Communications
Lecture 1

Signal and systems:

1. Signals:

A signal is a set of information or data. Examples include a telephone or a television signal, the monthly sales figures of a product, etc.

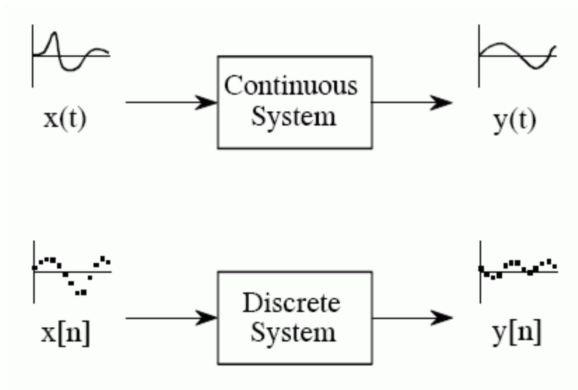
A signal is a description of how one parameter varies with another parameter. For instance, voltage changing over time, where $v(t)$, time is the independent variable and voltage is the dependent one.

2. Systems:

Signals can be processed by systems, which may modify them or extract additional information from them.

A system is any process that produces an output signal y in response to an input signal x.

A system may be made up of physical components, as in electrical, mechanical, or hydraulic systems (hardware realization), or it may be an algorithm that computes an output from an input signal (software realization).



Size of a signal:

The size of any entity is a quantity that indicates its strength.

If $v(t)$ and $i(t)$ are, respectively, the voltage and current across a resistor with resistance R, then the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

The total energy expended over the time interval $t_1 \leq t \leq t_2$ is

$$E = \int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt$$

and the average power over this time interval is

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t)dt$$

In many systems we will be interested in examining power and energy of signals over an infinite time interval, i.e., for $-\infty < t < +\infty$. In these cases, we define the total energy as the time interval increases without bound.

$$E = \lim_{T \to \infty} \int_{-T}^{T} p(t)dt = \lim_{T \to \infty} \int_{-T}^{T} \frac{1}{R} v^2(t)dt$$

and the time averaged power as

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} p(t)dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{R} v^2(t)dt$$

1. Signal Energy

Assume a signal $g(t)$ as a voltage across a one-ohm resistor. We define signal energy E_g of the signal $g(t)$ as the energy that the voltage $g(t)$ dissipates on the resistor. More formally, we define E_g (for a real signal) as

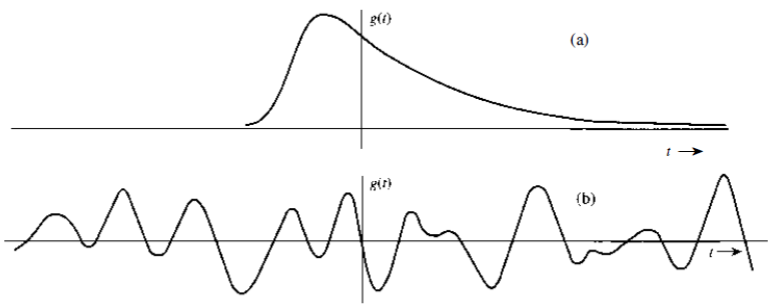
$$E_g = \int_{-\infty}^{\infty} g^2(t)dt$$

This definition can be generalized to a complex-valued signal $g(t)$ as

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

where $|x|$ denotes the magnitude of the (possibly complex) number $|x|$.

Examples of signals.
(a) Signal with finite energy.
(b) Signal with finite power.



2. Signal Power

To be a meaningful measure of signal size, the signal energy must be finite.

For energy to be finite, the signal amplitude must go to zero as $|t|$ approaches infinity (see the figure above). Otherwise the energy will not converge.

If the amplitude of $g(t)$ does not go to zero as $|t|$ approaches infinity, the signal energy is infinite.

In such a case we rely on the time average of the energy (if it exists), which is the average power P_g defined (for a real signal) by

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt$$

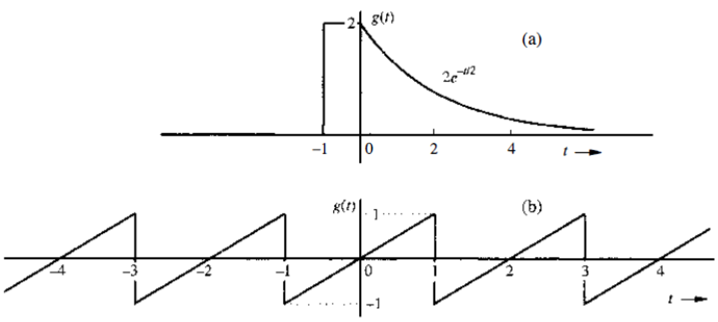
We can generalize this definition for a complex signal $g(t)$ as

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

Observe that the signal power P_g is the time average (mean) of the signal amplitude square, that is, the mean square value of $g(t)$. Indeed, the square root of P_g is the familiar rms (root mean square) value of $g(t)$.

A signal with finite energy is called an energy signal e.g. signal (a) in the figure above, while, a signal with infinite energy but finite power is called a power signal e.g. signal (b).

Example: Determine the suitable measures of the signals in the figure below



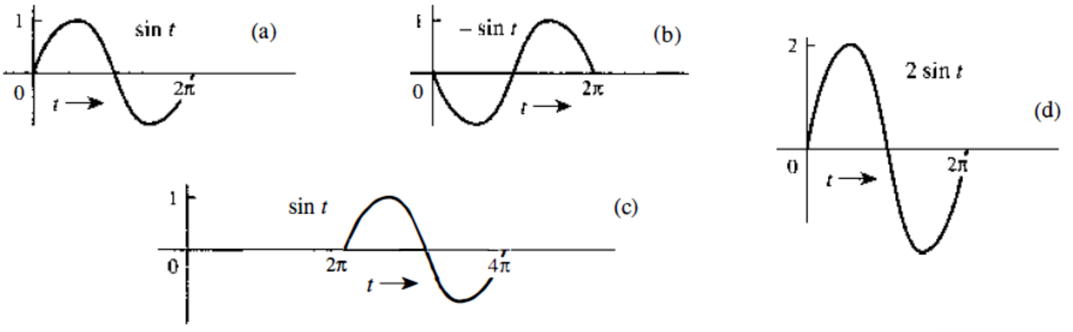
The signal (a) in the figure approaches 0 as $|t| \to \infty$. Therefore, the suitable measure for this signal is its energy E_g, given by

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 4e^{-t}dt = 4 + 4 = 8$$

The signal in (b) in the figure above does not approach 0 as $|t| \to \infty$. However, it is periodic, and therefore its power exists. We can therefore determine its power. For periodic signals, we can simplify the procedure by observing that a periodic signal repeats regularly each period (2 seconds in this case). Therefore, averaging $g^2(t)$ over an infinitely large interval is equivalent to averaging it over one period (2 seconds in this case). Thus

$$P_g = \frac{1}{2}\int_{-1}^{1} g^2(t)dt = \frac{1}{2}\int_{-1}^{1} t^2 dt = \frac{1}{3}$$

Example: Find the energies of the signals shown in the figure below. What is the effect on the energy if the signal is multiplied by k?



Let us denote the signal in question by $g(t)$ and its energy by E_g. For parts (a) and (b)

$$E_g = \int_0^{2\pi} \sin^2 t dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t dt = \pi + 0 = \pi$$

(c) $\quad E_g = \int_{2\pi}^{4\pi} \sin^2 t dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t dt = \pi + 0 = \pi$

(d) $\quad E_s = \int_0^{2\pi} (2\sin t)^2 dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t dt\right] = 4[\pi + 0] = 4\pi$

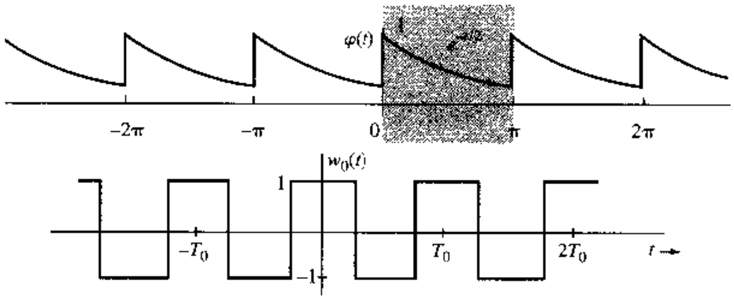
Sign change and time shift does not affect the signal energy. Doubling the signal amplitude multiplies its energy by 4. In the same way we can show that the energy of $kg(t)$ is $k^2 E_y$.

Example: Find the power of a sinusoid $y(t) = C\cos(\omega_0 t + \theta)$

Assume the sinusoid has a period of T_o

$$\begin{aligned} P_y &= \frac{1}{T_0}\int_0^{T_0} C^2 \cos^2(\omega_0 t + \theta)\, dt = \frac{C^2}{2T_0}\int_0^{T_0} [1 + \cos(2\omega_0 t + 2\theta)]\, dt \\ &= \frac{C^2}{2T_0}\left[\int_0^{T_0} dt + \int_0^{T_0} \cos(2\omega_0 t + 2\theta)\, dt\right] = \frac{C^2}{2T_0}\{T_0 + 0\} = \frac{C^2}{2} \end{aligned}$$

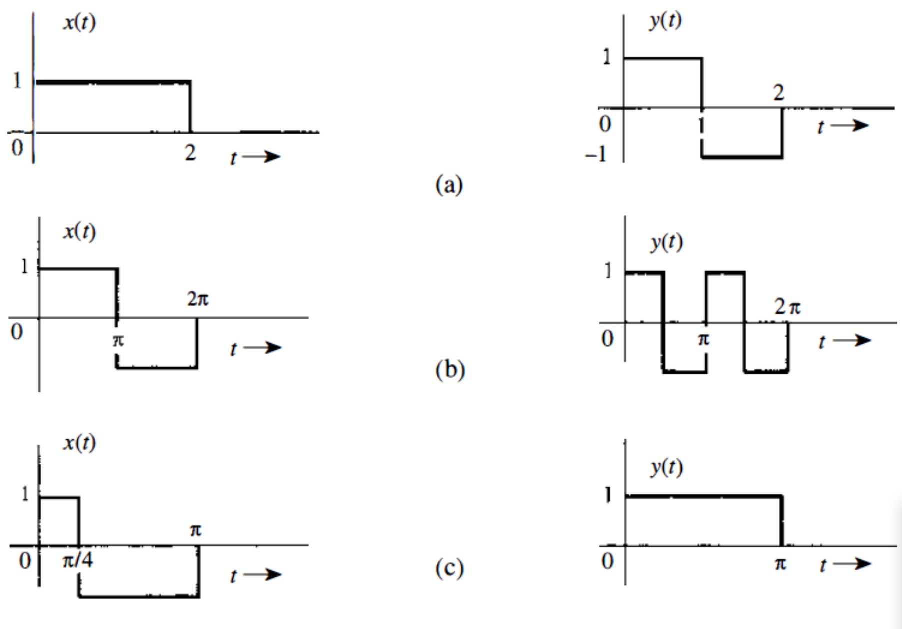
Example: Find the power and the rms value for the signals in the figure below



$$P = \frac{1}{\pi}\int_0^{\pi}\left(e^{-t/2}\right)^2 dt = \frac{1}{\pi}\int_0^{\pi} e^{-t}dt = \frac{1}{\pi}\left[1 - e^{-\pi}\right]$$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} w_0^2(t)dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt = 1$$

Example: Find the energies of the signals shown in the figure below



Example: Find the power and the rms value for the signal in the figure below