STUDENTS' PERFORMANCE ASSESSMENT

Form 08

Mathematics 2th year students

Instructor: Mahmoud Shakir Wahhab

Table 1, Plan of whole year assessments

Program Outcomes	Course Learning Objectives	Strategies for Achieving Outcomes	Assessment Method (results table after performing)
• Differential equation are basic importance in engineering mathematics because many physical laws are relations appear mathematically in the form of a differential equation • Formulate relevant research problems; conduct experimental and/or analytical study and analyzing results with modern mathematical/scientific methods and use of software tools.	• To create a congenial environment that promotes learning, growth, and imparts the ability to work with inter-disciplinary groups in professional, industry, and research organizations. • To broaden and deepen their capabilities in analytical and experimental research methods, analysis of data, and drawing relevant conclusions for scholarly writing and presentation.	1. Align goals and objectives to achieve common desire outcomes 2. Eliminate bad habits 3. Welcome Failure 4. Benefit the daily goal setting 5. Avoid procrastination	1. In-class and online quizzes 2. Homework 3. Peer feedback activities 4. Practice exams

Table 2, Assessment Rubrics

Rubric	4- Exceeds	3- Meets	2-Progressing	1-Below Average
Engineering Knowledge	Students can apply concepts of basic science and basic mathematics to solve engineering problems.	The student will just be able to understand the concepts of basic science and basic mathematics to solve engineering problems	The student will just be able to remember the concepts of basic science and basic mathematics to solve engineering problems	The student does not have an engineering sense
Problem Analysis	Student can analyze a given problem and identify the constraints and define the requirements for a given problem which are suitable for its solution	The student is just able to have a grasp of a problem statement and its constraints and can understand problem definition and the requirements for a given problem which are suitable for its solution.	Students need assistance to have a grasp of the problem statement and its constraints and can understand problem definition and the requirements for a given problem which are suitable for its solution.	The student is not able to recognize the basics of problem analysis
Design and Development of Solutions	The student can design a functional and realistic system consisting of multiple components or processes.	The student can understand and apply the engineering knowledge for the design of functional and realistic system consisting of multiple components or processes.	The student would require aid to and apply the engineering knowledge for the design functional and realistic system consisting of multiple components or processes	The student does not have the imagination to design an engineering part

Table 3, Students Works Rating

Students Outcome	Max Score
	High : 100
	Low : 50
	Mean :75
	SD : 2.5

Table 4, Student and Faculty Evaluations of Learning Outcomes

Students Outcomes	Students Rating	Instructor Rating	Instructor Comments
Not yet achieved	**Not yet achieved**	**Not yet achieved**	**Not yet achieved**

Table 5, Changes/Improvements

Assessment of Changes/Improvements Made this year	

Changes/Improvements That Will Be Made Next Time the Course is Offered	

Table 6, Final Evaluation

Outcome	Average	Notes
Not yet achieved	Not yet achieved	Not yet achieved

Appendices:

Materials: (Course notes should be here)

Faculty Curriculum Vitae:

Mahmoud Shakir Wahhab
Ass. Teacher in Electronic and control.
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University of south Ural, Russia
M, Sc. Mechatronics Engineering (2018)
Thesis: "Automation control system design of temperature stabilization of belt conveyor AC motor"

University of northern technical, Iraq
B, Sc. electronic and control (2006)

<u>Miscellaneous</u>
Computer Skills:

Matlab/Simulink/GUI
Ansys and Ansys workbench
Multisim
Solid Works
LabVIEW
Visual Basic
AUTOCAD (2D/3D)

Languages: Arabic– native language
English – Very good at reading and writing.
Russian - Good at reading and writing.
Turkish - Good at reading and writing.

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Second Stage

Mathematics

Review of integrals

Lecture (1)

Presented by :-
Mahmoud Shakir Wahhab

Review of integrals

- The most of the mathematical operations have inverse operations: the inverse operation of addition is subtraction, the inverse operation of multiplication is division, the inverse operation of differentiation is called integration.
- When a function f(x) is known we can differentiate it to obtain its derivative $\frac{df}{dx}$. The reverse process is to obtain the function f(x) from knowledge of its derivative. This process is called integration.

Indefinite Integral

The indefinite integral or general anti-derivative $\int \boldsymbol{f(x)\ dx}$ of a function $f\,(x)$ stands for all possible anti-derivatives of f (x) defined on an interval:

$$\int \boldsymbol{f(x)\ dx = F(x) + C}$$

where C is a constant and F(x) is an arbitrary anti-derivative of f (x).

Definite Integration

The definite integral is denoted by $\int_a^b f(x)dx$, where a is the lower limit of the integral and b is the upper limit of the integral.

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Note integration constants are not written in definite integrals since they always cancel in them:

$$\int_a^b f(x)dx = F(x)\Big|_a^b \quad = [F(b) + C] - [F(a) + C]$$

$$= F(b) - F(a)$$

Basic integration formulas

- The integral of the power function

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C , \qquad n \in R , n \neq -1$$

EXAMPLE

$$\int x^2 \, dx \longrightarrow \frac{x^3}{3} + C$$

EXAMPLE

$$\int \sqrt{x^3} \, dx \longrightarrow \int x^{\frac{3}{2}} \, dx \longrightarrow \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C \longrightarrow \frac{2}{5} \sqrt{x^5} + C$$

EXAMPLE

$$\int (3x - x^{-1})^2 \; dx$$

Solution

$$\int (9x^2 - 6 + x^{-2}) \; dx \longrightarrow 9\int x^2\,dx \; - 6\int dx + \int x^{-2}\,dx$$

$$9\,\frac{x^3}{3} - 6x + \frac{x^{-1}}{-1} + C \longrightarrow 3x^3 - 6x - \frac{1}{x} + C$$

EXAMPLE

$$\int \sqrt{3x^2 - 2x + 3}\ (3x - 1)\ dx$$

Solution

$$\int (3x^2 - 2x + 3)^{\frac{1}{2}} \cdot (3x - 1)\ dx \qquad \times \frac{2}{2}$$

$$\frac{1}{2} \int (3x^2 - 2x + 3)^{\frac{1}{2}} \cdot 2(3x - 1) dx \longrightarrow \frac{1}{2} \cdot \frac{(3x^2 - 2x + 3)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\frac{1}{3} \sqrt{(3x^2 - 2x + 3)^3} + C$$

EXAMPLE

$$\int_0^4 x\,\sqrt{x^2+9}\,dx$$

Solution

$$\int_0^4 (x^2+9)^{\frac{1}{2}}\cdot x\,dx \quad \times \frac{2}{2} \longrightarrow \frac{1}{2}\int_0^4 (x^2+9)^{\frac{1}{2}}\cdot 2x\;dx$$

$$\frac{1}{2}\cdot\left.\frac{(x^2+9)^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^4 \longrightarrow \frac{1}{3}\sqrt{(x^2+9)^3}\Big|_0^4 \longrightarrow \frac{1}{3}\left[\sqrt{(4^2+9)^3}-\sqrt{(0^2+9)^3}\right]$$

$$\frac{1}{3}\,(125-27)=\frac{98}{3}$$

- If in the indefinite integral of power function $n = -1$

EXAMPLES

$$\int \frac{1}{x}\,dx \longrightarrow \ln|x| + C$$

$$\int \frac{2}{4+7x}\,dx \longrightarrow 2\int \frac{1}{4+7x}\,dx \cdot \frac{7}{7} \longrightarrow \frac{2}{7}\int \frac{7}{4+7x}\,dx$$

$$\frac{2}{7}\ln|4+7x| + C$$

- The indefinite integral of the exponential function

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \;, \qquad a \neq 1$$

and if the base $a = e \; then$ $\int e^x \, dx = e^x + C$

EXAMPLE

$$\int 7^{2x} \, dx \longrightarrow \int 7^{2x} \, dx \cdot \frac{2}{2} \longrightarrow \frac{1}{2} \int 7^{2x} \cdot 2dx \longrightarrow \frac{1}{2} \cdot \frac{7^{2x}}{\ln 7} + C$$

EXAMPLE

$$\int e^{-5x}\ dx \longrightarrow \frac{1}{-5}\int e^{-5x} \cdot -5\, dx \longrightarrow \frac{e^{-5x}}{-5} + C$$

EXAMPLE

$$\int x^2\ e^{x^3}\ dx$$

$$\frac{1}{3}\int 3\, x^2 e^{x^3}\ dx \longrightarrow \frac{1}{3}\, e^{x^3} + C$$

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Second Stage

Mathematics

Integrals of Trigonometric Functions part (1)

Lecture (2)

Presented by :-
Mahmoud Shakir Wahhab

Integrals of Trigonometric Functions

As we developed the calculus of the trigonometric and exponential functions, we obtained formulas for the anti-derivatives of certain of these functions. For convenience, we summarize those formulas. Here are the formulas from

$$\int \sin x \; dx = -\cos x + C$$

$$\int \cos x \; dx = \sin x + C$$

$$\int sec^2\, x \, dx = \tan x + C$$

$$\int csc^2\, x \; dx = -\cot x + C$$

$$\int \tan x \; \sec x \; dx = \sec x + C$$

$$\int \cot x \csc x \; dx = -\csc x + C$$

$$\int \tan x \ dx \longrightarrow \int \frac{\sin x}{\cos x} dx \longrightarrow -\int \frac{-\sin x}{\cos x} dx \longrightarrow -\ln|\cos x| + C$$

$$\int \cot x \ dx \longrightarrow \int \frac{\cos x}{\sin x} dx \longrightarrow \ln|\sin x| + C$$

$$\int \sec x \ dx \longrightarrow \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} dx \longrightarrow \int \frac{\sec^2 x + \tan x \sec x}{\sec x + \tan x} dx$$

$$\ln|\sec x + \tan x| + C$$

$$\int \csc x \ dx \longrightarrow \int \csc x \cdot \frac{\csc x + \cot x}{\csc x + \cot x} dx \longrightarrow \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} dx$$

$$-\int \frac{-(\csc^2 x + \csc x \cot x)}{\csc x + \cot x} dx \longrightarrow -\ln|\csc x + \cot x| + C$$

EXAMPLES

$$\int \sin 7x \; dx \longrightarrow \int \sin 7x \cdot \frac{7}{7} \, dx \longrightarrow \frac{1}{7} \int \sin 7x \; \cdot 7 \, dx \longrightarrow \frac{-1}{7} \cos 7x + C$$

$$\int \frac{2\cos(\sqrt[3]{x})}{\sqrt[3]{x^2}} \, dx \longrightarrow 2 \int \cos(\sqrt[3]{x}) \cdot x^{\frac{-2}{3}} dx \longrightarrow 2 \int \cos(\sqrt[3]{x}) \cdot x^{\frac{-2}{3}} dx \cdot \frac{1/3}{1/3}$$

$$6 \int \cos(\sqrt[3]{x}) \cdot x^{\frac{-2}{3}} \cdot \frac{1}{3} dx \longrightarrow 6\sin(\sqrt[3]{x}) + C$$

$$\int \frac{\tan(4x)}{cos^2(4x)} \, dx \longrightarrow \int \tan(4x) \cdot \sec^2(4x) \; dx \longrightarrow \frac{1}{4} \int \tan(4x) \cdot \sec^2(4x) \; 4dx$$

$$\frac{1}{4} \, \frac{\tan^2(4x)}{2} + C \longrightarrow \frac{1}{8} \tan^2(4x) + \mathrm{C}$$

$$\int \csc x\ (\cot x + \csc x)\, dx \longrightarrow \int \csc x \cot x\ dx + \int \csc^2 x\ dx$$

$$-\csc x\ - \cot x + C$$

$$\int x\ e^{-\ln x}\, dx \longrightarrow \int x\ e^{\ln x^{-1}}\, dx \longrightarrow \int x \cdot \frac{1}{x}\, dx \longrightarrow \int dx \longrightarrow x + C$$

$$\int \frac{x^6 + \cos 7x}{x^7 + \sin 7x}\, dx \longrightarrow \frac{1}{7} \int \frac{7\,(x^6 + \cos 7x)}{x^7 + \sin 7x}\, dx \longrightarrow \frac{1}{7} \ln(x^7 + \sin 7x) + C$$

$$\int \frac{\tan\sqrt{x}}{\sqrt{x}}\,dx \longrightarrow \int \frac{\sin\sqrt{x}}{\cos\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx \longrightarrow -2\int \frac{-\sin\sqrt{x}}{\cos\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$-2\ln\left|\cos\sqrt{x}\right| + C$$

$$\int \frac{e^{-\tan x}}{\cos^2 x}\,dx \longrightarrow \int e^{-\tan x} \cdot \frac{1}{\cos^2 x}\,dx \longrightarrow \int e^{-\tan x} \cdot \sec^2 x\ dx$$

$$-e^{-\tan x} + C$$

$$\int \frac{2\ \csc^2 3x}{\sec 3x\ \sqrt{\sin 3x}}\, dx \longrightarrow \int \frac{2\ \frac{1}{\sin^2 3x}}{\frac{1}{\cos 3x}\cdot\ (\sin 3x)^{\frac{1}{2}}}\, dx$$

$$2\int \frac{1}{\sin^2 3x}\cdot\frac{1}{(\sin 3x)^{\frac{1}{2}}}\cdot \cos 3x\ dx \longrightarrow 2\int \sin^{\frac{-5}{2}} 3x\ \cdot \cos 3x\ dx \quad \times \frac{3}{3}$$

$$\frac{2}{3}\int \sin^{\frac{-5}{2}} 3x\ \cdot \cos 3x\ \cdot 3\ dx \longrightarrow \frac{2}{3}\cdot\frac{\sin^{\frac{-3}{2}} 3x}{\frac{-3}{2}} + C \longrightarrow \frac{-4}{9}\cdot\frac{1}{\sin^{\frac{3}{2}} 3x} + C$$

$$\frac{-4}{9\sqrt{(\sin 3x)^3}} + C$$

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Second Stage

Mathematics

Integrals of Trigonometric Functions part (2)

Lecture (3)

Presented by :-
Mahmoud Shakir Wahhab

Integrals Involving Powers of Sine and Cosine

Integrals of powers of sine are usually evaluated using trigonometric identities and the solution depends on whether the power is odd or even.

- ***For odd powers,*** the integrand is transformed by factoring out one sine and the remaining even powered sine is converted into cosine using the identity

$\sin^2 x = 1 - \cos^2 x$ $\qquad\qquad$ $\cos^2 x = 1 - \sin^2 x$

EXAMPLE

$$\int \sin^5 x \; dx$$

$$\int \sin^5 x \; dx \longrightarrow \int \sin x \cdot \sin^4 x \; dx \longrightarrow \int \sin x \; (\sin^2 x)^2 \, dx$$

$$\int \sin x \; (1 - \cos^2 x)^2 \, dx \longrightarrow \int \sin x \cdot (1 - 2\cos^2 x + \cos^4 x) \, dx$$

$$\int \sin x \; dx - 2\int \cos^2 x \cdot \sin x \; dx + \int \cos^4 x \cdot \sin x \; dx$$

$$-\cos x + 2\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + C \longrightarrow -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

EXAMPLE

$$\int \cos^3 2x \; dx$$

$$\int \cos^3 2x \; dx \longrightarrow \int \cos 2x \cdot \cos^2 2x \; dx \longrightarrow \int \cos 2x \cdot (1 - \sin^2 2x)\, dx$$

$$\int \cos 2x \; dx - \int \sin^2 2x \cdot \cos 2x \, dx$$

$$\frac{1}{2}\int \cos 2x \cdot 2 \, dx - \frac{1}{2}\int \sin^2 2x \cdot \cos 2x \cdot 2dx$$

$$\frac{1}{2}\sin 2x - \frac{1}{2}\,\frac{\sin^3 2x}{3} + C \longrightarrow \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x + C$$

For even powers, the half angle identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ is used to reduce the power of sine into an expression where direct integration formulas can already be applied.

For even powers, the half angle identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ is used to reduce the power of cosine into an expression where direct integration formulas can already be applied.

EXAMPLE

$$\int \sin^4 x \ dx$$

$$\int (\sin^2 x)^2 \ dx \longrightarrow \int \left[\frac{1}{2}\,(1 - cos2x)\right]^2 dx \longrightarrow \int \left(\frac{1}{2}\right)^2 \cdot (1 - cos2x)^2 \, dx$$

$$\frac{1}{4}\left(\int (1 - 2cos2x + \cos^2 2x)\, dx\right)$$

$$\int \cos^2 2x \ dx \longrightarrow \int \frac{1}{2}\,(1 + \cos 4x) dx \longrightarrow \frac{1}{2}\left[\int dx + \int \cos 4x \, dx\right]$$

$$\frac{1}{2}\left[\int dx + \int \cos 4x \, dx \cdot \frac{4}{4}\right] \longrightarrow \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) \longrightarrow \frac{1}{2}x + \frac{1}{8}\sin 4x$$

$$\therefore \int \cos^2 2x \;\, dx = \frac{1}{2}x + \frac{1}{8}\sin 4x$$

$$\frac{1}{4}\left(\int (1 - 2cos2x + \cos^2 2x)\, dx\right) \longrightarrow \frac{1}{4}\left(\int dx - 2\int cos2x\, dx + \int \cos^2 2x \, dx\right)$$

$$\frac{1}{4}\left(x - \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right) + C \longrightarrow \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

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Second Stage

Mathematics

Integrals of Trigonometric Functions part (3)

Lecture (4)

Presented by :-
Mahmoud Shakir Wahhab

Mixed powers of sine & cosine functions

$$\int \cos^m x \ \sin^n x \ dx \ (m, n \ positive \ integers)$$

A) m,n : one odd / one even

1. Factor out one power from the trigonometric function that has the odd power.

2. Use $\sin^2 x + \cos^2 x = 1$ to transform the remaining even power of the odd trigonometric function into the other trigonometric function.

EXAMPLE

$$\int \cos^5 x \ \sin^2 x \ dx$$

$$\int \cos^4 x \ \sin^2 x \cos x \ dx \longrightarrow \int (1 - \sin^2 x)^2 \sin^2 x \cos x \ dx$$

$$\int (1 - 2\sin^2 x + \sin^4 x) \sin^2 x \cos x \, dx$$

$$\int \sin^2 x \cos x \, dx - 2 \int \sin^4 x \cos x \, dx + \int \sin^6 x \cos x \, dx$$

$$\frac{\sin^3 x}{3} - 2 \cdot \frac{\sin^5 x}{5} + \frac{\sin^7 x}{7} + C \longrightarrow \frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$$

$$\int \cos^m x \, \sin^n x \, dx \; (m, n \; positive \; integers)$$

B) m,n : both odd

1. Choose one of the trigonometric functions and factor out one power.

2. Use $\sin^2 x + \cos^2 x = 1$ to transform the remaining even power of the chosen odd trigonometric function into the other trigonometric function.

EXAMPLE

$$\int \cos^3 x \, \sin^3 x \, dx$$

$$\int \cos^3 x \, \sin^3 x \, dx \longrightarrow \int \cos^3 x \sin^2 x \sin x \, dx$$

$$\int \cos^3 x \, (1 - \cos^2 x) \sin x \, dx \longrightarrow \int \cos^3 x \sin x \, dx - \int \cos^5 x \sin x \, dx$$

$$\frac{-1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + C$$

$$\int \cos^m x \ \sin^n x \ dx \ (m, n \ positive \ integers)$$

C) m,n : both even

Replace all even powers using the half-angle identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

EXAMPLE

$$\int \cos^2 x \, \sin^2 x \, dx$$

$$\int \cos^2 x \, \sin^2 x \, dx \longrightarrow \int \frac{1}{2}(1+\cos 2x)\cdot\frac{1}{2}(1-\cos 2x)dx$$

$$\frac{1}{4}\int (1-\cos^2 2x)dx$$

$$\int \cos^2 2x \, dx \longrightarrow \int \frac{1}{2}(1+\cos 4x)dx \longrightarrow \frac{1}{2}\left(\int dx + \int \cos 4x \, dx\right)$$

$$\frac{1}{2}\left(x+\frac{1}{4}\sin 4x\right) \longrightarrow \left(\frac{1}{2}x+\frac{1}{8}\sin 4x\right)$$

$$\frac{1}{4}\int(1-\cos^2 2x)dx \longrightarrow \frac{1}{4}\left(\int dx - \int \cos^2 2x\, dx\right)$$

$$\frac{1}{4}\left[x-\left(\frac{1}{2}x+\frac{1}{8}\sin 4x\right)\right]+C \longrightarrow \frac{1}{4}\left(\frac{1}{2}x-\frac{1}{8}\sin 4x\right)+C$$

$$\frac{1}{8}x-\frac{1}{32}\sin 4x+C$$

Thank you for listening

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Second Stage

Mathematics

Integrals of Trigonometric Functions part (4)

Lecture (5)

Presented by :-
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Guidelines for Evaluating Integrals Involving Secant and Tangent

- If there are **no secant factors and the power of the tangent is even and positive**, convert a tangent-squared factor into a secant-squared factor, then expand and repeat as necessary

EXAMPLE Evaluate $\int \tan^4 x\, dx$

$$\int \tan^2 x \cdot \tan^2 x\, dx \longrightarrow \int \tan^2 x\,(\sec^2 x - 1)dx \longrightarrow \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx$$

$$\int \tan^2 x \sec^2 x\, dx - \left(\int (\sec^2 x - 1)dx\right) \longrightarrow \int \tan^2 x \sec^2 x\, dx - \int \sec^2 x\, dx + \int dx$$

$$\frac{1}{3}\tan^3 x - \tan x + x + C$$

- If there are **no secant factors and the power of the tangent is odd and positive**, the integrand is transformed by factoring out one tangent and the remaining even powered tangent is converted into secant using the identity $\tan^2 x = \sec^2 x - 1$

EXAMPLE Evaluate $\int \tan^3 x \, dx$

$$\int \tan^2 x \cdot \tan x \, dx \longrightarrow \int (\sec^2 x - 1) \tan x \; dx$$

$$\int \sec^2 x \tan x \; dx - \int \tan x \; dx \longrightarrow \frac{1}{2} \tan^2 x + \ln|\cos x| + C$$

- If there are **no tangent factors and the power of the secant is even and positive.**

Factor out $\sec^2 x$ .Use the identity $\sec^2 x = \tan^2 x + 1$

EXAMPLE Evaluate $\int \sec^4 x \; dx$

$$\int \sec^2 x \cdot \sec^2 x \; dx \longrightarrow \int \sec^2 x \cdot (\tan^2 x + 1) dx$$

$$\int \tan^2 x \sec^2 x \, dx + \int \sec^2 x \, dx \longrightarrow \frac{1}{3} \tan^3 x + \tan x + C$$

- If there are **no tangent factors and the power of the secant is odd, use integration by part**

Integrating powers of products of $\tan x$ *and* $\sec x$

➢ The power of $\tan x$ and $\sec x$ are odd.

1- Factor out one power of $\tan x$ and one power of $\sec x$.

2- Use $\tan^2 x = \sec^2 x - 1$ to transform the remaining even power of tan to be in terms of sec

EXAMPLE Evaluate $\int \tan^3 x \ \sec^3 x \ dx$

Solution

$$\int \tan^2 x \ \sec^2 x \ \tan x \ \sec x \ dx \longrightarrow \int (\sec^2 x - 1) \sec^2 x \ \tan x \ \sec x \ dx$$

$$\int \sec^4 x \ \tan x \ \sec x \ dx - \int \sec^2 x \ \tan x \ \sec x \ dx \longrightarrow \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$$

➢The power of $\tan x$ and $\sec x$ are even.

1- Factor out $\sec^2 x$

2- Use $\sec^2 x = \tan^2 x + 1$ to transform the remaining even power of sec to be in terms of tan

EXAMPLE Evaluate $\int \tan^2 x \ \sec^4 x \ dx$

Solution

$$\int \tan^2 x \ \sec^2 x \sec^2 x \, dx \longrightarrow \int \tan^2 x \ \sec^2 x \, (\tan^2 x + 1) dx$$

$$\int \tan^4 x \ \sec^2 x \ dx + \int \tan^2 x \sec^2 x \, dx \longrightarrow \frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$$

➢ The power of $\tan x$ is odd and power of $\sec x$ is even.

1- Factor out one power of $\tan x$ and one power of $\sec x$.

2- Use $\tan^2 x = \sec^2 x - 1$ to transform the remaining even power of tan to be in terms of sec

EXAMPLE Evaluate $\int \tan^3 x \ \sec^4 x \ dx$

Solution

$$\int \tan^2 x \ \sec^3 x \ \tan x \ \sec x \ dx \longrightarrow \int (\sec^2 x - 1) \sec^3 x \ \tan x \ \sec x \ dx$$

$$\int \sec^5 x \ \tan x \ \sec x \ dx - \int \sec^3 x \ \tan x \ \sec x \ dx \longrightarrow \frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + C$$

➢ The power of $\tan x$ is even and power of $\sec x$ is odd.

use integration by part.

THANK YOU FOR LISTENING

ANY QUESTION

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Second Stage

Mathematics

Integrals of Inverse Trigonometric Functions

Lecture (6)

Presented by :-
Mahmoud Shakir Wahhab

Before studying the integrations of inverse trigonometric functions, we must remember the laws of derivatives of inverse trigonometric functions. Below are the derivatives of the six inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\cos^{-1}(u) = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \qquad |u| < 1$$

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1}(u) = \frac{-1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}, \qquad |u| > 1$$

$$\frac{d}{dx}\csc^{-1}(u) = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}, \qquad |u| > 1$$

We learned about the Inverse Trigonometric Functions, and it turns out that the derivatives of them are not trigonometric expressions, but algebraic. When memorizing these, remember that the functions starting with “c” are negative, and the functions with tan and cot don’t have a square root.

Also remember that sometimes you see the inverse trigonometric function written as $\arcsin x$ and sometimes you see $\sin^{-1} x$.

Integrals Involving the Inverse Trigonometric Functions

When we integrate to get Inverse Trigonometric Functions back, we have use tricks to get the functions to look like one of the inverse trigonometric. Here are the integration formulas involving the Inverse Trigonometric Functions, notice that we only have formulas for three of the inverse trigonometric functions.

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C \qquad \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{1+u^2} = \tan^{-1}(u) + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}(u) + C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right) + C$$

EXAMPLE

$$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-(x)^2}}\,dx \longrightarrow \sin^{-1} x\Big|_0^{\frac{1}{2}} \longrightarrow \left[\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)\right] \longrightarrow \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

EXAMPLE

$$\int \frac{dx}{\sqrt{1-16x^2}}$$

$$\int \frac{1}{\sqrt{1-(4x)^2}}\,dx \longrightarrow \frac{1}{4}\int \frac{4}{\sqrt{1-(4x)^2}}\,dx \longrightarrow \frac{1}{4}\sin^{-1}(4x) + C$$

EXAMPLE $\displaystyle\int \frac{dy}{9+y^2}$

$$\int \frac{1}{(3)^2 + (y)^2}\, dy \longrightarrow \frac{1}{3}\tan^{-1}\left(\frac{y}{3}\right) + C$$

EXAMPLE $\displaystyle\int \frac{2\cos t}{1+\sin^2 t}\, dt$

$$2\int \frac{\cos t}{1 + (\sin t)^2}\, dt \longrightarrow 2\tan^{-1}(\sin t) + C$$

EXAMPLE $$\int \frac{dx}{x\sqrt{4x^2-1}}$$

$$\int \frac{1}{x\sqrt{(2x)^2-1}}\,dx \longrightarrow \int \frac{2}{2x\sqrt{(2x)^2-1}}\,dx \longrightarrow \sec^{-1}|2x| + C$$

EXAMPLE $$\int \frac{dx}{x\sqrt{x^2-9}}$$

$$\int \frac{1}{x\sqrt{(x)^2-3^2}}\,dx \longrightarrow \frac{1}{3}\sec^{-1}\left(\frac{|x|}{3}\right) + C$$

EXAMPLE $\displaystyle\int \frac{dx}{25+4x^2}$

$$\int \frac{dx}{4\left(\frac{25}{4}+x^2\right)} \longrightarrow \frac{1}{4}\int \frac{1}{\left(\left(\frac{5}{2}\right)^2+x^2\right)}\,dx \longrightarrow \frac{1}{4}\cdot\frac{1}{\frac{5}{2}}\tan^{-1}\left(\frac{x}{\frac{5}{2}}\right)+C$$

$$\frac{1}{4}\cdot\frac{2}{5}\tan^{-1}\left(\frac{2x}{5}\right)+C \longrightarrow \frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right)+C$$

EXAMPLE $\int \frac{dx}{x\sqrt{16x^2 - 4}}$

$$\int \frac{dx}{x\sqrt{16\left(x^2 - \frac{1}{4}\right)}} \longrightarrow \int \frac{dx}{4x\sqrt{x^2 - \frac{1}{4}}} \longrightarrow \frac{1}{4}\int \frac{1}{x\sqrt{x^2 - \left(\frac{1}{2}\right)^2}}\,dx$$

$$\frac{1}{4} \cdot \frac{1}{\frac{1}{2}} \sec^{-1}\left(\frac{x}{\frac{1}{2}}\right) + C \longrightarrow \frac{1}{2}\sec^{-1}|2x| + C$$

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Second Stage

Mathematics

Integration by Substitution

(Indefinite Integral)

Lecture (7)

Presented by :-
Mahmoud Shakir Wahhab

There are several techniques for rewriting an integral so that it fits one or more of the basic formulas. One of the most powerful techniques is integration by substitution. With this technique, you choose part of the integrand to be u and then rewrite the entire integral in terms of u.

Why is substitution method used in integration?

The substitution method (also called u−substitution) is used when an integral contains some function and its derivative. In this case, we can set u equal to the function and rewrite the integral in terms of the new variable u. This makes the integral easier to solve.

Steps to solve integration by substitution method

1. Let u be a function of x (usually part of the integrand).

2. Solve for x and dx in terms of u and du.

3. Convert the entire integral to u-variable form and try to fit it to one or more of the basic integration formulas. If none fits, try a different substitution.

4. After integrating, rewrite the anti-derivative as a function of x.

EXAMPLE

$$\int \frac{x}{\sqrt{(x+1)^2}}\, dx$$

Solution

$$let\ u = x + 1 \longrightarrow x = u - 1 \longrightarrow dx = du$$

$$\int \frac{u-1}{u^2}\, du \longrightarrow \int \frac{u}{u^2}\, du - \int \frac{1}{u^2} du \longrightarrow \int \frac{1}{u}\, du - \int u^{-2}\, du$$

$$\ln|u| - \frac{u^{-1}}{-1} + C \longrightarrow \ln|u| + \frac{1}{u} + C \longrightarrow \ln|x+1| + \frac{1}{(x+1)} + C$$

EXAMPLE

$$\int \frac{1}{\sqrt{x}(1+x)}\,dx$$

Solution

$$let\ u = \sqrt{x} \longrightarrow u^2 = x \longrightarrow 2u\,du = dx$$

$$\int \frac{1}{\cancel{u}\,(1+u^2)} \cdot 2\cancel{u}\,du \longrightarrow 2\int \frac{1}{1+u^2}\,du$$

$$2\ \tan^{-1} u + C \longrightarrow 2\ \tan^{-1}\sqrt{x} + C$$

EXAMPLE

$$\int \frac{3\cos(\frac{\pi}{x})}{x^2}\, dx$$

Solution

$$let\ u = \frac{\pi}{x} \longrightarrow u = \pi x^{-1} \longrightarrow du = -\pi x^{-2} dx \longrightarrow du = \frac{-\pi dx}{x^2}$$

$$dx = \frac{-x^2 du}{\pi}$$

$$\int \frac{3\cos(u)}{\cancel{x^2}} \cdot \frac{-\cancel{x^2} du}{\pi} \longrightarrow \frac{-3}{\pi}\int \cos(u)\, du \longrightarrow \frac{-3}{\pi}\sin u + C$$

$$\frac{-3}{\pi}\sin\left(\frac{\pi}{x}\right) + C$$

EXAMPLE

$$\int \frac{\sin x}{1+\cos^2 x}\, dx$$

Solution

$$let\ u = \cos x \longrightarrow du = -\sin x\ dx \longrightarrow dx = \frac{du}{-\sin x}$$

$$\int \frac{\cancel{\sin x}}{1+u^2} \cdot \frac{du}{-\cancel{\sin x}} \longrightarrow -\int \frac{1}{1+u^2}\, du \longrightarrow -\tan^{-1} u + C$$

$$-\tan^{-1}(\cos x) + C$$

EXAMPLE $\int x^2\sqrt{1-x}\,dx$

Solution

$$let\ u = 1-x \longrightarrow x = 1-u \longrightarrow dx = -du$$

$$\int (1-u)^2\cdot\sqrt{u}\,(-du) \longrightarrow -\int (1-2u-u^2)\cdot u^{\frac{1}{2}}\,du$$

$$-\left\{\int u^{\frac{1}{2}}\,du - 2\int u^{\frac{3}{2}}\,du - \int u^{\frac{5}{2}}\,du\right\} \longrightarrow -\left\{\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - 2\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{7}{2}}}{\frac{7}{2}}\right\} + C$$

$$-\frac{2}{3}\sqrt{u^3}+\frac{4}{5}\sqrt{u^5}+\frac{7}{2}\sqrt{u^7}+C \longrightarrow -\frac{2}{3}\sqrt{(1-x)^3}+\frac{4}{5}\sqrt{(1-x)^5}+\frac{7}{2}\sqrt{(1-x)^7}+C$$

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Second Stage

Mathematics

Integration by Substitution

(definite Integral)

Lecture (8)

Presented by :-
Mahmoud Shakir Wahhab

- To evaluate definite integrals, it is often more convenient to determine the limits of integration for the variable u. This is often easier than converting back to the variable x and evaluating the anti-derivative at the original limits.

- If you are dealing with definite integrals (ones with limits of integration) you must be particularly careful when you substitute.

Definite Integral Using U-Substitution

- When evaluating a definite integral using u-substitution, one has to deal with the **limits of integration.**

- We can change the limits of integration when we make the substitution, calculate the anti-derivative in terms of u and evaluate using the new limits of integration

EXAMPLE Evaluate $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$ Using Substitution

Solution

$$let\ u = \sqrt{2x-1} \longrightarrow u^2 = 2x-1 \longrightarrow x = \frac{1}{2}(u^2+1) \longrightarrow dx = u\,du$$

Lower limit: When x=1 $\longrightarrow u = \sqrt{2(1)-1} = 1$

Upper limit: When x=5 $\longrightarrow u = \sqrt{2(5)-1} = 3$

$$\int_1^3 \frac{(u^2+1)}{2\,u} \cdot u\,du \longrightarrow \frac{1}{2}\int_1^3 (u^2+1)\,du \longrightarrow \frac{1}{2}\left\{\frac{u^3}{3} + u\Big|_1^3\right\}$$

$$\frac{1}{2}\left[(9+3) - \left(\frac{1}{3}+1\right)\right] \longrightarrow \frac{16}{3}$$

EAMPLE Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^3\theta \cos\theta \; d\theta$ Using Substitution

Solution

$let\ u = \sin\theta \longrightarrow du = \cos\theta \; d\theta \longrightarrow d\theta = \dfrac{du}{\cos\theta}$

Lower limit: When $\theta = \frac{\pi}{4} \longrightarrow u = \sin\left(\frac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$

Upper limit: When $\theta = \frac{\pi}{3} \longrightarrow u = \sin\left(\frac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} u^3 \cdot \cancel{\cos\theta} \cdot \frac{du}{\cancel{\cos\theta}} \longrightarrow \int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} u^3 \, du \longrightarrow \frac{1}{4}u^4 \Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \longrightarrow \frac{1}{4}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right]$$

$$= \frac{5}{64}$$

EAMPLE Evaluate $\int_{4\pi^2}^{9\pi^2} \frac{\sin(\sqrt{x}\,)}{\sqrt{x}}\,dx$ Using Substitution

Solution

$let\ u = \sqrt{x} \longrightarrow u^2 = x \longrightarrow 2u\,du = dx$

Lower limit: When $x = 4\pi^2 \longrightarrow u = 2\pi$

Upper limit: When $x = 9\pi^2 \longrightarrow u = 3\pi$

$$\int_{2\pi}^{3\pi} \frac{\sin(u)}{u} \cdot 2u\,du \longrightarrow 2\int_{2\pi}^{3\pi} \sin(u)\,du \longrightarrow 2[-\cos u|_{2\pi}^{3\pi}]$$

$$2[\cos(2\pi) - \cos(3\pi)] \longrightarrow 2(1+1) = 4$$

EAMPLE Evaluate $\int_{0}^{\pi} \cos(\theta)\sqrt{\sin\theta}\ d\theta$ Using Substitution

Solution

$let\ u = \sin\theta \longrightarrow du = \cos\theta\ d\theta \longrightarrow d\theta = \frac{du}{\cos\theta}$

Lower limit: When $\sin(0) \longrightarrow u = 0$

Upper limit: When $\sin(\pi) \longrightarrow u = 0$

$$\int_{0}^{0} \sqrt{u} \cdot \cos(\theta) \frac{du}{\cos\theta} \longrightarrow \int_{0}^{0} u^{\frac{1}{2}}\, du \longrightarrow \left. \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right|_{0}^{0} \longrightarrow \frac{2}{3}\left[(0)^{\frac{3}{2}} - (0)^{\frac{3}{2}}\right] = 0$$

EAMPLE Evaluate $\int_0^{\frac{\pi}{2}} \cos x \ \sin(\sin x) \ dx$ Using Substitution

Solution

$let\ u = \sin x \longrightarrow du = \cos x \ dx \longrightarrow dx = \dfrac{du}{\cos x}$

Lower limit: When $\sin(0) \longrightarrow u = 0$

Upper limit: When $\sin\left(\frac{\pi}{2}\right) \longrightarrow u = 1$

$$\int_0^1 \cos x \sin(u) \cdot \frac{du}{\cos x} \longrightarrow \int_0^1 \sin(u)\, du \longrightarrow (-\cos x)\Big|_0^1$$

$$\cos(0) - \cos(1) \longrightarrow 1 - \cos(1)$$

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Mathematics

Integration by Trigonometric Substitution

Lecture (9)

Presented by :-
Mahmoud Shakir Wahhab

- The technique of trigonometric substitution is very natural. The principle aim of trigonometric substitution is to remove square roots, however the method is useful for a wider class of examples.

- In each substitution the square-root is eliminated. We trade an integral with a square root for a new integral of some trigonometric function, so this is a good bargain for most examples. There are more advanced trigonometric substitutions, but we will focus on just these three basic cases.

- This type of substitution is usually indicated when the function you wish to integrate contains a polynomial expression that might allow you to use the fundamental identity $\sin^2 x + \cos^2 x = 1$ in one of three forms:

$\cos^2 x = 1 - \sin^2 x$ $\qquad$ $\sec^2 x = 1 + \tan^2 x$ $\qquad$ $\tan^2 x = \sec^2 x - 1$

- If your function contains $1 - x^2$ let $x = \sin\theta$, if it contains $1 + x^2$ let $x = \tan\theta$, if it contains $x^2 - 1$ let $x = \sec x$

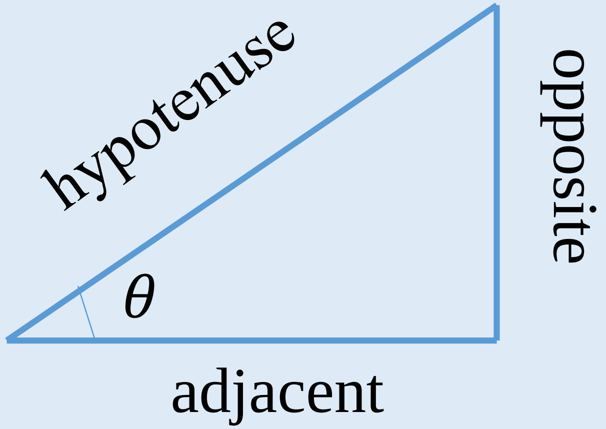
- If your function contains $a^2 - x^2$ let $x = \mathrm{a}\sin\theta$, if it contains $a^2 + x^2$ let $x = \mathrm{a}\tan\theta$, if it contains $x^2 - a^2$ let $x = \mathrm{a}\sec x$

The process for finding integrals using trigonometric substitution

- Try to fit your problem to one of the patterns $a^2 - x^2$, $a^2 + x^2$, $x^2 - a^2$
- Make the suitable substitution of x and dx.
- Simplify the integrand as needed and integrate.
- Reverse substitute until your result is in terms of x.
- Draw a triangle to get rid of expressions involving $\sin\theta$, $\cos\theta$, $\tan\theta$, $etc.$, which aren't easily handled by the substitution itself.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$



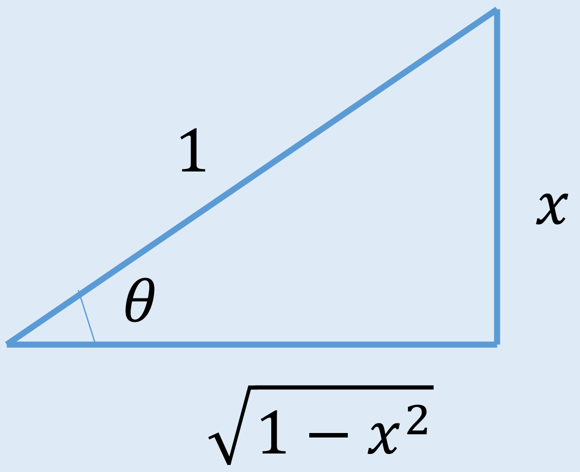
EXAMPLE Evaluate $\displaystyle\int \frac{dx}{x^2\sqrt{1-x^2}}$

Solution

$$x = \sin\theta \longrightarrow dx = \cos\theta \; d\theta$$

$$\int \frac{\cos\theta \; d\theta}{\sin^2\theta\sqrt{1-\sin^2\theta}} \longrightarrow \int \frac{\cos\theta \; d\theta}{\sin^2\theta \cdot \cos\theta} \longrightarrow \int \frac{1}{\sin^2\theta}\, d\theta$$

$$\int \csc^2\theta \; d\theta \longrightarrow -\cot\theta + C \longrightarrow \frac{-\sqrt{1-x^2}}{x} + C$$



EXAMPLE Evaluate $\int \sqrt{9-x^2}\,dx$

Solution

$x = 3\sin\theta \longrightarrow dx = 3\cos\theta\ d\theta$

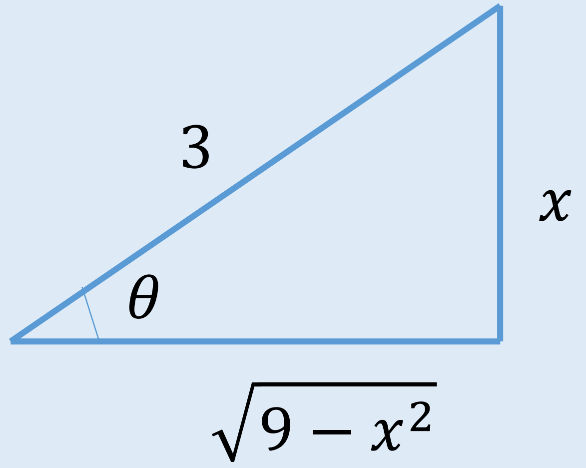
$\int \sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\ d\theta \longrightarrow \int \sqrt{9(1-\sin^2\theta)}\,3\cos\theta\,d\theta$

$9\int \cos\theta\cdot\cos\theta\,d\theta \longrightarrow 9\int \cos^2\theta\ d\theta \longrightarrow 9\int \frac{1}{2}(1+\cos 2\theta)d\theta$

$\frac{9}{2}\left[\int d\theta + \int \cos 2\theta\,d\theta\right] \longrightarrow \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$

$$\frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$\because \sin 2\theta = 2\sin\theta\cos\theta$$



$$\frac{9}{2}\left(\theta + \frac{1}{2}\cdot 2\sin\theta\cos\theta\right) + C \longrightarrow \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + C$$

$$x = 3\sin\theta \longrightarrow \sin\theta = \frac{x}{3} \longrightarrow \theta = \sin^{-1}\left(\frac{x}{3}\right)$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\cdot\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{x} \longrightarrow \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x\sqrt{9-x^2}}{2} + C$$

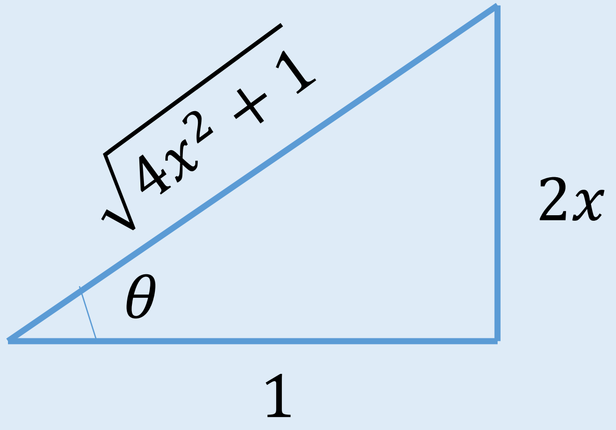
EXAMPLE Evaluate $\displaystyle\int \frac{dx}{\sqrt{4x^2+1}}$

Solution

$$2x = \tan\theta \longrightarrow x = \frac{1}{2}\tan\theta \longrightarrow dx = \frac{1}{2}\sec^2\theta \; d\theta$$

$$\int \frac{\frac{1}{2}\sec^2\theta \; d\theta}{\sqrt{4\cdot\frac{1}{4}\tan^2\theta + 1}} \longrightarrow \frac{1}{2}\int \frac{\sec^2\theta \; d\theta}{\sqrt{\tan^2\theta+1}} \longrightarrow \frac{1}{2}\int \frac{\sec^2\theta \; d\theta}{\sqrt{\sec^2\theta}}$$

$$\frac{1}{2}\int \sec\theta \; d\theta \longrightarrow \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$



$$\frac{1}{2}\ln\left|\sqrt{4x^2+1} + 2x\right| + C$$

EXAMPLE

Evaluate $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx$

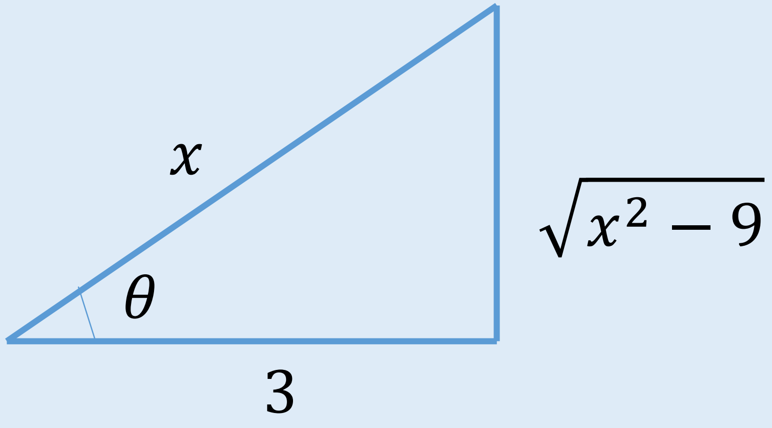
Solution

$$x = 3\sec\theta \longrightarrow dx = 3\sec\theta\tan\theta\ d\theta \longrightarrow \sec\theta = \frac{x}{3} \longrightarrow \theta = \sec^{-1}\left(\frac{x}{3}\right)$$

$$\int \frac{\sqrt{9\sec^2\theta - 9}}{3\sec\theta}\cdot 3\sec\theta\tan\theta\ d\theta \longrightarrow \int \sqrt{9(\sec^2\theta - 1)}\cdot\tan\theta\ d\theta \longrightarrow 3\int \tan^2\theta\, d\theta$$

$$3\left[\int(\sec^2\theta - 1)\ d\theta\right] \longrightarrow 3\left(\int \sec^2\theta\, d\theta - \int d\theta\right)$$

$$3(\tan\theta - \theta) + C \longrightarrow 3\left(\frac{\sqrt{x^2-9}}{3} - \sec^{-1}\left(\frac{x}{3}\right)\right) + C$$



$$\sqrt{x^2-9} - 3\sec^{-1}\left(\frac{x}{3}\right) + \mathrm{C}$$

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Mathematics

Integration by Partial Fractions
Part (1)

Lecture (10)

Presented by :-
Mahmoud Shakir Wahhab

- The method of partial fractions is used to integrate rational functions. That is, we want to compute $\int \frac{P\,(x)}{Q\,(x)}\,dx$ where P,Q are polynomials.

- Partial fractions is the name given to a technique of integration that may be used to integrate any ratio of polynomials. A ratio of polynomials is called a rational function.

How do you know when to use partial fractions in integration?

Partial fractions can only be done if the degree of the numerator is strictly less than the degree of the denominator. That is important to remember.

How many types of partial fractions are there?

The Fundamental Theorem of Algebra thus tells us that there are 4 **different** "simplest"‘ denominator **types**:

Type 1: Linear Factors:- Suppose that our denominator can be factorized completely into distinct linear factors. That is

$$Q(x) = (x - a_1)(x - a_2) \dots (x - a_n)$$

where the values $a_1, a_2 \dots a_n$ are all different

$$\frac{x+8}{x^2+x-2} = \frac{x+8}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

Type 2: Repeated Linear Factors:- Suppose that when we factorize Q(x) we obtain a repeated linear factor. That is, some term of the form $(x-a)^m$ where $m \geq 2$. In a partial fractions decomposition, such a factor produces m seperate contributions:

$$\frac{A}{(x-a)^m}+\frac{B}{(x-a)^{m-1}}+\frac{C}{(x-a)^{m-2}}+\frac{D}{(x-a)}$$

$$\frac{x+5}{x^3(x+2)^2} = \frac{A}{x^3}+\frac{B}{x^2}+\frac{C}{x}+\frac{D}{(x+2)^2}+\frac{E}{(x+2)}$$

Type 3: Quadratic Factors (irreducible factors of degree 2)

Suppose that the denominator Q(x) contains an irreducible quadratic term: a term of the form $ax^2 + bx + c \;\; where \;\; b^2 - 4ac < 0$

$$\frac{x^2 - x + 2}{x^3 + 4x} = \frac{x^2 - x + 2}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

Type 4: Repeated Quadratic Factors (repeated irreducible factors of degree 2)

If Q(x) contains a repeated factor $(ax^2+bx+c)^m$ where ax^2+bx+c is irreducible and $m \geq 2$

$$\frac{Ax+B}{(ax^2+bx+c)^m}+\frac{Cx+D}{(ax^2+bx+c)^{m-1}}+\cdots+\frac{Ex+F}{(ax^2+bx+c)}$$

$$\frac{x-1}{(x^2+2x+5)^2(x-3)^3}$$

$$=\frac{Ax+B}{(x^2+2x+5)^2}+\frac{Cx+D}{(x^2+2x+5)}+\frac{E}{(x-3)^3}+\frac{F}{(x-3)^2}+\frac{G}{(x-3)}$$

In mathematics we often combine two or more rational expressions into one.

$$\frac{4}{x+1}+\frac{3}{x-2} \rightarrow \frac{4(x-2)+3(x+1)}{(x+1)(x-2)} \rightarrow \frac{4x-8+3x+3}{x^2-2x+x-2} \rightarrow \frac{7x-5}{x^2-x-2}$$

Occasionally, however, the reverse procedure is necessary. The problem is to take a fraction whose denominator is a product of factors, and split it into a sum of simpler fractions.

EXAMPLE Evaluate $\int \frac{x+8}{x^2+x-2}$

Solution

$$\frac{x+8}{x^2+x-2} = \frac{x+8}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$A = \frac{x+8}{\cancel{(x+2)}(x-1)} \cdot \cancel{(x+2)}\Big|_{x=-2} \longrightarrow A = -2$$

$$B = \frac{x+8}{(x+2)\cancel{(x-1)}} \cdot \cancel{(x-1)}\Big|_{x=1} \longrightarrow B = 3$$

$$\int \frac{-2}{x+2}dx + \int \frac{3}{x-1}dx \longrightarrow -2\ln|x+2| + 3\ln|x-1| + C$$

$$3\ln|x-1| - 2\ln|x+2| + C \longrightarrow \ln|x-1|^3 - \ln|x+2|^2 + C \longrightarrow \ln\frac{|x-1|^3}{|x+2|^2} + C$$

EXAMPLE Evaluate $\int \frac{1}{(x-1)^2\,(x+1)}\,dx$

Solution

$$\frac{1}{(x-1)^2\,(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)}$$

$$A = \frac{1}{(x-1)^2\,\cancel{(x+1)}} \cdot \cancel{(x+1)}\Big|_{x=-1} \longrightarrow A = \frac{1}{4}$$

$$B = \frac{1}{\cancel{(x-1)^2}\,(x+1)} \cdot \cancel{(x-1)^2}\Big|_{x=1} \longrightarrow B = \frac{1}{2}$$

$$C = \frac{1}{(x-1)^2\,(x+1)} \cdot (x-1)\Big|_{x=1} \longrightarrow C = \infty$$

$$\frac{1}{(x-1)^2\,(x+1)} = \frac{1/4}{(x+1)} + \frac{1/2}{(x-1)^2} + \frac{C}{(x-1)}$$

$$\frac{1}{(x-1)^2\,(x+1)} = \frac{1/4(x-1)^2 + 1/2\,(x+1) + C(x-1)(x+1)}{(x-1)^2\,(x+1)}$$

$$1 = \frac{1}{4}(x^2 - 2x + 1) + \frac{1}{2}(x+1) + C(x^2 - 1)$$

$$1 = \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4} + \frac{1}{2}x + \frac{1}{2} + Cx^2 - C$$

$$1 = \frac{1}{4}x^2 + \frac{3}{4} + Cx^2 - C$$

$$x^2 \to \quad 0 = \frac{1}{4} + C \longrightarrow C = \frac{-1}{4} \qquad\qquad x^0 \to \quad 1 = \frac{3}{4} - C \longrightarrow C = \frac{-1}{4}$$

$$\int \frac{1/4}{(x+1)}dx + \int \frac{1/2}{(x-1)^2}dx + \int \frac{-1/4}{(x-1)}dx$$

$$\frac{1}{4}\int \frac{1}{(x+1)}dx + \frac{1}{2}\int (x-1)^{-2}\,dx - \frac{1}{4}\int \frac{1}{(x-1)}dx$$

$$\frac{1}{4}\ln|(x+1)| + \frac{1}{2}\frac{(x-1)^{-1}}{-1} - \frac{1}{4}\ln|(x-1)| + C \longrightarrow \frac{1}{4}\ln\left|\frac{x+1}{x-1}\right| - \frac{1}{2(x-1)} + C$$

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Mathematics

Integration by Partial Fractions
Part (2)

Lecture (11)

Presented by :-
Mahmoud Shakir Wahhab

Type 3: Quadratic Factors (irreducible factors of degree 2)

EXAMPLE Evaluate $\displaystyle\int \frac{x^2+x-3}{(x+1)(x^2-2x+3)}$

Solution

$$\frac{x^2+x-3}{(x+1)(x^2-2x+3)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-2x+3)}$$

$$A = \frac{x^2+x-3}{\cancel{(x+1)}(x^2-2x+3)} \cdot \cancel{(x+1)}\Big|_{x=-1} \longrightarrow A = \frac{-1}{2}$$

$$\frac{x^2+x-3}{(x+1)(x^2-2x+3)} = \frac{-1/2}{(x+1)} + \frac{Bx+C}{(x^2-2x+3)}$$

$$\frac{x^2+x-3}{(x+1)(x^2-2x+3)} = \frac{-1/2\,(x^2-2x+3)+(Bx+C)(x+1)}{(x+1)(x^2-2x+3)}$$

$$x^2+x-3 = -1/2\,(x^2-2x+3)+(Bx+C)(x+1)$$

$$x^2+x-3 = \frac{-1}{2}x^2+x-\frac{3}{2}+Bx^2+Bx+Cx+C \qquad \times 2$$

$$2x^2+2x-6 = -x^2+2x-3+2Bx^2+2Bx+2Cx+2C$$

$$x^2 \;\rightarrow\; 2=-1+2B \longrightarrow B=\frac{3}{2}$$

$$2x^2 + 2x - 6 = -x^2 + 2x - 3 + 2Bx^2 + 2Bx + 2Cx + 2C$$

$$x^0 \;\rightarrow\; -6 = -3 + 2C \longrightarrow C = \frac{-3}{2}$$

$$\int \frac{-1/2}{x+1}dx + \int \frac{3/2\,x - 3/2}{x^2 - 2x + 3}dx \longrightarrow \frac{-1}{2}\int \frac{1}{x+1}dx + \frac{3}{2}\int \frac{x-1}{x^2-2x+3}dx \quad \times\frac{2}{2}$$

$$\frac{-1}{2}\int \frac{1}{x+1}dx + \frac{3}{4}\int \frac{2(x-1)}{x^2-2x+3}dx \longrightarrow \frac{-1}{2}\ln|x+1| + \frac{3}{4}\ln|x^2-2x+3| + C$$

Type 4: Repeated Quadratic Factors (repeated irreducible factors of degree 2)

EXAMPLE Evaluate $$\int \frac{x^3 + x^2 + 5x + 4}{(x^2 + 4)^2}\, dx$$

Solution

$$\frac{x^3 + x^2 + 5x + 4}{(x^2 + 4)^2} = \frac{Ax + B}{(x^2 + 4)^2} + \frac{Cx + D}{(x^2 + 4)}$$

$$\frac{x^3 + x^2 + 5x + 4}{(x^2 + 4)^2} = \frac{(Ax + B) + (Cx + D)(x^2 + 4)}{(x^2 + 4)^2}$$

$$x^3 + x^2 + 5x + 4 = Ax + B + Cx^3 + 4Cx + Dx^2 + 4D$$

$$x^3 + x^2 + 5x + 4 \quad = Ax + B + Cx^3 + 4Cx + Dx^2 + 4D$$

$$x^3 \rightarrow \quad 1 = C \longrightarrow C = 1$$

$$x^2 \rightarrow \quad 1 = D \longrightarrow D = 1$$

$$x^1 \rightarrow \quad 5 = A + 4C \longrightarrow A = 1$$

$$x^0 \rightarrow \quad 4 = B + 4D \longrightarrow B = 0$$

$$\int \frac{x}{(x^2+4)^2}dx + \int \frac{x+1}{x^2+4}dx \longrightarrow \int x\,(x^2+4)^{-2}dx + \int \frac{x}{x^2+4}dx + \int \frac{1}{x^2+4}dx$$

$$\frac{1}{2}\int (x^2+4)^{-2} \cdot 2xdx + \frac{1}{2}\int \frac{2x}{x^2+4}dx + \frac{1}{2}\int \frac{2dx}{x^2+(2)^2}$$

$$\frac{-1}{2(x^2+4)} + \frac{1}{2}\ln|x^2+4| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

EXAMPLE Evaluate $\int \frac{x^3+6x-2}{x^4+6x^2}dx$

Solution

$$\frac{x^3+6x-2}{x^2(x^2+6)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+6}$$

$$A = \frac{x^3+6x-2}{x^2(x^2+6)} \cdot x^2 \bigg|_{x=0} \longrightarrow A = \frac{-1}{3}$$

$$\frac{x^3+6x-2}{x^2(x^2+6)} = \frac{-1/3}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+6}$$

$$\frac{x^3+6x-2}{x^2(x^2+6)} = \frac{-1/3\,(x^2+6) + Bx(x^2+6) + x^2(Cx+D)}{x^2(x^2+6)}$$

$$x^3 + 6x - 2 = \frac{-1}{3}x^2 - 2 + Bx^3 + 6Bx + Cx^3 + Dx^2 \qquad \times 3$$

$$3x^3 + 18x - 6 = -x^2 - 6 + 3Bx^3 + 18Bx + 3Cx^3 + 3Dx^2$$

$$x^3 \;\rightarrow\; 3 = 3B + 3C \longrightarrow B + C = 1 \longrightarrow 1$$

$$x^2 \;\rightarrow\; 0 = -1 + 3D \longrightarrow D = \frac{1}{3}$$

$$x \;\rightarrow\; 18 = 18B \longrightarrow B = 1$$

Substitute the value of B into Equation 1 $\longrightarrow$ $C = 0$

$$\frac{A}{x^2}+\frac{B}{x}+\frac{Cx+D}{x^2+6} \qquad A=\frac{-1}{3} \qquad B=1 \qquad C=0 \qquad D=\frac{1}{3}$$

$$\int \frac{-1/3}{x^2}dx+\int \frac{1}{x}dx+\int \frac{1/3}{x^2+6} \longrightarrow \frac{-1}{3}\int x^{-2}dx+\int \frac{1}{x}dx+\frac{1}{3}\int \frac{1}{x^2+6}dx$$

$$\frac{-1}{3}\cdot\frac{x^{-1}}{-1}+\ln|x|+\frac{1}{3}\cdot\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)+C$$

$$\frac{1}{3x}+\ln|x|+\frac{1}{3\sqrt{6}}\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)+C$$

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Mathematics

Integration by Long Division

Lecture (11)

Presented by :-
Mahmoud Shakir Wahhab

- When the degree of the numerator is greater than or equal to the degree of the denominator the fraction is said to be improper. In such cases it is first necessary to carry out long division.

- To do long division we get the answer one digit at a time, then multiply, subtract, and get the remainder. Then the answer is $(\text{quotient}) + \dfrac{(\text{remainder})}{(\text{divisor})}$.

EXAMPLE Evaluate $\int \frac{3x^3 - 8x^2 + 4x - 1}{x^2 - 3x + 2} dx$

Solution

$$\int (3x+1)dx + \int \frac{x-3}{x^2-3x+2} dx$$

$$\begin{array}{r|l} & 3x \; +1 \\ \hline x^2 - 3x + 2 & 3x^3 - 8x^2 + 4x - 1 \\ & \mp 3x^3 \pm 9x^2 \mp 6x \\ \hline & x^2 - 2x - 1 \\ & \mp x^2 \pm 3x \mp 2 \\ \hline & x - 3 \end{array}$$

$$\int \frac{x-3}{(x-2)(x-1)} dx$$

$$\frac{A}{(x-2)} + \frac{B}{(x-1)}$$

$$A = \frac{x-3}{(x-2)(x-1)} \cdot (x-2)\Big|_{x=2} \longrightarrow A = -1$$

$$B = \frac{x-3}{(x-2)(x-1)} \cdot (x-1)\Big|_{x=1} \longrightarrow B = 2$$

$$\int (3x+1)dx + \int \frac{-1}{x-2}dx + \int \frac{2}{x-1}dx$$

$$3\int x\,dx + \int dx - \int \frac{1}{x-2}dx + \int \frac{2}{x-1}dx$$

$$\frac{3x^2}{2} + x - \ln|x-2| + 2\ln|x-1| + C$$

EXAMPLE

Evaluate $\displaystyle\int \frac{x^4+9x^3+31x^2+49x+27}{x^3+5x^2+8x+4}\,dx$

Solution

$$\begin{array}{r|l} & x+4 \\ \hline x^3+5x^2+8x+4 & x^4+9x^3+31x^2+49x+27 \\ & \mp x^4 \mp 5x^3 \mp 8x^2 \mp 4x \\ \hline & 4x^3+23x^2+45x+27 \\ & \mp 4x^3 \mp 20x^2 \mp 32x \mp 16 \\ \hline & 3x^2+13x+11 \end{array}$$

$$(x+4)+\frac{3x^2+13x+11}{x^3+5x^2+8x+4}$$

$(1)(4)$

$(-1)(-4)$

$(2)(2)$

$(-2)(-2)$

$$(-1)^3+5(-1)^2+8(-1)+4$$

$$-1+5-8+4=0$$

$$\begin{array}{r|l}
 & x^2+4x+4 \\
\hline
x+1 & x^3+5x^2+8x+4 \\
 & \mp x^3 \mp x^2 \\
\cline{2-2}
 & 4x^2+8x+4 \\
 & \mp 4x^2 \mp 4x \\
\cline{2-2}
 & 4x+4 \\
 & \mp 4x \mp 4 \\
\cline{2-2}
 & 0
\end{array}$$

$$\frac{3x^2+13x+11}{(x+1)(x^2+4x+4)} \longrightarrow \frac{3x^2+13x+11}{(x+1)(x+2)^2}$$

$$\frac{3x^2+13x+11}{(x+1)(x+2)^2} = \frac{A}{(x+1)} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)}$$

$$A = \frac{3x^2+13x+11}{(x+1)(x+2)^2} \cdot (x+1)\Big|_{x=-1} \longrightarrow A = 1$$

$$B = \frac{3x^2+13x+11}{(x+1)(x+2)^2} \cdot (x+2)^2\Big|_{x=-2} \longrightarrow B = 3$$

$$\frac{3x^2+13x+11}{(x+1)(x+2)^2} = \frac{1}{(x+1)} + \frac{3}{(x+2)^2} + \frac{C}{(x+2)}$$

$$\frac{3x^2+13x+11}{(x+1)(x+2)^2} = \frac{(x+2)^2+3(x+1)+C(x+1)(x+2)}{(x+1)(x+2)^2}$$

$$3x^2+13x+11 = x^2+4x+4+3x+3+Cx^2+3Cx+2C$$

$$x^2 \quad \rightarrow \quad 3 = 1 + C \longrightarrow C = 2$$

$$\therefore \frac{x^4+9x^3+31x^2+49x+27}{x^3+5x^2+8x+4} = (x+4) + \frac{1}{(x+1)} + \frac{3}{(x+2)^2} + \frac{2}{(x+2)}$$

$$\int \frac{x^4 + 9x^3 + 31x^2 + 49x + 27}{x^3 + 5x^2 + 8x + 4} dx$$

$$= \int (x+4)\, dx + \int \frac{1}{(x+1)} dx + \int \frac{3}{(x+2)^2} dx + \int \frac{2}{(x+2)} dx$$

$$\int x\, dx + 4\int dx + \int \frac{1}{(x+1)} dx + 3\int (x+2)^{-2} + 2\int \frac{1}{(x+2)} dx$$

$$\frac{1}{2}x^2 + 4x + \ln|x+1| + 3\frac{(x+2)^{-1}}{-1} + 2\ln|x+2| + C$$

$$\frac{1}{2}x^2 + 4x + \ln|x+1| - \frac{3}{(x+2)} + 2\ln|x+2| + C$$

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Mathematics

Integration by Parts

Part (1)

Lecture (13)

Presented by :-
Mahmoud Shakir Wahhab

- Integration by parts is a technique used to solve integrals that fit the form:

$$\int u\,dv$$

- This method is to be used when normal integration and substitution do not work.

- The integrand must contain two separate functions. For example,

$\int x \cos x \, dx$ contains the two functions of $\cos x$ and x. Note that $1dx$ can be considered a function.

- The standard integration by parts formula is:

$$\int u\,dv = uv - \int v\,du$$

- The problems that most students encounter with this formula are the substitutions for *u*, *dv*, *v*, and *du*. Two possibilities exist for substitutions.

1- If one of the functions cannot be integrated, assign u to the function. For example

$$\int \ln x \; dx$$

$\ln x$ cannot be integrated so, $u = \ln x$ and $dv = 1dx$

2- If both functions can be integrated, then assign u to the function that eventually differentiates to zero. For example

$$\int x\ e^{x}\,dx$$

Both $e^{x}$and x can be integrated. Continually differentiating e^{x} will never yield zero. However, x will differentiate to 0. So, $u = x$ and $dv = e^{x}dx$

A special case arises when both functions never differentiate to 0. For example

$$\int e^{x}\sin x\ dx$$

$u = \sin x$ $\qquad$ $dv = e^{x}\,dx$

An acronym that is very helpful to remember when using integration by parts is ***LIATE***. Whichever function comes first in the following list should be u:

L	Logarithmic functions	$\ln(x)$, $\log_2(x)$, $etc.$
I	Inverse trigonometric functions	$\sin^{-1}(x)$, $\cot^{-1}(x)$, $etc.$
A	Algebraic functions	x , x^2 , $\sqrt{x^3}$, $etc.$
T	Trigonometric functions	$\sin(x)$, $\sec(x)$, $etc.$
E	Exponential functions	e^x , 7^x . etc

EXAMPLE Evaluate $\int \ln x \ dx$

Solution

$u = \ln x$ $dv = dx$

$du = \frac{1}{x}$ $v = x$

$$\int udv = uv - \int vdu$$

$$\int \ln x \ dx = \ln x \cdot (x) - \int x \cdot \frac{1}{x} dx \longrightarrow x \ln x - \int dx$$

$$\therefore \int \ln x \ dx = x \ln x - x + C$$

EXAMPLE Evaluate $\int \sin^{-1}(x)\ dx$

Solution

$u = \sin^{-1}(x)$ $dv = dx$

$du = \dfrac{1}{\sqrt{1-x^2}}$ $v = x$

$$\int u dv = uv - \int v du \longrightarrow \int \sin^{-1}(x) = \sin^{-1}(x)\cdot x - \int x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\sin^{-1}(x)\cdot x - \int (1-x^2)^{\frac{-1}{2}} \cdot x\, dx \longrightarrow \sin^{-1}(x)\cdot x - \frac{-1}{2}\int (1-x^2)^{\frac{-1}{2}} \cdot -2x\, dx$$

$$x\cdot \sin^{-1}(x) + \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{1/2} + C \longrightarrow x\cdot \sin^{-1}(x) + \sqrt{1-x^2} + C$$

EXAMPLE Evaluate $\int x\ e^x\,dx$

Solution

$u = x$ $\quad\quad\quad\quad$ $dv = e^x$

$du = 1$ $\quad\quad\quad\quad$ $v = e^x$

$$\int udv = uv - \int vdu$$

$$\int x\ e^x\,dx = xe^x - \int e^x \cdot 1dx$$

$$\int x\ e^x\,dx = xe^x - e^x + C$$

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Mathematics

Integration by Parts
Part (2)

Lecture (14)

Presented by :-
Mahmoud Shakir Wahhab

Some integrals must repeat several times before simplification is complete.

EXAMPLE Evaluate $\int x^2 \, e^x \, dx$

Solution

$u = x^2$ $\quad dv = e^x$

$du = 2x$ $\quad v = e^x$

$$\int udv = uv - \int vdu \longrightarrow \int x^2 \, e^x \, dx = x^2 e^x - \int e^x \cdot 2xdx$$

$u = x$ $\quad dv = e^x$

$du = 1$ $\quad v = e^x$

$$\int udv = uv - \int vdu \longrightarrow \int e^x \cdot x \, dx = xe^x - \int e^x \, dx \longrightarrow xe^x - e^x + C$$

$$\int x^2 \ e^x \, dx = x^2 e^x - 2\int e^x \, x dx$$

$$x^2 e^x - 2(xe^x - e^x) + C \longrightarrow x^2 e^x - 2xe^x + 2e^x + C$$

Method 2

$$\int x^2 \ e^x \, dx$$

$$\begin{array}{ccc} x^2 & & e^x \\ & + & \\ 2x & & e^x \\ & - & \\ 2 & & e^x \\ & + & \\ 0 & & e^x \end{array}$$

$$x^2 e^2 \ -2xe^x \ +2e^x + C$$

A special case arises when both functions never differentiate to 0.

EXAMPLE Evaluate $\int e^x \sin x \; dx$

Solution

$u = \sin x$ $\qquad$ $dv = e^x$

$du = \cos x$ $\qquad$ $v = e^x$

$$\int u dv = uv - \int v du \longrightarrow \int e^x \sin x \; dx = \sin x \; e^x - \int e^x \cos x \; dx$$

$$\int e^x \cos x \, dx$$

$u = \cos x$ $\qquad$ $dv = e^x$

$du = -\sin x$ $\qquad$ $v = e^x$

$$\int e^x \cos x \, dx \longrightarrow e^x \cos x - \int -e^x \sin x \, dx \longrightarrow e^x \cos x + \int e^x \sin x \, dx$$

$$\because \int e^x \sin x \; dx = \sin x \; e^x - \int e^x \cos x \; dx$$

$$\int e^x \sin x \; dx = \sin x \; e^x - \left(e^x \cos x + \int e^x \sin x \, dx \right)$$

$$\int e^x \sin x \; dx = \sin x \; e^x - e^x \cos x - \int e^x \sin x \, dx$$

$$2 \int e^x \sin x \; dx = \sin x \; e^x - e^x \cos x \longrightarrow \int e^x \sin x \; dx = \frac{\sin x \; e^x - e^x \cos x}{2}$$

$$\int e^x \sin x \; dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$$

EXAMPLE Evaluate $\int x \ln x \ dx$

Solution

$u = \ln x \qquad\qquad dv = xdx$

$du = \dfrac{1}{x} \qquad\qquad v = \dfrac{1}{2}x^2$

$$\int udv = uv - \int vdu \longrightarrow \int x \ln x \ dx = \ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$$

$$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx \longrightarrow \frac{1}{2}x^2 \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + C$$

$$\int x \ln x \ dx \longrightarrow \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

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