators for automobiles
2. Heat exchangers of a wide variety, used in different industries
3. Cooling of electric motors, transformers, etc.
4. Cooling of electronic equipment, chips, I.C. boards etc.

The selection of fins is made on the basis of ***thermal performance*** and ***cost***. the fins is stronger when the fluid is a gas rather than a liquid. The selection of suitable fin geometry requires a compromise among:

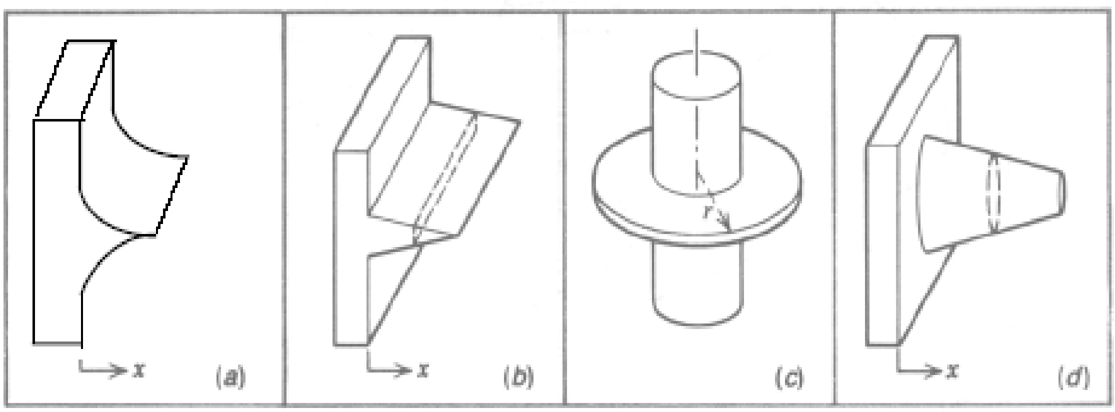
• *A cost and weight are available space*
• *Pressure drop of the heat transfer fluid*
• *Heat transfer characteristics of the extended surface.*

Types of fins:

There are innumerable types of fins used in practice some of the more common types are shown in Fig.



uniform Fin configurations (a) Rectangular Fin, (b)& (c)Pin Fin



non-uniform Fin configurations

(a) Parabolic (b) Triangular (c) Annular fin (d) Pin fin.

Consider a fin of rectangular cross section attached to the base surface, as shown in Fig.

Let L be the length of fin,

w, its width and

t its thickness.

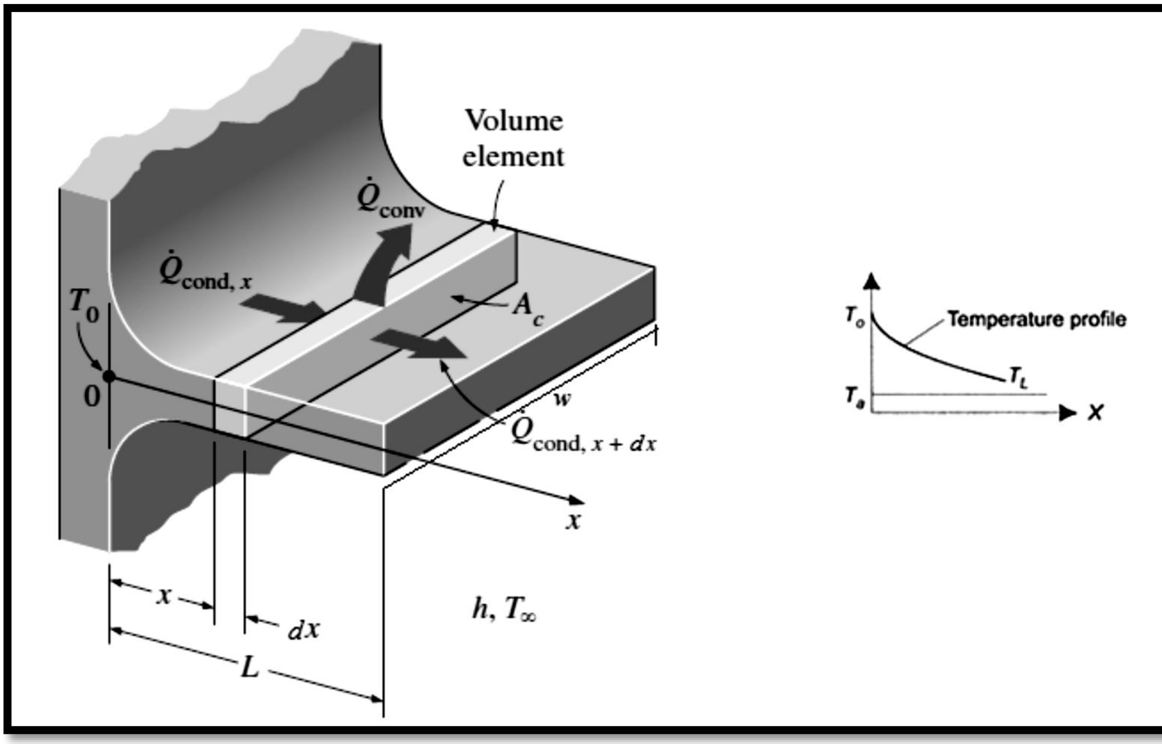
Let P be the perimeter = 2 (w + t).

Let Ac be the area of cross section and

To the temperature at the base, as shown.

Assumptions

1. Steady state conduction, with no heat generation in the fin
2. Thickness t is small compared to length L and width w, i.e. one-dimensional conduction in the X-direction only.
3. Thermal conductivity, k of the fin material is constant.
4. Uniform heat transfer coefficient h, over the entire length of fin.
5. No bond resistance in the joint between the fin and the base wall, and
6. Negligible radiation effect.



Consider the one-dimensional fin exposed to a surrounding fluid at a temperature $T\infty$ as shown in Figure .The temperature of the base of the fin is $T0$. We approach the problem by making an energy balance on an element of the fin of thickness dx as shown in the figure. Thus

Energy in left face=energy out right face+energy lost by convection

The defining equation for the convection heat-transfer coefficient is recalled as

$q = hA\,(T_w - T_\infty)$

where the area in this equation is the surface area for convection. Let the cross-sectional area of the fin be A and the perimeter be P. Then the energy quantities are

$$\text{Energy in left face} = q_x = -kA\frac{dT}{dx}$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{dT}{dx}\right]_{x+dx}$$

$$= -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right)$$

$$\text{Energy lost by convection} = hP\,dx\,(T - T_\infty)$$

Here it is noted that the differential surface area for convection is the product of the perimeter of the fin and the differential length dx. When we combine the quantities, the energy balance yields

$$-kA_c\frac{\partial T}{\partial x} = \left(-kA_c\frac{\partial T}{\partial x} - kA_c\frac{\partial^2 T}{\partial x^2}dx\right) + h\,(P\,dx)(T - T_a)$$

$$kA_c\frac{\partial^2 T}{\partial x^2}dx - h\,(P\,dx)(T - T_a) = 0$$

$$\frac{\partial^2 T}{\partial x^2} - m^2.(T - T_a) = 0 \ldots\ldots\ldots\ldots b$$

Where

$$m = \sqrt{\frac{h\,.P}{k\,.A_c}}$$

Note that m has units of: (m^{-1}) and is a constant, since for a given operating conditions of a fin, generally h and k are assumed to be constant. Now, define excess temperature

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

Let $\theta = T - T_\infty$. Then Equation

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0$$

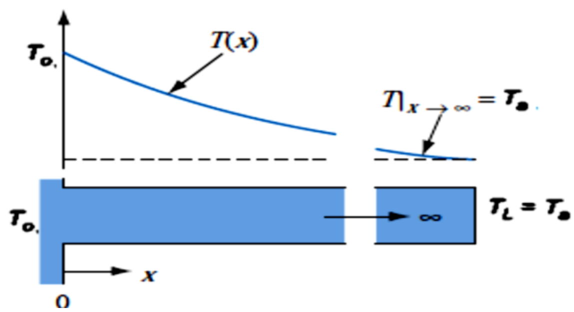
One boundary condition is

$$\theta = \theta_0 = T_0 - T_\infty \qquad \text{at } x = 0$$

The other boundary condition depends on the physical situation. Several cases may be considered:

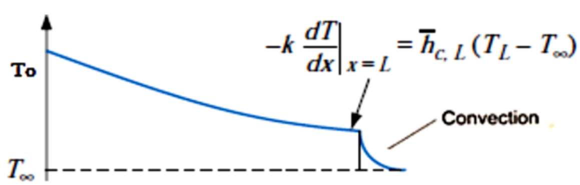
CASE 1 The fin is very long, and the temperature at the end of the fin is essentially that of the surrounding fluid.

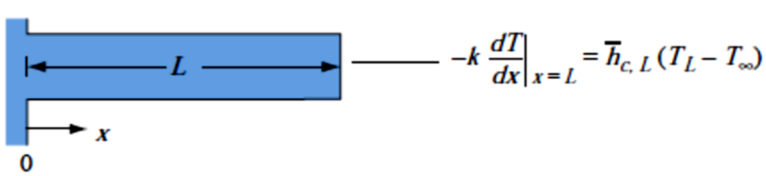
$$\theta(\infty) = (T_\infty - T_\infty) = 0 \qquad at \qquad x = \infty$$



CASE 2 The fin is of finite length and loses heat by convection from its end.

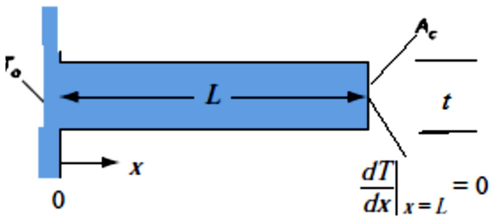
$$-k\frac{d\theta(x)}{dx}\bigg|_{x=L} = h\theta(L) \qquad at \qquad x = L$$

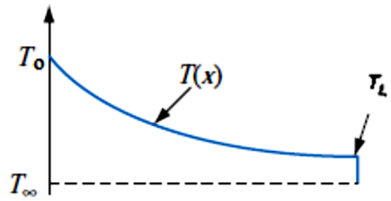




CASE 3 The end of the fin is insulated so that *dT/dx*=0 at *x*=*L*.

$$\frac{d\theta(x)}{dx} = 0 \qquad at \qquad x=L$$





If we let $m^2 = hP/kA$, the general solution for Equation

$$\frac{\partial^2\theta}{\partial x^2} - m^2.\theta = 0\,.$$

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

For case 1 the boundary conditions are

$$\theta = \theta_0 \quad \text{at } x = 0$$
$$\theta = 0 \quad \text{at } x = \infty$$

$$\theta(x) = \theta_o.e^{-mx}$$

$$\frac{\theta(x)}{\theta_o} = e^{-mx}$$

and the solution becomes

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

The solution for case 2 is more involved algebraically, and the result is

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m\,(L - x) + (h/mk)\sinh m\,(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

For case 3 the boundary conditions are

$$\theta = \theta_0 \quad \text{at } x = 0$$
$$\frac{d\theta}{dx} = 0 \quad \text{at } x = L$$

Thus

$$\theta_0 = C_1 + C_2$$
$$0 = m(-C_1 e^{-mL} + C_2 e^{mL})$$

Solving for the constants C_1 and C_2, we obtain

$$\frac{\theta}{\theta_0} = \frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}}$$
$$= \frac{\cosh[m(L-x)]}{\cosh mL}$$

The hyperbolic functions are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

All of the heat lost by the fin must be conducted into the base at x=0.

Using the equations for the temperature distribution,

we can compute the heat loss from :

$$q = -kA \left. \frac{dT}{dx} \right]_{x=0}$$

An alternative method of integrating the convection heat loss could be used:

$$q = \int_0^L hP(T - T_\infty)\,dx = \int_0^L hP\,\theta\,dx$$

In most cases, however, the first equation is easier to apply. For case 1,

$$q = -kA\,(-m\theta_0 e^{-m(0)}) = \sqrt{hPkA}\,\theta_0$$

For case 3,

$$q = -kA\theta_0 m \left(\frac{1}{1+e^{-2mL}} - \frac{1}{1+e^{+2mL}} \right)$$
$$= \sqrt{hPkA}\,\theta_0 \tanh mL$$

The heat flow for case 2 is

$$q = \sqrt{hPkA}\,(T_0 - T_\infty)\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

In this development it has been assumed that the substantial temperature gradients occur only in the x direction.

This assumption will be satisfied if the fin is sufficiently thin. For most fins of practical interest the error introduced by this assumption is less than 1 percent.

The overall accuracy of practical fin calculations will usually be limited by uncertainties in values of the convection coefficient h.

Calculation of Heat Lost by the Fin

The heat lost by the fin can be calculated either by

$$q = -kA \left.\frac{dT}{dx}\right]_{x=0}$$

or by

$$q = \int_0^L hP(T - T_\infty)\,dx = \int_0^L hP\,\theta\,dx$$

FOR CASE 1

$$\frac{\theta}{\theta_0} = e^{-mx}, \quad \frac{d\theta}{dx} = -m\theta_0 e^{-mx}, \quad \left.\frac{d\theta}{dx}\right|_{x=0} = -m\theta_0$$

$$\therefore \quad -kA(-m\theta_0) = kAm\theta_0, \quad For\ q = kA\sqrt{\frac{hP}{kA}}\theta_0 \Rightarrow$$

$$q = \sqrt{hPkA}\,\theta_0$$

FOR CASE 2

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

The heat flow for this case is

$$q = \sqrt{hPkA}\,\theta_0 \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

FOR CASE 3

$$\frac{\theta}{\theta_0} = \frac{\cosh[m(L-x)]}{\cosh mL}, \quad \frac{d\theta}{dx} = \theta_0\left(\frac{-me^{-mx}}{1+e^{-2mx}} + \frac{me^{mx}}{1+e^{2mx}}\right)$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = \theta_0 m\left(\frac{1}{1+e^{2mx}} - \frac{1}{1+e^{-2mx}}\right) \Rightarrow \frac{d\theta}{dx} = \theta_0 m\left(-\frac{e^{mL} - e^{-mL}}{e^{mx} + e^{-mL}}\right)$$

$$\therefore \quad q = -kA\theta_0 m\left(-\frac{e^{mL} - e^{-mL}}{e^{mx} + e^{-mL}}\right) \Rightarrow$$

$$q = \sqrt{hPkA}\,\theta_0 \tanh mL$$

Example:

An aluminum fin [k =200 W/m · °C] 3 mm thick and 7.5 cm long protrudes from a wall, The base is maintained at 300°C, and the ambient temperature is 50°C with h=10 W/m^2 · °C. Calculate the heat loss from the fin per unit depth of material.

Solution:

Let neglecting the heat lost from the end

$$q = \sqrt{hPkA}\,\theta_0 \tanh mL$$

$$P = 2(W + t) = 2(1 + 0.003) = 2.006m$$

$$A = W.t = 1 \times 0.003 = 0.003m^2$$

$$\theta_0 = T_0 - T_\infty = 300 - 50 = 250^0 C$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10 \times 2.006}{200 \times 0.003}} \cong 5.774$$

$\therefore$

$$q = \sqrt{10 \times 2.006 \times 200 \times 0.003} \times 250 \times \tanh(5.774 \times 0.075)$$

$$q = 357\, W/m\ depth$$

Fin Effectiveness

The effectiveness of the fin in transferring heat is given by the *fin efficiency* (η_f).

$$\eta_f = \frac{\text{actual heat transferred}}{\text{heat that would be transferred if entire fin area were at base temperature}} = \frac{q_{fin}}{q_{max}}$$

$$q_{max} = h.A_f.\theta_0, \quad A_f = P.L\,, A_f = surface\ area\ of\ the\ fin$$

For case 3

$$q_{fin} = \sqrt{hPkA}\,\theta_0 \tanh mL \quad \Rightarrow$$

$$\eta_f = \frac{\sqrt{hPkA}\,\theta_0 \tanh mL}{hPL\theta_0} = \frac{\tanh mL}{mL}$$

Fin Performance (Є)

$$\varepsilon = \frac{q\ with\ fin}{q\ without\ fin} = \frac{\eta_f A_f h\theta_0}{hA_b\theta_0}$$

$$A_f = surface\ area\ of\ the\ fin = P.L$$

$$A_b = base\ area\ of\ the\ fin = A$$

Corrected Length (Lc)

Lc is used in all equations that apply for the case of the fin with an insulated tip (**<u>case</u> 3)**

$$Lc = L + \frac{t}{2} \quad \text{(For general)}$$

$$Am = L.t$$

Example: The outer surface of an oil heater at a uniform temperature of 150°C is to be filled with straight rectangular fins having a uniform thermal conductivity of 25 W/m.K. The ambient air temperature is 20°C and the heat transfer coefficient is 570 W/m^2.K. Determine the thickness and fin efficiency if the length of each fin is 10mm and each to remove 900W per meter length of primary surface.

Solution.

$$T_0 = 150°\text{C}, \quad k = 25 W/m.K, \quad h = 570 W/m^2.K, \quad L = 10mm, \quad T_\infty = 20°\text{C}$$

$$q = \sqrt{\text{hPkA}}\,\theta_0 \tanh mLc$$

$$P = 2(w+t) = 2(t+1), \quad A = wt, \quad w = 1$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{570\times 2(t+1)}{25\times t}} = 6.75\sqrt{\frac{t+1}{t}}$$

$$q = \sqrt{570\times 2(t+1)\times 25\times t}\,(150-20) tanh\left(0.0675\sqrt{\frac{t+1}{t}}\right)$$

$$900 = 21964.5\sqrt{t^2+t}\, tanh\left(0.0675\sqrt{\frac{t+1}{t}}\right)$$

$$by\ trial\ and\ error \Rightarrow t \cong 2.07mm$$

$$\eta_f = \frac{\tanh mLc}{mLc} = 60.7\%$$

PRINCIPLE OF CONVECTION AND BOUNDARY LAYER

Principles of Convection

Convection was considered only insofar as it related to the boundary conditions imposed on a conduction problem. The steady of convection will be focused on the methods of calculating convection heat transfer specially, the ways of predicting the value of the convection heattransfer coefficient *h* (depends on the density, viscosity and fluid velocity in addition to its thermal properties) that requires an energy balance along with an analysis of the fluid dynamics of the problems concerned.

To steady the convection, we may discuss:

1- Fluid dynamics and boundary-layer analysis.
2- 2- Energy balance on the flow system and determine the influence of the flow on the temperature gradients in the fluid.
3- The heat-transfer rate from a heated surface to a fluid.

Nusselt Number

Since we know that adjacent to the solid surface the fluid layer is stationary and the heat transfer in this fluid layer is by conduction

$$q_{cond} = -k_f(dT/dy) \ldots\ldots b$$

and the heat transferred by convection subsequently must be equal to this fluid layer, we can equate Eqs. a and b:

$$h.(T_s - T_a) = -k_f(dT/dy)$$

$$h = (-k_f(dT/dy))/(T_s - T_a)$$

i.e. the problem of finding out the value of "*h*" reduces to the task of finding out the temperature gradient (dT/dy) at y = 0 i.e. at the surface. Since the heat transfer coefficient depends on flow conditions, its value on a surface varies from point to point. However, we generally take an average value of "*h*" by properly averaging the local value of heat transfer coefficient over the entire surface.

It is also common practice to non-dimensionalise the heat transfer coefficient with 'Nusselt number. Nusselt number is defined as:

$$Nu = \frac{h.\delta}{k_f}$$

where δ is a characteristic dimension and k_f is the fluid thermal conductivity.

To get a physical interpretation of the Nusselt number, consider a thin layer of fluid with thickness δ and with a temperature difference of ΔT between the two surfaces. Then, we have:

$$\frac{q_{conv}}{q_{cond}} = \frac{h\,.\Delta T}{k_f\,.\,\frac{\Delta T}{\delta}} = \frac{h.\delta}{k_f} = Nu$$

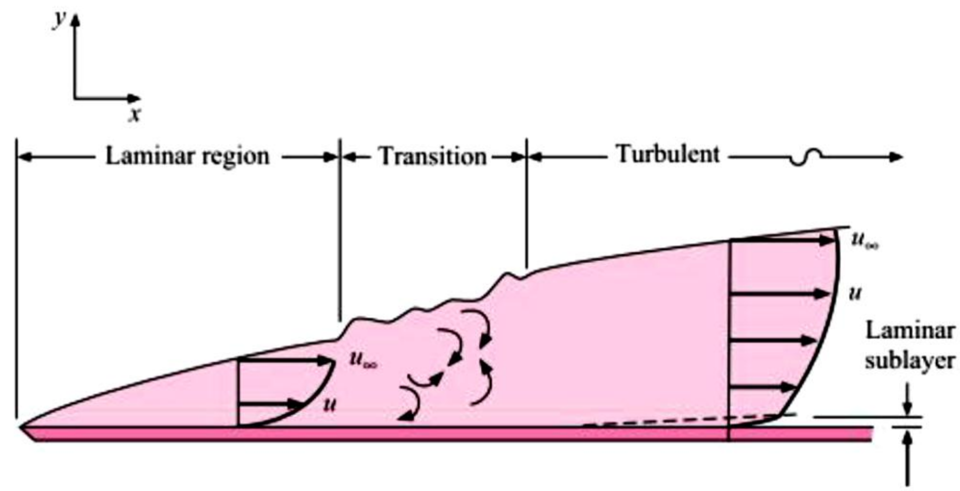
In other words, Nusselt number tells us how much the heat transfer is enhanced due to convection as compared to only conduction. Or, higher the Nusselt number, larger the heat transfer by convection.

Viscous Flow

Beginning at the leading edge of the plate, a region develops where the influence of viscous forces is felt. These viscous forces are described in terms of a shear stress τ between the fluid layers. If this stress is assumed to be proportional to the normal velocity gradient, we have the defining equation for the viscosity,

$$\tau = \mu \frac{du}{dy}$$

The constant of proportionality μ is called the *dynamic viscosity*. A typical set of units is newton-seconds per square meter; however, many sets of units are used for the viscosity, and care must be taken to select the proper group that will be consistent with the formulation at hand. The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the *boundary layer*



For a flow over a flat plate as shown in Figure, different flow region develops by the influence of viscous forces.

- The viscous forces are described in terms of a shear stress τ between the fluid layers.
- The region of flow that develops from the leading edge of the plate in which the effects

of viscosity are observed is called the boundary layer.

There are three stages

1- Laminar, flow of the fluid is laminar.

2- Transition, small disturbances in flow.

3- Turbulent, random flow which is characterized by eddies.

For flat plate the transition from laminar to turbulent flow occurs when

$$\frac{u_\infty x}{\nu} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5$$

Where

u_∞ = free-stream velocity, m/s

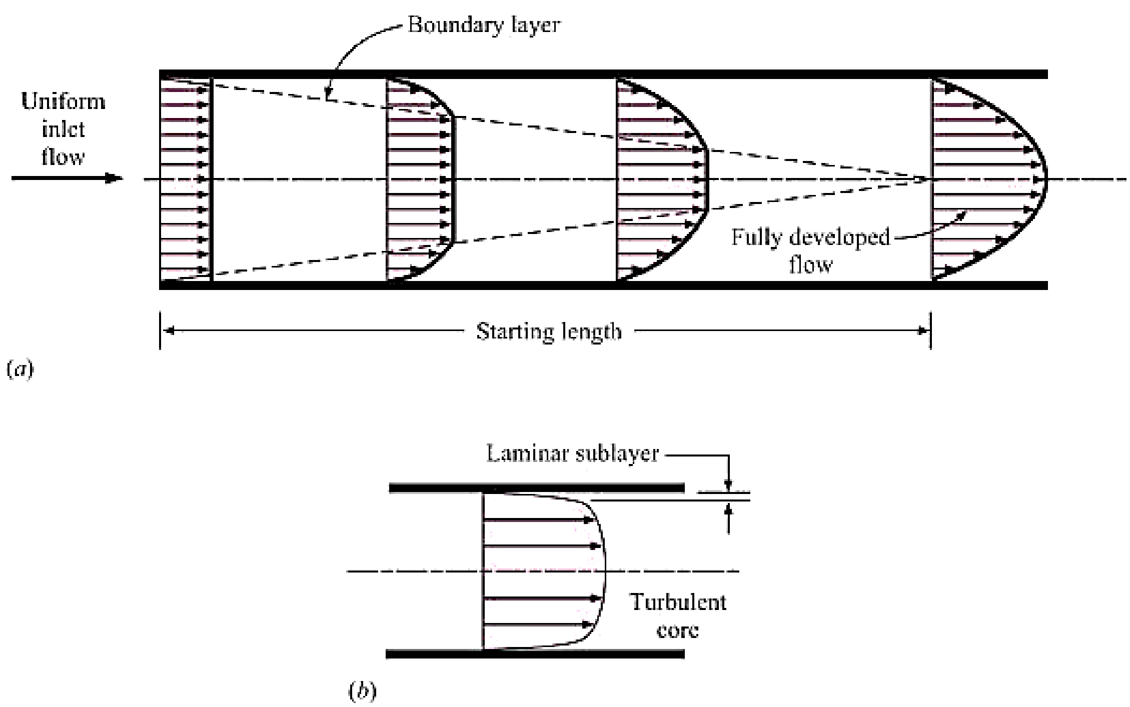
x = distance from leading edge, m

$\nu = \mu / \rho$ = kinematic viscosity, m^2/s

As shown in the figure, the velocity profile in laminar section is approximately parabolic, while the turbulent profile has a portion near the wall that is very nearly linear.

This linear portion is called a *laminar sublayer*

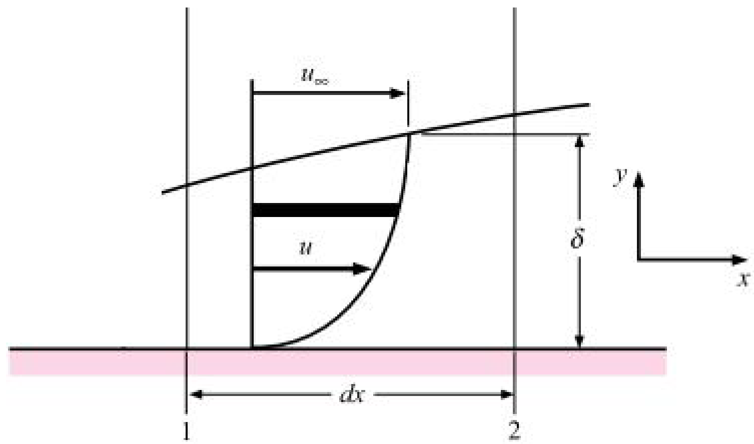
Flow in Pipes



A boundary layer develops at the entrance, as shown. Eventually the boundary layer fills the entire tube, and the flow is said to be fully developed. If the flow is laminar, a parabolic velocity profile is noted. When the flow is turbulent, a blunter profile is observed.

$$\mathrm{Re} = \frac{\rho\, u\, d}{\mu} \langle \; 2300 \; \textit{for laminar flow}$$

Laminar Boundary Layer on a Flat Plate



Consider the boundary-layer flow system shown in Figure.

The free-stream velocity outside the boundary layer is $u\infty$, and the boundary-layer thickness is δ. The thickness of the boundary layer at any distance on the flat plate can be estimated by using the following equation

$$\frac{\delta}{x} = \frac{4.64}{\mathrm{Re}_x^{1/2}}$$

where

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu}$$

Example

Air at 27◦C and 1 atm flows over a flat plate at a speed of 2 m/s. Calculate the boundary-layer thickness at distances of 20 cm and 40 cm from the leading edge of the plate. The viscosity of air at 27◦C is 1.85×10^{-5} kg/m· s.

Solution

The density of air is calculated from

$$\rho = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(300)} = 1.177\ \text{kg/m}^3 \quad [0.073\ \text{lb}_m/\text{ft}^3]$$

The Reynolds number is calculated as

$$\text{At } x = 20\ \text{cm:} \qquad \text{Re} = \frac{(1.177)(2.0)(0.2)}{1.85 \times 10^{-5}} = 25{,}448$$

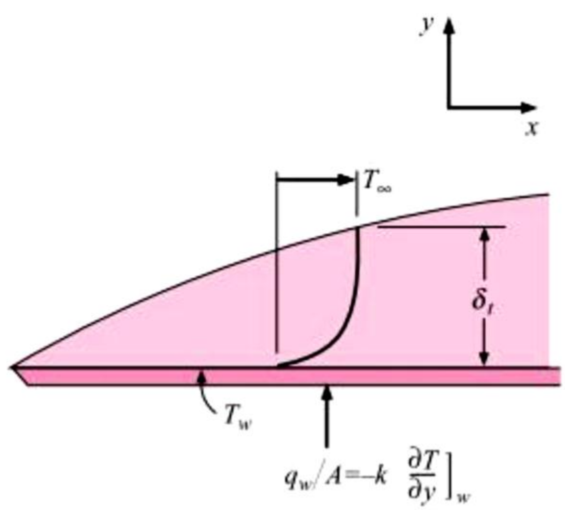
$$\text{At } x = 40\ \text{cm:} \qquad \text{Re} = \frac{(1.177)(2.0)(0.4)}{1.85 \times 10^{-5}} = 50{,}897$$

The boundary-layer thickness is calculated from Equation (5-21):

$$\text{At } x = 20\ \text{cm:} \qquad \delta = \frac{(4.64)(0.2)}{(25{,}448)^{1/2}} = 0.00582\ \text{m} \quad [0.24\ \text{in}]$$

$$\text{At } x = 40\ \text{cm:} \qquad \delta = \frac{(4.64)(0.4)}{(50{,}897)^{1/2}} = 0.00823\ \text{m} \quad [0.4\ \text{in}]$$

The thermal Boundary Layer



A thermal boundary layer is defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat-exchange process between the fluid and the wall. The temperature of the wall is Tw, the temperature of the fluid outside the thermal boundary layer is $T\infty$, and the thickness of the thermal boundary layer is designated as δt. At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. Thus the local heat flux per unit area, q'' , is

$$\frac{q}{A} = q'' = -k \left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$$

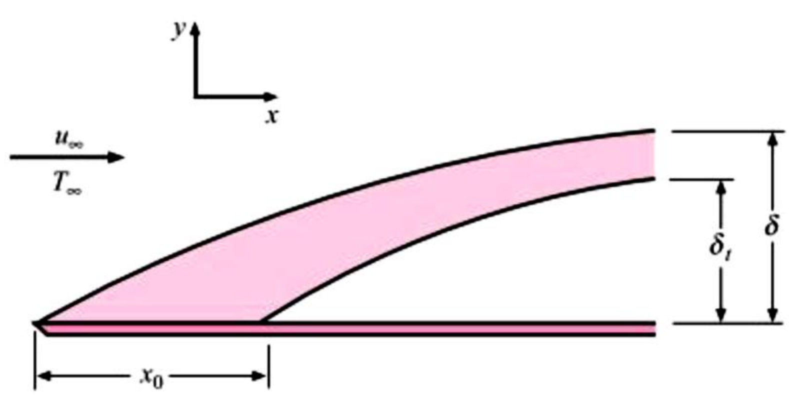
From Newton's law of cooling

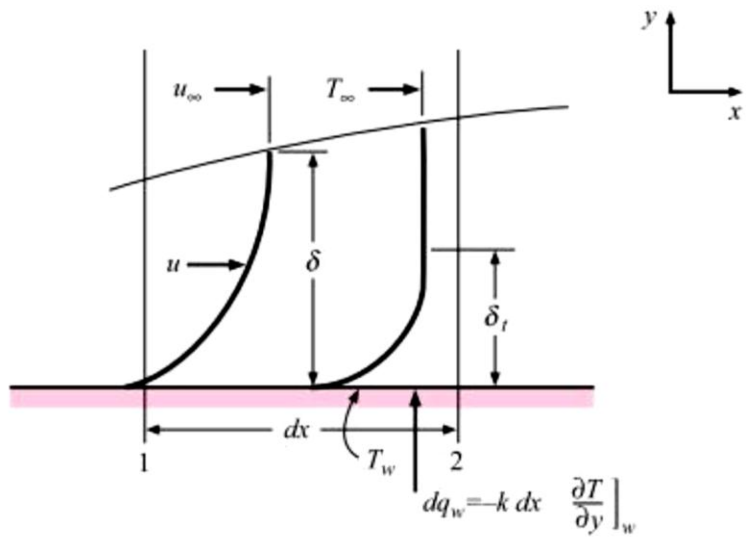
$$q'' = h(T_w - T_\infty)$$

Where h is the convection heat-transfer coefficient. Combining these equations, we have

$$h = \frac{-k(\partial T/\partial y)_{\text{wall}}}{T_w - T_\infty}$$

$\left.\frac{\partial T}{\partial y}\right]_{\text{wall}}$: is the temperature gradient at the wall thickness of thermal boundary layer.





The thickness boundary layer can be calculated by using the following equation

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

Where

Pr = Prandtl number

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p\mu}{k}$$

x0= is the distance from the leading edge where the heating begins.

When the plate is heated over the entire length, x0 = 0, then

$$\frac{\delta_t}{\delta} = \frac{1}{1.026}\mathrm{Pr}^{-1/3}$$

Heat Transfer Coefficient

The heat transfer coefficient at any x position on the flat plate can be found by the equation

$$h_x = 0.332k\ \mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

The last equation can be written in dimensionless form as

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

For a plate heated over its entire length, $x0 = 0$ and

$$\mathrm{Nu}_x = 0.332\mathrm{Pr}^{1/3}\ \mathrm{Re}_x^{1/2}$$

Where

Nux= Nusselt number $=h_x\, x/k$

The above equation expresses the local values of the heat-transfer coefficient in terms of the distance from the leading edge of the plate and the fluid properties. For the case where x0 = 0 the average heat-transfer coefficient and Nusselt number may be obtained by integrating over the length of the plate:

$$\bar{h} = \frac{\int_0^L h_x\,dx}{\int_0^L dx} = 2h_{x=L}$$

Where

$\bar{h}$ is the average heat transfer coefficient.

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 2\ \mathrm{Nu}_{x=L}$$

or

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 0.664\ \mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}$$

where

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

The above relations are used for laminar flow constant wall temperature. Note that, all the physical properties are evaluated at film temperature Tf, which is

$$T_f = \frac{T_w + T_\infty}{2}$$