Course File In Machine Design

For 3rd Year Students

Prepared By

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Subject	Year of student	Hours In Week			Units
		Theory	Practical	Total	
Machine Design	3rd Year Students	2	3	5	6

The project:-

The student can be able to:

1-Define the basic principles to design different mechanical parts.

2-Study variable loads and thermal stresses to design mechanical instruments and equipment's.

ITEM	WEEK	SYLLABUS
1	1-2	METALS &SIMPLE STRESSES
2	3-4	VARIABLE LOADS &STRESS CONCENTRATIONS
3	5-7	BOLTS,RIVETS AND WELDED JOINTS
4	8-9	PRIMARY LOADS IN SCREWS
5	10-11	POWER SCREW DESIGN
6	12-14	SHAFT DESIGN
7	15	KEY WAYS AND SPLINED SHAFTS
8	16	COUPLINGS
9	17	BELT AND CHAIN DRIVE DESIGN
10	18-20	ROLLING BEARINGS
11	21-22	SLIDING BEARINGS
12	23-24	SPRINGS
13	25-26	PRESSURE VESSELS
14	27-28	STSTIC &DYNAMIC SEALS
15	29-30	SPUR GEAR DESIGN

REFRENCES

1- MACHINE DESIGN, KHURMI 1956-2005 –INDIA

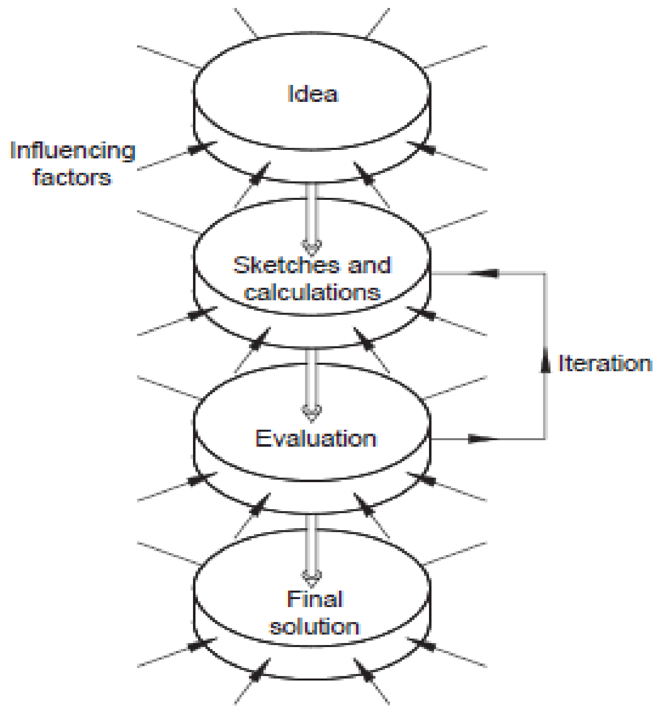
2- MECHANICAL DESIGNS, PETER R.CHILDS, UK.

Tutorial part

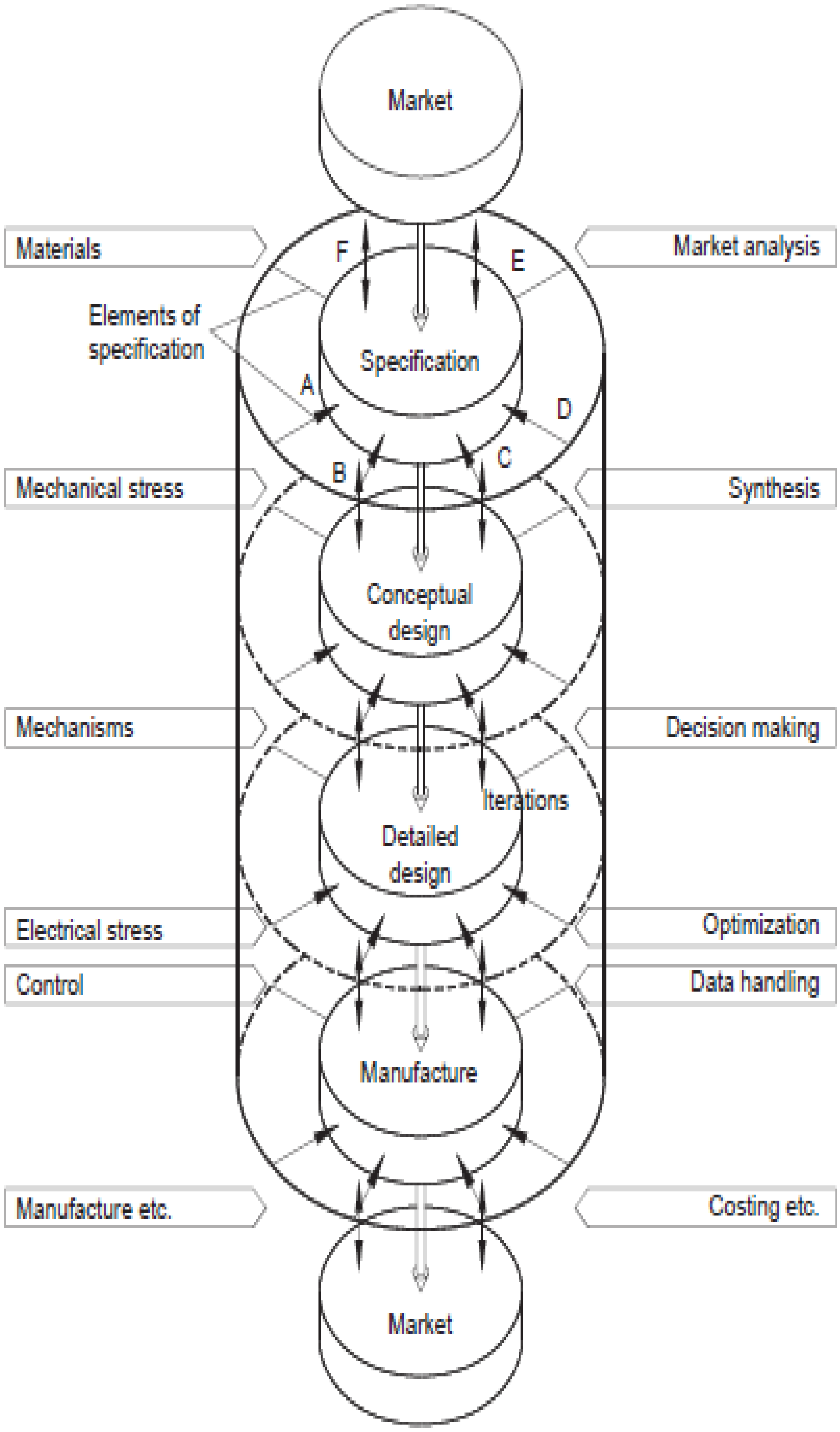
week	Subject
1-15	**Design-CAD and specified Design software**

Lecture 1

Machine design is defined as :-
The **total activity** necessary to product or process to meet a market need.



The traditional and familiar 'inventor's' approach to design.



The total design process (after Pugh, 1990).

SUFFIXES

a	axial
b	bending
c	compressive
f	endurance
s	strength properties of material
t	tensile
u	ultimate
y	yield

ABBREVIATIONS

AISI	American Iron and Steel Institute
ASA	American Standards Association
AMS	Aerospace Materials Specifications
ASM	American Society for Metals
ASME	American Society of Mechanical Engineers
ASTM	American Society for Testing Materials
BIS	Bureau of Indian Standards
BSS	British Standard Specifications
DIN	Deutsches Institut für Normung
ISO	International Standards Organization
SAE	Society of Automotive Engineers
UNS	Unified Numbering system

Strain. The axial loading shown in Fig. 1.6 also produces an axial strain (ε), given by Eq.

$$\varepsilon = \frac{\delta}{L}$$

where (δ) is change in length of the bar and (L) is length of the bar.

STRENGTH OF MACHINES

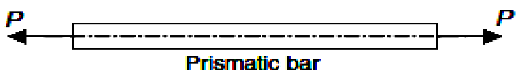


FIGURE 1.6 Axial loading.

Strain is a dimensionless quantity and does not have a unit if the change in length ε and the length (L) are in the same units. However, if the change in length (δ) is in inches or millimeters, and the length (L) is in feet or meters, then the strain (ε) will have a unit.

Stress-Strain Diagrams. If the stress (σ) is plotted against the strain (ε) for an axially loaded bar, the stress-strain diagram for a ductile material in Fig. 1.7 results, where A is proportional limit, B elastic limit, C yield point, D ultimate strength, and F fracture point.

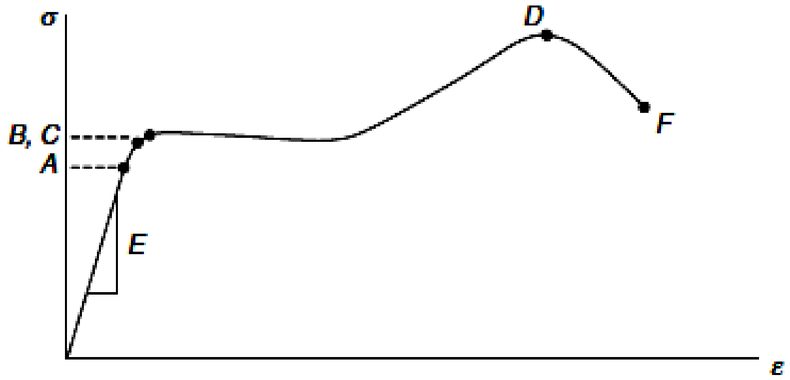


FIGURE 1.7 Stress-strain diagram (ductile material).

The stress-strain diagram is linear up to the proportional limit, and has a slope (E) called the modulus of elasticity. In this region the equation of the straight line up to the proportional limit is called Hooke's law, and is given by Eq. (1.3).

$$\sigma = E\,\varepsilon$$

The numerical value for the modulus of elasticity (E) is very large, so the stress-strain diagram is almost vertical to point A, the proportional limit. However, for clarity the horizontal placement of point A has been exaggerated on both Figs. 1.7 and 1.8.

Deformation. As a consequence of the axial loading shown in Fig. 1.9, there is a corresponding lengthening of the bar (δ), given by Eq.

$$\delta = \frac{PL}{AE}$$

where δ = change in length of bar (positive for tension, negative for compression)
P = axial force (positive for tension, negative for compression)
L = length of bar
A = cross-sectional area of bar
E = modulus of elasticity of bar material

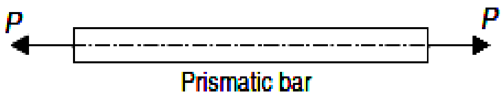


FIGURE 1.9 Axial loading.

Society of Mechanical Engineers (ASME). The working stress may be considered as either the yield strength or the tensile strength divided by a number called the *factor of safety*.

$$\sigma_w = \frac{\sigma_0}{N_0} \quad \text{or} \quad \sigma_w = \frac{\sigma_u}{N_u} \tag{1-5}$$

where σ_w = working stress
σ_0 = yield strength
σ_u = tensile strength
N_0 = factor of safety based on yield strength
N_u = factor of safety based on tensile strength

DIRECT SHEAR

The overlapping bars in Fig. 1.11 are held together by a single rivet as shown.

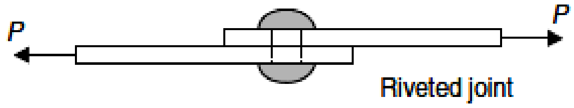


FIGURE 1.11 Direct shear loading.

Stress. If the rivet is cut in half at the overlap to expose the cross-sectional area (A) of the rivet, then Fig. 1.12 shows the resulting free-body-diagram.

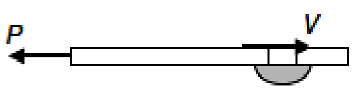


FIGURE 1.12 Free-body-diagram.

TORSION

Figure 1.19 shows a circular shaft acted upon by opposing torques (T), causing the shaft to be in torsion. This type of loading produces a shear stress in the shaft, thereby causing one end of the shaft to twist about the axis relative to the other end.

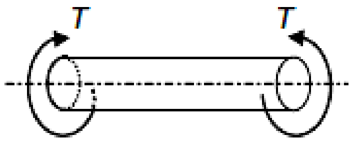


FIGURE 1.19 Torsion.

Stress. The two opposing torques (T) produce a twisting load along the axis of the shaft, resulting in a shear stress distribution (τ) as given by Eq. (

$$\tau = \frac{Tr}{J} \qquad 0 \le r \le R$$

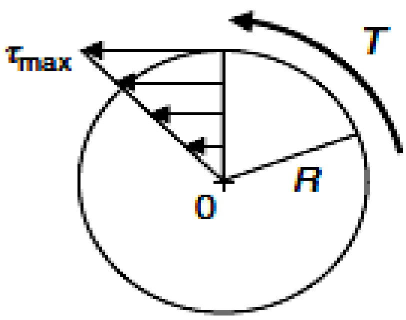


FIGURE 1.20 Shear stress distribution.

BENDING

Figure 1.23 shows a simply-supported beam with a concentrated force (F) located at its midpoint. This force produces both a bending moment distribution and a shear force distribution in the beam. At any location along the length (L) of the beam, the bending moment produces a normal stress (σ) and the shear force produces a shear stress (τ).

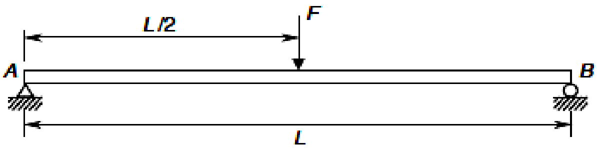


FIGURE 1.23 Bending.

It is assumed that the bending moment and shear force is known. If not, bending moment and shear force distributions, as well as deflection equations, are provided in Chap. 2 for a variety of beam configurations and loadings. Note that beam deflections represent the *deformation* caused by bending. Also, there is no explicit expression for strain owing to bending, because again, there are so many possible variations in beam configuration and loading.

Stress Owing to Bending Moment. Once the bending moment (M) has been determined at a particular point along a beam, then the normal stress distribution (σ) can be determined from Eq. (1.35) as

$$\sigma = \frac{My}{I} \tag{1.35}$$

where (y) is distance from the neutral axis (centroid) to the point of interest and (I) is area moment of inertia about an axis passing through the neutral axis.

The distribution given by Eq. (1.35) is linear as shown in Fig. 1.24, with the maximum normal stress (σ_{max}) occurring at the top of the beam, the minimum normal stress (σ_{min}) occurring at the bottom of the beam, and zero at the neutral axis ($y = 0$).

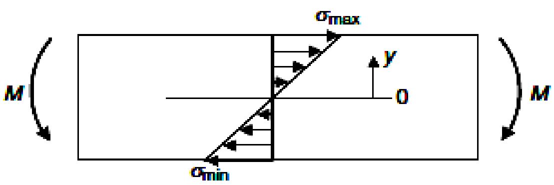


FIGURE 1.24 Bending stress distribution.

COMBINED LOADINGS

Plane Stress Element. The geometry of a differential plane stress element is shown in Fig. 4.1, where the dimensions (Δx) and (Δy) are such that the stresses, whether normal (σ) or shear (τ), can be considered constant over the cross-sectional areas of the edges

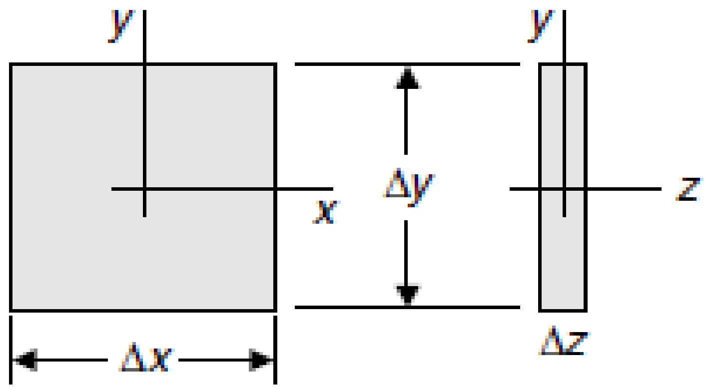


FIGURE 4.1 Geometry of a plane stress element.

Uniaxial Stress Element. For the fundamental loadings of axial, thermal, and bending, a uniaxial stress element is produced and shown in Fig. 4.3,

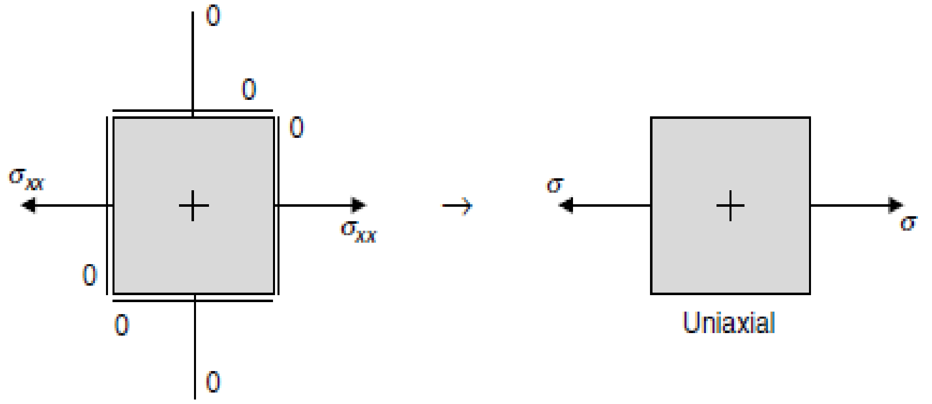


FIGURE 4.3 Uniaxial stress element.

where there is only normal stress (σ) along the axis of interest; and the other stresses, the normal stress (σ_{yy}) and the four shear stresses (τ_{xy}), are zero.

4.2 AXIAL AND TORSION

The first combination of loadings to be considered is axial and torsion. This is a very common loading for shafts carrying both a torque (T) and an end load (P), as shown in Fig. 4.6.

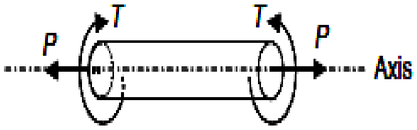


FIGURE 4.6 Axial and torsion loading.

Stress Element. **The general stress element shown in Fig. 4.2 becomes the stress element shown in Fig. 4.7, where the normal stress (σ_{xx}) is the axial stress, the normal stress (σ_{yy}) is zero, and the shear stress (τ_{xy}) is the shear stress due to torsion.**

COMBINED LOADINGS

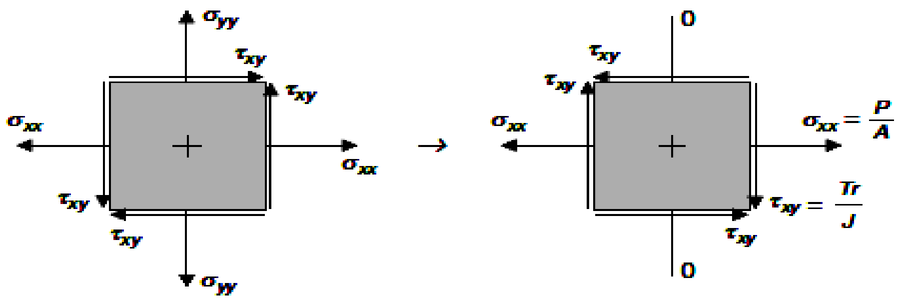


FIGURE 4.7 **Stress element for axial and torsion.**

The shear stress due to torsion (τ_{xy}) is shown downward on the right edge of the stress element because the torque (T) shown in Fig. 4.6 is counterclockwise looking in from the right side to the left side.

AXIAL AND BENDING

The second combination of loadings to be considered is axial and bending. This is a somewhat common loading for structural elements constrained axially. Shown in Fig. 4.10 is a simply-supported beam with a concentrated force (F) at its midpoint, and a compressive axial load (P).

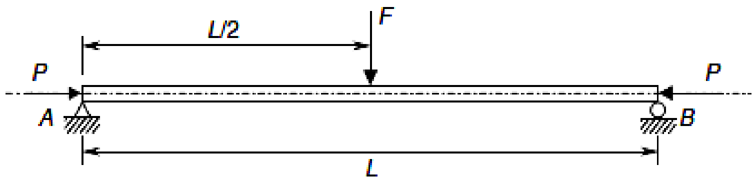


FIGURE 4.10 Axial and bending loads.

In this section, the bending moment (M) and shear force (V) are assumed to be known for whatever beam and loading is of interest. (See Chap. 2 on *Beams*.)

Stress Element. The general stress element shown in Fig. 4.2 becomes the stress element shown in Fig. 4.11, where the normal stress (σ_{xx}) is a combination of the axial stress and bending stress, the normal stress (σ_{yy}) is zero, and the shear stress (τ_{xy}) is the shear stress due to bending.

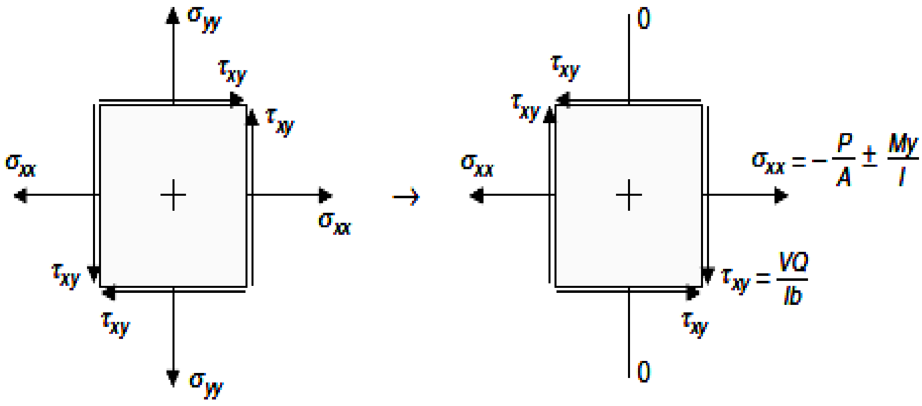


FIGURE 4.11 Stress element for axial and bending loads.

where $Q = A\bar{y}$, first moment of area (A)

A = area out beyond point of interest, specified by distance (y)

$\bar{y}$ = distance to centroid of area (A) defined above

I = area moment of inertia about an axis passing through neutral axis

b = width of beam at point of interest

AXIAL AND THERMAL

The third combination of loading to be considered is an axial load and a thermal load. This type of loading can occur when a machine element is put under a tensile, or compressive, preload during assembly in a factory environment, then subjected to an additional thermal load either due to a temperature drop in the winter or a temperature rise in the summer. Recall that if the machine element is not constrained, then under a temperature change the element merely gets longer or shorter and no stress is developed.

Figure 4.14 shows a thin-walled pipe, or tube, with flanges constrained between two fixed supports. (Note that typically pipe designations are based on inside diameter, whereas tubing is based on outside diameter.) Suppose that the original length of the pipe was shorter than the distance between the supports so that a tensile preload is developed in the pipe when it is installed. Also, suppose that what is of interest is the additional load that will be produced when the pipe is subjected to a temperature drop during the winter.

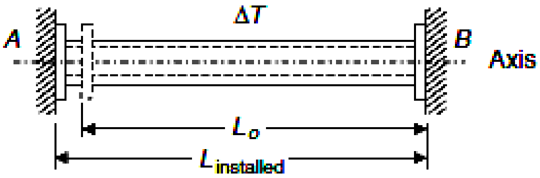


FIGURE 4.14 Axial and thermal loading.

The axial stress due to the lengthening of the pipe during installation is given by Eq. where the axial strain (ε) is multiplied by the modulus of elasticity (E).

$$\sigma_{\text{axial}} = E\,\varepsilon_{\text{axial}} = E\left(\frac{\Delta L}{L}\right) = E\left(\frac{L_{\text{installed}} - L_o}{L_o}\right)$$

COMBINED LOADINGS

The thermal stress due to a temperature drop (ΔT) is given by Eq. (4.6) where the thermal strain (ε_T) is multiplied by the modulus of elasticity (E)

$$\sigma_{\text{thermal}} = E\varepsilon_T = E\alpha\,(\Delta T) \tag{4.2}$$

and (α) is the coefficient of thermal expansion of the pipe.

Combining these two normal stresses, both of which are constant over the cross section of the pipe, gives the single stress (σ_{xx}) shown in Eq. (4.3),

$$\sigma_{xx} = \sigma_{\text{axial}} + \sigma_{\text{thermal}} = E\varepsilon_{\text{axial}} + E\varepsilon_T = E\left[\frac{\Delta L}{L} + \alpha(\Delta T)\right] \tag{4.3}$$

where

$$\frac{\Delta L}{L} = \frac{L_{\text{installed}} - L_o}{L_o} \tag{4.4}$$

Stress Concentration Factors and Notch Sensitivity

Lecture 2

Machine Design

Radiometric Thermoelasticity

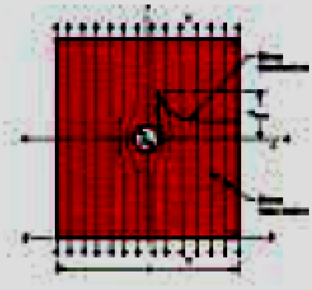
Automobile Connecting Rod | Hook and Clevis | Crack Tip

When materials are stressed the change in atomic spacing creates temperature differences in the material. Cameras which sense differences in temperature can be used to display the stress field in special materials.

Photoelasticity
(Continued)

When a photoelastic material is strained and viewed with a polariscope, distinctive colored fringe patterns are seen. Interpretation of the pattern reveals the overall strain distribution.

Geometric Stress Concentration Factors



$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

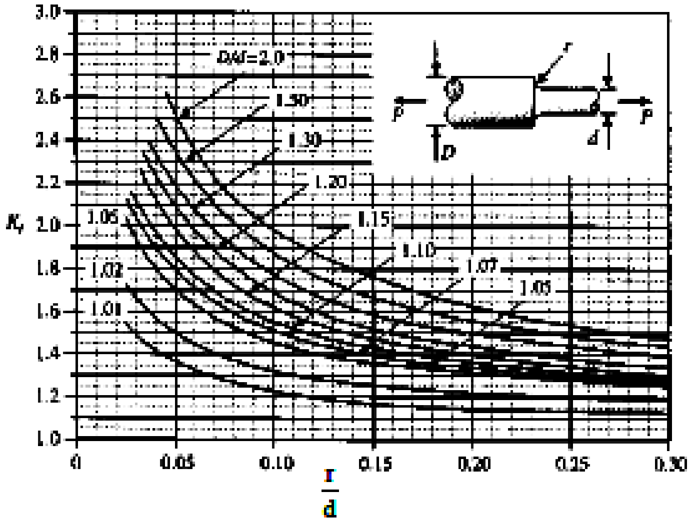
$$\sigma_{nom} = \frac{F}{A_0}$$

$$A_0 = (w - d)t$$

Geometric stress concentration factors can be used to estimate the stress amplification in the vicinity of a geometric discontinuity.

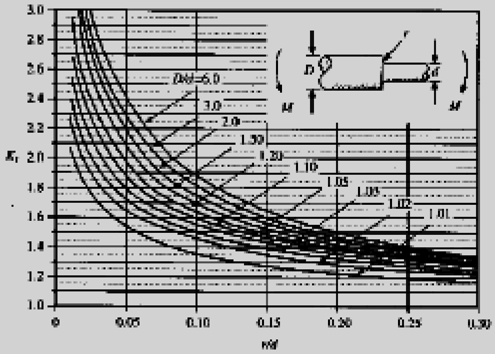
Geometric Stress Concentration Factors

(Tension Example)



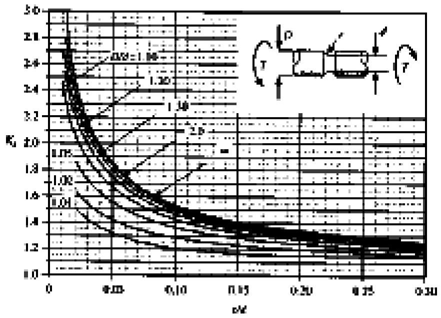
Geometric Stress Concentration Factors

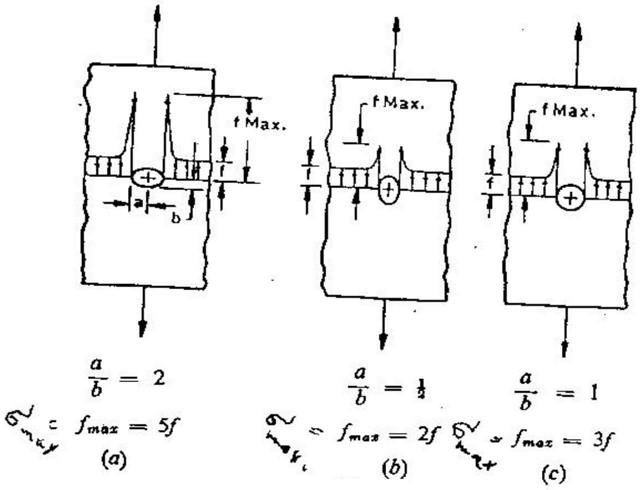
(Bending Example)



Geometric Stress Concentration Factors

(Torsion Example)





The maximum stress is given by:

$$\sigma_{max} = \sigma_0(1 + 2\frac{a}{b})$$

where:
σ_{max} : Max. Stress. (N/m^2)
σ_0 : Nominal stress. (N/m^2)
a & B : radius. (mm)

Or theoretical stress concentration factor

$$Kt = \frac{\sigma_{max}}{\sigma_0} = (1 + 2\frac{a}{b})$$

Kt : Stress concentration factor.

1. when $\frac{a}{b}$ is large

$\frac{a}{b} = 2 \quad \therefore \ \sigma_{max} = 5\sigma_0$

2. when $\frac{a}{b}$ is small

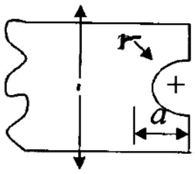
$\frac{a}{b} = \frac{1}{2} \quad \therefore \ \sigma_{max} = 2\sigma_0$.

3. when $\frac{a}{b} = 1$

$\therefore \ \sigma_{max} = 3\sigma_0$.

Stress concentration in notched tension member

$$\sigma_{max} = \sigma_0(1+\frac{a}{r})$$



Where
a: notch depth
r: radius at the bottom of the notch.

*Methods for reducing stress concentration

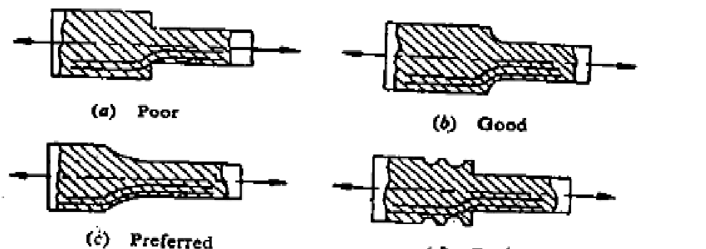


Fig. 6·8

In Fig. 6·8 (a) we see that the stress lines tend to bunch up and cut very close to the sharp reentrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 6·8 (b), and (c) to give more equally spaced flow lines.

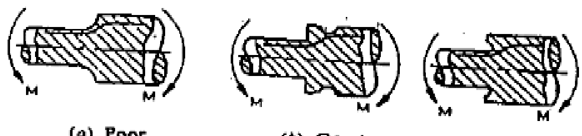


Fig. 6·9

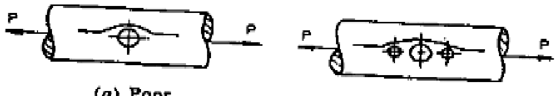


Fig. 6·10

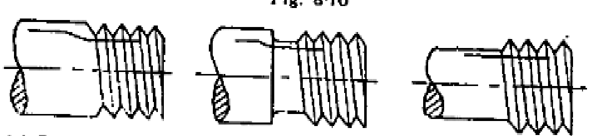
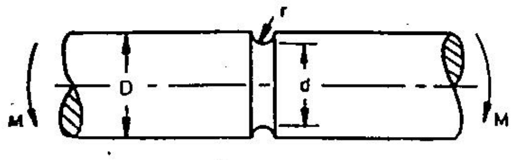


Fig. 6·11

Theoretical stress concentration factor (K_t) for a grooved shaft in bending



$$Z = \frac{\pi d^3}{32}$$

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	*0·04*	*0·08*	*0·10*	*0·12*	*0·16*	*0·20*	*0·24*	*0·28*	*0·30*
1·01	1·74	1·68	1·47	1·41	1·38	1·32	1·27	1·23	1·22	1·20
1·02	2·28	1·89	1·60	1·53	1·48	1·40	1·34	1·30	1·26	1·25
1·03	2·46	2·04	1·68	1·61	1·55	1·47	1·40	1·35	1·31	1·28
1·05	2·75	2·22	1·80	1·70	1·63	1·53	1·46	1·40	1·35	1·33
1·12	3·20	2·50	1·97	1·83	1·75	1·62	1·52	1·45	1·38	1·34
1·30	3·40	2·70	2·04	1·91	1·82	1·67	1·57	1·48	1·42	1·38
1·50	3·48	2·74	2·11	1·95	1·84	1·69	1·58	1·49	1·43	1·40
2·00	3·55	2·78	2·14	1·97	1·86	1·71	1·59	1·50	1·44	1·41
∞	3·60	2·85	2·17	1·98	1·88	1·71	1·60	1·51	1·45	1.42

Table 6·8

Example 6·1. ***Find the** maximum stress induced in the following cases taking stress **concentration** into account.*

*(1) A **rectangular** plate 60 mm × 10 mm with a hole 12 mm diameter as **shown in** Fig 6·13 (a) and subjected to a tensile load of 1,200 kg.*

*(2) **A stepped shaft** as shown in Fig. 6·13 (b) and carrying a tensile load of 1,200 kg.* *(Oxford University.)*

Solution.

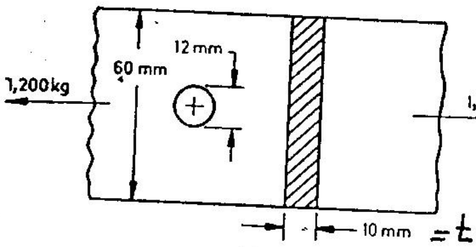
Case 1

Given. Width of the plate, $b = 60$ mm

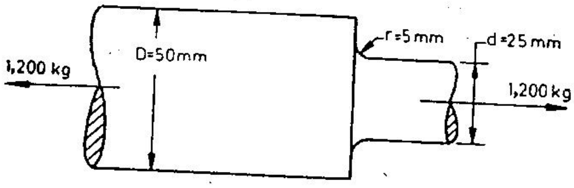
Thickness of the plate, $t = 10$ mm

Diameter of the hole, $d = 12$ mm

Tensile load, $W = 1{,}200$ kg



(*a*)



(*b*)

We know that cross-sectional area of the plate,

$$A = (b-d)t$$
$$= (60-12)10 = 480 \text{ mm}^2$$

$\therefore$ Nominal stress $= \frac{W}{A} = \frac{1{,}200}{480} = 2{\cdot}5 \text{ kg/mm}^2$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0{\cdot}2$$

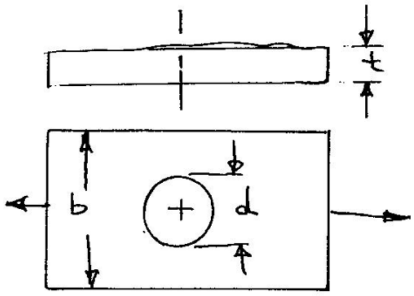
6.1

From Table 5.1, we find that theoretical stress concentration factor,

$$K_t = 2{\cdot}5$$

$\therefore$ Maximum stress $= K_t \times$ Nominal stress

$= 2{\cdot}5 \times 2{\cdot}5 = 6{\cdot}25 \text{ kg/mm}^2$ **Ans.**



$A = (b-d)t$

Case 2

Given. Maximum diameter of the shaft,

$$D = 50 \text{ mm}$$

Table 6.1

$\frac{d}{b}$	0·05	0·1	0·15	0·2	0·25	0·3	0·35	0·4	0·45	0·5	0·55
K_t	2·83	2·69	2·59	2·5	2·43	2·37	2·32	2·26	2·22	2·17	2·13

Minimum diameter of the shaft,

$$d = 25 \text{ mm}$$

Radius of fillet, $r = 5$ mm

Tensile load, $W = 1{,}200$ kg

We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times 25^2 = 491 \text{ mm}^2$$

$\therefore$ Nominal stress

$$= \frac{W}{A}$$

$$= \frac{1{,}200}{491} = 2{\cdot}4 \text{ kg/mm}^2$$

Ratio of maximum diameter to minimum diameter,

$$\frac{D}{d} = \frac{50}{25} = 2$$

Ratio of radius of fillet to minimum diameter,

$$\frac{r}{d} = \frac{5}{25} = 0{\cdot}2$$

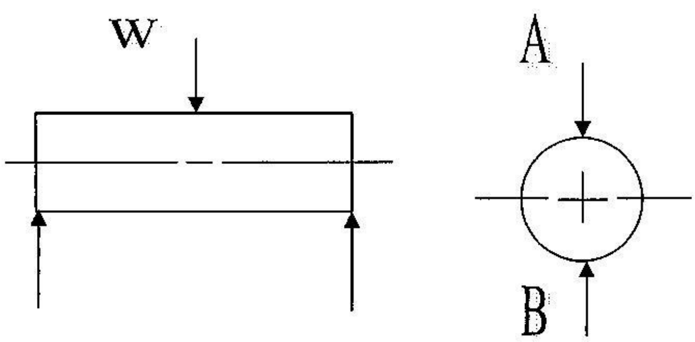
From Table 5·3, we find that theoretical stress concentration factor, $K_t = 1{\cdot}64$

$\therefore$ Maximum stress $= K_t \times$ Nominal stress

$$= 1{\cdot}64 \times 2{\cdot}4 = 3{\cdot}94 \text{ kg/mm}^2$$ **Ans.**

Variable Stresses In Machine Parts

It has been found by experiments that when a material is subjected to repeated stresses it fails below yield point stresses, such type of failure is known as fatigue.



in the above figure shown consider a beam of cross section and carrying a load (W) which is cyclic. The stress in A is compression stress and in B is tension stress and then the point A occupies point B. From the above we see that for each revolution of the beam stresses are reversed from compression to tension, these stresses are known as reversed or cycle stresses. In order to study the effect of fatigue a standard polished specimen as shown in the figure below is subjected to cyclic loading in a fating testing machine, the bending stresses varies continuously from maximum compression and represented by a time-stress diagram as shown in figure below:

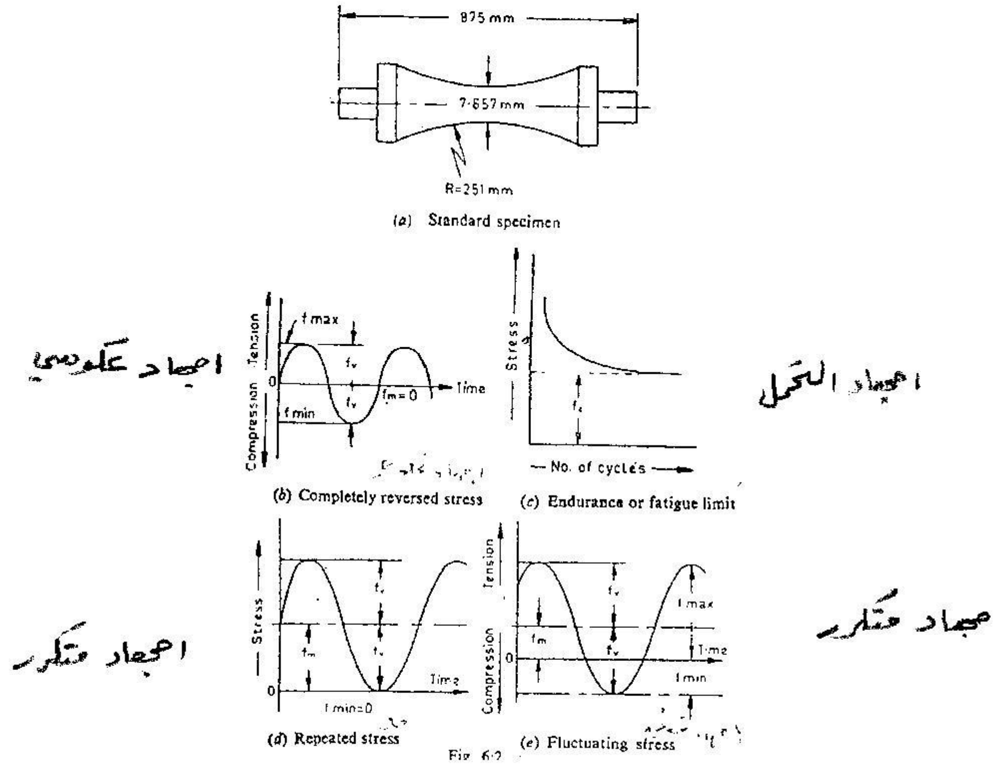


Fig (6-2)

The variable stresses may by considered as a combination of steady (or mean or average) stress and a completely reversed stress compo nent σ_v

1. **Average or mean stress**

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. **Reversed stress component or alternating or variable stress**

$$\sigma_r = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading the stress varies from maximum to zero ($\sigma_{min} = 0$) in each cycle as shown in fig 6-2 (d)

$$\sigma_m = \sigma_r = \frac{\sigma_{max}}{2}$$

3. **Stress ratio** $R = \frac{\sigma_{max}}{\sigma_{min}}$

For completely reversed stresses, R = -1
For repeated stresses, R = 0 } conditions

FACTOR OF SAFETY FOR FATIGUE LOADING

When a component is subjected to fatigue loading the endurance limit is the criterion for failure therefore the factor of safety should be based on endurance limit:

FACTOR OF SAFETY (F.O.S)= $\frac{\textit{Endurance limit stress}}{\textit{Design of working stress}}$

There are several ways in which problems involving combination of variable stresses may be solved and the following are important from the subject point to view:

1-Gerber method

2- Goodman method

3- Soderberg method

The relations proposed by Gerber, Goodman and Soderberg are as follows:

$\boldsymbol{\sigma_v = \sigma_e\left[1-(\frac{\sigma_{av}}{\sigma_u})2\right]}$............ ***Gerber method test data for ductile materials***

$\boldsymbol{\sigma_v = \sigma_e\left[1-\frac{\sigma_{av}}{\sigma_u}\right]}$ ....***Goodman method preferred in mechanical engineering design***

$\boldsymbol{\sigma_v = \sigma_e\left[\frac{1}{f.o.s}-\frac{\sigma_{av}}{\sigma_y}\right]}$ ......***Soderberg method is used in structural parts subjected to fatigue loads contain region of high stress concentration***

Example-1:

Find the variable stress and variable load for a plate (t=1.15 cm and 120mm wide) take W_{av}=17500 Kg/cm^2, if σ_s= 800 Kg/cm^2 and design stress as 1500 Kg/cm^2

(take F.O.S = 1.5)

$$\sigma_v = \sigma_e\left(\frac{1}{F.S}-\frac{\sigma_{av}}{\sigma_y}\right)$$

$$F.O.S = \frac{\sigma_e}{design\ stress}$$

$$\sigma_{av} = \frac{W_{av}}{A} = \frac{17500}{1.15x12} = 1268.1\ {}^{Kg}/_{cm^2}$$

$$\sigma_e = F.O.Sx\ design\ stress = 1.5x1500 = 2250\,{}^{Kg}/_{cm^2}$$

$$\sigma_y = 2\sigma s = 2x1500 = 3000\,{}^{Kg}/_{cm^2}$$

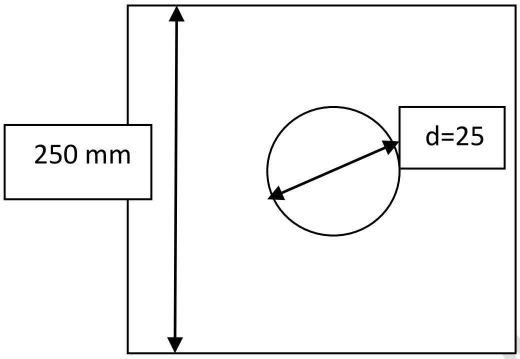
$\sigma_v = 2250(\frac{1}{1.5}-\frac{1268.1}{3000})=540\,{}^{Kg}/_{cm^2}$

$$\sigma_v = \frac{W_v}{A}$$

Wv= 540x13.8=7452 Kg *Ans.*

***Example-2:* Find the variable stress for the plate shown in the figure below (t=2cm, plate width=250mm and the hollow diameter=25 mm), take $W_{av.}$=17500 Kg/cm^2 if σ_y=3000 Kg/cm^2 and design stress as 1500Kg/cm^2(take f.o.s=1.5) then find the stress concentration factor.**

Solution:



$$\sigma_v = \sigma_e\left(\frac{1}{F.S} - \frac{\sigma_{av}}{\sigma_y}\right)$$

$$\sigma_{av} = \frac{W_{av}}{A} = \frac{W_{av}}{(b-d)t} = \frac{17500}{(25-2.5)*2} = 45\ {Kg}/{cm^2}$$

$$f.o.s = \frac{\sigma_e}{\sigma_{des.}}$$

σ_e= f.o.s*$\sigma_{des.}$=1.5*1500=2250 ${Kg}/{cm^2}$

$\sigma_v = 2250\left(\frac{1}{1.5} - \frac{45}{3000}\right)$=1451.25 ${Kg}/{cm^2}$

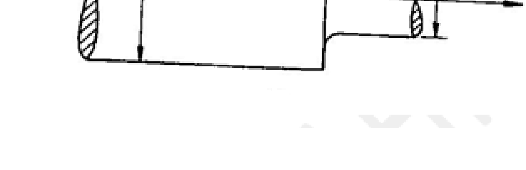
From table 6.3 to find the stress concentration factor

d/b=25/250=0.1

K_t=2.69

***Example-3:* Find the variable stress for the stepped shaft shown in the figure below (D=50mm, d=25 mm and the fillet radius=5 mm), take $W_{av.}$=16000 Kg/mm^2 if σ_s=1500 Kg/mm^2 , σ_e=2500 Kg/mm^2(take f.o.s=2) then find the stress concentration factor.**

Solution:



$$\sigma_v = \sigma_e\left(\frac{1}{F.S} - \frac{\sigma_{av}}{\sigma_y}\right)$$

$$\sigma_{av} = \frac{W_{av}}{A} = \frac{16000}{\frac{\pi}{4}d^2} = 490.8\ {}^{Kg}\!/_{cm^2}$$

$$\sigma_y = 2\sigma_s = 2 * 1500 = 3000\,{}^{Kg}\!/_{cm^2}$$

$$\sigma_v = 2500\left(\frac{1}{2} - \frac{490.8}{3000}\right) = 850\ {}^{Kg}\!/_{cm^2}$$

To find stress concentrate using factor for the stepped shaft by using the table 6.3

D/d=50/25=2

r/d=5/25=0.2

Kt=1.64

Example 6·2. *Determine the thickness of a 120 mm wide uniform plate for safe continues operation if the plate is to be subjected to a tensile load that has a maximum value of 25,000 kg and a minimum value of 10,000 kg. The properties of the plate material are as follow :—*

Endurance limit stress = 2,250 kg/cm², and

Yield point stress = 3,000 kg/cm²

The factor of safety based on yield point may be taken as 1·5.

(Gujarat University, 1976)

Solution

Given. Width of the plate,

$$b = 120 \text{ mm} = 12 \text{ cm}$$

Maximum tensile load,

$$W_{max} = 25,000 \text{ kg}$$

Minimum tensile load,

$$W_{min} = 10,000 \text{ kg}$$

Endurance limit stress,

$$\sigma_e = 2,250 \text{ kg/cm}^2$$

Yield point stress, $\sigma_y = 3,000 \text{ kg/cm}^2$

Factor of safety, $F.S. = 1·5$

Thickness of the plate

Let t = Thickness of the plate

∴ Cross-sectional area of the plate,

$$A = b \times t = 12t \text{ cm}^2$$

We know that average load,

$$W_{av} = \frac{W_{max}+W_{min}}{2} = \frac{25,000+10,000}{2} = 17,500 \text{ kg}$$

∴ Average stress,

$$\sigma_{av} = \frac{W_{av}}{A} = \frac{17,500}{12t} \text{ kg/cm}^2$$

Variable load,

$$W_v = \frac{W_{max}-W_{min}}{2} = \frac{25,000-10,000}{2} = 7,500 \text{ kg}$$

∴ Variable stress,

$$\sigma_v = \frac{W_v}{A} = \frac{7,500}{12t} \text{ kg/cm}^2$$

Now using the relation

$$\sigma_v = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_{av}}{\sigma_y}\right] \text{ with usual notations}$$

$$\frac{7,500}{12t} = 2,250\left[\frac{1}{1·5} - \frac{17,500}{12t \times 3,000}\right]$$

$$\frac{7,500}{12t} + \frac{17,500 \times 2,250}{12t \times 3,000} = \frac{2,250}{1·5}$$

$$\frac{7,500}{12t} + \frac{13,125}{12t} = \frac{2,250}{1·5}$$

$$\frac{20,625}{12t} = \frac{2,250}{1·5}$$

$$\therefore \quad t = \frac{20,625 \times 1·5}{12 \times 2,250} = 1·15 \text{ cm} \quad \textbf{Ans.}$$

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Example :plate with hollow

Example 6·3. (**S.I. Units**) *Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 N/mm² and a tensile yield strength of 350 N/mm². The member is subjected to a varying axial load from W_{min} = -300×10^3 N to W_{max} = 700×10^3 N and has a stress concentration factor = 1·8. Use factor of safety as 2·0.*

(*London University*)

Solution :

Given. Endurance strength,

$$\sigma_e = 265 \text{ N/mm}^2$$

Tensile yield strength,

$$\sigma_y = 350 \text{ N/mm}^2$$

Minimum load, $W_{min} = -300 \times 10^3$ N

Maximum load,

$$W_{max} = 700 \times 10^3 \text{ N}$$

Stress-concentration factor,

$$K_f = 1 \cdot 8$$

Factor of safety,

$$F.S. = 2$$

Diameter of a circular rod

Let d = Diameter of a circular rod in mm.

∴ Cross-sectional area of a circular rod,

$$A = \frac{\pi}{4} d^2 \text{ mm}^2$$

We know that average stress,

$$\sigma_{av} = \frac{\text{Average load}}{\text{cross-sectional area}}$$

$$= \frac{W_{max} + W_{min}}{2A}$$

$$= \frac{700 \times 10^3 + (-300 \times 10^3)}{2A}$$

$$= \frac{200 \times 10^3}{A} \text{ N/mm}^2$$

and variable stress,

$$\sigma_v = \frac{\text{Variable load}}{\text{Cross-sectional area}}$$

$$= \frac{W_{max} - W_{min}}{2A}$$

$$= \frac{700 \times 10^3 - (-300 \times 10^3)}{2A}$$

$$= \frac{500 \times 10^3}{A} \text{ N/mm}^2$$

Now using the relation

$$K_f . \sigma_v = \sigma_e \left[\frac{1}{F.S.} - \frac{\sigma_{av}}{\sigma_y} \right] \text{ with usual notations}$$

$$\frac{1 \cdot 8 \times 500 \times 10^3}{A} = 265 \left[\frac{1}{2} - \frac{200 \times 10^3}{A \times 350} \right]$$

$$\frac{1 \cdot 8 \times 500 \times 10^3}{A} + \frac{200 \times 10^3 \times 265}{A \times 350} = \frac{265}{2}$$

∴ $A = 8 \times 10^3$ mm

or $d = \sqrt{\frac{4A}{\pi}}$

$$A = \sqrt{\frac{4 \times 8 \times 10^3}{\pi}} = 100 \text{ mm } \textbf{Ans.}$$

Application of Soderbergs equation:

$$\frac{\pi d^3}{16} = \frac{f.o.s}{\sigma_{sy}}\sqrt{\left(\frac{\sigma_y}{\sigma_e}.K_f M\right)^2 + T^2}$$

NOTE:The above relations apply to a solid shaft, for hollow shaft the left hand side of the above equation must be multiplied by (1-k^4) where k is the ratio of inner diameter to outer diameter.

∴ **Ratio of journal diameter to shaft diameter,**

$$\frac{D}{d} = 1{\cdot}2$$

and **radius of the fillet,** $r = \frac{1}{10}$ shaft diameter $= 0{\cdot}1d$

$$\therefore \quad \frac{r}{d} = 0{\cdot}1$$

From Table 5·3, for $\frac{D}{d} = 1{\cdot}2$ and $\frac{r}{d} = 0{\cdot}1$, the theoretical stress concentration factor,

$$K_t = 1{\cdot}62$$

The two points at which failure may occur are at the end of the keyway and at the shoulder fillet. The critical section will be the one with larger product of $K_t M$. Since the notch sensitivity factor q is dependent upon the unknown dimensions of the notch and since the curves for notch sensitivity factor (Fig. 6·14) are not applicable to keyways, therefore the product $K_t M$ shall be the basis of comparison for the two sections.

∴ Bending moment at the end of the key way,

$$K_t M = 1{\cdot}6 \times 250\ [10-(2{\cdot}5+1)]$$
$$= 2{,}600 \text{ kg-cm}$$

($\because$ K_t for key ways $= 1{\cdot}6$)

and bending moment at the shoulder fillet,

$$K_t M = 1{\cdot}62 \times 250(10-2{\cdot}5)$$
$$= 3{,}037{\cdot}5 \text{ kg-cm}$$

Since $K_t M$ at the shoulder fillet is large, therefore considering the shoulder fillet as the critical section and using the relation

$$\frac{\pi d^3}{16} = \frac{F.S.}{f_{sy}}\sqrt{\left(\frac{f_y}{f_e}K_f\ M\right)^2 + T^2} \qquad \text{with usual notations}$$

$$\frac{\pi d^3}{16} = \frac{3}{\frac{3{,}000}{2}}\sqrt{\left(\frac{3{,}000}{1{,}800}\times 3{,}037{\cdot}5\right)^2 + (2{,}500)^2} = 11{\cdot}3$$

(Substituting $f_{sy} = \frac{1}{2} f_y$)

or $$d^3 = \frac{11{\cdot}3 \times 16}{\pi} = 57{\cdot}54$$

∴ $$d = \sqrt[3]{57{\cdot}54} = 3{\cdot}86 \text{ or } 4 \text{ cm} \quad \textbf{Ans.}$$

We know that $$\frac{D}{d} = 1{\cdot}2 \text{ (Given)}$$

∴ $$D = 1{\cdot}2d = 1{\cdot}2 \times 4 = 4{\cdot}8 \text{ cm} \quad \textbf{Ans.}$$

Since the ratio of radius of fillet to the shaft diameter, *i.e.*

$$\frac{r}{d} = 0{\cdot}1$$

∴ $$r = 0{\cdot}1d = 0{\cdot}1 \times 4 = 0{\cdot}4 \text{ cm} \quad \textbf{Ans.}$$

Note. Since r is known, therefore from Fig. 6·14, the notch sensitivity factor, q may be obtained. For $r = 0{\cdot}4$ cm or 4 mm, we have

$$q = 0{\cdot}93$$

∴ Fatigue stress concentration factor

$$K_f = 1+q\ (K_t-1)$$
$$= 1+0{\cdot}93(1{\cdot}62-1) = 1{\cdot}58$$

Using this value of K_f instead of K_t a new value of d may be calculated. We see that magnitudes of K_f and K_t are very close, therefore recalculation will not give any improvement in the results already obtained.

Example:Changing The Dimension Of The Stepped Shaft

LECTURE 3

A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as *single threaded* (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a *double threaded* (or double-start) screw is formed. Similarly, triple and quadruple (*i.e.* multiple-start) threads may be formed. The helical grooves may be cut either *right hand* or *left hand*. A screwed joint is mainly composed of two elements *i.e.* a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening.

Threaded fasteners

A fastener is a de
two or more com
fastening include n
addition, welding
form permanent jc

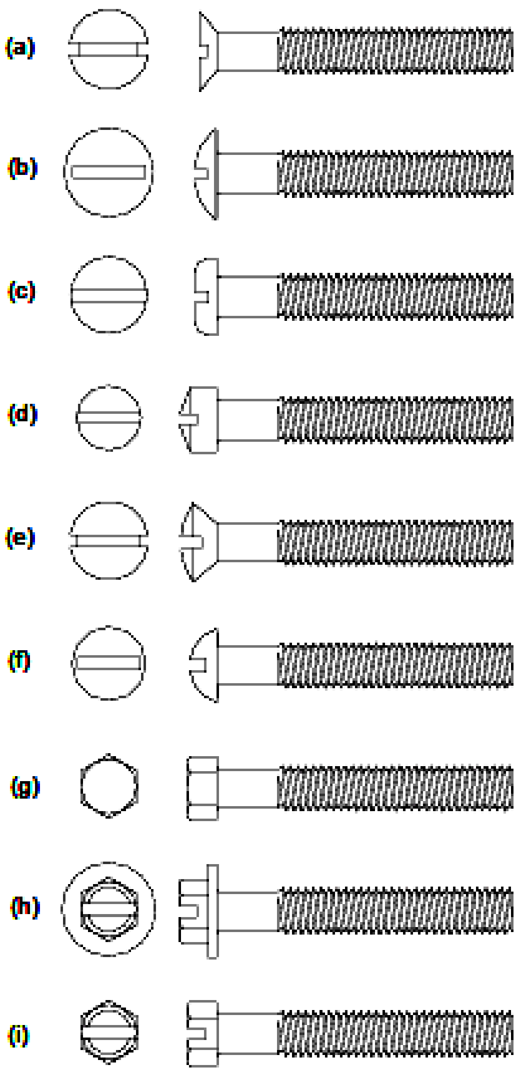


Figure Various machine screw styles. (a) Flat countersunk head. (b) Slotted truss head. (c) Slotted pan head. (d) Slotted fillister head. (e) Slotted oval countersunk. (f) Round head. (g) Hex. (h) Hex washer. (i) Slotted hexagon head.

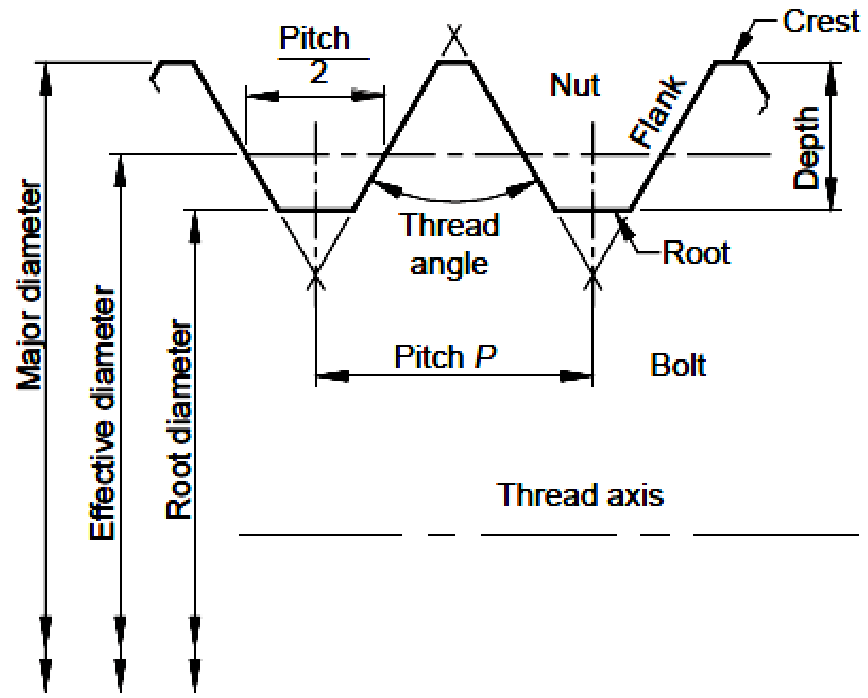


Figure Specialist terminology used for describing threads.

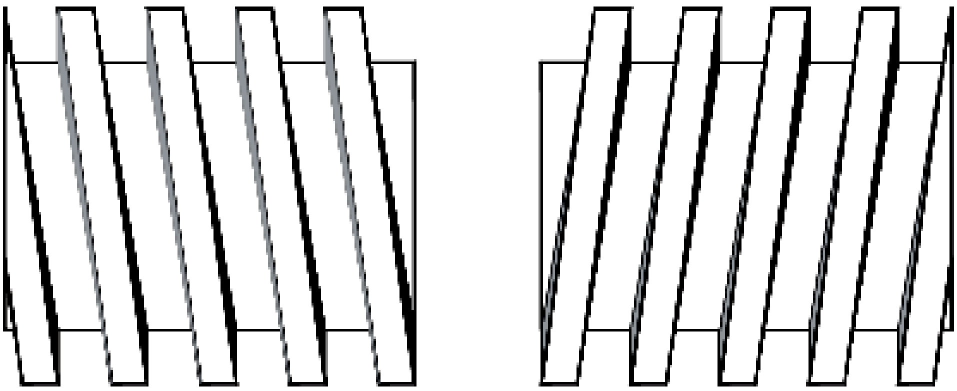


Figure (a) Right-hand thread; (b) Left-hand thread.

Table 12.2 Selected dimensions for a selection of British Standard ISO Metric Precision Hexagon Bolts. BS 3692:1967

Nominal size and thread diameter	Pitch of thread (coarse pitch series)	Width across flats		Height of head		Tapping drill	Clearance drill
		Max	Min	Max	Min		
M1.6	0.35	3.2	3.08	1.225	0.975	1.25	1.65
M2	0.4	4.0	3.88	1.525	1.275	1.60	2.05
M2.5	0.45	5.0	4.88	1.825	1.575	2.05	2.60
M3	0.5	5.5	5.38	2.125	1.875	2.50	3.10
M4	0.7	7.0	6.85	2.925	2.675	3.30	4.10
M5	0.8	8.0	7.85	3.650	3.35	4.20	5.10
M6	1	10.0	9.78	4.15	3.85	5.00	6.10
M8	1.25	13.0	12.73	5.65	5.35	6.80	8.20
M10	1.5	17.0	16.73	7.18	6.82	8.50	10.20
M12	1.75	19.0	18.67	8.18	7.82	10.20	12.20
M14	2	22.0	21.67	9.18	8.82	12.00	14.25
M16	2	24.0	23.67	10.18	9.82	14.00	16.25
M18	2.5	27.0	26.67	12.215	11.785	15.50	18.25
M20	2.5	30.0	29.67	13.215	12.785	17.50	20.25
M22	2.5	32.0	31.61	14.215	13.785	19.50	22.25
M24	3	36.0	35.38	15.215	14.785	21.00	24.25
M27	3	41.0	40.38	17.215	16.785	24.00	27.25
M30	3.5	46.0	45.38	19.26	18.74	26.50	30.50
M33	3.5	50.0	49.38	21.26	20.74	29.50	33.50
M36	4	55.0	54.26	23.26	22.74	32.00	36.50
M39	4	60.0	59.26	25.26	24.74	35.00	39.50
M42	4.5	65.0	64.26	26.26	25.74	37.50	42.50
M45	4.5	70.0	69.26	28.26	27.74	40.50	45.50
M48	5	75.0	74.26	30.26	29.74	43.00	48.75
M52	5	80.0	79.26	33.31	32.69	47.00	52.75
M56	5.5	85.0	84.13	35.31	34.69	50.50	56.75
M60	5.5	90.0	89.13	38.31	37.69	54.50	60.75
M64	6	95.0	94.13	40.31	39.69	58.00	64.75
M68	6	100.0	99.13	43.31	42.96	62.00	68.75

All dimensions in mm.

unified coarse pitch thread series (UNC) and the unified fine pitch thread series (UNF). Pertinent dimensions for selected UNC and UNF threads are given in Tables 12.3 and 12.4.

Table 12.3 American Standard thread dimensions for UNC screw threads

Size designation	Nominal major diameter (in)	Threads per inch	Tensile stress area (in^2)
0	0.0600		
1	0.0730	64	0.00263
2	0.0860	56	0.00370
3	0.0990	48	0.0487
4	0.1120	40	0.00604
5	0.1250	40	0.00796
6	0.1380	32	0.00909
8	0.1640	32	0.0140
10	0.1900	24	0.0175
12	0.2160	24	0.0242
Fractional sizes			
1/4	0.2500	20	0.0318
5/16	0.3125	18	0.0524
3/8	0.3750	16	0.0775
7/16	0.4375	14	0.1063
1/2	0.5000	13	0.1419
9/16	0.5625	12	0.182
5/8	0.6250	11	0.226
3/4	0.7500	10	0.334
7/8	0.8750	9	0.462
1	1.000	8	0.606
1⅛	1.125	7	0.763
1¼	1.250	7	0.969
1⅜	1.375	6	1.155
1½	1.500	6	1.405
1¾	1.750	5	1.90
2	2.000	4.5	2.50

Table 12.4 American Standard thread dimensions for UNF screw threads

Size designation	Nominal major diameter (in)	Threads per inch	Tensile stress area (in^2)
0	0.0600	80	0.00180
1	0.0730	72	0.00278
2	0.0860	64	0.00394
3	0.0990	56	0.00523
4	0.1120	48	0.00661
5	0.1250	44	0.00830
6	0.1380	40	0.01015
8	0.1640	36	0.01474
10	0.1900	32	0.0200
12	0.2160	28	0.0258
Fractional sizes			
1/4	0.2500	28	0.0364
5/16	0.3125	24	0.0580
3/8	0.3750	24	0.0878
7/16	0.4375	20	0.1187
1/2	0.5000	20	0.1599
9/16	0.5625	18	0.203
5/8	0.6250	18	0.256
3/4	0.7500	16	0.373
7/8	0.8750	14	0.509
1	1.000	12	0.663
1⅛	1.125	12	0.856
1¼	1.250	12	1.073
1⅜	1.375	12	1.315
1½	1.500	12	1.581

Testing, however, shows that the tensile strength is better defined using an area based on an average of the minor and pitch diameters.

$$A_t = \frac{\pi}{16}(d_p + d_r)^2$$

For UNS threads,

$$d_p = d - \frac{0.649519}{N} \quad \text{and} \quad d_r = d - \frac{1.299038}{N}$$

For ISO threads

$$d_p = d - 0.649519p$$

and

$$d_r = d - 1.226869p$$

The stress in a threaded rod due to a tensile load is

$$\sigma_t = \frac{F}{A_t}$$

the fatigue resistance of bolted connections. The recommended preload for reusable connections can be determined by

$$F_i = 0.75 A_t \sigma_p$$

and for permanent joints by

$$F_i = 0.9 A_t \sigma_p$$

where A_t is the tensile stress area of the bolt (m^2); σ_p, proof strength of the bolt (N/m^2).

Material properties for steel bolts are given in SAE standard J1199 and by bolt manufacturers.

If detailed information concerning the proof strength is unavailable then it can be approximated by

$$\sigma_p = 0.85\sigma_y$$

Once the preload has been determined the torque required to tighten the bolt can be estimated from

$$T = KF_i d$$

where T is wrench torque (N m); K, constant; F_i, preload (N); d, nominal bolt diameter (m).

The value of K depends on the bolt material and size. In the absence of data from manufacturers or detailed analysis, values for K are given in Table 12.5 for a variety of materials and bolt sizes.

Table 12.5 Values for the constant K for determining the torque required to tighten a bolt

Conditions	K
¼ to 1 inch mild steel bolts	0.2
Non-plated black finish steel bolts	0.3
Zinc plated steel bolts	0.2
Lubricated steel bolts	0.18
Cadmium plated steel bolts	0.16

Source: Oberg *et al.*, 1996.

Example 1

An M10 bolt has been selected for a re-useable application. The proof stress of the low carbon steel bolt material is 310 MPa. Determine the recommended preload on the bolt and the torque setting.

Solution

From Table 12.2, the pitch for a coarse series M10 bolt is 1.5 mm.

$$d_p = 10 - 0.649519 \times 1.5 = 9.026\ \text{mm}$$

$$d_r = 10 - 1.226869 \times 1.5 = 8.160\ \text{mm}$$

$$A_t = \frac{\pi}{16}(9.026 + 8.16)^2 = 57.99\ \text{mm}^2$$

For a reusable connection, the recommended preload is

$$F_i = 0.75 A_t \sigma_p = 13.48\ \text{kN}$$

From Table 12.5, $K = 0.2$. The torque required to tighten the bolt is given by

$$T = K F_i d = 0.2 \times 13.48 \times 10^3 \times 0.01$$
$$= 26.96\ \text{N m}$$

$$d_p = d - 0.649519p$$

and

$$d_r = d - 1.226869p$$

The stress in a threaded rod due to a tensile load is

$$\sigma_t = \frac{F}{A_t}$$

Example-2:

An M10 bolt has been selected for a permanent application. The proof stress of low carbon steel bolt material is 310 Mpa. Determine the recommended preload on the bolt and the torque setting, select the number of teeth 8. Use UNS

Solution:

$d_p = d - \frac{0.65}{N} = 10 - \frac{0.65}{8}$=9.92 mm

$d_r = d - \frac{1.3}{N} = 10 - \frac{1.3}{8}$= 9.84 mm

$$A_t = \frac{\pi}{16}\left(d_p + d_r\right)^2 = 76.6\, mm^2$$

$F_i = 0.9\, A_t \sigma_p = 21.37 * 10^3$N

From table 9 the value of constant , K=0.2

$T = K.F_i d$ N.m

T= 0.2*21.37*10^3*0.01=4.274 N.m ANS.

Example 3:

Determine the safe tensile load for a bolt of M30 coarse series made of mild steel for a permanent application assuming a safe tensile stress of 42 Mpa, then find the ultimate tensile load if (f.o.s=3) , then find the torque wrench.

Solution:

Since the standard bolt is M30

d=30 mm

From table 8 the pitch of the bolt coarse series=3.5

$d_p = d - 0.65p$ =30-0.65*3.5=27.7 mm

$d_r = d - 1.3p$ = 30- 1.3*3.5=25.7 mm

$A_t = \frac{\pi}{16}\left(d_p + d_r\right)^2$=π/16(27.7+25.7)^2=560.31 mm^2

Safe tensile load= $A_t\ \sigma_t$ =560.31*42=23.53 KN

Ultimate tensile load= safe tensile load*f.os=23.53*3=70.59 KN

F_i=0.9 $A_t\sigma_t$= 21.179 KN   K=0.2 from table 9=0.2

T=K F_i d= 0.2*21.179*0.03=0.12 N.m   ANS.

Practical joints normally fall between the two extremes of hard and soft joints. The clamped components of a typical hard joint have a stiffness of approximately three times that of the bolt. An externally applied load will be shared by the bolt and the clamped components according to the relative stiffnesses, which can be modelled by

$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e$$

$$F_c = F_i - \frac{k_c}{k_b + k_c} F_e$$

where F_b is final force in the bolt (N); F_i, initial clamping load (N); k_b, stiffness of the bolt (N/m); k_c, stiffness of the clamped components (N/m); F_e, externally applied load (N); F_c, final force on the clamped components (N).

k_c=3k_b

***Example 4:** A set of six M8 bolts is used to provide a clamping force of 20kNbetween two components in a machine. If the joint is subjected to an additional load of 18 kN after the initial preload of 8.5 kN per bolt has been applied determine the stress in the bolts. The stiffness of the clamped component can be assumed three times that of the material. The proof stress of low carbon steel bolt material is 310Mpa.*

Solution

Taking $k_c = 3k_b$,

$$F_b = F_i + \frac{k_b}{k_b + k_c}F_e = F_i + \frac{k_b}{k_b + 3k_b}F_e$$

$$= F_i + \frac{1}{4}F_e = 8500 + \frac{18000/6}{4}$$

$$= 9250 \text{ N}$$

$$F_c = F_i - \frac{k_c}{k_b + k_c}F_e = F_i - \frac{3k_b}{k_b + 3k_b}F_e$$

$$= F_i - \frac{3}{4}F_e = 8500 - \frac{3(18000/6)}{4}$$

$$= 6250 \text{ N}$$

As F_c is greater than zero, the joint remains tight. The tensile stress area for the M8 bolt can be determined from

$$d_p = 8 - 0.649519 \times 1.25 = 7.188 \text{ mm}$$

$$d_r = 8 - 1.226869 \times 1.25 = 6.466 \text{ mm}$$

$$A_t = \frac{\pi}{16}(7.188 + 6.466)^2 = 36.61 \text{ mm}^2$$

The stress in each bolt is given by

$$\sigma = \frac{F_b}{A_t} = \frac{9250}{36.61 \times 10^{-6}} = 252.7 \text{ MPa}$$

This is 82 per cent of the proof stress. The bolts are therefore safe.

$$d_p = d - 0.649519p$$

and

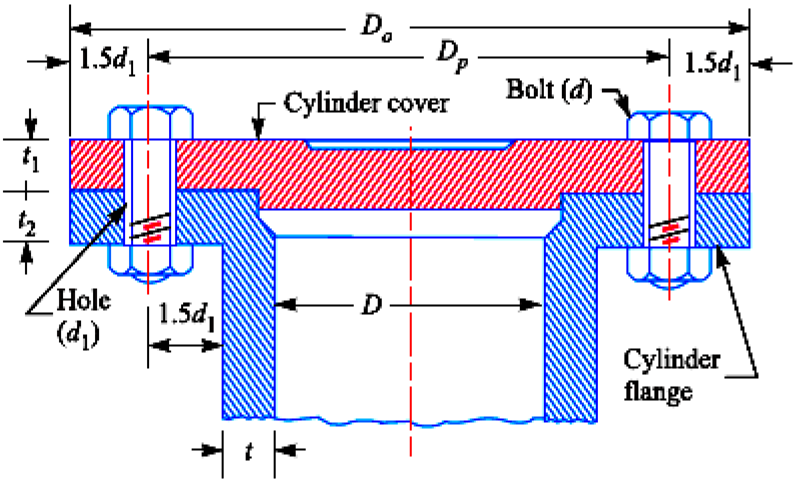
$$d_r = d - 1.226869p$$

The stress in a threaded rod due to a tensile load is

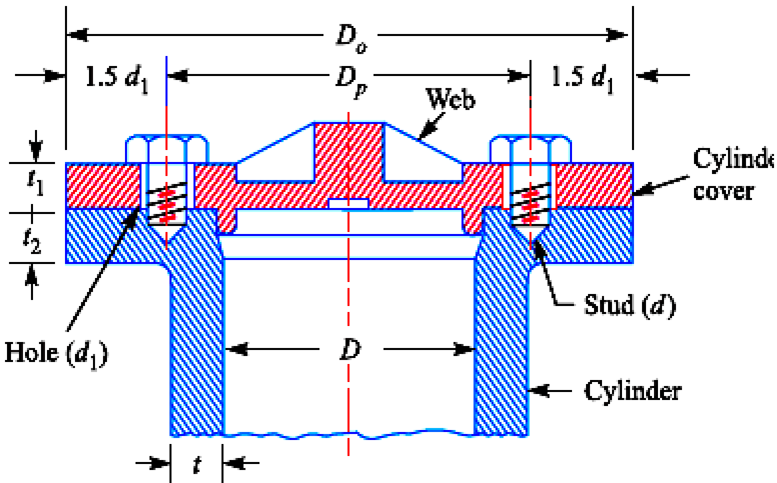
$$\sigma_t = \frac{F}{A_t}$$

Design of Cylinder Covers

The cylinder covers may be secured by means of bolts or studs but studs are preferred. This possible arrangement of securing the cover with bolts and studs is shown in the following figure.



(a) Arrangement of securing the cylinder cover with bolts.



(b) Arrangement of securing the cylinder cover with studs.

in order to find the size and number of bolts the following procedure may be adopted:

Let D = Diameter of the cylinder,
p = Pressure in the cylinder,
dc = Core diameter of the bolts or studs,
n = Number of bolts or studs, and
σ_t = Permissible tensile stress for the bolt or stud material.

the force acting on the cylinder cover:

$$P = \frac{\pi}{4} D_1^2 . P_p$$

This force is required by number of bolts or studs provided on the cover

Force on each bolt or cover

$$P = \frac{\pi}{4} D_1^2 . P_p . \frac{1}{n} \qquad \text{-------1}$$

and resistance offered by each bolt:

$$= \frac{\pi}{4} d_c^2 \sigma_t \qquad \text{-----------2}$$

equating eq.1 and eq.2

$$\frac{\pi}{4} D_1^2 . P_p . \frac{1}{n} = \frac{\pi}{4} d_c^2 \sigma_t$$

The P.C.D of the bolt or stud is taken as: $D_o = D_P + 3d_1$

$D_p = D_1 + 3d_1$ ***and the next outside diameter of the cover is kept as:***

$D_o = D_P + 3d_1$ ***and the initial tension due to tightening of bolt:***

$P_i = 2840d$ ***N for bolts and threads which need tight***

$$P_i = 2840d \ N$$

P_i=142 d Kg for bolts and studs which not need tight as cylinders $P_i = 142d \ Kg$

Resultant axial load on the bolt $P = p_1 + KP_2$

$P=P_1+KP_2$ where K value find from the following table

Type of joint	$K = \frac{a}{1+a}$
Metal to metal joint with through bolts	0·00 to 0·10
Hard copper gasket with long through bolts	0·25 to 0·50
Soft copper gasket with long through bolts	0·50 to 0·75
Soft packing with through bolts	0·75 to 1·00
Soft packing with studs	1·00

Example 11.8. *The cylinder head of a steam engine is subjected to a steam pressure of 0.7 N/mm². It is held in position by means of 12 bolts. A soft copper gasket is used to make the joint leak-proof. The effective diameter of cylinder is 300 mm. Find the size of the bolts so that the stress in the bolts is not to exceed 100 MPa.*

Solution. Given: $p = 0.7$ N/mm² ; $n = 12$; $D = 300$ mm ; $\sigma_t = 100$ MPa $= 100$ N/mm²

We know that the total force (or the external load) acting on the cylinder head *i.e.* on 12 bolts,

$$= \frac{\pi}{4}(D)^2\, p = \frac{\pi}{4}(300)^2\, 0.7 = 49\,490 \text{ N}$$

∴ External load on the cylinder head per bolt,

$$P_2 = 49\,490 / 12 = 4124 \text{ N}$$

Let d = Nominal diameter of the bolt, and

d_c = Core diameter of the bolt.

We know that initial tension due to tightening of bolt,

$$P_1 = 2840\, d \text{ N} \qquad \text{... (where } d \text{ is in mm)}$$

From Table 11.2, we find that for soft copper gasket with long through bolts, the minimum value of $K = 0.5$.

∴ Resultant axial load on the bolt,

$$P = P_1 + K \cdot P_2 = 2840\, d + 0.5 \times 4124 = (2840\, d + 2062) \text{ N}$$

We know that load on the bolt (P),

$$2840\, d + 2062 = \frac{\pi}{4}(d_c)^2\, \sigma_t = \frac{\pi}{4}(0.84d)^2\, 100 = 55.4\, d^2 \qquad \text{...(Taking } d_c = 0.84\, d)$$

$$\therefore \quad 55.4\, d^2 - 2840d - 2062 = 0$$

or

$$d^2 - 51.3d - 37.2 = 0$$

$$\therefore \quad d = \frac{51.3 \pm \sqrt{(51.3)^2 + 4 \times 37.2}}{2} = \frac{51.3 \pm 52.7}{2} = 52 \text{ mm}$$

..(Taking + ve sign)

Example 2 *A steam engine of effective diameter 300 mm is subjected to a steam pressure of 1.5 N/mm². The cylinder head is connected by 8 bolts having yield point 330 MPa and endurance limit at 240 MPa. The bolts are tightened with an initial preload of 1.5 times the steam load. A soft copper gasket is used to make the joint leak-proof. Assuming a factor of safety 2, find the size of bolt required. The stiffness factor for copper gasket may be taken as 0.5.*

Solution. Given : D = 300 mm ; p = 1.5 N/mm² ; n = 8 ; σ_y = 330 MPa = 330 N/mm²; σ_e = 240 MPa = 240 N/mm² ; $P_1 = 1.5\,P_2$; $F.S.$ = 2 ; K = 0.5

We know that steam load acting on the cylinder head,

$$P_2 = \frac{\pi}{4}(D)^2\,p = \frac{\pi}{4}(300)^2\,1.5 = 106\ 040\ \text{N}$$

∴ Initial pre-load,

$$P_1 = 1.5\,P_2 = 1.5 \times 106\ 040 = 159\ 060\ \text{N}$$

We know that the resultant load (or the maximum load) on the cylinder head,

$$P_{max} = P_1 + K.P_2 = 159\ 060 + 0.5 \times 106\ 040 = 212\ 080\ \text{N}$$

This load is shared by 8 bolts, therefore maximum load on each bolt,

$$P_{max} = 212\ 080 / 8 = 26\ 510\ \text{N}$$

and minimum load on each bolt,

$$P_{min} = P_1 / n = 159\ 060/8 = 19\ 882\ \text{N}$$

We know that mean or average load on the bolt,

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{26\,510 + 19\,882}{2} = 23\,196 \text{ N}$$

and the variable load on the bolt,

$$P_v = \frac{P_{max} - P_{min}}{2} = \frac{26\,510 - 19\,882}{2} = 3314 \text{ N}$$

Let d_c = Core diameter of the bolt in mm.

∴ Stress area of the bolt,

$$A_s = \frac{\pi}{4}(d_c)^2 = 0.7854\,(d_c)^2 \text{ mm}^2$$

We know that mean or average stress on the bolt,

$$\sigma_m = \frac{P_m}{A_s} = \frac{23\,196}{0.7854\,(d_c)^2} = \frac{29\,534}{(d_c)^2} \text{ N/mm}^2$$

and variable stress on the bolt,

$$\sigma_v = \frac{P_v}{A_s} = \frac{3314}{0.7854\,(d_c)^2} = \frac{4220}{(d_c)^2} \text{ N/mm}^2$$

According to *Soderberg's formula, the variable stress,

$$\sigma_v = \sigma_e\left(\frac{1}{F.S} - \frac{\sigma_m}{\sigma_y}\right)$$

$$\frac{4220}{(d_c)^2} = 240\left(\frac{1}{2} - \frac{29\,534}{(d_c)^2\,330}\right) = 120 - \frac{21\,480}{(d_c)^2}$$

or $$\frac{4220}{(d_c)^2} + \frac{21\,480}{(d_c)^2} = 120 \quad \text{or} \quad \frac{25\,700}{(d_c)^2} = 120$$

∴ $(d_c)^2 = 25\,700 / 120 = 214$ or $d_c = 14.6$ mm

From Table 11.1 (coarse series), the standard core diameter is d_c = 14.933 mm and the corresponding size of the bolt is M18. **Ans.**

LECTURE 4

RIVETED JOINTS DESIGN

INTRODUCTION

In engineering practice it is often required that two sheets or plates are joined together and carry the load in such ways that the joint is loaded. Many times such joints are required to be leak proof so that gas contained inside is not allowed to escape. A riveted joint is easily conceived between two plates overlapping at edges, making holes through thickness of both, passing the stem of rivet through holes and creating the head at the end of the stem on the other side. A number of rivets may pass through the row of holes, which are uniformly distributed along the edges of the plate. With such a joint having been created between two plates, they cannot be pulled apart. If force at each of the free edges is applied for pulling the plate apart the tensile stress in the plate along the row of rivet hole and shearing stress in rivets will create resisting force. Such joints have been used in structures, boilers and ships.

Objectives

After studying this unit, you should be able to

- describe the types of riveted joint,
- calculate the strength of riveted joints,

HEAD FORMING

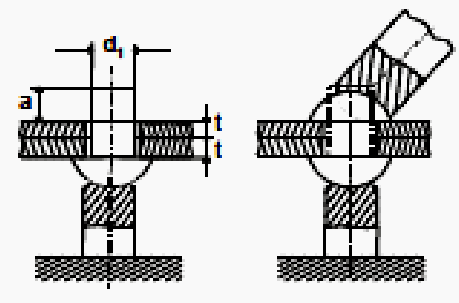


Figure Typical Head Forming of Rivet

MATERIALS USED FOR RIVETS

For steel plates the rivets are normally made in low carbon steel. However, the rivets in copper add to resistance against corrosion and aluminum rivets can be used to reduce the overall weight of the structure. The low carbon steel is standardized in composition particularly for boiler applications.

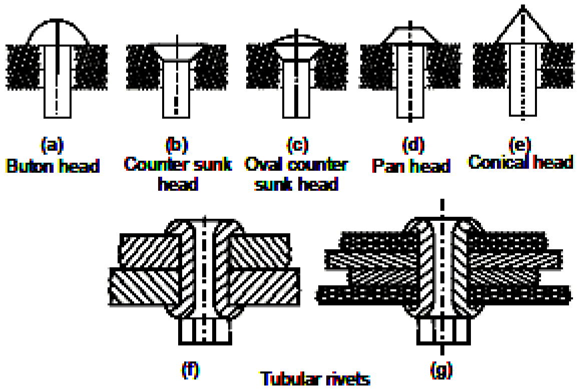


Figure : Different Types of Rivet Heads

The classification of riveted joints is based on the following :

Strong Joints

In these joints strength is the only criterion. Joints in engineering structure such as beams, trusses and machine frames are strong joints.

Tight Joints

These joints provide strength as well as are leak proof against low pressures. Joints in reservoirs, containers and tanks fall under this group.

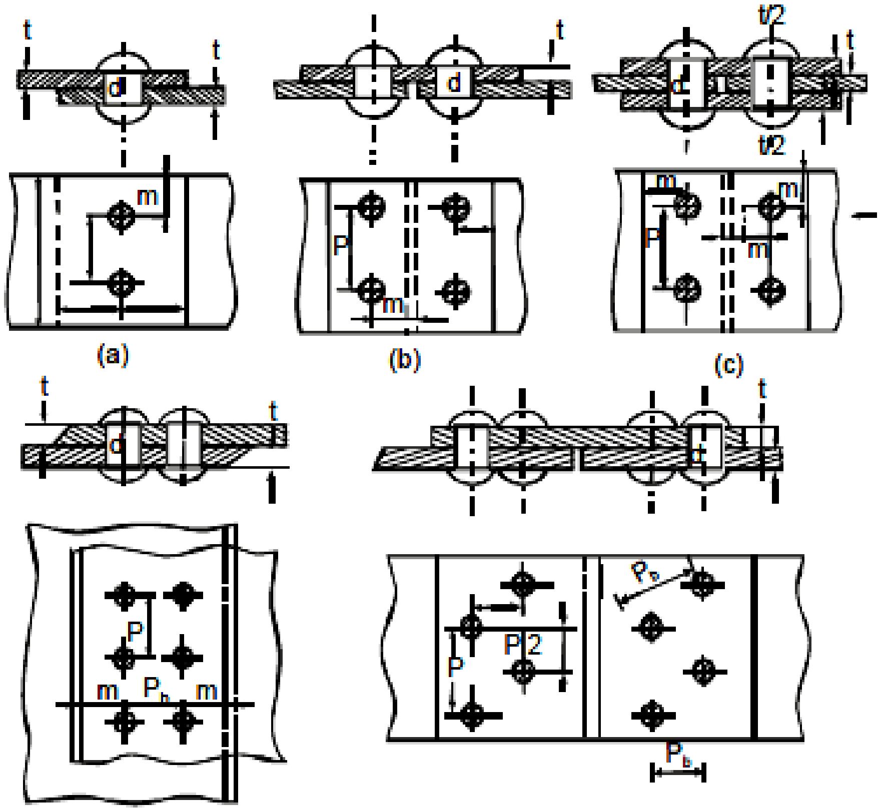


Figure : Types of Riveted Joints : (a) Single Riveted Lap Joint; (b) Single Riveted-single Cover Butt Joint; (c) Single Riveted Double Cover Butt Joint; (d) Double Riveted Lap Joint; (e) Double Riveted Single Cover Butt Joint; and (f) Double Riveted Double Cover Butt Joint

NOMENCLATURE

Several dimensions become obviously important in a riveted joint and a design will consist in calculating many of them. These dimensions and their notations as to be used in this text are described below.

Pitch

As seen from Figures (a), (b) and (c) pitch, denoted by p, is the center distance between two adjacent rivet holes in a row.

Back Pitch

The center distance between two adjacent rows of rivets is defined as back pitch. It is denoted by p_b and is shown in Figures (d) and (e).

Diagonal Pitch

The smallest distance between centers of two rivet holes in adjacent rows of a zigzag riveted joint is called diagonal pitch. Denoted by p_d, the diagonal pitch is shown in Figure (e).

Margin

It is the distance between centre of a rivet hole and nearest edge of the plate. It is denoted by m as shown in Figures (b), (c) and (d).

MODES OF FAILURE OF A RIVETED JOINT

a) **Tearing of Plate at the Section Weakened by Holes**
Figure 3.6 shows this mode of failure. The plate at any other section is obviously stronger, and hence does not fail. If tensile force *P* is to cause tearing, it will occur along weakest section, which carries the row of rivets. If only one pitch length *p* is considered; it is weakened by one hole diameter *d*. The area that resists the tensile force is
$A_t = (p - d)\, t$
If the permissible stress for plate in tension is $_t$, then tensile strength of the joint or tensile load carrying capacity of the joint

$$P_t = (p - d)t\sigma_t$$

If *P* is the applied tensile force per pitch length then the joint will not fail if

$$P_t \geq P$$

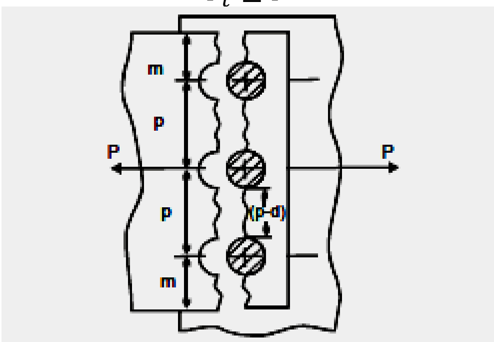


Figure 3.6 : Tearing of Plate at the Section Weakened by Holes

(b) **Shearing of Rivet**
Figure 3.7 shows how a rivet can shear. The failure will occur when all the rivets in a row shear off simultaneously. Considers the strength provided by the rivet against this mode of failure, one consider number of rivets in a pitch length which is obviously one. Further, in a lap joint failure due to shear may occur only along one section of rivet as shown in Figure 3.7(a). However, in case of double cover butt joint failure may take place along two sections in the manner shown in Figure 3.7(b). So in case of single shear th0e area resisting shearing of a rivet,
$A_s = \frac{\pi}{4} d^2$
(Since the difference between diameter of hole and diameter of rivet is very small, diameter of hole is used for diameter of the rivet).

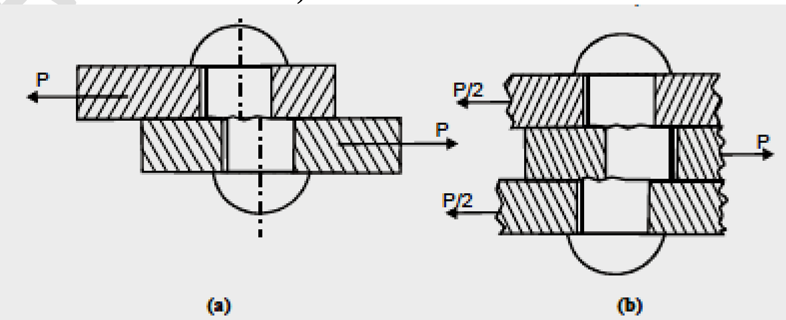


Figure 3.7 : Shearing of Rivet : (a) Single Shear; and (b) Double Shear

(Since the difference between diameter of hole and diameter of rivet is very small, diameter of hole is used for diameter of the rivet).

$$P_s = \sigma_s \frac{\pi}{4} d^2$$

The failure will not occur if

$$P_s \geq P$$

We may also write if *n* is the number of rivets per pitch length

$$P_s = n\sigma_s \frac{\pi}{4} d^2$$

The permissible stress in double shear is 1.75 times that in single shear. Hence in double shear

$$P_s = n * 1.75 * \sigma_s * \frac{\pi}{4} d^2$$

c) **Crushing of Plate and Rivet**

Due to rivet being compressed against the inner surface of the hole, there is a possibility that either the rivet or the hole surface may be crushed. The area, which resists this action, is the projected area of hole or rivet on diametral plane. The area per rivet is

$$A_c = dt$$

If permissible crushing or bearing stress of rivet or plate is $\sigma_c$the crushing strength of the joint or load carrying capacity of the joint against crushing is,

$$P_c = dt\sigma_c$$

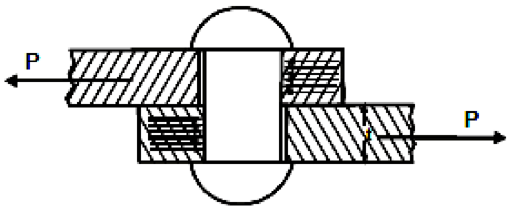


Figure 3.8 : Crushing of Rivet

The failure in this mode will not occur if

$$\boldsymbol{P_c \geq P}$$

where *P* is applied load per pitch length, and there is one rivet per pitch. If number of rivets is *n* in a pitch length then right hand side in Eq. is multiplied by *n*.

(d) **Shearing of Plate Margin near the Rivet Hole**

Figure 3.9 shows this mode of failure in which margin can shear along planes *ab* and *cd*. If the length of margin is *m*, the area resisting this failure is,

$$\boldsymbol{A_{ms} = 2mt}$$

If permissible shearing stress of plate is $\boldsymbol{\sigma_s}$ then load carrying capacity of the joint against shearing of the margin is,

$$\boldsymbol{P_{ms} = 2mt\sigma_s}$$

The failure in this case will not occur if

$$\boldsymbol{P_{ms} \geq P}$$

where *P* is the applied load per pitch length.

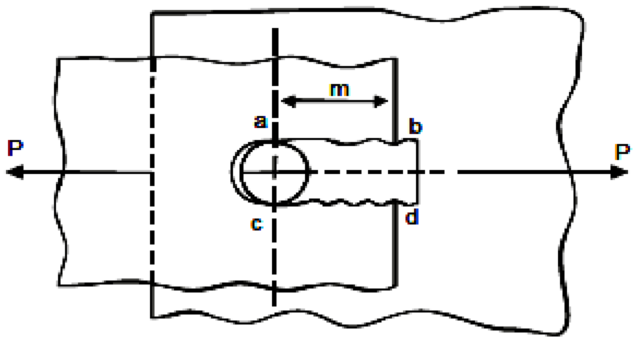


Figure 3.9 : Shearing of Margin

$$\eta = \frac{\text{Least of } P_t, P_s, P_c \text{ and } P_{ms}}{P_t\ \sigma_t}$$

EFFICIENCY OF RIVETED JOINTS

If only a pitch length of solid or hole free plate is considered then its load carrying capacity will be

$$P_1 = pt\sigma_t$$

*P*1 will apparently be greater then *Pt*, *Ps*, *Pc* or *Pms*. The ratio of any of *Pt*, *Ps*, *Pc* or *Pms* to *P*1 is defined as the efficiency of the joint in that particular mode. Ideally *Pt*, *Ps*, *Pc* and *Pms* all must be equal, but actually it may not be the case. The efficiency of the joint will be determined by least of *Pt*, *Ps*, *Pc*, and *Pms*. Thus efficiency of the joint is,

$$\gamma = \frac{least\ of\ p_t, p_s, p_c and p_{ms}}{pt\sigma_t}$$

Table 3.1 : Efficiencies of Commercial Boiler Joints

Type of Joint	Average Efficiency %	Maximum Efficiency %
Lap Joints		
Single riveted	45-60	63.3
Double riveted	63-70	77.5
Triple riveted	72-80	86.5
Butt Joints		
Single riveted	55-60	63.3
Double riveted	70-83	86.6
Triple riveted	80-90	95.0
Quadruple riveted	85-94	98.1

CALCULATION OF HOLE DIA AND PITCH

For an ideal joint the rivet should be equally strong against shearing and crushing. Hence, from Eqs. (3.3) and (3.7), making $P_s = P_c$

$$\frac{\pi}{4} d^2 \sigma_s = d t \sigma_c$$

$$d = 1.274 \frac{\sigma_c}{\sigma_s} t \quad \text{in single shear}$$

If rivet is in double shear,

$$d = 0.637 \frac{\sigma_c}{\sigma_s} t$$

Also equating right hand sides of Eqs

$$(p - d) t \sigma_t = \frac{\pi}{4} d^2 \sigma_s$$

$$p = \frac{\pi d^2}{4 t \sigma_t} \sigma_s + d$$

Equating right hand sides of Eqs.

$$2 m t \sigma_s = d t \sigma_c$$

$$m = \frac{d}{2} \frac{\sigma_c}{\sigma_s}$$

$$d = 6 \sqrt{t}$$

This is known as Unwin's formula

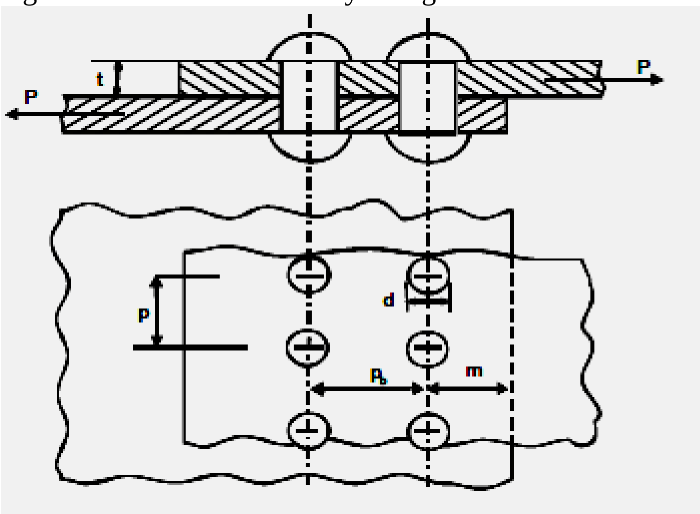
Example.1

Design a double riveted lap joint for MS plates 9.5 mm thick. Calculate the efficiency of the joint. The permissible stresses are :

$\sigma_t = 90Mpa$ $\sigma_s = 75\ Mpa$ $\sigma_c = 150Mpa$

Solution

The joint to be designed is shown schematically in Figure



(a) **Dia. of Rivet Hole *d* :** It is determined by Unwin's formula, Eq. (3.18)

$$d = 6\sqrt{t}$$

or $$d = 6\sqrt{9.5} = 18.5 \text{ mm} \qquad \ldots \text{(i)}$$

(b) **Pitch of the Joint, *p* :** In a double riveted joint there are 4 rivets in a pitch length. The rivet diameter will be taken as diameter of the hole as difference between them is small. The rivets can fail in shear or due to crushing. We will first determine the shearing and crushing strength of a rivet and equate the smaller of two to the plate tearing strength to determine *p*.

Shearing strength of one rivet

$$= \frac{\pi}{4} d^2 \tau_s = \frac{\pi}{4} (18.5)^2\ 75 = 20.16 \text{ kN} \qquad \ldots \text{(a)}$$

Crushing strength of one rivet

$$= \sigma_c\ dt = 150 \times 18.5 \times 9.5 = 26.36 \text{ kN} \qquad \ldots \text{(b)}$$

From (a) and (b) it is seen that the rivet is weaker in shear.

$\therefore$ We will equate tearing strength of plate with shearing strength of rivets in a pitch length. There are two rivets in the pitch length.

$\therefore$ $$\sigma_t\ (p - d)\ t = 2 \times \frac{\pi}{4} d^2\ \tau_s$$

or $$p = \frac{\pi}{2} \frac{d^2}{t} \frac{\tau_s}{\sigma_t} + d = \frac{\pi}{2} \frac{(18.5)^2}{9.5} \frac{75}{90} + 18.5$$

or $$p = 65.55 \text{ mm} \quad \text{say} \quad 65.7 \text{ mm} \qquad \ldots \text{(ii)}$$

The pitch should be such that head forming operation is not hindered. The practice dictates that $p \geq 3\,d$ so that head forming is permitted.
$3\,d = 55.5$ mm, and hence the value of p obtained in (ii) is acceptable.

(c) **The back Pitch p_b :** It must be between 2.5 d to 3.0 d. For chain riveting the higher value is preferred for the reason of head forming

$$p_b = 3d = 3 \times 18.5 = 55.5 \text{ mm} \qquad \ldots \text{(iii)}$$

(d) **Margin, m :** m is determined by equating shearing strength of rivet (smaller of shearing and crushing strengths of rivet). Remember that there are two rivets per pitch length :

$$\therefore \qquad 2mt\,\tau_s = 2\,\frac{\pi}{4}\,d^2\,\tau_s$$

$$\therefore \qquad m = \frac{\pi}{4}\,\frac{d^2}{t} = \frac{\pi}{4}\,\frac{(18.5)^2}{9.5} = 28.3 \text{ mm} \qquad \ldots \text{(iv)}$$

The minimum acceptable value of m is 1.5 d = 27.5 mm hence

m = 28.3 mm is acceptable.

Thus the design is completed with

d = 18.5 mm, p = 65.7 mm, p_b = 55.5 mm, m = 28.3 mm

The diameter is standardized, apparently based on drill size. Normally fractions like 18.5 mm may not be accepted. The rivet diameters are less than hole diameter by 1 mm. Yet the head formation process increases rivet diameter. We are not yet describing standard hole and rivet diameters. We postpone it for the time being.

(e) **Efficiency of Joint**

Tensile strength of plate without holes, per pitch length

$$P_1 = \sigma_t\ pt = 90 \times 65.7 \times 9.5 = 56.2 \text{ kN} \qquad \ldots \text{(c)}$$

Shearing strength of rivets in a pitch length

$$P_s = 2 \times \tau_s \times \frac{\pi}{4}\,d^2 = 2 \times 75 \times \frac{\pi}{4}\,(18.5)^2 = 40.3 \text{ kN} \qquad \ldots \text{(d)}$$

Crushing strength of rivets in a pitch length

$$P_c = 2 \times \sigma_c \times d \times t = 2 \times 150 \times 18.5 \times 9.5 = 52.7 \text{ kN} \qquad \ldots \text{(e)}$$

The tearing strength of plate with one hole in a pitch length

$$P_t = \sigma_t\,(p - d)\,t = 90\,(65.7 - 18.5)\,9.5 = 40.36 \text{ kN} \qquad \ldots \text{(f)}$$

The shearing strength of margin

$$P_{ms} = 2\ \tau_s\ mt = 2 \times 75 \times 28.3 \times 9.5 = 40.32 \text{ kN} \quad \ldots (g)$$

Out of all P_s, P_c, P_t and P_{ms}, the lowest is P_s

$$\eta = \frac{P_s}{P_t} = \frac{40.3}{56.2} = 71.7\% \quad \ldots (h)$$

The design values are

$d = 18.5$ mm, $p = 65.7$ mm, $p_b = 55.5$ mm, $m = 28.3$ mm, $\eta = 71.7\%$

Example 9.1. *A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.*

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : $t = 15$ mm ; $d = 25$ mm ; $p = 75$ mm ; $\sigma_{tu} = 400$ MPa $= 400$ N/mm^2 ; $\tau_u = 320$ MPa $= 320$ N/mm^2 ; $\sigma_{cu} = 640$ MPa $= 640$ N/mm^2

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300\ 000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2\ 320 = 314\ 200 \text{ N} \quad \ldots(\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480\ 000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\ 000/4 = 75\ 000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\ 000 = (p - d)\ t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\ \sigma_{ta}$$

$$\therefore \quad \sigma_{ta} = 75\ 000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75\ 000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2\ \tau_a = 982\ \tau_a$$

$$\therefore \quad \tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75\ 000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\ \sigma_{ca}$$

$$\therefore \quad \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

H W

Example 9.2. ***Find the efficiency of the following riveted joints :***

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.

2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm. Assume Permissible tensile stress in plate = 120 MPa

Permissible shearing stress in rivets = 90 MPa
Permissible crushing stress in rivets = 180 MPa

Solution. Given : $t = 6$ mm ; $d = 20$ mm ; $\sigma_t = 120$ MPa $= 120$ N/mm^2 ; $\tau = 90$ MPa $= 90$ N/mm^2 ; $\sigma_c = 180$ MPa $= 180$ N/mm^2

1. *Efficiency of the first joint*

Pitch, $p = 50$ mm ...(Given)

First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(i) Tearing resistance of the plate

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (50 - 20)\, 6 \times 120 = 21\ 600 \text{ N}$$

(ii) Shearing resistance of the rivet

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} (20)^2\, 90 = 28\ 278 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21\ 600 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 21\ 600 \text{ N}$$

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\ 000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{21\ 600}{36\ 000} = 0.60 \text{ or } 60\% \quad \textbf{Ans.}$$

2. *Efficiency of the second joint*

Pitch, $p = 65$ mm ...(Given)

(i) Tearing resistance of the plate,

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (65 - 20)\, 6 \times 120 = 32\ 400 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\, 90 = 56\ 556 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43\ 200 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 32\ 400 \text{ N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\ 800 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{32\ 400}{46\ 800} = 0.692 \text{ or } 69.2\% \quad \textbf{Ans.}$$

Example 9.3. *A double riveted double cover butt joint in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. The permissible stresses are :*

$\sigma_t = 120$ MPa; $\tau = 100$ MPa; $\sigma_c = 150$ MPa

Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

Solution. Given : $t = 20$ mm ; $d = 25$ mm ; $p = 100$ mm ; $\sigma_t = 120$ MPa $= 120$ N/mm^2 ; $\tau = 100$ MPa $= 100$ N/mm^2 ; $\sigma_c = 150$ MPa $= 150$ N/mm^2

First of all, let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivet.

(i) Tearing resistance of the plate

We know that tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (100 - 25)\, 20 \times 120 = 180\ 000 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4}\, (25)^2\, 100 = 196\ 375 \text{ N}$$

(iii) Crushing resistance of the rivets

Since the joint is double riveted, therefore the strength of two rivets is taken. We know that crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 25 \times 20 \times 150 = 150\ 000 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t,\ P_s \text{ and } P_c$$
$$= 150\ 000 \text{ N}$$

Efficiency of the joint

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 100 \times 20 \times 120$$
$$= 240\ 000 \text{ N}$$

$\therefore$ Efficiency of the joint

$$= \frac{\text{Least of } P_t,\ P_s \text{ and } P_c}{P} = \frac{150\ 000}{240\ 000}$$
$$= 0.625 \text{ or } 62.5\%$$ **Ans.**

Design of Boiler Joints

The boiler has a longitudinal joint as well as circumferential joint. The longitudinal joint is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used. The circumferential joint is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used. Since a boiler is made up of number of rings, therefore the longitudinal joints are staggered for convenience of connecting rings at places where both longitudinal and circumferential joints occur.

Assumptions in Designing Boiler Joints

The following assumptions are made while designing a joint for boilers :

1. The load on the joint is equally shared by all the rivets. The assumption implies that the shell and plate are rigid and that all the deformation of the joint takes place in the rivets themselves.
2. The tensile stress is equally distributed over the section of metal between the rivets.
3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. *Thickness of boiler shell.* First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, *i.e.*

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

where t = Thickness of the boiler shell,

P = Steam pressure in boiler,

D = Internal diameter of boiler shell,

σ_t = Permissible tensile stress, and

η_l = Efficiency of the longitudinal joint.

The following points may be noted :

(*a*) The thickness of the boiler shell should not be less than 7 mm.

(*b*) The efficiency of the joint may be taken from the following table.

Table 9.1. Efficiencies of commercial boiler joints.

Lap joints	*Efficiency (%)*	**Maximum efficiency*	*Butt joints (Double strap)*	*Efficiency (%)*	**Maximum efficiency*
Single riveted	45 to 60	63.3	Single riveted	55 to 60	63.3
Double riveted	63 to 70	77.5	Double riveted	70 to 83	86.6
Triple riveted	72 to 80	86.6	Triple riveted (5 rivets per pitch with unequal width of straps)	80 to 90	95.0
			Quadruple riveted	85 to 94	98.1

* The maximum efficiencies are valid for ideal equistrength joints with tensile stress = 77 MPa, shear stress = 62 MPa and crushing stress = 133 MPa.

Indian Boiler Regulations (I.B.R.) allow a maximum efficiency of 85% for the best joint.

(*c*) According to I.B.R., the factor of safety should not be less than 4. The following table shows the values of factor of safety for various kind of joints in boilers.

Table 9.2. Factor of safety for boiler joints.

Type of joint	*Factor of safety*	
	Hand riveting	*Machine riveting*
Lap joint	4.75	4.5
Single strap butt joint	4.75	4.5
Single riveted butt joint with two equal cover straps	4.75	4.5
Double riveted butt joint with two equal cover straps	4.25	4.0

2. *Diameter of rivets.* After finding out the thickness of the boiler shell (t), the diameter of the rivet hole (d) may be determined by using Unwin's empirical formula, *i.e.*

$$d = 6\sqrt{t} \quad \text{(when } t \text{ is greater than 8 mm)}$$

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing. The following table gives the rivet diameter corresponding to the diameter of rivet hole as per IS : 1928 – 1961 (Reaffirmed 1996).

Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).

Basic size of rivet mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole diameter (min) mm	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

According to IS : 1928 – 1961 (Reaffirmed 1996), the table on the next page (Table 9.4) gives the preferred length and diameter combination for rivets.

3. *Pitch of rivets.* The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

(*a*) The pitch of the rivets should not be less than $2d$, which is necessary for the formation of head.

(*b*) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where t = Thickness of the shell plate in mm, and

C = Constant.

The value of the constant C is given in Table 9.5.

Table 9.4. Preferred length and diameter combinations for rivets used in boilers as per IS : 1928–1961 (Reaffirmed 1996).

(All dimensions in mm)

Length	*Diameter*													
	12	14	16	18	20	22	24	27	30	33	36	39	42	48
28	×	–	–	–	–	–	–	–	–	–	–	–	–	–
31.5	×	×	–	–	–	–	–	–	–	–	–	–	–	–
35.5	×	×	×	–	–	–	–	–	–	–	–	–	–	–
40	×	×	×	×	–	–	–	–	–	–	–	–	–	–
45	×	×	×	×	×	–	–	–	–	–	–	–	–	–
50	×	×	×	×	×	×	–	–	–	–	–	–	–	–
56	×	×	×	×	×	×	×	–	–	–	–	–	–	–
63	×	×	×	×	×	×	×	×	–	–	–	–	–	–
71	×	×	×	×	×	×	×	×	×	–	–	–	–	–
80	×	×	×	×	×	×	×	×	×	–	–	–	–	–
85	–	×	×	×	×	×	×	×	×	×	–	–	–	–
90	–	×	×	×	×	×	×	×	×	×	–	–	–	–
95	–	×	×	×	×	×	×	×	×	×	×	–	–	–
100	–	–	×	×	×	×	×	×	×	×	×	–	–	–
106	–	–	×	×	×	×	×	×	×	×	×	×	–	–
112	–	–	×	×	×	×	×	×	×	×	×	×	–	–
118	–	–	–	×	×	×	×	×	×	×	×	×	×	–
125	–	–	–	–	×	×	×	×	×	×	×	×	×	×
132	–	–	–	–	–	×	×	×	×	×	×	×	×	×
140	–	–	–	–	–	×	×	×	×	×	×	×	×	×
150	–	–	–	–	–	–	×	×	×	×	×	×	×	×
160	–	–	–	–	–	–	×	×	×	×	×	×	×	×
180	–	–	–	–	–	–	–	×	×	×	×	×	×	×
200	–	–	–	–	–	–	–	–	×	×	×	×	×	×
224	–	–	–	–	–	–	–	–	–	×	×	×	×	×
250	–	–	–	–	–	–	–	–	–	–	–	–	×	×

Preferred numbers are indicated by ×.

Table 9.5. Values of constant *C*.

Number of rivets per pitch length	*Lap joint*	*Butt joint (single strap)*	*Butt joint (double strap)*
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

Note : If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than p_{max}, then the value of p_{max} is taken.

4. ***Distance between the rows of rivets.*** The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows :

(*a*) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets (p_b) should not be less than

$0.33\,p + 0.67\,d$, for zig-zig riveting, and

$2\,d$, for chain riveting.

(*b*) For joints in which the number of rivets in outer rows is ***half*** the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than

$0.33\,p + 0.67$ or $2\,d$, whichever is greater.

The distance between the rows in which there are full number of rivets shall not be less than $2d$.

(*c*) For joints in which the number of rivets in outer rows is ***half*** the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than $0.2\,p + 1.15\,d$. The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than $0.165\,p + 0.67\,d$.

Note : In the above discussion, p is the pitch of the rivets in the outer rows.

5. ***Thickness of butt strap.*** According to I.B.R., the thicknesses for butt strap (t_1) are as given below :

(*a*) The thickness of butt strap, in no case, shall be less than 10 mm.

(*b*) $t_1 = 1.125\,t$, for ordinary (chain riveting) single butt strap.

$t_1 = 1.125\,t\left(\dfrac{p-d}{p-2d}\right)$, for single butt straps, every alternate rivet in outer rows being omitted.

$t_1 = 0.625\,t$, for double butt-straps of equal width having ordinary riveting (chain riveting).

$t_1 = 0.625\,t\left(\dfrac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(*c*) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75\,t$, for wide strap on the inside, and

$t_2 = 0.625\,t$, for narrow strap on the outside.

6. ***Margin.*** The margin (m) is taken as $1.5\,d$.

Note : The above procedure may also be applied to ordinary riveted joints.

Design of Circumferential Lap Joint for a Boiler

The following procedure is adopted for the design of circumferential lap joint for a boiler.

1. *Thickness of the shell and diameter of rivets.* The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2. *Number of rivets.* Since it is a lap joint, therefore the rivets will be in single shear.

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

where n = Total number of rivets.

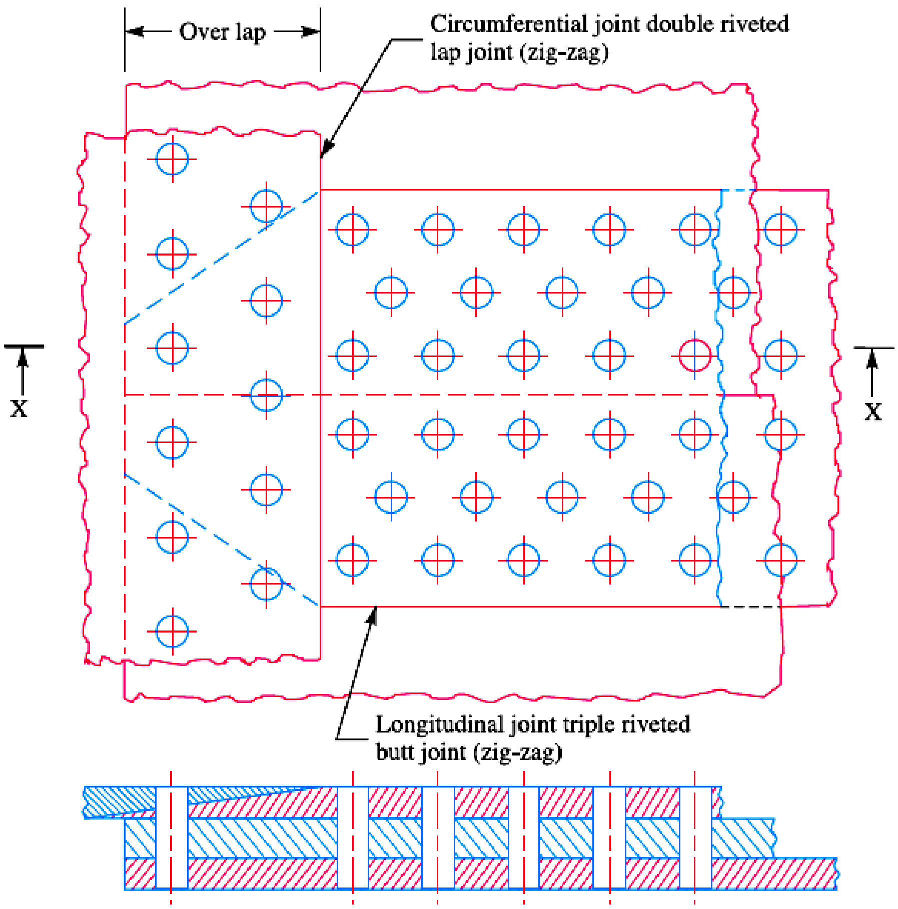
Knowing the inner diameter of the boiler shell (D), and the pressure of steam (P), the total shearing load acting on the circumferential joint,

$$W_s = \frac{\pi}{4} \times D^2 \times P \qquad \ldots(ii)$$

From equations (i) and (ii), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \qquad n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}$$



Example 9.4. *A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume*

$\sigma_t = 80$ *MPa* ; $\tau = 60$ *MPa* ; *and* $\sigma_c = 120$ *MPa*

State how the joint will fail and find the efficiency of the joint.

Solution. Given : $t = 13$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\sigma_c = 120$ MPa $= 120$ N/mm^2

1. *Diameter of rivet*

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm}$$

From Table 9.3, we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the rivet hole (d) is 23 mm and the corresponding diameter of the rivet is 22 mm. **Ans.**

2. *Pitch of rivets*

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig. 9.6 (*c*)], therefore there are two rivets per pitch length, *i.e.* $n = 2$. Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)t \times \sigma_t = (p - 23)\ 13 \times 80 = (p - 23)\ 1040 \text{ N} \quad \text{...}(i)$$

and shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (23)^2\ 60 = 49\ 864 \text{ N} \quad \text{...}(ii)$$

...(∵ There are two rivets in single shear)

From equations (*i*) and (*ii*), we get

$$p - 23 = 49864 / 1040 = 48 \quad \text{or} \quad p = 48 + 23 = 71 \text{ mm}$$

The maximum pitch is given by,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for 2 rivets per pitch length, the value of C is 2.62.

$$\therefore \quad p_{max} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}$$

Since p_{max} is more than p, therefore we shall adopt

$$p = 71 \text{ mm} \quad \textbf{Ans.}$$

3. *Distance between the rows of rivets*

We know that the distance between the rows of rivets (for zig-zag riveting),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 71 + 0.67 \times 23 \text{ mm}$$
$$= 38.8 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

4. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 23 = 34.5 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

Failure of the joint

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\,t \times \sigma_t = (71 - 23)13 \times 80 = 49\,920 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\,(23)^2\,60 = 49\,864 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\,760 \text{ N}$$

The least of P_t, P_s and P_c is $P_s = 49\,864$ N. Hence the joint will fail due to shearing of the rivets. **Ans.**

Efficiency of the joint

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73\,840 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{P_s}{P} = \frac{49\,864}{73\,840} = 0.675 \text{ or } 67.5\% \quad \textbf{Ans.}$$

Example 9.5. *Two plates of 7 mm thick are connected by a triple riveted lap joint of zig-zag pattern. Calculate the rivet diameter, rivet pitch and distance between rows of rivets for the joint. Also state the mode of failure of the joint. The safe working stresses are as follows :*

$\sigma_t = 90$ *MPa ;* $\tau = 60$ *MPa ; and* $\sigma_c = 120$ *MPa.*

Solution. Given : $t = 7$ mm ; $\sigma_t = 90$ MPa = 90 N/mm^2 ; $\tau = 60$ MPa = 60 N/mm^2 ; $\sigma_c = 120$ MPa = 120 N/mm^2

1. *Diameter of rivet*

Since the thickness of plate is less than 8 mm, therefore diameter of the rivet hole (d) is obtained by equating the shearing resistance (P_s) to the crushing resistance (P_c) of the rivets. The triple riveted lap joint of zig-zag pattern is shown in Fig. 9.7 (*b*). We see that there are three rivets per pitch length (*i.e.* $n = 3$). Also, the rivets in lap joint are in single shear.

We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 3 \times \frac{\pi}{4} \times d^2 \times 60 = 141.4\, d^2 \text{ N} \quad ...(i)$$

$$...(\because n = 3)$$

$$P_c = n \times d \times t \times \sigma_c = 3 \times d \times 7 \times 120 = 2520\, d \text{ N} \quad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$141.4\, d^2 = 2520\, d \quad \text{or} \quad d = 2520 / 141.4 = 17.8 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 19 mm and the corresponding diameter of rivet is 18 mm. **Ans.**

2. Pitch of rivets

Let p = Pitch of rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 19)\, 7 \times 90 = 630\,(p - 19) \text{ N} \quad ...(iii)$$

and shearing resistance of the rivets,

$$P_s = 141.4\, d^2 = 141.4\,(19)^2 = 51\,045 \text{ N} \quad ...[\text{From equation } (i)] \;...(iv)$$

Equating equations *(iii)* and *(iv)*, we get

$$630\,(p - 19) = 51\,045$$

$$p - 19 = 51\,045 / 630 = 81 \text{ or } p = 81 + 19 = 100 \text{ mm}$$

According to I.B.R., maximum pitch,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for lap joint and 3 rivets per pitch length, the value of C is 3.47.

$$\therefore \quad p_{max} = 3.47 \times 7 + 41.28 = 65.57 \text{ say } 66 \text{ mm}$$

Since p_{max} is less than p, therefore we shall adopt $p = p_{max} = 66$ mm **Ans.**

3. Distance between rows of rivets

We know that the distance between the rows of rivets for zig-zag riveting,

$$p_b = 0.33\, p + 0.67\, d = 0.33 \times 66 + 0.67 \times 19 = 34.5 \text{ mm} \quad \textbf{Ans.}$$

Mode of failure of the joint

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (66 - 19)\, 7 \times 90 = 29\,610 \text{ N}$$

Shearing resistance of rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 3 \times \frac{\pi}{4}\,(19)^2\, 60 = 51\,045 \text{ N}$$

and crushing resistance of rivets.

$$P_c = n \times d \times t \times \sigma_c = 3 \times 19 \times 7 \times 120 = 47\,880 \text{ N}$$

From above we see that the least value of P_t, P_s and P_c is P_t = 29 610 N. Therefore the joint will fail due to tearing off the plate.

Example 9.6. *Two plates of 10 mm thickness each are to be joined by means of a single riveted double strap butt joint. Determine the rivet diameter, rivet pitch, strap thickness and efficiency of the joint. Take the working stresses in tension and shearing as 80 MPa and 60 MPa respectively.*

Solution. Given : t = 10 mm ; σ_t = 80 MPa = 80 N/mm^2 ; τ = 60 MPa = 60 N/mm^2

1. *Diameter of rivet*

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{10} = 18.97 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 19 mm and the corresponding diameter of the rivet is 18 mm. **Ans.**

2. *Pitch of rivets*

Let p = Pitch of rivets.

Since the joint is a single riveted double strap butt joint as shown in Fig. 9.8, therefore there is one rivet per pitch length (*i.e.* n = 1) and the rivets are in double shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 19)10 \times 80 = 800\,(p - 19) \text{ N} \quad ...(i)$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \quad ...(\because \text{Rivets are in double shear})$$

$$= 1 \times 1.875 \times \frac{\pi}{4}(19)^2\, 60 = 31\,900 \text{ N} \quad ...(\because n = 1) \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$800\,(p - 19) = 31\,900$$

$\therefore$ $p - 19 = 31\,900 / 800 = 39.87$ or $p = 39.87 + 19 = 58.87$ say 60 mm

According to I.B.R., the maximum pitch of rivets,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint and 1 rivet per pitch length, the value of C is 1.75.

$\therefore$ $p_{max} = 1.75 \times 10 + 41.28 = 58.78$ say 60 mm

From above we see that $p = p_{max}$ = 60 mm **Ans.**

3. *Thickness of cover plates*

We know that thickness of cover plates,

$$t_1 = 0.625\, t = 0.625 \times 10 = 6.25 \text{ mm} \quad \textbf{Ans.}$$

Efficiency of the joint

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (60 - 19)\, 10 \times 80 = 32\,800 \text{ N}$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 1 \times 1.875 \times \frac{\pi}{4}(19)^2\, 60 = 31\,900 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t \text{ and } P_s = 31\,900 \text{ N}$$

Strength of the unriveted plate per pitch length

$$P = p \times t \times \sigma_t = 60 \times 10 \times 80 = 48\,000 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t \text{ and } P_s}{P} = \frac{31\,900}{48\,000} = 0.665 \text{ or } 66.5\% \quad \textbf{Ans.}$$

Example 9.7. *Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm². Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; and shear stress in the rivet 56 MPa.*

Solution. Given : $D = 1.5$ m = 1500 mm ; $P = 0.95$ N/mm² ; $\eta_l = 75\% = 0.75$; $\sigma_t = 90$ MPa = 90 N/mm² ; $\sigma_c = 140$ MPa = 140 N/mm² ; $\tau = 56$ MPa = 56 N/mm²

1. ***Thickness of boiler shell plate***

We know that thickness of boiler shell plate,

$$t = \frac{P.D}{2\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{0.95 \times 1500}{2 \times 90 \times 0.75} + 1 = 11.6 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

2. ***Diameter of rivet***

Since the thickness of the plate is greater than 8 mm, therefore the diameter of the rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.8 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21 mm and the corresponding diameter of the rivet is 20 mm. **Ans.**

3. ***Pitch of rivets***

Let p = Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 21)12 \times 90 = 1080\,(p - 21)\text{N} \quad ...(i)$$

Since the joint is double riveted double strap butt joint, as shown in Fig. 9.9, therefore there are two rivets per pitch length (*i.e.* $n = 2$) and the rivets are in double shear. Assuming that the rivets in double shear are 1.875 times stronger than in single shear, we have

Shearing strength of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4}(21)^2 \times 56 \text{ N}$$
$$= 72\,745 \text{ N} \quad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$1080\,(p - 21) = 72\,745$$

$\therefore$ $p - 21 = 72\,745 / 1080 = 67.35$ or $p = 67.35 + 21 = 88.35$ say 90 mm

According to I.B.R., the maximum pitch of rivets for longitudinal joint of a boiler is given by

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for a double riveted double strap butt joint and two rivets per pitch length, the value of C is 3.50.

$\therefore$ $p_{max} = 3.5 \times 12 + 41.28 = 83.28$ say 84 mm

Since the value of p is more than p_{max}, therefore we shall adopt pitch of the rivets,

$$p = p_{max} = 84 \text{ mm} \quad \textbf{Ans.}$$

4. ***Distance between rows of rivets***

Assuming zig-zag riveting, the distance between the rows of the rivets (according to I.B.R.),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 84 + 0.67 \times 21 = 41.8 \text{ say } 42 \text{ mm} \quad \textbf{Ans.}$$

5. ***Thickness of cover plates***

According to I.B.R., the thickness of each cover plate of equal width is

$$t_1 = 0.625\,t = 0.625 \times 12 = 7.5 \text{ mm} \quad \textbf{Ans.}$$

6. ***Margin***

We know that the margin,

$$m = 1.5\,d = 1.5 \times 21 = 31.5 \text{ say } 32 \text{ mm} \quad \textbf{Ans.}$$

Let us now find the efficiency for the designed joint.

Tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (84 - 21)12 \times 90 = 68\ 040 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 1.875 \times \frac{\pi}{4} (21)^2 \times 56 = 72\ 745 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 21 \times 12 \times 140 = 70\ 560 \text{ N}$$

Since the strength of riveted joint is the least value of P_t, P_s or P_c, therefore strength of the riveted joint,

$$P_t = 68\ 040 \text{ N}$$

We know that strength of the un-riveted plate,

$$P = p \times t \times \sigma_t = 84 \times 12 \times 90 = 90\ 720 \text{ N}$$

∴ Efficiency of the designed joint,

$$\eta = \frac{P_t}{P} = \frac{68\ 040}{90\ 720} = 0.75 \text{ or } 75\%$$ **Ans.**

Since the efficiency of the designed joint is equal to the given efficiency of 75%, therefore the design is satisfactory.

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LECTURE 5

Design of Welded Joints

Welding and welded connections

Welding is the process of joining two pieces of metal by creating a strong metallurgical bond between them by heating or pressure or both. It is distinguished from other forms of mechanical connections, such as riveting or bolting, which are formed by friction or mechanical interlocking. It is one of the oldest and reliable methods of joining.

Objectives

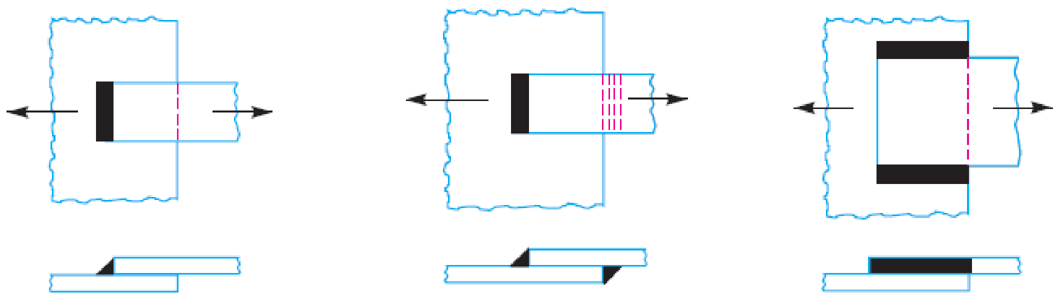
After studying this unit, you should be able to

- describe the types of welded joint,
- calculate the strength of welded joints,

Types of Welded Joints

Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be **1.** Single transverse fillet, **2.** Double transverse fillet, and **3.** Parallel fillet joints.
The fillet joints are shown in Fig. 10.2. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

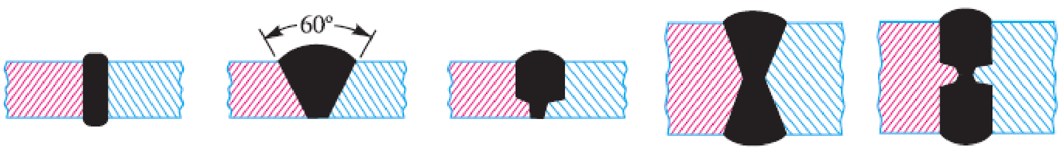


(a) Single transverse. (b) Double transverse. (c) Parallel fillet.

Fig. 10.2. Types of lap or fillet joints.

Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. 10.3. In butt welds, the plate edges do not require beveling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be beveled to V or U-groove on both sides.

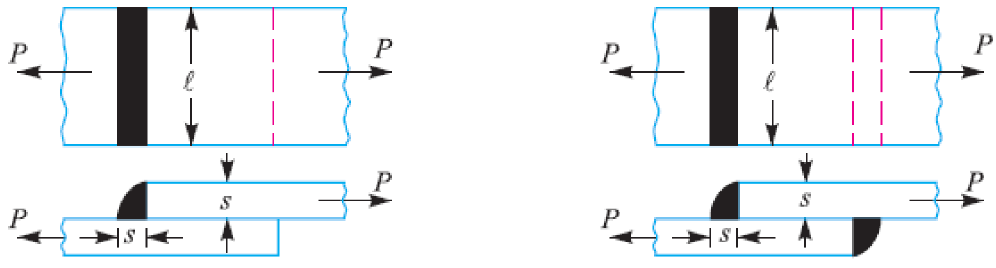


(a) Square butt joint. (b) Single V-butt joint. (c) Single U-butt joint. (d) Double V-butt joint. (e) Double U-butt joint.

Fig. 10.3. Types of butt joints.

Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. 10.6 (*a*) and (*b*) respectively.



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

Fig. 10.6. Transverse fillet welds.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*. The enlarged view of the fillet is shown in Fig. 10.7. The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e. BD*) is known as ***throat thickness.*** The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),

s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

From Fig. 10.7, we find that the throat thickness,

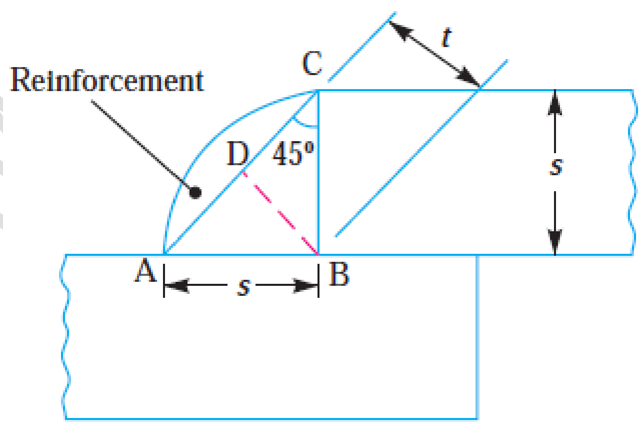


Fig. 10.7. Enlarged view of a fillet weld.

$t = s \times \sin 45° = 0.707\, s$

*Minimum area of the weld or throat area,

A = Throat thickness ×Length of weld

$= t \times l = 0.707\, s \times l$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = Throat area × Allowable tensile stress = $0.707\ s \times l \times \sigma_t$

and tensile strength of the joint for double fillet weld,

$P = 2 \times 0.707\ s \times l \times \sigma_t = 1.414\ s \times l \times \sigma_t$

Note: Since the weld is weaker than the plate due to slag and blow holes, therefore the weld is given a reinforcement which may be taken as 10% of the plate thickness.

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig. 10.8 (*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

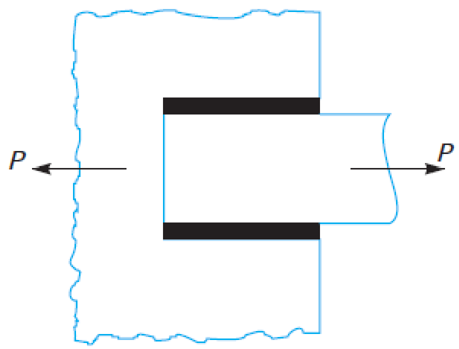
$A = 0.707\ s \times l$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

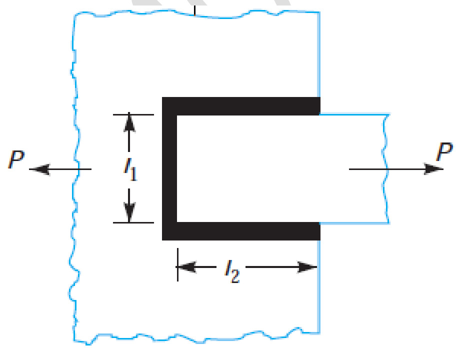
P = Throat area × Allowable shear stress = $0.707\ s \times l \times \tau$

and shear strength of the joint for double parallel fillet weld,

$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\ s \times l \times \tau$



(*a*) Double parallel fillet weld.



(*b*) Combination of transverse and parallel fillet weld.

Fig. 10.8

Notes: 1. If there is a combination of single transverse and double parallel fillet welds as shown in Fig. 10.8 (*b*), then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds.

Mathematically,

$P = 0.707s \times l1 \times \sigma t + 1.414\ s \times l2 \times \tau$

where $l1$ is normally the width of the plate.

2. In order to allow for starting and stopping of the bead, 12.5 mm should be added to the length of each weld obtained by the above expression.

3. For reinforced fillet welds, the throat dimension may be taken as 0.85 *t*.

Special Cases of Fillet Welded Joints

The following cases of fillet welded joints are important from the subject point of view.

1. *Circular fillet weld subjected to torsion.* Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig. 10.9.

Let d = Diameter of rod,

r = Radius of rod,
T = Torque acting on the rod,
s = Size (or leg) of weld,
t = Throat thickness,
**J* = Polar moment of inertia of the weld section= $\frac{\pi t d^3}{4}$

We know that shear stress for the material,

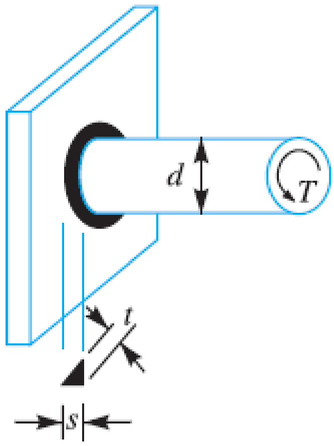


Fig. 10.9. Circular fillet weld subjected to torsion.

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{Tr}{J} = \frac{T * {}^{d}/_{2}}{J} \qquad J = \frac{\pi t d^3}{4}$$

$$\tau = \frac{T * {}^{d}/_{2}}{\frac{\pi t d^3}{4}}$$

$$\tau = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.
Length of throat, *t* = *s* sin 45° = 0.707 *s*
and maximum shear stress,

$$\tau_{max} = \frac{2T}{\pi * 0.707 s * d^2} \quad = \tau_{max} = \frac{2.83T}{\pi * s * d^2}$$

Example 1. ***A 50 mm diameter solid shaft is welded to a flat plate by 10 mm fillet weld as shown in Fig. 10.12. Find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.***
Solution. **Given : *d* = 50 mm ; *s* = 10 mm ; |*max* = 80 MPa = 80 N/mm2**

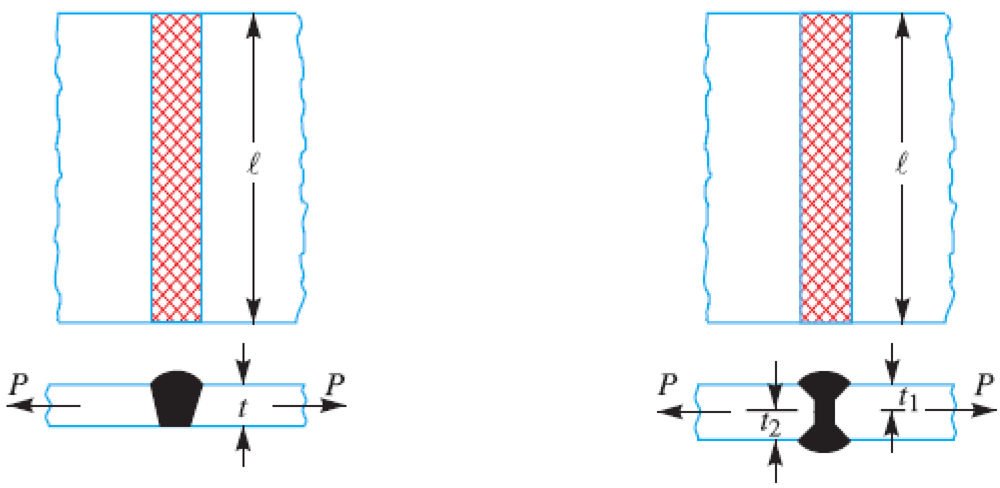
Let *T* = Maximum torque that the welded joint can sustain.
We know that the maximum shear stress

$$\tau_{max} = \frac{2T}{\pi * 0.707 s * d^2} = \frac{2.83T}{\pi * s * d^2}$$

$$80 = \frac{2.83T}{\pi * 10 * 50} = 2.22 * 10^6 \text{ N.mm} \qquad \textbf{Ans.}$$

Strength of Butt Joints

The butt joints are designed for tension or compression. Consider a single V-butt joint as shown
in Fig. 10.14 (*a*).



(*a*) Single *V*-butt joint. (*b*) Double *V*-butt joint.

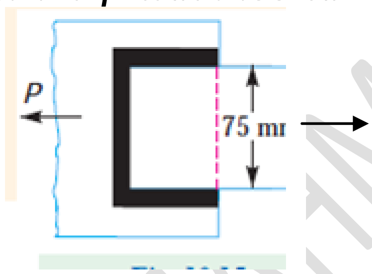
Fig. 10.14. Butt joints.

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is
equal to thickness of plates.
4 Tensile strength of the butt joint (single-*V* or square butt joint),
$P = t \times l \times \sigma t$
where *l* = Length of weld. It is generally equal to the width of plate.
and tensile strength for double-*V* butt joint as shown in Fig. 10.14 (*b*) is given by
$P = (t1 + t2)\ l \times \sigma t$
where *t*1 = Throat thickness at the top, and
*t*2 = Throat thickness at the bottom.

Example 2. ***A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.***
***Solution.* *Given:* *Width = 100 mm ;**

Thickness = 10 mm ; P = 80 kN = 80 × 103 N ;
τ = 55 MPa = 55 N/mm2
Let l =Length of weld, and
s = Size of weld = Plate thickness = 10 mm ... (Given)
We know that maximum load which the plates can carry for double parallel fillet weld (P),
80 × 103 = 1.414 × s × l × τ = 1.414 × 10 × l × 55 = 778 l
∴ l = 80 × 103 / 778 = 103 mm
Adding 12.5 mm for starting and stopping of weld run, we have
l = 103 + 12.5 = 115.5 mm ***Ans.***

Example 3.
A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in the figure below:-



The maximum tensile and shear stress are 70 Mpa and 56 Mpa respectively . Find the length of each parallel fillet weld if the joint is subjected to both static and fatigue loading.
Given data:
Transverse and parallel fillet weld
Width=w=75 mm
S=12.5 mm
σ_t ***=70 Mpa*** ***τ=56 Mpa***
length of parallel fillet weld= l_2 ***=---***
i-static loading
ii- fatigue loading
Solution;
Let l_1 ***=length of transverse fillet weld***
∴ l_1 ***=75-12.5=62.5 mm***
Now , since total loading acting on the plates
$P = \sigma_t . A$
$P=70 * 75 * 12.5 = 65.63 * 10^3$ ***N***
Let , P_1 ***=load carried by single transverse fillet weld***

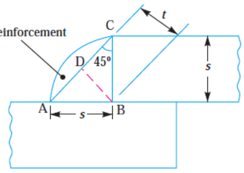


Fig. 10.7. Enlarged view of a fillet weld.

Failure starts from minimum area
Since minimum thickness =throat thickness
∴ t = throat thickness
Sin45=t/s
T=sin45*S
∴ minimum area will be at throat
A= t*l
$A=0.707 .S.l_1$--------for transverse fillet weld
∴ minimum area for double transverse parallel fillet weld
$A=2.t.l_2$ … Two areas
$A=2*0.707*S*l_2$
$=1.414*S*l_2$ for double parallel fillet weld

∴ load carried out by single transverse fillet weld
$P_1=A*\sigma_t=0.707*S*l_1*\sigma_t$
$P_1= 0.707*12.5*62.5*70= 38.664*10^3$ N
Similarly load carried by parallel fillet weld
$P_2=A*\tau$
$=1.414 .S. l_2. \tau$
$= 1.414*12.5*l_2*56$
$= 989.8\ l_2$ N
Hence total load on weld or plates
$P=P_1+P_2$
∴ $65.63*10^3=(38.66*10^3)+(990\ l_2)$
$L_2=27.2$ mm

∴ adding 12.5 mm for start and stop of weld
$l_2= 27.2+12.5=39.7$ mm≅40 mm for static loading ANS.

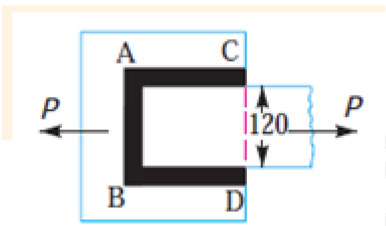
Example 4.

Determine the weld run for a plate of size 120 mm wide and 15 mm thick to be welded to another plate by means of :

i> *a single transverse fillet weld*

ii> *double parallel fillet weld when, the joint is subjected to variable loads.*
Take:
σ_t *=70 Mpa* τ *=56 Mpa*

Given data:
Width=w=120 mm
Thickness=S=15 mm
First of all let's draw the weld graph



i-length of transverse fillet weld (l_1)= --
ii-length of parallel fillet weld (l_2)= --
variable loads =fatigue loading
Solution
Since length of single transverse fillet weld
l_1=120-12.5=107.5 mm
total load acting on the plates

$P = \sigma_t . A$ =120*15*70=126*10^3 N

∴ ***Minimum thickness in weld =throat thickness***
Failure always takes place in minimum area
A= t*l_1
A=0.707*S*l_1

Load carried by transverse fillet weld

$$P_1 = A * \sigma_{t1}$$

i- *for stress concentration factor for transverse fillet weld=1.5*

ii- = = = *for double parallel fillet weld=2.7*

$$\sigma_{t1} = \frac{70}{1.5} = 46.7\ N/mm^2$$

$$\tau_1 = \frac{56}{2.7} = 20.74 N/mm^2$$

*P_1=0.707*S*l_1*46.7=53.24 *10^3 N*
Now load carried by double parallel fillet weld:
$P_2 = 2A * \tau$

P_2=2*0.707*S*l_2* τ
P_2=2*0.707**15*l_2*20.74=440 l_2 N

l_2=165.4 mm
adding 12.5 mm in l_2 in order to consider start and stop of weld
l_2=165.4+12.5=177.9 mm ANS

Example/ 5

A plate 100mm wide and 12.5mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 Mpa. Consider the joint first under ststic loading the under fatigue loading. (Take K_t=2.7)

Sol. Given data: W=100mm S=12.5 mm P=50 kN τ =56Mpa

P=1.414.S.l. τ

$50*10^3$=1.414*12.5*l*56 l=50.5 mm

Adding 12.5mm for starting and stopping the weld run

12.5+50.5=63 mm

Now to find the length for fatigue loading

$$\sigma_{kt} = \frac{\sigma_s}{K_t} = \frac{56}{2.7} = 20.74 \ N/_{mm^2}$$

$50*10^3$=1.414*12.5*l*20.74 l=136.2 mm ans.

adding 12.5 for starting and stopping the weld run
136.2+12.5=148.7 mm

Example 6 *The fillet welds of equal legs are used to fabricate a `T' as shown in Fig. 10.17 (a) and (b), where s is the leg size and l is the length of weld.*

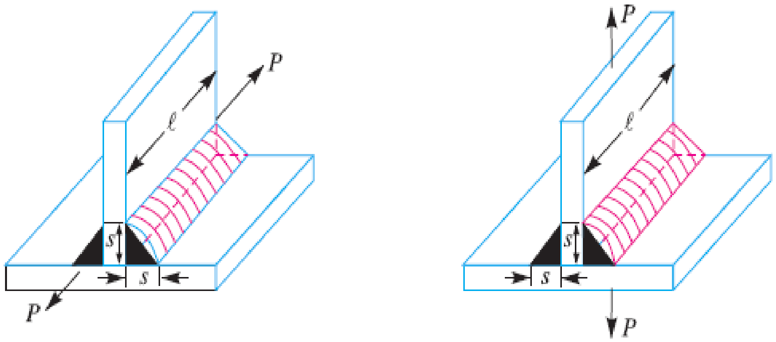


Fig. 10.17

Locate the plane of maximum shear stress in each of the following loading patterns:

1. Load parallel to the weld (neglect eccentricity), and

2. Load at right angles to the weld (transverse load).

Find the ratio of these limiting loads.

Solution. Given : Leg size = s ; Length of weld = l

1. ***Plane of maximum shear stress when load acts parallel to the weld (neglecting eccentricity)***

Let θ = Angle of plane of maximum shear stress, and

t = Throat thickness BD.

From the geometry of Fig. 10.18, we find that

$$BC = BE + EC$$

$$= BE + DE \qquad ...(\because EC = DE)$$

or

$$s = BD\cos\theta + BD\sin\theta$$

$$= t\cos\theta + t\sin\theta$$

$$= t(\cos\theta + \sin\theta)$$

$$\therefore \qquad t = \frac{s}{\cos\theta + \sin\theta}$$

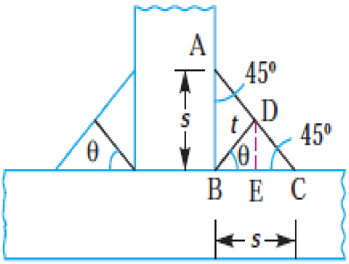


Fig. 10.18

We know that the minimum area of the weld or throat area,

$$A = 2\,t \times l = \frac{2s \times l}{(\cos\theta + \sin\theta)} \qquad ...(\because \text{ of double fillet weld})$$

and shear stress, $$\tau = \frac{P}{A} = \frac{P(\cos\theta + \sin\theta)}{2s \times l} \quad ...(i)$$

For maximum shear stress, differentiate the above expression with respect to θ and equate to zero.

$$\therefore \quad \frac{d\tau}{d\theta} = \frac{P}{2s \times l}(-\sin\theta + \cos\theta) = 0$$

or $$\sin\theta = \cos\theta \quad \text{or} \quad \theta = 45^\circ$$

Substituting the value of $\theta = 45^\circ$ in equation *(i)*, we have maximum shear stress,

$$\tau_{max} = \frac{P(\cos 45^\circ + \sin 45^\circ)}{2s \times l} = \frac{1.414\,P}{2s \times l}$$

or $$P = \frac{2s \times l \times \tau_{max}}{1.414} = 1.414 s \times l \times \tau_{max} \text{ Ans.}$$

Example/7

Find the throat thickness and minimum area of throat thickness for a transverse fillet weld for a plate 100 mm wide and 12.5 mm thick, then find the reinforcement of the weld zone. Draw enlarged view for the fillet weld

Solution/

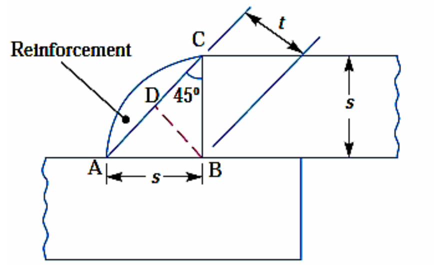


Fig. 10.7. Enlarged view of a fillet weld.

$t = \sin 45 x\ S$ =sin 45x12.5=0.707x12.5=8.8375 mm

Reinforcement=0.85 t=0.85*8.83=7.5 mm

Lecture 6

Shaft Design

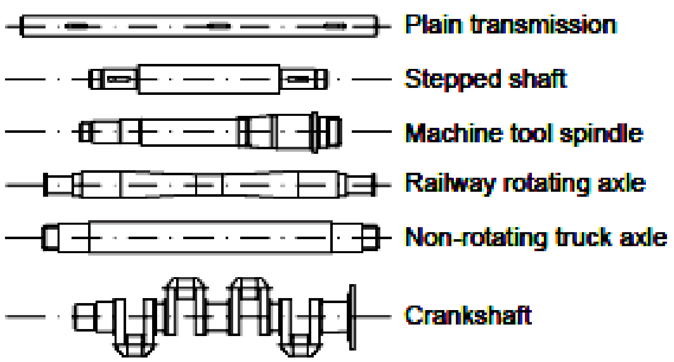
Shafts

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.
In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

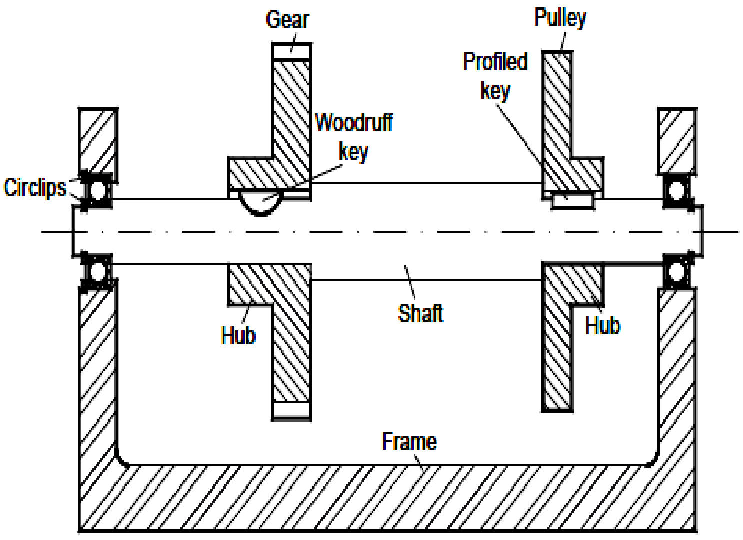
Notes: 1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.
2. An *axle*, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.
3. A *spindle* is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

Objectives

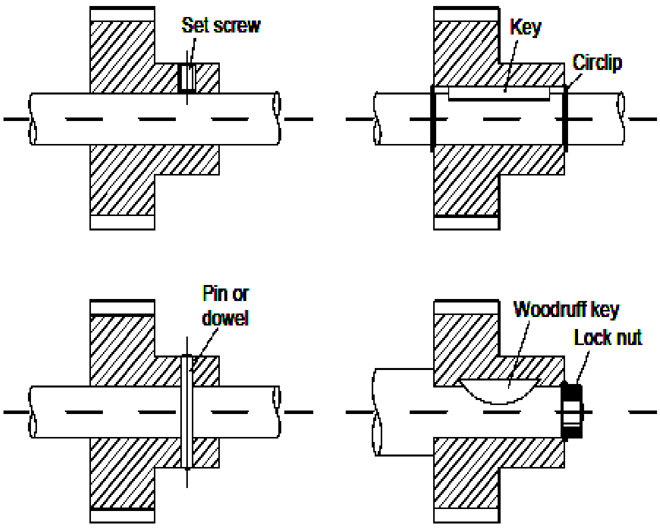
After studying this unit, you should be able to
- **describe the types of shafts**
- **calculate the strength and loads of shafts**



Typical shaft arrangements (adapted from Reshetov, 1978).



Typical shaft arrangement incorporating constant diameter sections and shoulders for locating added components.



Alternative methods of shaft–hub connection.

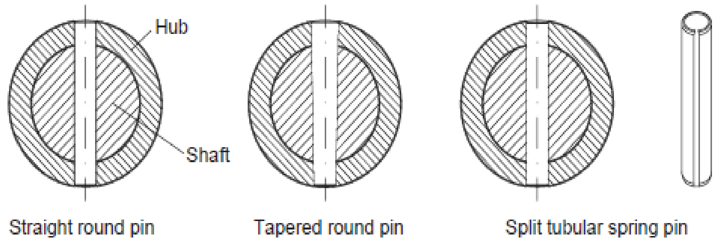


Figure : Pins for torque transmission and component location.

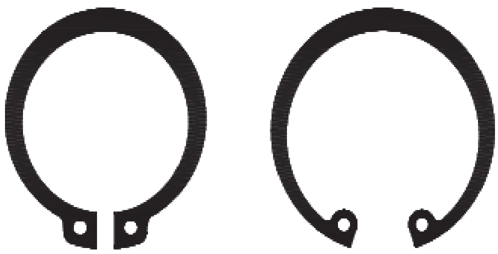


Figure Snap rings or circlips.

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Types of Shafts

The following two types of shafts are important from the subject point of view :

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and

all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.
2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :
25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm
with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.
The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :
1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements
like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsion and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as
(*a*) 112 MPa for shafts without allowance for keyways.
(*b*) 84 MPa for shafts with allowance for keyways.

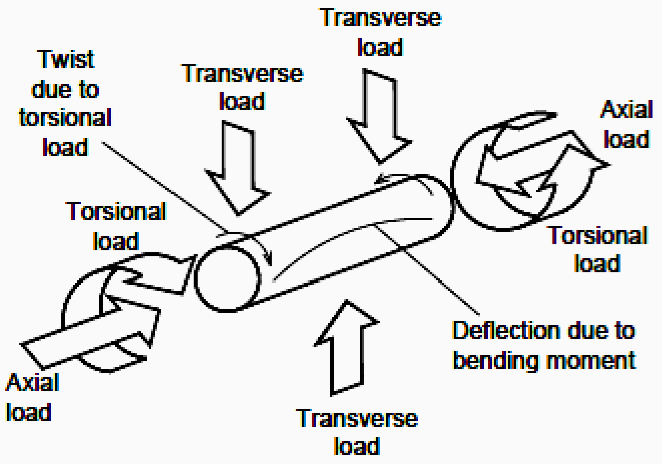
For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress, $\sigma_t = 0.6\ \sigma_{el}$ or $0.36\ \sigma_u$, whichever is less. The maximum permissible shear stress may be taken as
(*a*) 56 MPa for shafts without allowance for key ways.
(*b*) 42 MPa for shafts with allowance for keyways.
For shafts purchased under definite physical specifications, the permissible shear stress (τ) may
be taken as 30 per cent of the elastic limit in tension (σel) but not more than 18 per cent of the ultimate tensile strength (σu). In other words, the permissible shear stress, $\tau = 0.3\ \sigma el$ or $0.18\ \sigma u$, whichever is less.

Design of Shafts

The shafts may be designed on the basis of
1. Strength, and 2. Rigidity and stiffness.
In designing shafts on the basis of strength, the following cases may be considered :
(*a*) Shafts subjected to twisting moment or torque only,
(*b*) Shafts subjected to bending moment only,
(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsion and bending loads.

We shall now discuss the above cases, in detail, in the following pages.



Typical shaft loading and deflection (adapted from Beswarick, 1994a).

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{...}(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre
$= d/2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...(ii)}$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o/2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...(iii)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft
$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \qquad \text{...(iv)}$$

From the equations (*iii*) or (*iv*), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and
R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW $= 20 \times 10^3$ W; $\tau = 42$ MPa $= 42$ N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or $d = 48.7$ say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

$\therefore$ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

$\therefore$ $d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$

or $d = 159.4$ say 160 mm **Ans.**

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

$\therefore$ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad ...(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o\text{)}$$

and $y = d_o / 2$

Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Note: We have already discussed in Art. 14.1 that the axles are used to transmit bending moment only. Thus, axles are designed on the basis of bending moment only, in the similar way as discussed above.

Example 14.4. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50\text{ kN} = 50 \times 10^3\text{ N}$; $L = 100\text{ mm}$; $x = 1.4\text{ m}$; $\sigma_b = 100\text{ MPa} = 100\text{ N/mm}^2$

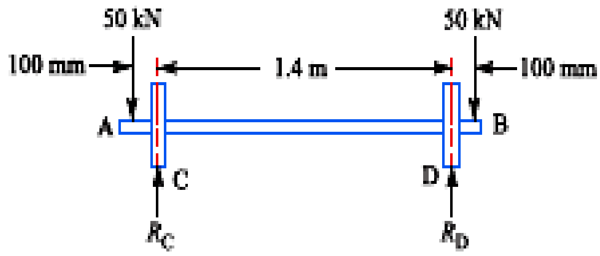


Fig. 14.1

The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6\text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \quad \text{or} \quad d = 79.8 \text{ say } 80\text{ mm}$$ **Ans.**

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \quad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \quad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \quad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \quad ...(iv)$$

The expression $\frac{1}{2}\left[(M+\sqrt{M^2+T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment**. By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M+\sqrt{M^2+T^2}\right] = \frac{\pi}{32}\times\sigma_b\times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*iii*) and (*v*) may be written as

$$T_e = \sqrt{M^2+T^2} = \frac{\pi}{16}\times\tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \frac{1}{2}\left(M+\sqrt{M^2+T^2}\right) = \frac{\pi}{32}\times\sigma_b\,(d_o)^3\,(1-k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

Solution. Given : M = 3000 N-m = 3 × 10⁶ N-mm ; T = 10 000 N-m = 10 × 10⁶ N-mm ; σ_{tu} = 700 MPa = 700 N/mm² ; τ_u = 500 MPa = 500 N/mm²

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2+T^2} = \sqrt{(3\times10^6)^2+(10\times10^6)^2} = 10.44\times10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44\times10^6 = \frac{\pi}{16}\times\tau\times d^3 = \frac{\pi}{16}\times 83.3\times d^3 = 16.36\,d^3$$

$$\therefore \quad d^3 = 10.44\times10^6/16.36 = 0.636\times10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

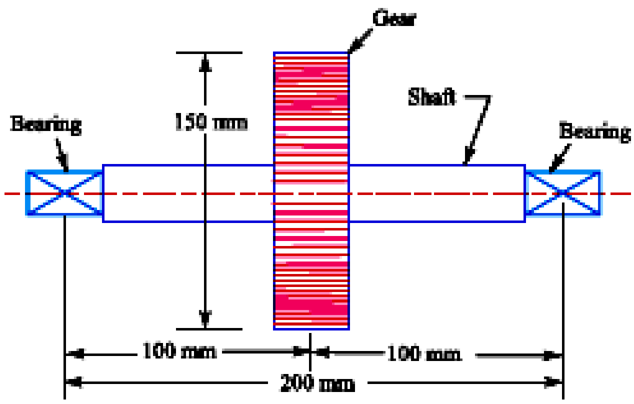
Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Example 14.6. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm² ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.



We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$
$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

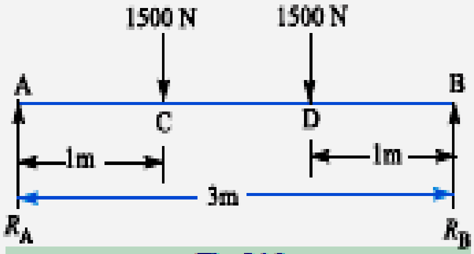


Fig. 14.3

$\therefore$ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$
$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

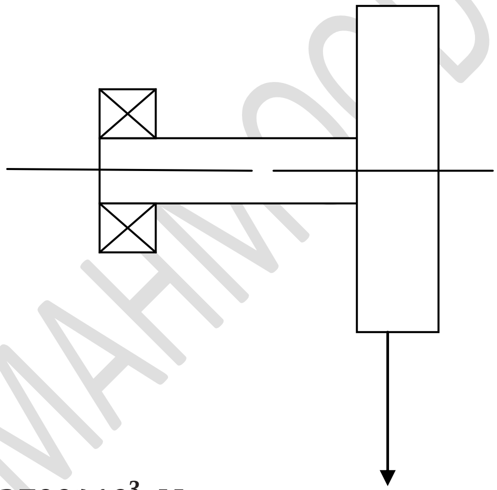
$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\ d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

$\therefore$ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm **Ans.**

Example:

A line shaft is driven by means of motor placed vertically below it. A pulley on the line shaft is (1.5 m) in diameter and has belt tensions of (5.4 kN and 1.8 kN) on tight side and slack of belt respectively. Both these tensions are assumed to be vertical. If the pulley to be over hanged from the shaft, the distance of the center line of the pulley from the center line of the shaft is (400 mm), find the diameter of the shaft (take σ_s=42 Mpa)

Solution



D=150 m= 1500 mm

Tension side=5.4 kN

Slack side= 1.8 kN

σ_s***=42 Mpa***

Torque transmitted by shaft

$T=(T_1-T_2).R=(5400-1800)*1500/2=2700*10^3$ ***N.mm***

The total load on the shaft

$W=T_1+T_2=5400+1800=7200$ ***N***

Bending moment acting on the shaft

$M_A=W*400=7200*400=2880*10^3$ ***N.mm***

Shaft is subjected to both twisting and bending moment

$$T_e=\sqrt{T^2+M^2}=\sqrt{(2700*10^3)^2+(2880*10^3)^2}$$

$=3950*10^3$ ***N.mm***

$$T_e=\frac{\pi}{16}\sigma_s d^3$$

$3950*10^3=\frac{\pi}{16}*42*d^3$

D=78 mm ans.

Lecture 7

Key Design

Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Objectives

After studying this unit, you should be able to

- ***Describe the types of keys and couplings***

- ***calculate the strength of keys and couplings***

Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys, 2. Saddle keys, 3. Tangent keys, 4. Round keys, and 5. Splines.

We shall now discuss the above types of keys, in detail, in the followings:

1. ***Rectangular sunk key.*** A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

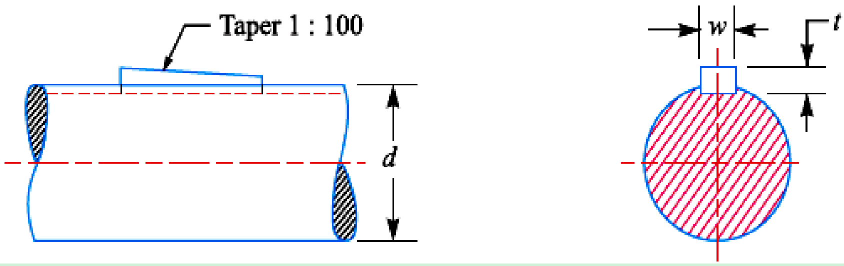


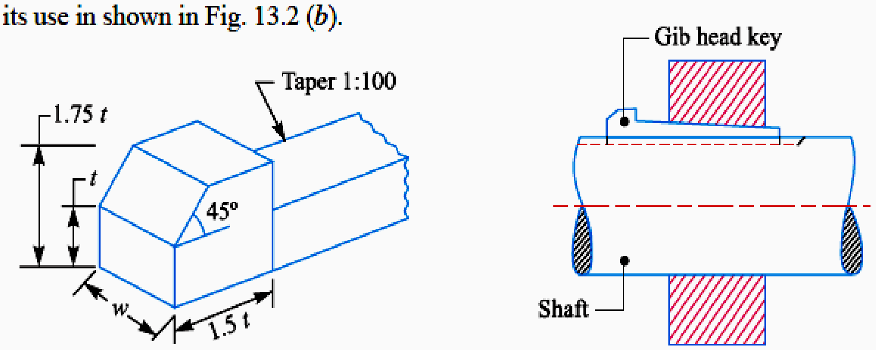
Fig. 13.1. Rectangular sunk key.

2. ***Square sunk key.*** The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. *Parallel sunk key.* The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. *Gib-head key.* It is a rectangular sunk key with a head at one end known as ***gib head.*** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 13.2 (*a*) and its use in shown in Fig. 13.2 (*b*).



(*a*) (*b*)

Fig. 13.2. Gib-head key.

The usual proportions of the gib head key are :

Width, $w = d / 4$;

and thickness at large end. $t = 2w / 3 = d / 6$

5. ***Feather key.*** A key attached to one member of a pair and which permits relative axial movement is known as ***feather key.*** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

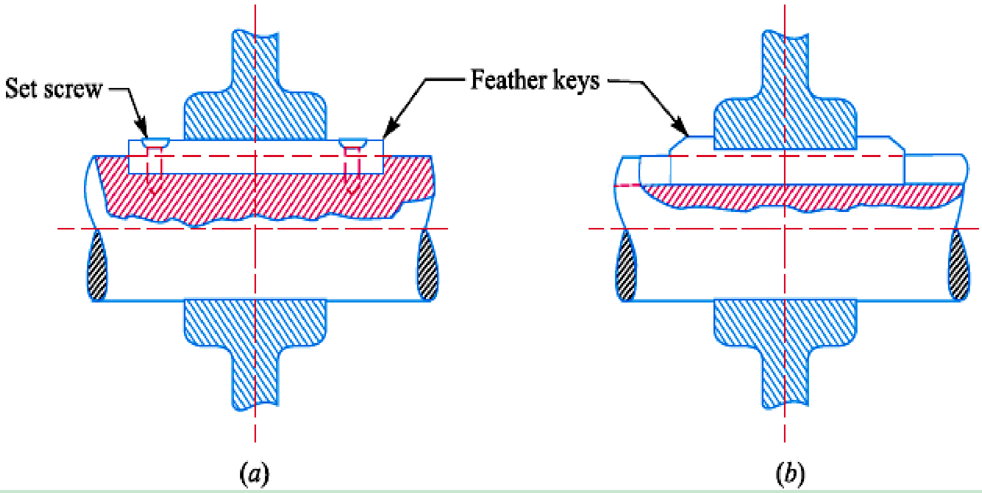


Fig. 13.3. Feather key.

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

6. ***Woodruff key.*** The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 13.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

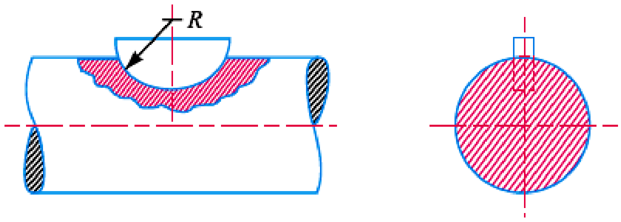


Fig. 13.4. Woodruff key.

13.6 Round Keys

The round keys, as shown in Fig. 13.7(*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

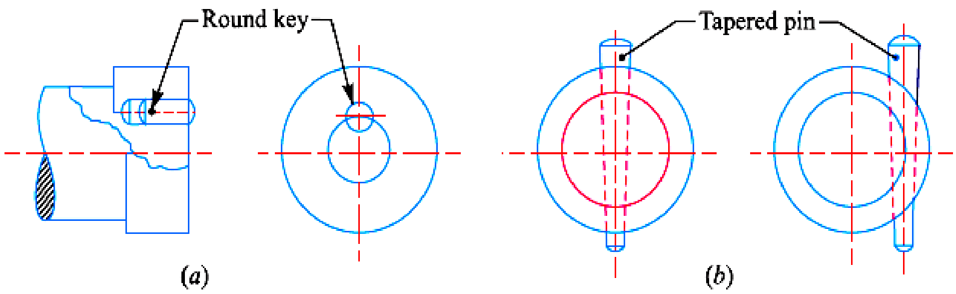
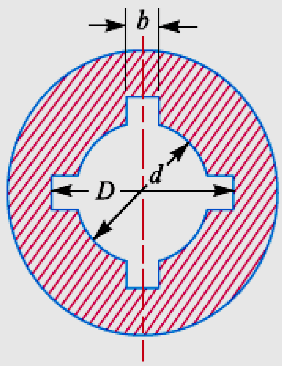


Fig. 13.7. Round keys.

13.7 Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. 13.8. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

Fig. 13.8. Splines.

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces ($F1$) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key. The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the

shaft within the hub. The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 13.9.
In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

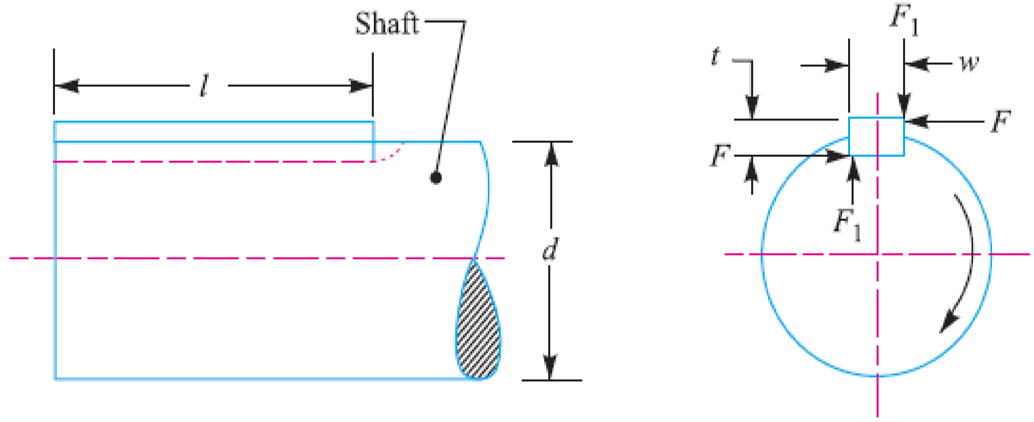


Fig. 13.9. Forces acting on a sunk key.

Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 13.9.
Let T = Torque transmitted by the shaft,
F = Tangential force acting at the circumference of the shaft,
d = Diameter of shaft,
l = Length of key,
w = Width of key.
t = Thickness of key, and
τ and σc = Shear and crushing stresses for the material of key.
A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing. Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \text{...(i)}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...(ii)}$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...[Equating equations (i) and (ii)]}$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad \text{...(iii)}$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation (*iii*), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{...(iv)}$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{...(v)}$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \quad \text{... (Taking } w = d/4\text{)} \quad \text{...(vi)}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \quad \text{... [From equation (vi)]}$$

w

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\ 800\ l \text{ N-mm} \quad ...(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\ (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\ 800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\ l \text{ N-mm} \quad ...(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm}$$ **Ans.**

Example1:

Design the rectangular key for a shaft of 50mm diameter, the shearing and crushing stresses for the key material are 42Mpa. And 70 Mpa respectively.

Given data: dia.=d=50 mm

σ_s= 42 Mpa and σ_c=70 Mpa

Let L=length of the key

L=1.25d=1.25*50=62.5 mm

Now considering the shearing failure of key to find the length of the key

$$T = L.W.\sigma_s.\frac{d}{2}$$

$$T = \frac{\pi}{16}.\sigma_{s1}.d^3$$

$$L.W.\sigma_s.\frac{d}{2} = \frac{\pi}{16}.\sigma_{s1}.d^3$$

$$W = \frac{2\pi\sigma_{s1}d^2}{16\sigma_s.L} = \frac{\pi d^2}{8.L} = \frac{\pi * 50^2}{8 * 62.5} = 15.7 = 16\,mm$$

Now considering crushing stress failure to find the thickness of the key

$$\frac{\pi}{16}.\sigma_{s1}.d^3 = \frac{t}{2}.L.\sigma_c.\frac{d}{2}$$

$$t = \frac{\pi.\sigma_{s1}.d^2 * 2 * 2}{16.L.\sigma_c}$$

$$= \frac{\pi * 42 * 50^2 * 4}{16 * 62.5 * 70} = 18.84 = 20\,mm$$ *ans.*

Example2:

A45mm dia. Shaft is made of steel with yield shear strength of 400 Mpa.. A parallel key of size 14mm wide and 9 mm thick made of steel with a yield strength of 340 Mpa is used. Find the length of key if the shaft is loaded to transmit the maximum permissible torque. Use maximum stress theory and assume F.O.S=2

Solution;

Given data:

d= 45 mm

σ_{sy} for key=340 Mpa

σ_{sy} for shaft= 400 Mpa

W=14 mm and t=9 mm

According to max. Shear stress theory

For shaft $\sigma_{sp} = \frac{\sigma_{sy}}{2*FOS} = \frac{400}{2*2} = 100\ N/mm^2$

For key $\sigma_{sp} = \frac{\sigma_{sy}}{2*FOS} = \frac{340}{2*2} = 85\ N/mm^2$

Now considering shear failure for key to find the length of the key:

$$T = \frac{\pi}{16}.\sigma_{s1}.d^3 = \frac{\pi}{16} * 100 * 45^3 = 1.8 * 10^3\ mm$$

$$1.8 * 10^3 = L.W.\sigma_s.\frac{d}{2} = L * 14 * 85 * \frac{45}{2}$$

L=67.2 mm

Now considering crushing failure to find the length of the key

$1.8*10^3 = \frac{t}{2}.L.\sigma_c.\frac{d}{2}$

To find the crushing stress

$$\frac{W}{t} = \frac{\sigma_c}{2\sigma_s}$$

σ_c =264.5 N/mm^2

∴*L=67.2 mm*

Note: if we get two different lengths we choose the larger one ans.

Example 3

A 15 KW 960 rpm motor has a mild steel shaft of 40 mm diameter and an extension of 75 mm long for the shaft. The permissible shear and crushing stresses for the mild steel key are 56 Mpa and 112 Mpa. Design square key in the motor shaft extension, then check shear strength against normal strength of the shaft.

Solution:

Given data:

P=15 KW N=960rpm d=40 mm L=75 mm σ_s*=56 Mpa*

σ_c*=112 Mpa*

$T = \frac{60P}{2\pi N} = \frac{60*15*10^3}{2\pi*960}$ $=149*10^3$ N.mm

$149*10^3 = L.W.\sigma_s.\frac{d}{2}$

$149*10^3=75*W*56*40/2$

W=1.4 the width is too small therefore take W=d/4=40/4=10 mm

Strength of key due to shear failure= $L.W.\sigma_s.\frac{d}{2}$

=75*10*56*40/2=84000 N

Strength of shaft due to shear failure= $\frac{\pi}{16}.\sigma_{s1}.d^3$

$=\frac{\pi}{16}.*56*.40^3$ ***=703360 N***

$\therefore$ ***84000/703360=1.47 ans.***

Example 13.2. *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.} \right)$$

$$\therefore \quad l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $\frac{\text{Thickness of key } (t)}{2}$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Example 13.3. *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Solution. Given : P = 15 kW = 15 × 10³ W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ; τ = 56 MPa = 56 N/mm² ; σ_c = 112 MPa = 112 N/mm²

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$\therefore$ $w = 149 \times 10^3 / 84 \times 10^3 = 1.8$ mm

This width of keyway is too small. The width of keyway should be at least d / 4.

$\therefore$ $w = \frac{d}{4} = \frac{40}{4} = 10$ mm **Ans.**

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \quad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

∴ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\ 0.8125 = 571\,844 \text{ N}$$

ınd shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47$$ **Ans.**

Lecture 8

Shaft Coupling Design

A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator. Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Objectives

After studying this unit, the student should be able to

- describe the types of couplings
- calculate the strength couplings

Types of Shafts Couplings

1. *Rigid coupling*.

sleeve or Muff-coupling

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft.

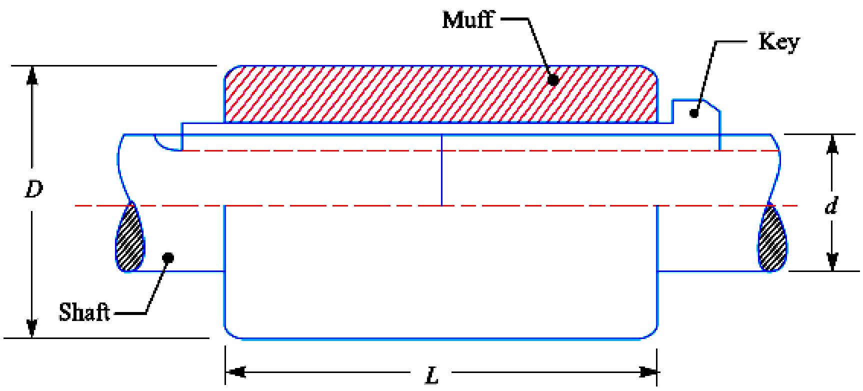


Fig. 13.10. Sleeve or muff coupling.

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. *Design for key*

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Clamp or Compression Coupling

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\,d$

where d = Diameter of the shaft.

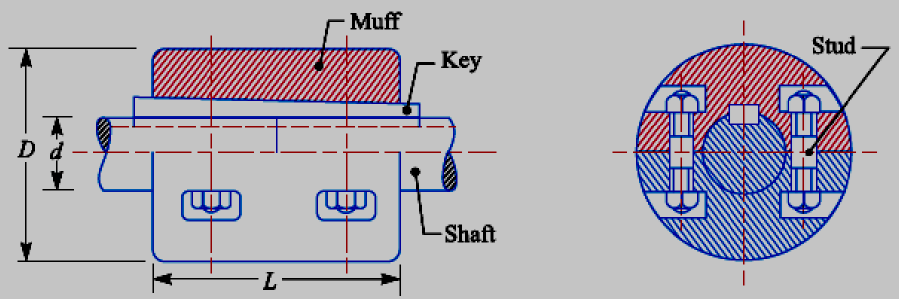


Fig. 13.11. Clamp or compression coupling.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling

2. *Design of clamping bolts*

Let T = Torque transmited by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

∴ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 13.5. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 100$ r.p.m. ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $n = 6$; $\sigma_t = 70$ MPa $= 70$ N/mm^2 ; $\mu = 0.3$

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3 \text{ or } d = 71.4 \text{ say } 75 \text{ mm}$$ **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm}$$ **Ans.**

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm}$$ **Ans.**

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22$ mm **Ans.**

Thickness of key, $t = 14$ mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

Shaft Coupling

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Flange Coupling

Rigid coupling.

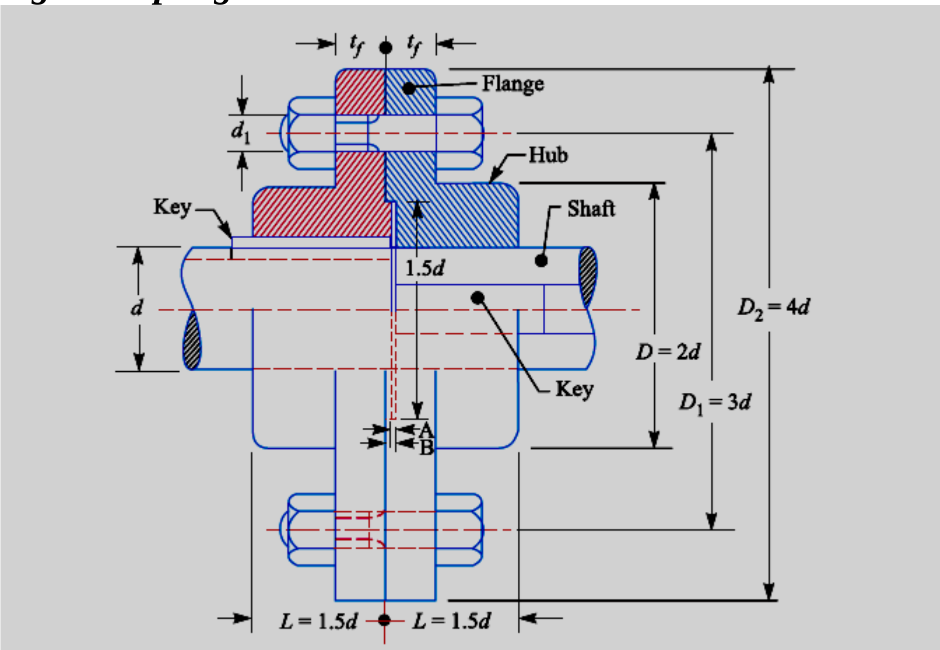


Fig. 13.12. Unprotected type flange coupling.

1. *Unprotected type flange coupling.*

The usual proportions for an unprotected type cast iron flange coupling as shown in the following Fig.

If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$$D = 2\,d$$

Length of hub,

$$L = 1.5\,d$$

Pitch circle diameter of bolts,

$$D_1 = 3d$$

Outside diameter of flange,

$$D_2 = D1 + (D1 - D) = 2\,D1 - D = 4\,d$$

Thickness of flange,

$$tf = 0.5\,d$$

Number of bolts = 3, for d upto 40 mm
= 4, for d upto 100 mm
= 6, for d upto 180 mm

2. Protected type flange coupling. In a protected type flange coupling, as shown in Fig. 13.13, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

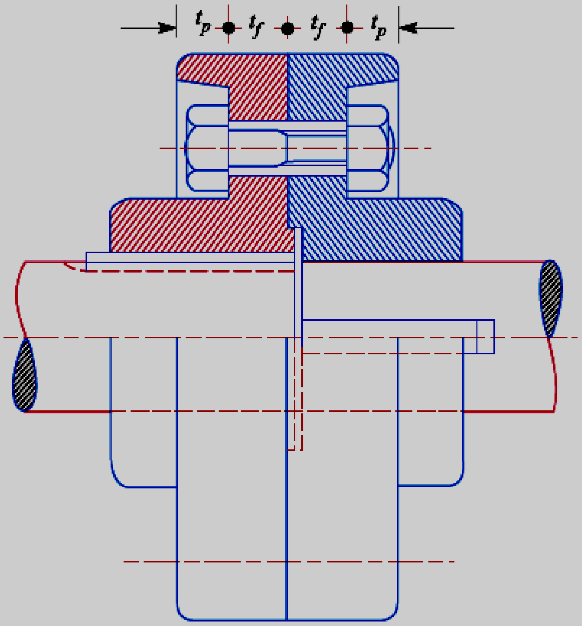


Fig. 13.13. Protective type flange coupling.

The thickness of the protective circumferential flange (t_p) is taken as 0.25 *d*. The other proportions of the coupling are same as for unprotected type flange coupling.

3. Marine type flange coupling. In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. 13.14. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft. The number of bolts may be chosen from the following table.

Table 13.2. Number of bolts for marine type flange coupling.
[According to IS : 3653 – 1966 (Reaffirmed 1990)]

Shaft diameter (mm)	35 to 55	56 to 150	151 to 230	231 to 390	Above 390
No. of bolts	4	6	8	10	12

The other proportions for the marine type flange coupling are taken as follows :
Thickness of flange = d / 3
Taper of bolt = 1 in 20 to 1 in 40
Pitch circle diameter of bolts, D_1 = 1.6 d
Outside diameter of flange, D_2 = 2.2 d

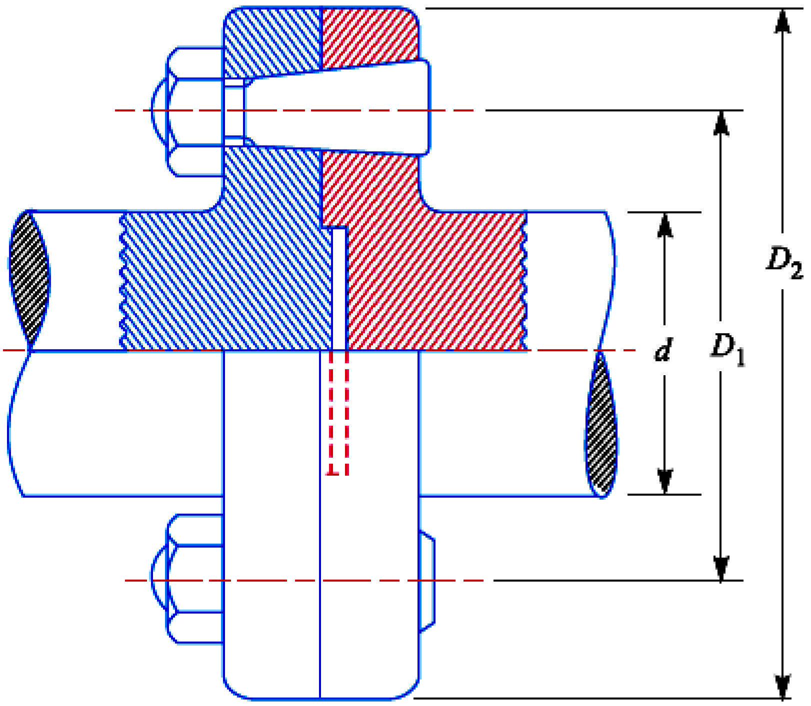


Fig. 13.14. Marine type flange coupling.

Design of Flange Coupling

Consider a flange coupling as shown in Fig. 13.12 and Fig. 13.13.
Let d = Diameter of shaft or inner diameter of hub,
D = Outer diameter of hub,
d_1 = Nominal or outside diameter of bolt,
D_1 = Diameter of bolt circle,
n = Number of bolts,
t_f = Thickness of flange,
τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively
τ_c = Allowable shear stress for the flange material i.e. cast iron,
σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.
The flange coupling is designed as discussed below :

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (*T*) as that of a solid shaft.

$$\therefore \qquad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (*L*) is taken as 1.5 *d*.

$T = \frac{\pi}{16}\tau\left\{\frac{D^4-d^4}{D}\right\}$ $\qquad$ $T = \frac{\pi}{16}\tau\{1-K^4\}$

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (*n*) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 ***d***. We know that

Load on each bolt $= \frac{\pi}{4}(d_1)^2\,\tau_b$

∴ Total load on all the bolts

$$= \frac{\pi}{4}(d_1)^2\,\tau_b \times n$$

and torque transmitted, $T = \frac{\pi}{4}(d_1)^2\,\tau_b \times n \times \frac{D_1}{2}$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f)\,\sigma_{cb}$$

∴ Torque, $T = (n \times d_1 \times t_f \times \sigma_{cb})\,\frac{D_1}{2}$

From this equation, the induced crushing stress in the bolts may be checked.

Example 1. ***Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :***

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupling

.

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 900$ r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k = 40$ MPa = 40 N/mm^2; $\sigma_{cb} = \sigma_{ck} = 80$ MPa = 80 N/mm^2 ; $\tau_c = 8$ MPa = 8 N/mm^2

The protective type flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3$ or $d = 30.1$ say 35 mm **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm} \quad \textbf{Ans.}$$

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3 / 63\,147 = 3.4$ N/mm^2 = 3.4 MPa

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, $w = 12$ mm **Ans.**

and thickness of key, $t = w = 12$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 52.5$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\ \tau_k$$

$\therefore$ $\tau_k = 215 \times 10^3/11\,025 = 19.5\ \text{N/mm}^2 = 19.5$ MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$\therefore$ $\sigma_{ck} = 215 \times 10^3/\ 5512.5 = 39\ \text{N/mm}^2 = 39$ MPa

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\,d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\ \tau_c$$

$\therefore \quad \tau_c = 215 \times 10^3/134\,713 = 1.6\ \text{N/mm}^2 = 1.6\ \text{MPa}$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$n = 3$

and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$\therefore \quad (d_1)^2 = 215 \times 10^3/4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$D_2 = 4\,d = 4 \times 35 = 140$ mm **Ans.**

Thickness of the protective circumferential flange,

$t_p = 0.25\,d = 0.25 \times 35 = 8.75$ say 10 mm **Ans.**

Example 2. Design a cast iron flange flange coupling (un protective type) to connect two shafts (8 cm dia.) runs at (250 r.p.m) and transmits a torque of (430 kg.m)

The following permissible stresses may be used:

Permissible shear stress for shaft , bolt, and key material =500 kg/cm^2

Permissible crushing stress for bolt and key material= 1500 kg/cm^2

Permissible shear stress for cast iron =80 kg/cm^2

(Draw a neat sketch of the coupling).

Solution:

Given Dia. of the shaft d=8 cm

Speed of the shaft N=250 r.p.m

Torque transmitted T= 43000 kg.cm

Permissible shear stress for shaft, bolt, and key material

$\sigma_s = 500\ kg/cm^2$

$\sigma_c = 15\,00\ kg/cm^2$

σ_s ***for cast iron (C.I)=80 kg/cm^2***

1- Design for hub

D=2d =2x8=16 cm

$$T = \frac{\pi}{16}\ \sigma_S\left(\frac{D^4 - d^4}{D}\right)$$

$$43000 = \frac{\pi}{16}\ \sigma_S\left(\frac{16^4-8^4}{16}\right)$$

σ_S***=57 kg/cm^2 (57<80 it's safe)***

Since the induced stress is less than (80 kg/cm^2), therefore it's safe

2- Design of key

from table (12) the proportion of key for (8 cm) dia. shaft are:

W=25mm=2.5cm

t=14 mm=1.4 cm

l=1.5d=1.5*8=12 cm

$$T = L.W.\sigma_S.\frac{d}{2}$$

$$43000 = 12x2.5x\sigma_S x\frac{8}{2}$$

43000=240 σ_S

σ_S ***=179.2 Kg/cm^2 (179.2 <500 Its safe)***

$$T = L.\frac{t}{2}\sigma_c.\frac{d}{2}$$

$43000 = 12.\frac{1.4}{2}\sigma_c.\frac{8}{2}=134.4\sigma_c$

$\sigma_c=320.8\approx321\ Kg/cm^2$

321<1500 its safe

3- Design for flange: L= 1.5d= 1.5x8= 12 cm.

The thickness of the flange is taken as:

$t_f=0.5\ d = 0.5x8= 4\ cm$

$T = \frac{\pi D^2}{2}\sigma_S.t_f$

$43000 = \frac{\pi 16^2}{2}\sigma_S x4$

$\sigma_S= 26.7\ kg/cm^2$

Since the induced shear stress in the flange is less than (80kg/cm²), therefore its safe

4-Design for bolts

Since the diameter of shaft is (8 cm) therefore No. of bolts =4

Pitch circle dia. = $D_1= 3d =3x8=24\ cm$

$T = \frac{\pi}{4}.d_1^2.\sigma_s.\frac{D_1}{2}.n$

$43000 = \frac{\pi}{4}.d_1^2\ x500x4x\frac{24}{2}\ =37680.d_1^2$

d= 1.06 cm therefore M x

from table ( ) the nearest standard size of bolt d_1*= M Ans.*

The outside diameter = $D_2= 4d=4x8= 32\ cm$ *Ans.*

Example 3:

A marine type flange coupling is used to transmit (3.75 MW) at (150 r.p.m). The allowable shear stress of the shaft and bolt may be taken as (50 N/mm^2). Determine the shaft diameter and diameter of the bolts. cm^2 (draw a neat sketch of the coupling). Solution:

Given horse power transmitted=P=3.75MW=3.75 x10^6 W

Speed= N=150 r.p.m

. σ_s=50 N/mm^2

Let d=dia. of the bolt

Using the relation:

$$T = \frac{Px60}{2\pi N}$$

$$T = \frac{3.75x10^6x60}{2\pi x150} = 0.24x10^6 \ N.m$$

$$= 0.24x10^9 \ N.mm$$

Torque transmitted:

$$T = \frac{\pi}{16} * \sigma_s * d^3$$

$$0.24x10^9 = \frac{\pi}{16} * 50 * d^3$$

d=290.2=290 mm

let d$_1$= dia. of the bolts

from table 13.2 (3a) we find the number of bolts for the shaft diameter of 300 mm

n=10 bolts

$$T = \frac{\pi}{4}.d_1^2.\sigma_s.n.\frac{D_1}{2}$$ ***D$_1$=1.6D= 1.6X290=646 mm***

$$0.24x10^9 = \frac{\pi}{4}.d_1^2x50x10x.\frac{646}{2} = 126777.5 \ .d_1^2$$

d=43.5≈44 mm from table the nearest number to 50.46 is M x

D$_2$=2d= 2x290 =580 mm and t$_f$= d/3= 290/3=96.7 mm ***Ans.***

Example :Design a protective type flange coupling for a M.S shaft transmitting

90 kW at 250rpm. The allowable shear stress in the shaft and the key is 60Mpa, and the allowable shear stress for coupling bolts is 30 Mpa. (Take permissible shear stress for hub (flange) =14 Mpa).

Solution:

1- Design for shaft

$$T = \frac{Px60}{2\pi N} = \frac{90*10^3*60}{2\pi*250} = 3439.5 = 3440*10^3 N - mm$$

Now according to strength criteria:

$$T = \frac{\pi}{16}\tau . d^3$$

$$3440*10^3 = \frac{\pi}{16}*60*d^3 = 11.8d^3 = 12d^3$$

$$d^3 = 28666.7$$

d= 65.9

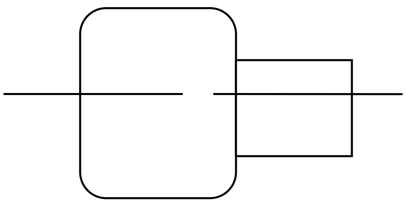
=66 mm

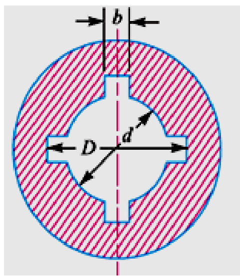
d=66 mm

2-Design for hub

We assume the hub as a hollow shaft subjected to shear:

Let: D=Diameter of hub





∴ D=2d=2*66=132 mm

And the length of hub:

L=1.5d=1.5*66=99≈100mm

$T = \frac{\pi}{16}\tau\left\{\frac{D^4-d^4}{D}\right\} = \frac{\pi}{16}*\left\{\frac{132^4-66^4}{132}\right\}\tau$***=2156220 τ***

τ ***=1.5 N/mm***^{2}

Shear stress in the hub<permissible shear stress for hub material

∴ hub is safe under shear

3- Design for key

Since d=66 mm =diameter of shaft

∴ from table (3) we select w=20 mm and t=12 mm

Now considering the key under shear stress failure

$$T = L.w.\tau\frac{d}{2}$$

$3440*10^3=100*20*\tau*66/2=66000\ \tau$

$\tau = 52.12\ N/mm^2$

Shear stress for key <permissible shear stress for key material

$52.13 < 60\ N/mm^2$

Now considering the key under crushing stress

$$T = \frac{t}{2}.L.\sigma_c.\frac{d}{2}$$

$3440*10^3=12/2*100*\sigma_c*66/2=19800\ \sigma_c*$

$\sigma_c=115.9\ N/mm^2$

Crushing stress for key<permissible crushing stress for key material

$115.9<120\ N/mm^2$

∴ key is safe under crushing stress

4- Design of flange

Let: $t_f=0.5d=0.5*75=37.5\approx 38$ ***mm***

And: $t_p=0.25d=0.25*75=18.75\approx 19$ ***mm***

$$T = \frac{\pi D^2}{2}\tau.t_f$$

$3440*10^3=\pi D^2/2*38*\tau$

τ=2.56 N/mm^2

shear stress for flange< permissible shear stress for flange material

2.56< 14

5- Design of bolt

Let: D_1=PCD for bolts

*D_1=3d=3*75=225 mm*

Considering bolts under shear:

Shearing area= $\frac{\pi}{4} * d_c^2 \tau \frac{D_1}{2}$

$T = \frac{\pi}{4} . d_c^2 . n * \tau$ *n=number of bolts*

Since the diameter of the shaft=75 mm

From table 3a n=6 bolts

$3440*10^3=\pi/4*d_c^2*6*30*225/2$

d_c= 14.7 mm from table 8 the standard size for the bolt is M18 coarse series

*and D_2=outer diameter=4d=4*75=300 mm Ans.*

Lecture -9

Chain Drives Design

The chains are made up of number of rigid links which are hinged together by pin joints in order to provide the necessary flexibility for wrapping round the driving and driven wheels. These wheels have projecting teeth of special profile and fit into the corresponding recesses in the links of the chain as shown in Fig .below. The chains are used for velocities up to 25 m / s and for power up to 110 kW. In some cases, higher power transmission is also possible.

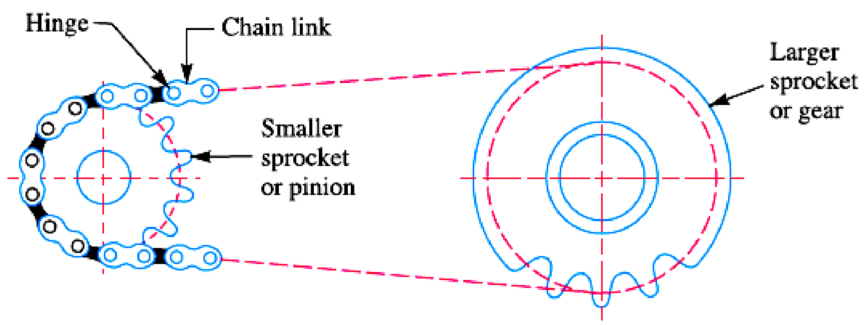


Fig. . Sprockets and chain.

Objectives

After studying this unit, the student should be able to:

- ***describe the types of chains***
- ***calculate the strength of chains***

Types Of Chains

The power transmitting chains are of the following three types.

1. Block or bush chain. A block or bush chain is shown in Fig. 21.6. This type of chain was used in the early stages of development in the power transmission.

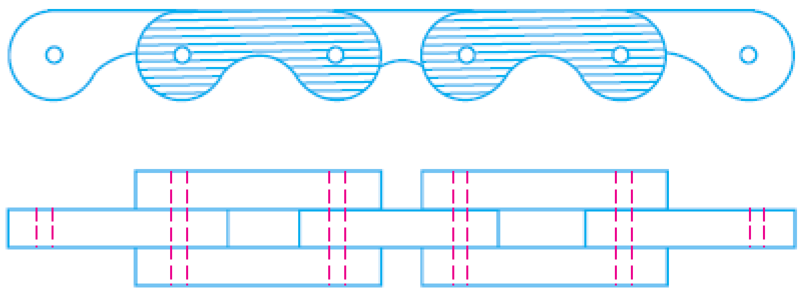


Fig. 21.6. Block or bush chain.

2. Bush roller chain. A bush roller chain as shown in Fig. 21.7, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the

sprocket wheel teeth against wear. The pins, bushes and rollers are made of alloy steel.

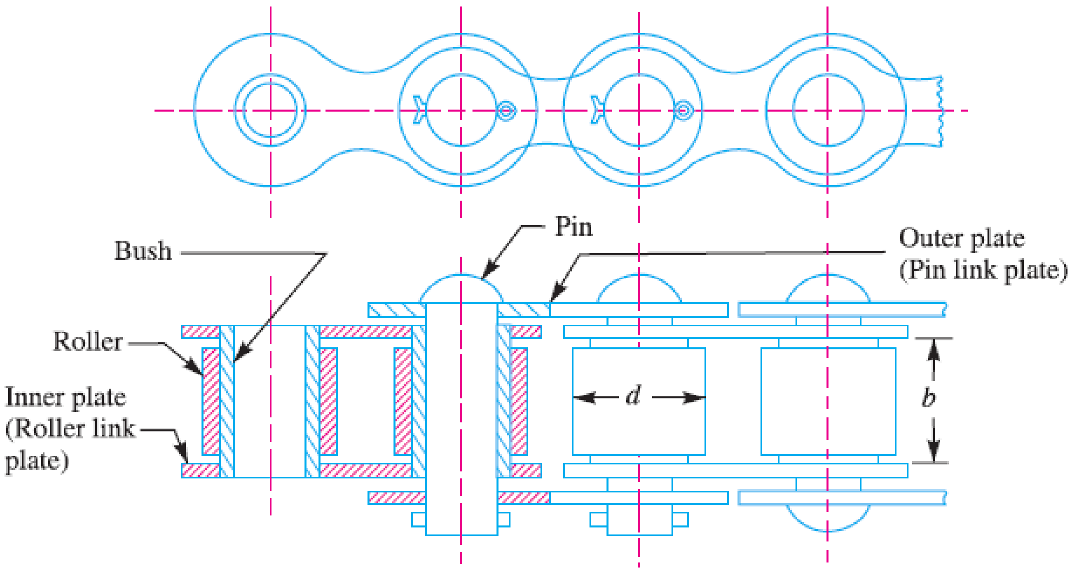


Fig. 21.7. Bush roller chain.

***3. Silent chain.** A silent chain (also known as inverted tooth chain) is shown in Fig. 21.9.*

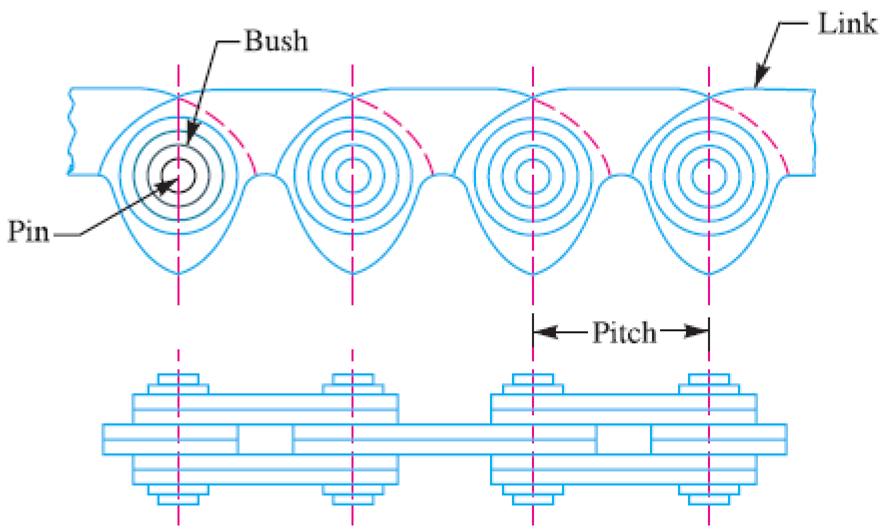
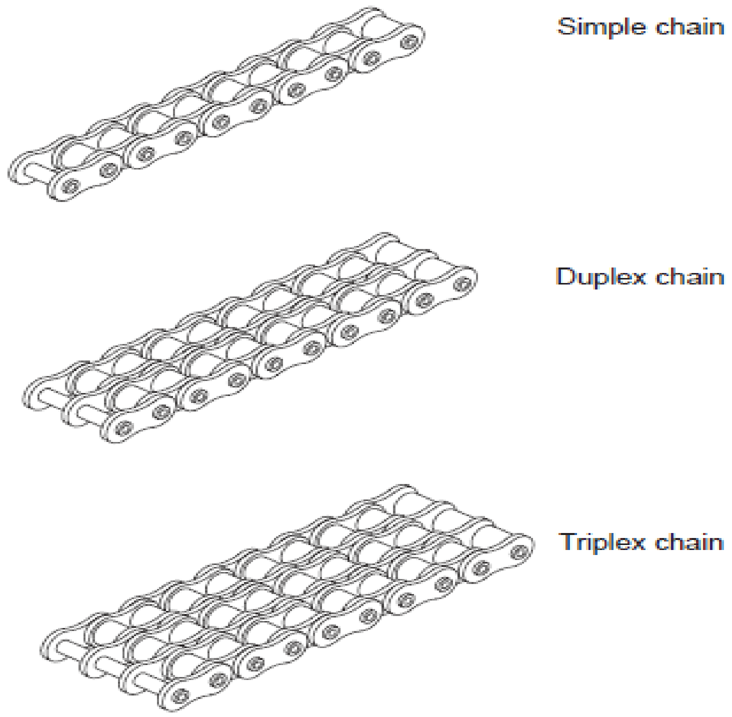


Fig. 21.9. Silent chain.



Terms Used in Chain Drive

The following terms are frequently used in chain drive.

1. Pitch of chain. It is the distance between the hinge centre of a link and the corresponding hinge centre of the adjacent link, as shown in Fig. 21.2. It is usually denoted by p.

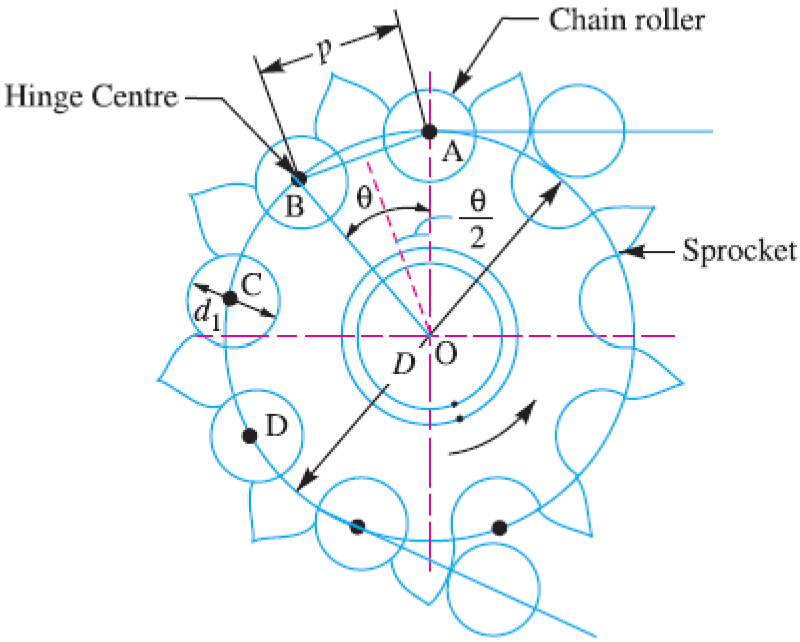


Fig. 21.2. Terms used in chain drive.

***2. Pitch circle diameter of chain sprocket.** It is the diameter of the circle on which the hinge centers of the chain lie, when the chain is wrapped round a sprocket as shown in Fig. 21.2. The points A, B, C, and D are the hinge centers of the chain and the circle drawn through these centers is called pitch circle and its diameter (D) is known as pitch circle diameter.*

Relation Between Pitch and Pitch Circle Diameter

Let D = Diameter of the pitch circle, and
T = Number of teeth on the sprocket.

$$\theta = \frac{360^o}{T}$$

$$D = P\ cosec\{\frac{180^o}{T}\}$$

The sprocket outside diameter (Do), for satisfactory operation is given by

$$Do = D + 0.8\ d1$$

where d_1 = Diameter of the chain roller.

Velocity Ratio of Chain Drives

The velocity ratio of a chain drive is given by

$$V.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

where N_1 = Speed of rotation of smaller sprocket in r.p.m.,
N_2 = Speed of rotation of larger sprocket in r.p.m.,
T_1 = Number of teeth on the smaller sprocket, and
T_2 = Number of teeth on the larger sprocket.

Length of Chain and Centre Distance

An open chain drive system connecting the two sprockets is shown in Fig. 21.3.

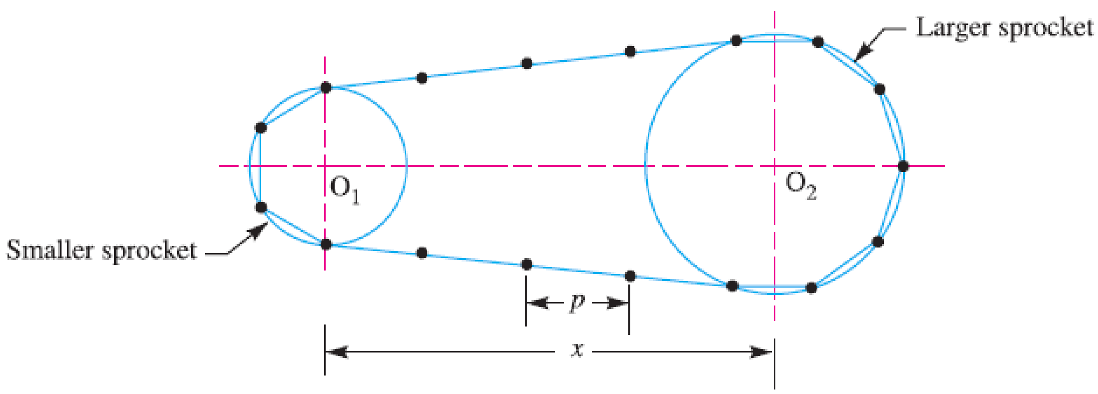


Fig. 21.3. Length of chain.

Let T_1 = Number of teeth on the smaller sprocket,
T_2 = Number of teeth on the larger sprocket,
p = Pitch of the chain, and
x = Centre distance.

The length of the chain (L) must be equal to the product of the number of chain links (K) and the pitch of the chain (p). Mathematically,

$$L = K.p$$

$$K = \frac{T_1+T_2}{2} + \frac{2x}{P} + [\frac{T_2-T_1}{2\pi}]^2 \frac{P}{x}$$

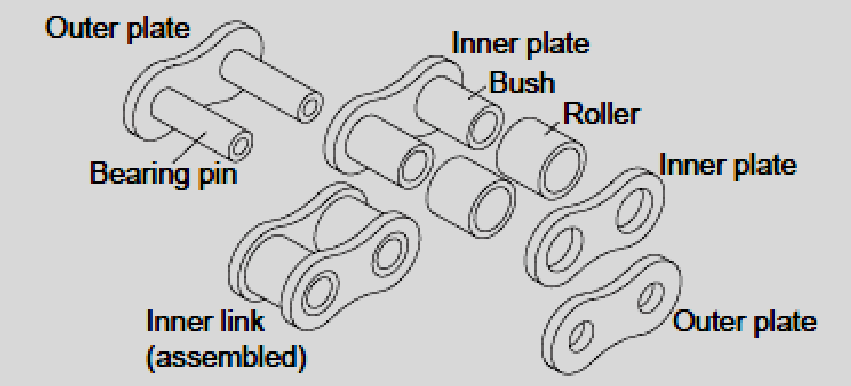


Figure 3 Roller chain components.

Design Procedure of Chain Drive:

The chain drive is designed as discussed below:

1. First of all, determine the velocity ratio of the chain drive.

$$V.R = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Where N_1 = *Speed of rotation of smaller sprocket in r.p.m.,*
N_2 = *Speed of rotation of larger sprocket in r.p.m.,*
T_1 = *Number of teeth on the smaller sprocket, and*
T_2 = *Number of teeth on the larger sprocket.*

2. Select the minimum number of teeth on the smaller sprocket or pinion from Table 21.5.

Table 21.5. Number of teeth on the smaller sprocket.

Type of chain	*Number of teeth at velocity ratio*					
	1	2	3	4	5	6
Roller	31	27	25	23	21	17
Silent	40	35	31	27	23	19

3. Find the number of teeth on the larger sprocket.

$$T_2 = T_1 . \frac{N_1}{N_2}$$

4. Determine the design power by using the service factor, such that
Design power = Rated power × Service factor (K_S)

The service factor (K_S) is the product of various factors, such as load factor (K_1), lubrication factor (K_2) and rating factor (K_3). The values of these factors are taken as follows:

1. Load factor (K_1) = 1, for constant load
= 1.25, for variable load with mild shock
= 1.5, for heavy shock loads

2. Lubrication factor (K_2) = 0.8, for continuous lubrication
= 1, for drop lubrication
= 1.5, for periodic lubrication

3. Rating factor (K_3) = 1, for 8 hours per day
= 1.25, for 16 hours per day
= 1.5, for continuous service

$$K_S = K_1 . K_2 . K_3 .$$

5. Choose the type of chain, number of strands for the design power and r.p.m. of the smaller sprocket from Table 21.4.

Table 21.4. Power rating (in kW) of simple roller chain.

Speed of smaller sprocket or pinion (r.p.m.)	*Power (kW)*				
	06 B	08 B	10 B	12 B	16 B
100	0.25	0.64	1.18	2.01	4.83
200	0.47	1.18	2.19	3.75	8.94
300	0.61	1.70	3.15	5.43	13.06
500	1.09	2.72	5.01	8.53	20.57
700	1.48	3.66	6.71	11.63	27.73
1000	2.03	5.09	8.97	15.65	34.89
1400	2.73	6.81	11.67	18.15	38.47
1800	3.44	8.10	13.03	19.85	–
2000	3.80	8.67	13.49	20.57	–

6. Note down the parameters of the chain, such as pitch, roller diameter, minimum width of roller etc. from Table 21.1.

Table 21.1. Characteristics of roller chains according to IS: 2403 — 1991.

ISO Chain number	Pitch (p) mm	Roller diameter (d_1) mm Maximum	Width between inner plates (b_1) mm Maximum	Transverse pitch (p_1)mm	Breaking load (kN) Minimum		
					Simple	Duplex	Triplex
05 B	8.00	5.00	3.00	5.64	4.4	7.8	11.1
06 B	9.525	6.35	5.72	10.24	8.9	16.9	24.9
08 B	12.70	8.51	7.75	13.92	17.8	31.1	44.5
10 B	15.875	10.16	9.65	16.59	22.2	44.5	66.7
12 B	19.05	12.07	11.68	19.46	28.9	57.8	86.7
16 B	25.4	15.88	17.02	31.88	42.3	84.5	126.8
20 B	31.75	19.05	19.56	36.45	64.5	129	193.5
24 B	38.10	25.40	25.40	48.36	97.9	195.7	293.6
28 B	44.45	27.94	30.99	59.56	129	258	387
32 B	50.80	29.21	30.99	68.55	169	338	507.10
40 B	63.50	39.37	38.10	72.29	262.4	524.9	787.3
48 B	76.20	48.26	45.72	91.21	400.3	800.7	1201

7. Find pitch circle diameters and pitch line velocity of the smaller sprocket.

Pitch circle diameter of smaller sprocket or pinion:

$$d_1 = P\ coces\left(\frac{180}{T_1}\right) \quad mm$$

$$d_2 = Pcosec\left(\frac{180}{T_2}\right) \quad mm$$

and pitch line velocity:

$$V_1 = \frac{\pi d_1 N_1}{60} \quad m/s$$

8. Determine the load (W) on the chain by using the following relation, i.e.

$$W = \frac{Rated\ power}{Pitch\ line\ velocity} \quad N$$

9. Calculate the factor of safety by dividing the breaking load (W_B) to the load on the chain (W). This value of factor of safety should be greater than the value given in Table 21.2.

$$Factor\ of\ safety = \frac{W_B}{W}$$

Table 21.2. Factor of safety (*n*) for bush roller and silent chains.

Type of chain	Pitch of chain (mm)	Speed of the sprocket pinion in r.p.m.								
		50	200	400	600	800	1000	1200	1600	2000
Bush roller chain	12 – 15	7	7.8	8.55	9.35	10.2	11	11.7	13.2	14.8
	20 – 25	7	8.2	9.35	10.3	11.7	12.9	14	16.3	–
	30 – 35	7	8.55	10.2	13.2	14.8	16.3	19.5	–	–
Silent chain	12.7 – 15.87	20	22.2	24.4	28.7	29.0	31.0	33.4	37.8	42.0
	19.05 – 25.4	20	23.4	26.7	30.0	33.4	36.8	40.0	46.5	53.5

***10.** Fix the centre distance between the sprockets.*

The minimum centre distance between the smaller and larger sprockets should be 30 to 50 times the pitch. Let us take it as 30 times the pitch.

∴ Centre distance between the sprockets, = 30 p

In order to accommodate initial sag in the chain, the value of centre distance is reduced by 2 to 5 mm.

***11.** Determine the length of the chain.*

$L = K\,.p \qquad mm$

We know that the number of chain links

$$K = \frac{T_1+T_2}{2} + \frac{2x}{P} + \left[\frac{T_2-T_1}{2\pi}\right]^2 \frac{P}{x}$$

***Example 21.1.** Design a chain drive to actuate a compressor from 15 kW electric motor running at 1000 r.p.m., the compressor speed being 350 r.p.m. The minimum centre distance is 500 mm. The compressor operates 16 hours per day. The chain tension may be adjusted by shifting the motor on slides.*

***Solution.** Given : Rated power = 15 kW ; N1 = 1000 r.p.m ; N2 = 350 r.p.m.*

We know that the velocity ratio of chain drive,

$$V.R = \frac{N_1}{N_2} = \frac{1000}{350} = 2.86\ say\ 3$$

From Table 21.5, we find that for the roller chain, the number of teeth on the smaller sprocket or pinion (T_1) for a velocity ratio of 3 are 25.

number of teeth on the larger sprocket or gear,

$$T_2 = T_1.\frac{N_1}{N_2} = 25x\frac{1000}{350} = 71.5\ say\ 72$$

We know that the design power
= Rated power × Service factor (K_S)
The service factor (K_S) is the product of various factors K_1, K_2 and K_3. The values of these
factors are taken as follows:
Load factor (K_1) for variable load with heavy shock
= 1.5
Lubrication factor (K_2) for drop lubrication
= 1
Rating factor (K_3) for 16 hours per day
= 1.25
∴Service factor, $K_S = K_1.K_2.K_3$ = 1.5 × 1 × 1.25 = 1.875
and design power = 15 × 1.875 = 28.125 kW
From Table 21.4, we find that corresponding to a pinion speed of 1000 r.p.m. the power transmitted for chain No. 12 is 15.65 kW per strand. Therefore, a chain No. 12 with two strands can be used to transmit the required power. From Table 21.1, we find that Pitch, p = 19.05 mm
Roller diameter, d = 12.07 mm
Minimum width of roller, w = 11.68 mm
Breaking load, WB = 59 kN = 59 × 10^3 N
We know that pitch circle diameter of the smaller sprocket or pinion,

$$d_1 = P\ coces\left(\frac{180}{T_1}\right) = 19.05\ coces\left(\frac{180}{25}\right)mm = 19.05x7.98 = 152mm = 0.152m$$

and pitch circle diameter of the larger sprocket or gear

$$d_2 = Pcosec\left(\frac{180}{T_2}\right) = 19.05cosec\left(\frac{180}{72}\right)mm = 19.05x22.9 = 436mm = 0.436m$$

Pitch line velocity of the smaller sprocket,

$$V_1 = \frac{\pi d_1 N_1}{60} = \frac{\pi x0.152x1000}{60} = 7.96\ m/s$$

∴ Load on the chain,

$$W = \frac{Rated\ power}{Pitch\ line\ velocity} = \frac{15}{7.96} = 1.844\ kN = 1844N$$

$$Factor\ of\ safety = \frac{W_B}{W} = \frac{59X10^3}{1844} = 32$$

This value is more than the value given in Table 21.2, which is equal to 11.
The minimum centre distance between the smaller and larger sprockets should be 30 to 50 times
the pitch. Let us take it as 30 times the pitch.
∴ Centre distance between the sprockets,
= 30 p = 30 × 19.05 = 572 mm
In order to accomodate initial sag in the chain, the value of centre distance is reduced by 2 to 5 mm.
∴ Correct centre distance
x = 572 – 4 = 568 mm

$$K = \frac{T_1+T_2}{2} + \frac{2x}{P} + [\frac{T_2-T_1}{2\pi}]^2 \frac{P}{x} = \frac{25+72}{2} + \frac{2x568}{19.05} + [\frac{72-25}{2\pi}]^2 \frac{19.05}{568x}$$

=48.5+59.6+1.9=110 links

∴ Length of the chain,
L = K.p = 110 × 19.05 = 2096 mm = 2.096 m Ans.

***Exercise 21.2** Design a roller chain to transmit power from a 20 kW motor to a reciprocating pump which operates with constant loading and continuous lubrication . The pump is to operate continuously 24 hours per day. The speed of the motor is 600 r.p.m. and that of the pump is 200 r.p.m. Find: 1. number of teeth on each sprocket; 2. pitch and width of the chain.*
Solution:
given data:
type of chain: roller chain
rated power: 20 kW
rating factor: continuously 24 hrs/day= K_3= 1.5 for continues service
N_1=600 r.p.m
N_2= 200 r.p.m
The chain drive is designed as discussed below:
1. Firt of all, determine the velocity ratio of the chain drive

$$V.R = \frac{N_1}{N_2} = \frac{600}{200} = 3$$

2.Select the minimum number of teeth on the smaller sprocket or pinion from table 21.5 the number of teeth for smaller sprocket for roller chain=T_1=25

3. The number of teeth on the larger sprocket:
T_2= T_1.N_1/N_2= 25x600/200=75

4. To find the design power:
Design power = Rated power x Service factor (K_s)
Ks= $K_1K_2K_3$

K_1= const. loading=1
K_2= Lubrication factor= continuos lubrication=0.8
K_3=Rating factor=K_3= Continuos service=1.5
∴ Design power= 1 x 0.8 x 1.5 x 20 = 24 kW
5. The type of chain, number of strand for the design power and r.p.m of the smaller sprocket from table 21.4:
since the smaller sprocket is 600 r.p.m and the nearest number in the table is 700 r.p.m and the power rated is 27.73 there fore the chain number is 16B

6. Now noting down the following parameters from table 21.1
P=25.4 mm
d_1=15.88 mm
b_1=17.02 mm
and the breaking load for simple chain type is =42.5 kN
and the breaking load = 126.8 kN answer.

Exercise 21.3

Design a chain drive to run a blower at 600 r.p.m. The power to the blower is available from a 8 kW motor at 1500 r.p.m. The centre distance is to be kept at 800 mm.(assume constant loading, drop lubrication, 16 Hrs/day)
solution:
given data:
rated power=8 kW
N_1= 1500 R.P.M
N_2= 600 R.P.M
K_1=1=Constant loading
K_2=1= drop lubrication
K_3= 1.25= 16 hrs/day

1- velocity ratio
V.R= N_1/N_2=1500/600=2.5=3

2- from table 21.5
T_1= 25

3-$T_2=T_1 . N_1/N_2$

=25 x 1500/600=63

4- design power= rated power x service factor(K_s)

design power= 8 x 1x 1 x 1.25 =10 kW

4-the type of chain from table 21.4
N_1= 1500 r.p.m is nearest number to 1400 r.p.m
the nearest number for power rating (8 kW) is 6.81 for chain type 08B

5- from table 21.1

P= 12.7 kW
d_1=8.51 mm
b_1= 7.75 mm
and breaking load for simple chain = 17.8 kW

7- $d_1 = P\ coces\left(\frac{180}{T_1}\right) = 12.7\ coces\left(\frac{180}{25}\right) = 12.7\ x\ cosec\ 7.2 = 12.7x\frac{1}{sin\ 7.2} = 100.33\ mm = 0.1033\ m$

$d_2 = P\ coces\left(\frac{180}{T_2}\right) = 12.7\ coces\left(\frac{180}{63}\right)$=255.02 mm= 0.255 m

and pitch line velocity for smaller sprocket

$V_1 = \frac{\pi.d_1.N_1}{60} = \frac{\pi\ x\ 0.1033\ x1500}{60}$=8.1 m/s

8- load W= rated power/pitch line velocity
= 8 x 1000/8.1= 987.65 N
9- F.O.S = W_B/W= 17.8 x 10^3/ 987= 18.03

10-the center distance

X=30P = 30 x 12.7 =381 mm
to accommodate the initial sag in the chain

381-4= 377 mm

L=K.P

$K = \frac{T_1+T_2}{2} + \frac{2x}{P} + [\frac{T_2-T_1}{2\pi}]^2\frac{P}{x} = \frac{25+63}{2} + \frac{2\ x\ 377}{12.7} + [\frac{63-25}{2\pi}]^2\frac{12.7}{377} =$ *129.7=130 links*

L= 130 x 12.7 =1651 mm= 1.651 m ***answer.***

Exercise 21.4

A chain drive using bush roller chain transmits 5.6 kW of power. The driving shaft on an electric motor runs at 1440 r.p.m. and velocity ratio is 5. The centre distance of the drive is restricted to 550 ± 2% mm and allowable pressure on the pivot joint is not to exceed 10 N/mm2. The drive is required to operate continuously with periodic lubrication and driven machine is such that load can be regarded as fairly constant with jerk and impact. Design the chain drive by calculating leading dimensions, number of teeth on the sprocket and specify the breaking strength of the chain. Assume a factor of safety of 13.
solution:

given data:
rated power= 5.6 kW
N_1=1440 r.p.m
V.R =5
X= 550 ±2%
Pressure= 10 N/mm^2

load factor=K_1=constant load=1
lubrication factor=K_2= periodic=1.5
rating factor= K_3= continuos service=1.5
type of chain= roller chain
f.o.s= 13
1- V.R = N_1/N_2
5= 1440/N_2
N_2= 288 r.p.m

2- number of teeth for smaller sprocket from table 21.5
T_1= 21

3- Number of larger sprocket:
$T_2 = T_1.N_1 / N_2$

= 21 x 1440/ 288
T_2= 105

4- design power= rated power x service factor(K_S)
= 5.6 x 1 x 1.5 x1.5= 12.6 kW

5-the type of chain from table 21.4
speed of smaller sprocket=1400 r.p.m
and nearest power 6.81 kW IS 6.81 kW
chain type = 08B

6- now noting down the parameters from table 21.1 for chain number 08B IS
P= 12.7 mm
d_1= 8.51 mm
b_1=7.75 mm
W_B= 17.8 for simple chain

7- the pitch circle dia. for smaller sprocket and pitch circle velocity

$$d_1 = P\ coces\left(\frac{180}{T_1}\right) = 12.7\ coces\left(\frac{180}{21}\right) = 85.23\ mm = 0.0852\ m$$

$$d_2 = P\ coces\left(\frac{180}{T_2}\right) = 12.7\ coces\left(\frac{180}{105}\right) = 424.749\ mm = 0.424\ m$$

$$V_1 = \frac{\pi.d_1.N_1}{60} = \frac{\pi.x\ 0.0852\ x\ 1440}{60} = 6.42\ m/s$$

8- W= rated power/ pitch line velocity

= 17.8/ 6.42= 2.77 N

W_B= 13/6.42=2.02 x 10^3 N

9- f.o.s= W_B/W

13= 17.8/ W
W= W_B/13= 17.8/13=1.36 kN

10- X=30P

=30 x 12.7=381 mm
381 -2=379 mm

L=K.P

$$K=\frac{T_1+T_2}{2}+\frac{2x}{P}+[\frac{T_2-T_1}{2\pi}]^2\frac{P}{x}=\frac{21+105}{2}+\frac{2x\ 379}{12.7}+[\frac{105-21}{2\pi}]^2\frac{12.7}{379}=$$

∴ L= mm= m answer

Lecture10

Flat Belt Drives

Introduction

The belts or *ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

Objectives

After studying this unit, the student should be able to:

- ***describe the types of belts***
- ***calculate the strength of belts***

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. Light drives. ***These are used to transmit small powers at belt speeds up to about 10 m/s as in agricultural machines and small machine tools.***

2. Medium drives. ***These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.***

3. Heavy drives. ***These are used to transmit large powers at belt speeds above 22 m/s as in compressors and generators.***

Types of Belts

1. *Flat belt*.   2. *V- belt*. 3. *Circular belt or rope*.

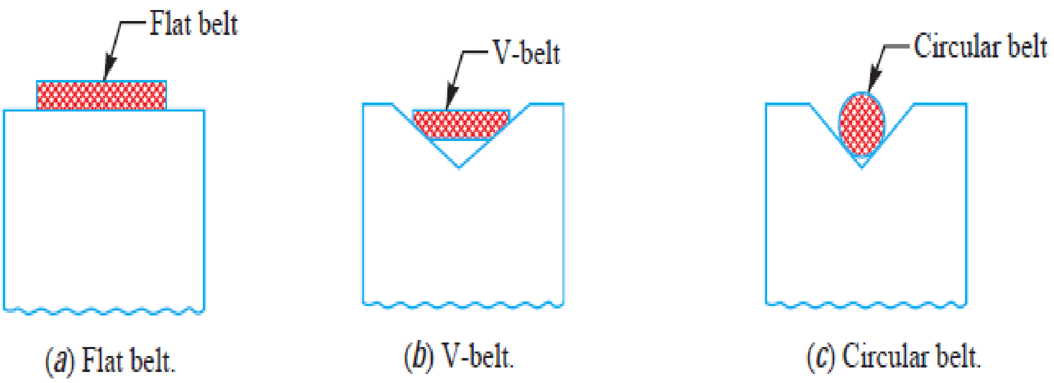


Fig. Types of belts

Material used for Belts

1. *Leather belts*.

2. *Cotton or fabric belts*.

3. *Rubber belt*.

4. *Balata belts*.

Types of Flat Belt Drives

1. ***Open belt drive.***

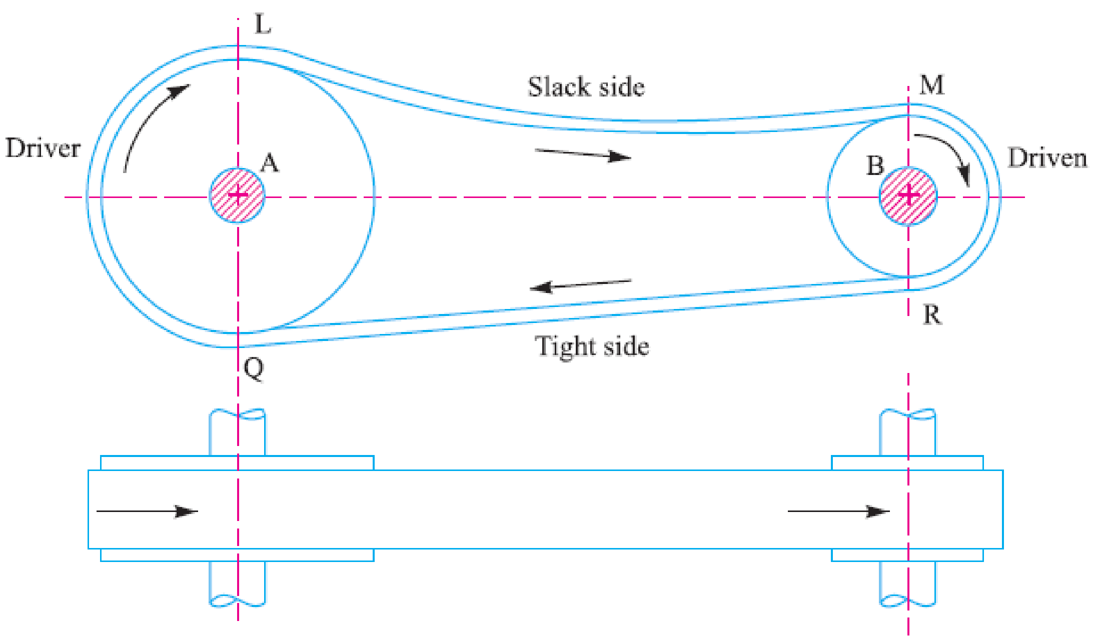


Fig. Open belt drive.

2. ***Crossed or twist belt drive.***

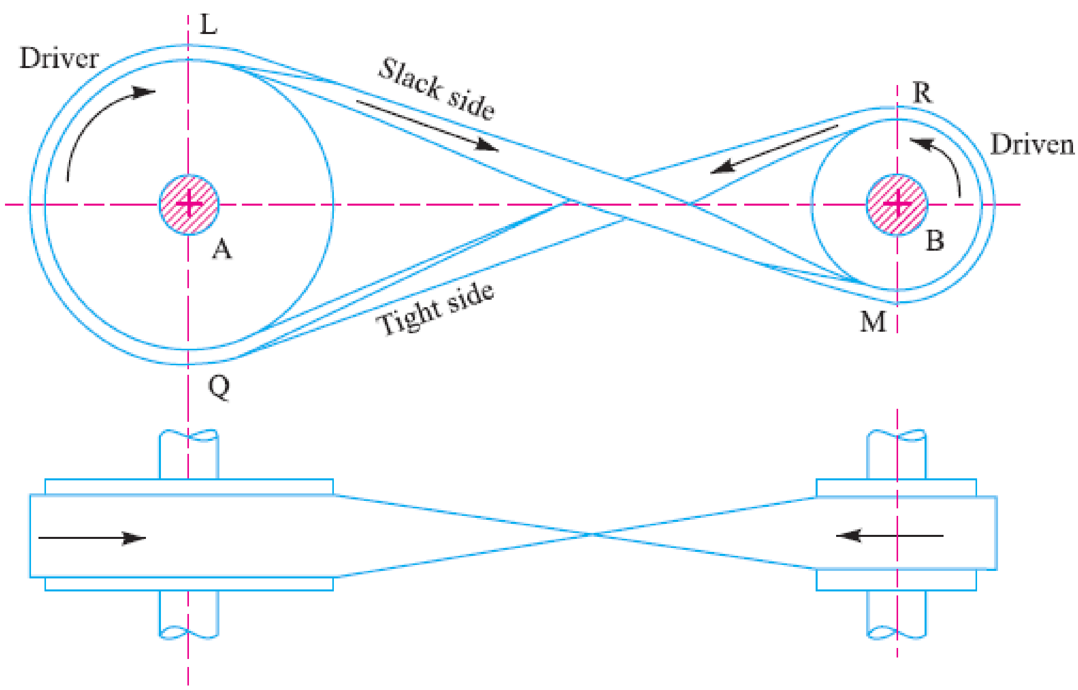
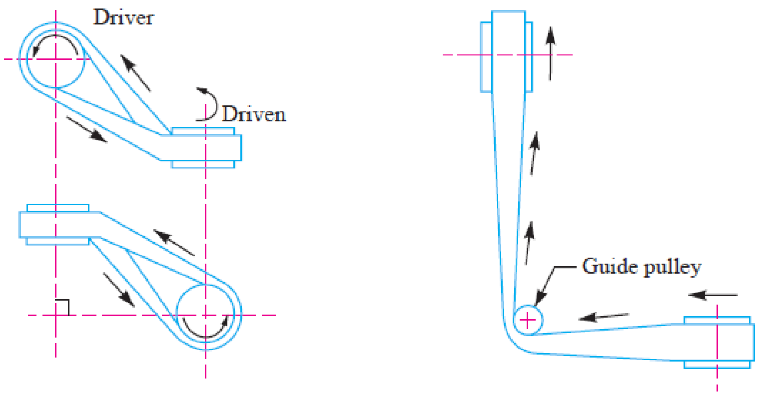


Fig. Crossed or twist belt drive.

3. *Quarter turn belt drive.*



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

4. *Belt drive with idler pulleys*.

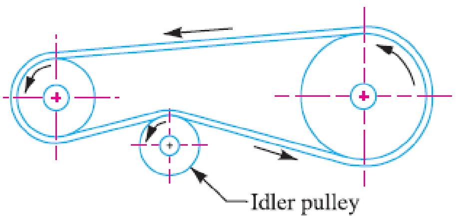


Fig. Belt drive with single idler pulley.

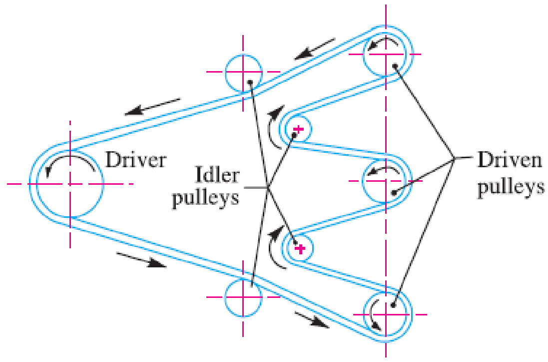


Fig. Belt drive with many idler pulleys.

5. *Compound belt drive.*

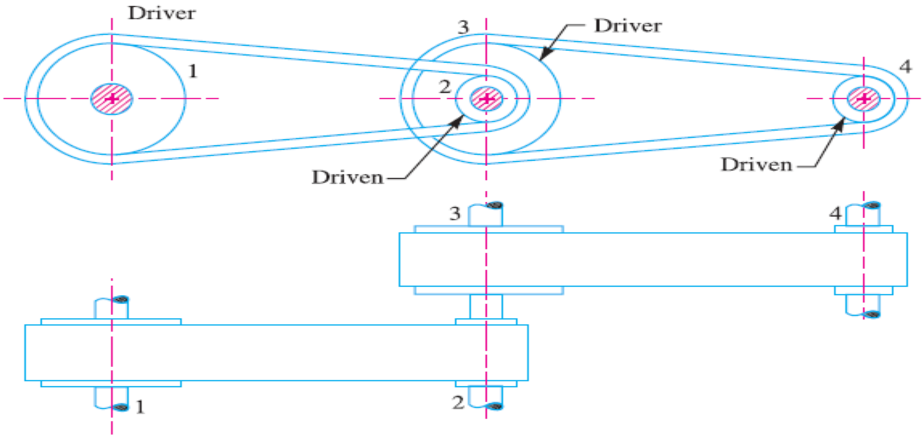


Fig. Compound belt drive.

6. ***Stepped or cone pulley drive.***

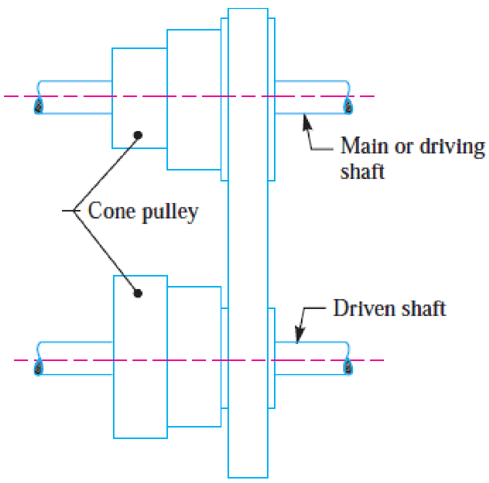


Fig. Stepped or cone pulley drive.

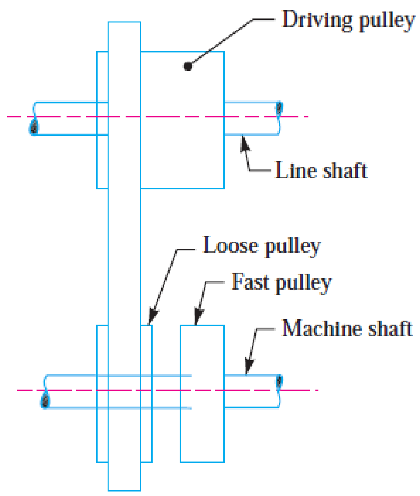


Fig. . Fast and loose pulley drive.

Velocity Ratio of a Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 ***= Diameter of the driver,***

d_2 ***= Diameter of the follower,***

N_1 ***= Speed of the driver in r.p.m.,***

N_2 ***= Speed of the follower in r.p.m.,***

$\therefore$ ***Length of the belt that passes over the driver, in one minute***

$= \pi d_1 N_1$

Similarly, length of the belt that passes over the follower, in one minute
$= \pi d_2 N_2$
Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore
$\pi d_1 N_1 = \pi d_2 N_2$

and velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. *The velocity ratio of a belt drive may also be obtained as discussed below:*
We know that the peripheral velocity of the belt on the driving pulley,

$$V_1 = \frac{\pi d_1 n_1}{60} \quad m/s$$

and peripheral velocity of the belt on the driven pulley,

$$V_2 = \frac{\pi d_2 n_2}{60}$$

When there is no slip, then $V_1 = V_2$

$$\frac{\pi d_1 n_1}{60} = \frac{\pi d_2 n_2}{60}$$

2. *In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by:*

$$\frac{N_4}{N_1} = \frac{d_1 d_3}{d_2 d_4}$$

$$\frac{\text{speed of last driven}}{\text{speed of first driven}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Slip of the Belt

In sometimes, the frictional grip becomes insufficient of belts and pulleys. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage. The result of the belt slipping is to reduce the velocity ratio of the system.

Let S_1 *% = Slip between the driver and the belt, and*
S_2 *% = Slip between the belt and follower,*

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

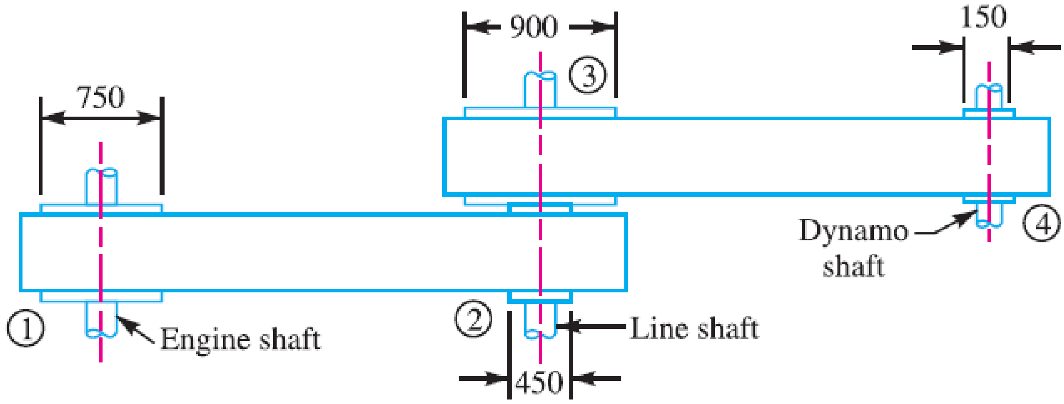
If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

Example 18.1. *An engine running at 150 r.p.m. drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft is 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley*

keyed to a dynamo shaft. Find the speed of dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Solution. ***Given :*** $N_1 = 150$ ***r.p.m. ;*** $d_1 = 750$ ***mm ;*** $d_2 = 450$ ***mm ;*** $d_3 = 900$ ***mm ;*** $d_4 = 150$ ***mm ;*** $s_1 = s_2 = 2\%$***. The arrangement of belt drive is shown in Fig.***



All dimensions in mm.

Fig.

Let N_4 ***= Speed of the dynamo shaft.***

1. When there is no slip

We know that:

$$\frac{N_4}{N_1} = \frac{d_1 d_3}{d_2 d_4}$$

$$\frac{N_4}{150} = \frac{750x900}{450x150} = 10$$

$N_4 = 150X10 = 1500$ ***r.p.m***

2. When there is a slip of 2% at each drive

$$\frac{N_4}{N_1} = \frac{d_1 * d_3}{d_2 * d4}\left(1 - \frac{S_1}{100}\right)(1 - \frac{S_2}{100})$$

$$\frac{N_4}{150} = \frac{750 * 900}{450 * 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right)$$

$N_4 = 9.6x150 = 1440$ ***r.p.m*** ***Ans.***

Length of an Open Belt Drive

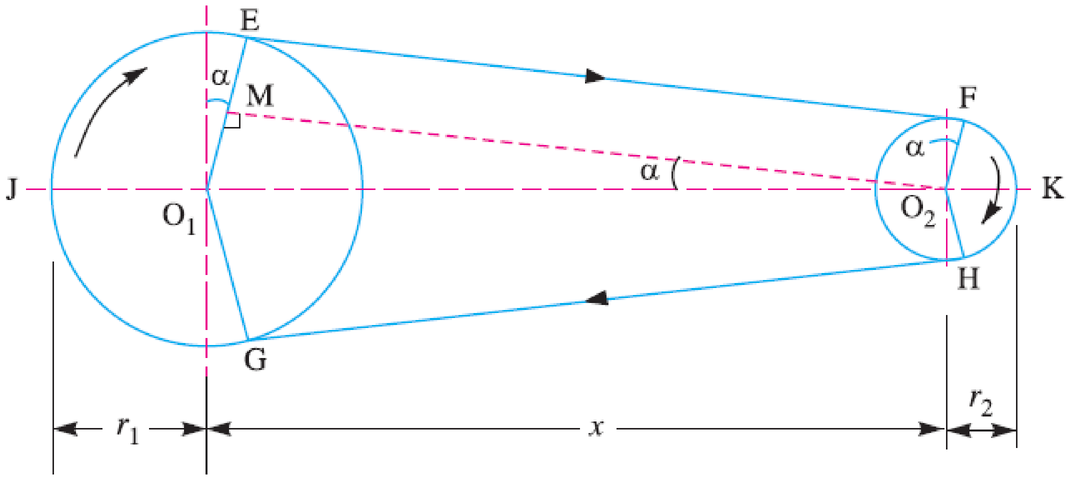


Fig. 1 Open belt drive.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,
x = Distance between the centers of two pulleys (i.e. O1O2), and
L = Total length of the belt.

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \text{ in terms of pulley radii}$$

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \text{ in term s of pulley diameter}$$

Length of a Cross Belt Drive

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \text{ in terms of pulley radii}$$

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \text{ in term s of pulley diameter}$$

Power Transmitted by a Belt

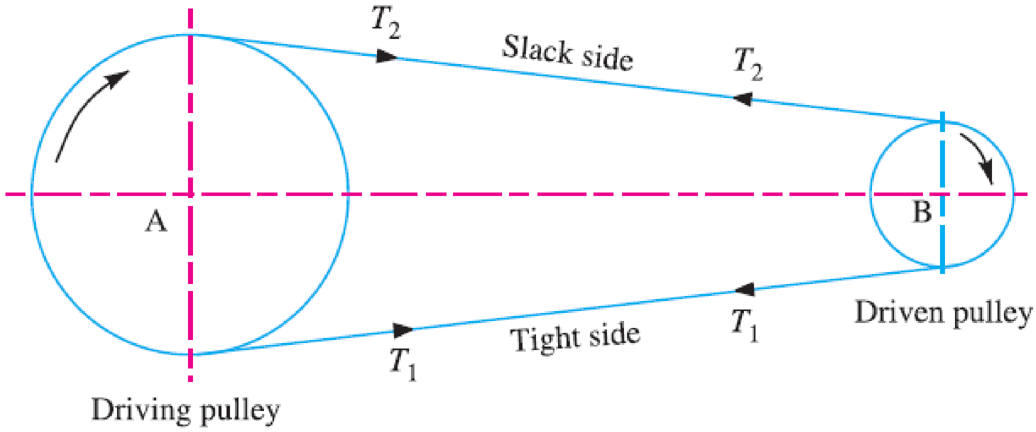


Fig. Power transmitted by a belt.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in Newton's, r_1 and r_2 = Radii of the driving and driven pulleys respectively in meters, and v = Velocity of the belt in m/s. The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (i.e. $T_1 - T_2$).

∴ Work done per second = $(T_1 - T_2)\ v$ N-m/s

and power transmitted = $(T_1 - T_2)\ v$ W ... (1 N-m/s = 1W)

A little consideration will show that torque exerted on the driving pulley is $(T_1 - T_2)\ r_1$.

Similarly, the torque exerted on the driven pulley is $(T_1 - T_2)\ r_2$.

Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. below.

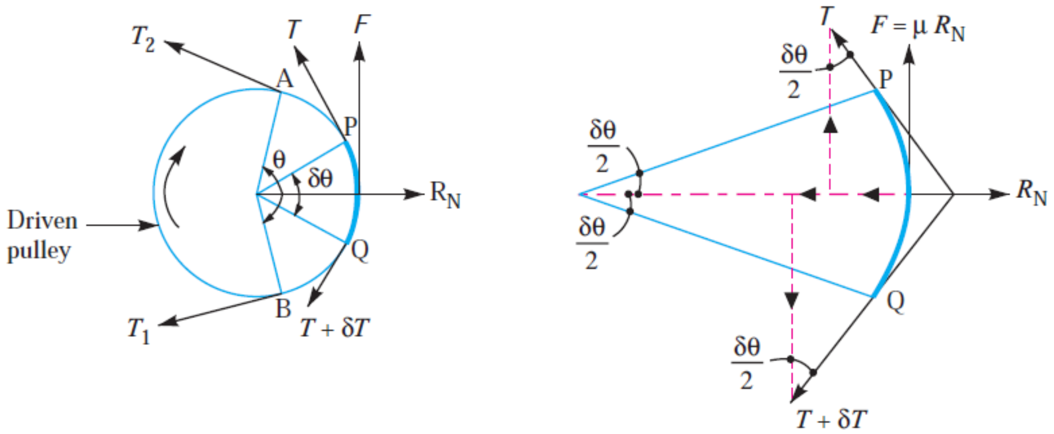


Fig. 18.16. Ratio of driving tensions for flat belt.

Let T_1 = Tension in the belt on the tight side,
T_2 = Tension in the belt on the slack side, and
(θ = Angle of contact in radians (i.e. angle subtended by the arc AB, along which the belt touches the pulley, at the centre).
Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$(at the centre of the pulley as shown in Fig. 18.16. The belt PQ is in equilibrium under the following forces:

1. Tension T in the belt at P,
2. Tension (T + δT) in the belt at Q,
3. Normal reaction RN, and
4. Frictional force F = $\mu \times$ RN, where μ is the coefficient of friction between the belt
and pulley.

$$2.3\, log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

Notes : 1. While determining the angle of contact, it must be remembered that it is the angle of contact at the smaller pulley, if both the pulleys are of the same material. We know that

$sin\alpha = \frac{r_1 - r_2}{x}$ ***for open belt drive***

$sin\alpha = \frac{r_1 + r_2}{x}$ ***for cross-belt drive***

$\theta = (180^o - 2\alpha)\frac{\pi}{180}$ ***rad*** ***for open belt drive***

$\theta = (180^o + 2\alpha)\frac{\pi}{180}$ ***rad*** ***for cross-belt drive***

2. When the pulleys are made of different material (i.e. when the coefficient of friction of the pulleys or the angle of contact are different), then the design will refer to the pulley for which $\mu.\theta$ is small.

Example 18.2. Two pulleys, one 450 mm diameter and the other 200 mm diameter, on parallel shafts 1.95 m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?

Solution.

Solution. *Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25*

The arrangement of crossed belt drive is shown in Fig.

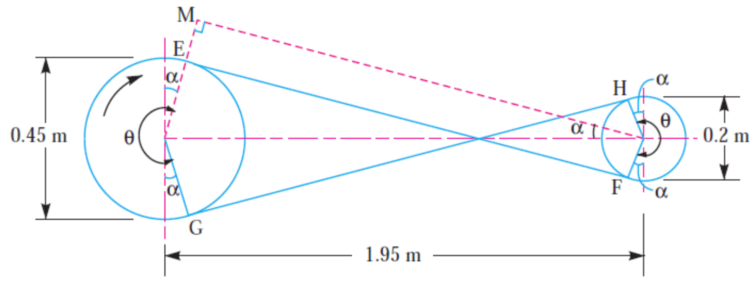


Fig. 18.17

Length of the belt

We know that length of the belt , $L = \pi(r_1 + r_2) + 2x + \frac{(r_1+r_2)^2}{x}$

$L = \pi(0.225 + 0.1) + 2x1.95 + \frac{(0.225+0.1)^2}{1.95}$ =4.974 m *ans.*

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$sin\alpha = \frac{r_1 + r_2}{x}$$

$$sin\alpha = \frac{0.225+0.1}{1.95} = 9.6^o$$

$$\theta = (180^o + 2\alpha)\frac{\pi}{180}$$

$$\theta = (180^o + 2x9.6) = 199.2^o$$

=199.2xπ/180=3.477

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$$2.3\ log\left(\frac{T_1}{T_2}\right) = \mu\theta = 0.25x3.477 = 0.8693$$

$log\left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378$ *or T_1/T_2=2.387* *(taking antilog of 0.378)*

$$T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419\ N$$

We know that the velocity of belt,

$$V_1 = \frac{\pi d_1 n_1}{60} = \frac{\pi x 0.45 x 200}{60} = 4.713\ m/s$$

P= $(T_1 - T_2)v$= (1000-419)4.713=2738 W=2.738 kW ***Ans.***

Example 8.2

A fan is belt driven by an electric motor running at 1500 rpm. The pulley diameters for the fan and motor are 500 and 355mm, respectively. A flat belt has been selected with a width of 100mm, thickness of 3.5mm, coefficient of friction of 0.8, density of 1100 kg/m3 and permissible stress of 11MN/m2.The centre distance is 1500mm. Determine the power capacity of the belt.

Solution

The arcs of contact for the driving and driven pulleys are:

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$= \pi - 2\sin^{-1}\frac{500 - 355}{2 \times 1500}$$

$$= 3.045 \text{ rad},$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D - d}{2C}$$

$$= \pi + 2\sin^{-1}\frac{500 - 355}{2 \times 1500}$$

$$= 3.238 \text{ rad}.$$

The maximum tension in the tight side is given as a function of the maximum permissible stress in the belt by:

$$F_1 = \sigma_{max} A = 11 \times 10^6 (3.5 \times 10^{-3} \times 100 \times 10^{-3}) = 3850 \text{ N}.$$

The belt velocity is

$$V = 1500 \times \frac{2\pi}{60} \times \frac{0.355}{2} = 27.88 \text{ m/s}.$$

The mass per unit length is

$$m = \rho A = 1100 \times 0.1 \times 0.0035 = 0.385 \text{ kg/m}.$$

The centrifugal load is given by

$$F_c = 0.385(27.88)^2 = 299.3 \text{ N}$$

Using Eq. 8.10

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} = \frac{3850 - 299.3}{F_2 - 299.3} = e^{0.8 \times 3.045}$$

$$F_2 = 610.0 \text{ N}.$$

The power capacity is given by $(F_1 - F_2)\ V = (3850 - 610)27.88 = 90.3$ kW.

Answer

Lecture11

Sliding Contact Bearings

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided.

Objectives

After studying this unit, you should be able to

- ***describe the types of sliding contact bearings***

- ***calculate the design procedure of sliding contact bearings***

Classification of Bearings

1. Depending upon the direction of load to be supported. The bearings under this group are classified as:

(a) Radial bearings, and (b) Thrust bearings.

In radial bearings, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 26.1 (a) and (b).

In thrust bearings, the load acts along the axis of rotation as shown in Fig. 26.1 (c).

Note : These bearings may move in either of the directions as shown in Fig. 26.1.

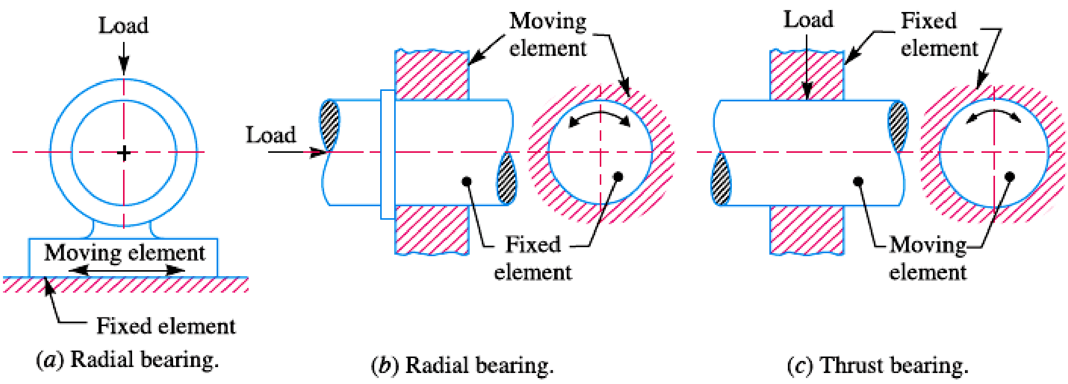


Fig. 26.1. Radial and thrust bearings.

2. Depending upon the nature of contact.

The bearings under this group are classified as :

(a) Sliding contact bearings, and (b) Rolling contact bearings.

In sliding contact bearings, as shown in Fig. 26.2 (a), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

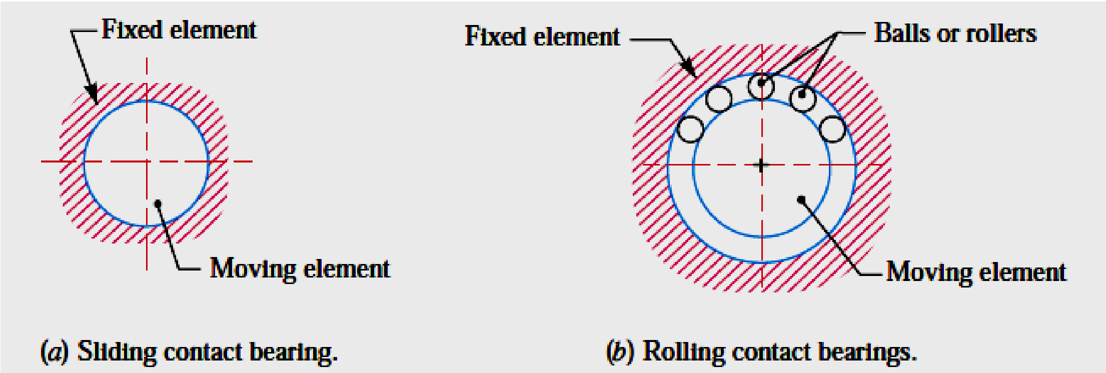


Fig. 26.2. Sliding and rolling contact bearings.

In rolling contact bearings, as shown in Fig. 26.2 (b), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 26.1 (a), may be called slipper or guide bearings. Such type of bearings are usually found in cross-head of steam engines.

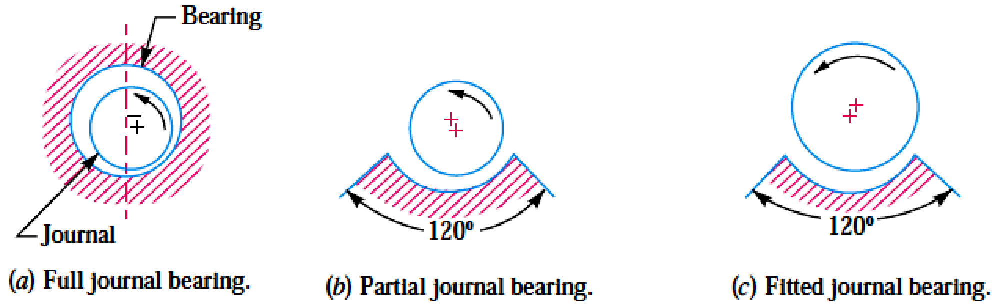


Fig. 26.3. Journal or sleeve bearings.

1. Thick film bearings. *The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.*

2. Thin film bearings. *The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other at least part of the time. Such type of bearings are also called boundary lubricated bearings.*

3. Zero film bearings. *The zero film bearings are those which operate without any lubricant present.*

4. Hydrostatic or externally pressurized lubricated bearings. *The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.*

Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (no rotating) bearing and a rotating journal.

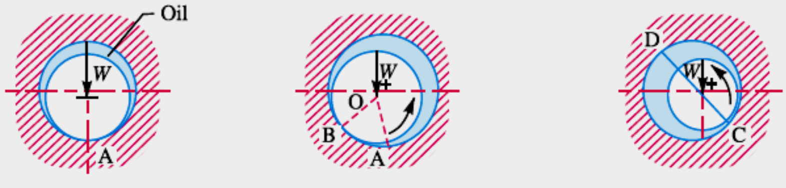


Fig. 26.4. Wedge film journal bearing.

Properties of Sliding Contact Bearing Materials

Table 26.1. Properties of metallic bearing materials.

Bearing material	*Fatigue strength*	*Comfor-mability*	*Embed-dability*	*Anti scoring*	*Corrosion resistance*	*Thermal conductivity*
Tin base babbit	Poor	Good	Excellent	Excellent	Excellent	Poor
Lead base babbit	Poor to fair	Good	Good	Good to excellent	Fair to good	Poor
Lead bronze	Fair	Poor	Poor	Poor	Good	Fair
Copper lead	Fair	Poor	Poor to fair	Poor to fair	Poor to fair	Fair to good
Aluminium	Good	Poor to fair	Poor	Good	Excellent	Fair
Silver	Excellent	Almost none	Poor	Poor	Excellent	Excellent
Silver lead deposited	Excellent	Excellent	Poor	Fair to good	Excellent	Excellent

Lubricants

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups :

1. Liquid, 2. Semi-liquid, and 3. Solid.

Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

1. *Determine the bearing length by choosing a ratio of l / d from Table 20*

Table 26.3. Design values for journal bearings.

Machinery	Bearing	Maximum bearing pressure (p) in N/mm^2	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm^2	$\frac{c}{d}$	$\frac{l}{d}$
Automobile and air-craft engines	Main Crank pin Wrist pin	5.6 – 12 10.5 – 24.5 16 – 35	0.007 0.008 0.008	2.1 1.4 1.12	—	0.8 – 1.8 0.7 – 1.4 1.5 – 2.2
Four stroke-Gas and oil engines	Main Crank pin Wrist pin	5 – 8.5 9.8 – 12.6 12.6 – 15.4	0.02 0.04 0.065	2.8 1.4 0.7	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Two stroke-Gas and oil engines	Main Crank pin Wrist pin	3.5 – 5.6 7 – 10.5 8.4 – 12.6	0.02 0.04 0.065	3.5 1.8 1.4	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Marine steam engines	Main Crank pin Wrist pin	3.5 4.2 10.5	0.03 0.04 0.05	2.8 2.1 1.4	0.001	0.7 – 1.5 0.7 – 1.2 1.2 – 1.7
Stationary, slow speed steam engines	Main Crank pin Wrist pin	2.8 10.5 12.6	0.06 0.08 0.06	2.8 0.84 0.7	0.001	1 – 2 0.9 – 1.3 1.2 – 1.5
Stationary, high speed steam engine	Main Crank pin Wrist pin	1.75 4.2 12.6	0.015 0.030 0.025	3.5 0.84 0.7	0.001	1.5 – 3 0.9 – 1.5 13 – 1.7
Reciprocating pumps and compressors	Main Crank pin Wrist pin	1.75 4.2 7.0	0.03 0.05 0.08	4.2 2.8 1.4	0.001	1 – 2.2 0.9 – 1.7 1.5 – 2.0
Steam locomotives	Driving axle Crank pin Wrist pin	3.85 14 28	0.10 0.04 0.03	4.2 0.7 0.7	0.001	1.6 – 1.8 0.7 – 1.1 0.8 – 1.3

Machinery	Bearing	Maximum bearing pressure (p) in N/mm^2	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm^2	$\frac{c}{d}$	$\frac{l}{d}$
Railway cars	Axle	3.5	0.1	7	0.001	1.8 – 2
Steam turbines	Main	0.7 – 2	0.002 – 0.016	14	0.001	1 – 2
Generators, motors, centrifugal pumps	Rotor	0.7 – 1.4	0.025	28	0.0013	1 – 2
Transmission shafts	Light, fixed Self -aligning Heavy	0.175 1.05 1.05	0.025- 0.060	7 2.1 2.1	0.001	2 – 3 2.5 – 4 2– 3
Machine tools	Main	2.1	0.04	0.14	0.001	1–4
Punching and shearing machines	Main Crank pin	28 56	0.10	—	0.001	1–2
Rolling Mills	Main	21	0.05	1.4	0.0015	1–1.5

2. Check the bearing pressure, p = W / l.d from Table 20 for probable satisfactory value.
3. Assume a lubricant from Table 21 and its operating temperature (t_0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum high temperature installations such as steam turbines.

Table 26.2. Absolute viscosity of commonly used lubricating oils.

S. No.	*Type of oil*	*Absolute viscosity in kg / m-s at temperature in °C*											
		30	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*90*
1.	SAE 10	0.05	0.036	0.027	0.0245	0.021	0.017	0.014	0.012	0.011	0.009	0.008	0.005
2.	SAE 20	0.069	0.055	0.042	0.034	0.027	0.023	0.020	0.017	0.014	0.011	0.010	0.0075
3.	SAE 30	0.13	0.10	0.078	0.057	0.048	0.040	0.034	0.027	0.022	0.019	0.016	0.010
4.	SAE 40	0.21	0.17	0.12	0.096	0.78	0.06	0.046	0.04	0.034	0.027	0.022	0.013
5.	SAE 50	0.30	0.25	0.20	0.17	0.12	0.09	0.076	0.06	0.05	0.038	0.034	0.020
6.	SAE 60	0.45	0.32	0.27	0.20	0.16	0.12	0.09	0.072	0.057	0.046	0.040	0.025
7.	SAE 70	1.0	0.69	0.45	0.31	0.21	0.165	0.12	0.087	0.067	0.052	0.043	0.033

Note : We see from the above table that the viscosity of oil decreases when its temperature increases.

4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table 26.3, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c / d from Table 20.
Diametral clearance ratio. It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= \frac{c}{d} = \frac{D-d}{d}$$

6. Determine the coefficient of friction (μ) .
In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{C}\right) + K$$

... (when Z is in kg / m-s and p is in N / mm²)
where Z, N, p, d and c have usual meanings as discussed in previous article, and
k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l / d).
= 0.002 for l / d ratios of 0.75 to 2.8.
The operating values of ZN / p should be compared with values given in Table 26.3 to ensure safe margin between operating conditions and the point of film break down.
7. Determine the heat generated by using the relation as :
The heat generated in a bearing is due to the fluid friction and friction of the parts having
relative motion. Mathematically, heat generated in a bearing,
$Q_g = \mu.W.V$ N-m/s or J/s or watts ...(i)
where μ = Coefficient of friction,
W = Load on the bearing in N,

= Pressure on the bearing in N/mm² × Projected area of the bearing in mm² = p (l × d),
V = Rubbing velocity in m/s = $V = \frac{\pi.D.N}{60}$
where , d: diameter is in metres, and
N = Speed of the journal in r.p.m.
8. Determine the heat dissipated by using the relation as

Heat dissipated by the bearing,
$Q_d = C.A\ (t_b - t_a)$ *J/s or W ... (Q 1 J/s = 1 W) ...(ii)*
where
C = Heat dissipation coefficient in W/m2/°C,
A = Projected area of the bearing in m2 = l × d,
t_b *= Temperature of the bearing surface in °C, and*
t_a *= Temperature of the surrounding air in °C.*
The value of C have been determined experimentally by O. Lasche. The values depend upon the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m²/°C), for journal bearings may be taken as follows :
For unventilated bearings (Still air)
= 140 to 420 W/m²/°C
For well ventilated bearings
= 490 to 1400 W/m²/°C
It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t_0) and the temperature of the outside air (t_a). In other words,
$t_b - t_a = \frac{1}{2}(t_o - t_a)$
Notes : 1. For well designed bearing, the temperature of the oil film should not be more than 60°C, otherwise the viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temperature of the oil film is often called as the operating temperature of the bearing.
2. In case the temperature of the oil film is higher, then the bearing is cooled by circulating water through coils built in the bearing.
3. The mass of the oil to remove the heat generated at the bearing may be obtained by equating the heat generated to the heat taken away by the oil. We know that the heat taken away by the oil,
Qt = m.S.t J/s or watts
where
m = Mass of the oil in kg / s,
S = Specific heat of the oil. Its value may be taken as 1840 to 2100 J / kg / °C,
t = Difference between outlet and inlet temperature of the oil in °C.
9. Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.
Example 26.1. Design a journal bearing for a centrifugal pump from the following data :

Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; diameter of journal 100 mm, Type of oil is SAE 10, for which the absolute viscosity at 55°C = 0.017 kg / m-s; Ambient temperature of oil = 15.5°C ; Maximum bearing pressure for the pump = 1.5 N / mm^2.

Solution. *Given : W = 20 000 N ; N = 900 r.p.m. ; t_0 = 55°C ; Z = 0.017 kg/m-s ; t_a = 15.5°C ; d=100 mm ,p = 1.5 N/mm2 ; t = 10°C ; C = 1232 W/m2/°C*

The journal bearing is designed as discussed in the following steps :

1. First of all, let us find the length of the journal (l). From Table 26.3, we find that the ratio of l / d for centrifugal pumps varies from 1 to 2.

Let us take l / d = 1.6.

*∴ l = 1.6 d = 1.6 × 100 = 160 mm **Ans.***

2. We know that bearing pressure,

$P = \frac{w}{l.d} = \frac{20\,000}{160x100} = 1.25$ N/mm^2

Since the given bearing pressure for the pump is 1.5 N/mm^2, therefore the above value of p is safe and hence the dimensions of l and d are safe.

3. $\frac{ZN}{P} = \frac{0.017X900}{1.25} = 12.24$

From Table 26.3, we find that the operating value of

$$\frac{ZN}{p} = 28$$

the minimum value of the bearing modulus at which the oil film will break is given by:

$$k = \frac{1}{3}\left(\frac{ZN}{P}\right) = \frac{1}{3}x28 = 9.33$$

Since the calculated value of bearing characteristic number $\frac{ZN}{p} = 12.24$ is more than 9.33 therefore the bearing will operate under hydrodynamic conditions.

4. From Table 26.3, we find that for centrifugal pumps, the clearance ratio (c/d) = 0.0013

5. We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{C}\right) + K$$

$\mu = \frac{33}{10^8}(12.24)\left(\frac{1}{0.0013}\right) + 0.002$=0.0051

6. Heat generated

Qg = µ.W.V= $\mu.w.\frac{\pi.D.N}{60} = 0.0051x20000\left(\frac{\pi x0.1x900}{60}\right) = 480.7\ W$

7. Heat dissipated

$Q_d = C.A\ (t_b - t_a) = C.l.d\ (t_b - t_a)$

t_b-t_a=1/2(t_o-t_a)=1/2(55^o-15.5^o)=19.75^{o}C

Q_d=1232x0.1x19.75=389.3 W

*We see that the **heat generated is greater than the heat dissipated which indicates that the bearing is warming up**. Therefore, either the bearing should be redesigned by taking t0 = 63°C or the bearing should be cooled artificially.*
We know that the amount of artificial cooling required

= Heat generated – Heat dissipated =Q_g-Q_d=480.7-389.3=91.4 W

Mass of lubricating oil required for artificial cooling
Let m = Mass of the lubricating oil required for artificial cooling in kg / s.
We know that the heat taken away by the oil,

Q_t= m.s.t=m.x1900x10=19000m W

... [Q Specific heat of oil (S) = 1840 to 2100 J/kg/°C]

Equating this to the amount of artificial cooling required, we have
19 000 m = 91.4
4 m = 91.4 / 19 000 = 0.0048 kg / s = 0.288 kg / min Ans.

***Example 26.2.** The load on the journal bearing is 150 kN due to turbine shaft of 300 mm diameter running at 1800 r.p.m. Determine the following:*
1. Length of the bearing if the allowable bearing pressure is 1.6 N/mm^2, and
2. Amount of heat to be removed by the
lubricant per minute if the bearing temperature is 60°C and viscosity of the oil at 60°C is 0.02 kg/m-s and the bearing clearance is 0.25 mm.
***Solution.** Given : W = 150 kN = 150 × 103 N ;*
d = 300 mm = 0.3 m ; N = 1800 r.p.m. ;
p = 1.6 N/mm^2 ; Z = 0.02 kg / m-s ; c = 0.25 mm
1. Length of the bearing
Let l = Length of the bearing in mm.
We know that projected bearing area,
A = l × d = l × 300 = 300 l mm^2
and allowable bearing pressure (p),

1.6=W/A= 150x10^3/300l=500/l

l = 500/1.6 = 312.5 mm Ans.

2. Amount of heat to be removed by the lubricant
We know that coefficient of friction for the bearing,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{C}\right) + K$$

$$\mu = \frac{33}{10^8}\left(\frac{0.02x1800}{1.6}\right)\left(\frac{300}{0.25}\right) + 0.002$$

=0.009+0.002=0.011

$V = \frac{\pi.D.N}{60} = \frac{\pi x 0.3x1800}{60}$ = *28.3 m/s*

Amount of heat to be removed by the lubricant,

Qg = μ.W.V = 0.011 × 150 × 103 × 28.3 = 46 695 J/s or W
= 46.695 kW Ans.

Example 26.3. ***A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm2. The speed of the journal is 900 r.p.m. and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m-s. The room temperature is 35°C. Find :***
1. The amount of artificial cooling required, and 2. The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J / kg / °C.
Solution. *Given : d = 50 mm = 0.05 m ; l = 100 mm = 0.1 m; p = 1.4 N/mm2 ; N = 900 r.p.m. ; d / c = 1000 ; Z = 0.011 kg / m-s ; t0 = 75°C ; ta = 35°C ; t = 10°C ; S = 1850 J/kg / °C*
1. Amount of artificial cooling required
We know that the coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{C}\right) + K = \frac{33}{10^8}\left(\frac{0.011x900}{1.4}\right)(1000) + 0.002$$

= 0.002 33 + 0.002 = 0.004 33

Load on the bearing,

W = p × d.l = 1.4 × 50 × 100 = 7000 N

and rubbing velocity,

$V = \frac{\pi.D.N}{60} = \frac{\pi x 0.05x900}{60}$ = *2.36 m/s*
∴ Heat generated,
Qg = μ.W.V = 0.004 33 × 7000 × 2.36 = 71.5 J/s
Let t_b = Temperature of the bearing surface.

We know that

$t_b - t_a = \frac{1}{2}(t_o - t_a) = \frac{1}{2}(75 - 35) = 20^oC$

Since the value of heat dissipation coefficient (C) for unventilated bearing varies from 140 to 420 W/m2/°C, therefore let us take C = 280 W/m2 /° C
We know that heat dissipated,
$Q_d = C.A\ (t_b - t_a) = C.l.d\ (t_b - t_a)$

= 280 × 0.05 × 0.1 × 20 = 28 W = 28 J/s

∴ Amount of artificial cooling required
= Heat generated – Heat dissipated = $Q_g - Q_d$
= 71.5 – 28 = 43.5 J/s or W Ans.

2. Mass of the lubricating oil required
Let m = Mass of the lubricating oil required in kg / s.
We know that heat taken away by the oil,
Q_t = m.S.t = m × 1850 × 10 = 18 500 m J/s
Since the heat generated at the bearing is taken away by the lubricating oil, therefore equating
Q_g = Qt or 71.5 = 18 500 m
∴ m = 71.5 / 18 500 = 0.003 86 kg / s = 0.23 kg / min Ans.

Example 26.4. A 150 mm diameter shaft supporting a load of 10 kN has a speed of 1500 r.p.m. The shaft runs in a bearing whose length is 1.5 times the shaft diameter. If the diametral clearance of the bearing is 0.15 mm and the absolute viscosity of the oil at the operating temperature is 0.011 kg/m-s, find the power wasted in friction.
Solution. Given : d = 150 mm = 0.15 m ; W = 10 kN = 10 000 N ; N = 1500 r.p.m. ; l = 1.5 d ; c = 0.15 mm ; Z = 0.011 kg/m-s
We know that length of bearing,
l = 1.5 d = 1.5 × 150 = 225 mm
∴ Bearing pressure,
P=W/A=
=W/L.d
=10 000/225x150
=0.296 N/mm^2

We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{C}\right) + K$$

$$\mu = \frac{33}{10^8}\left(\frac{0.011x1500}{0.296}\right)\left(\frac{150}{0.15}\right) + 0.002$$

=0.018+0.002=0.002

$$V = \frac{\pi.D.N}{60}$$

$$V = \frac{\pi.x0.15x1500}{60}$$

=11.78 m/s

We know that heat generated due to friction,

$Qg = \mu.W.V = 0.02 \times 10\,000 \times 11.78 = 2356\ W$

∴ ***Power wasted in friction***

$= Qg = 2356\ W = 2.356\ kW$ ***Ans.***

Lecture 12

Rolling Contact Bearings

Introduction

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called antifriction bearings.

Objectives

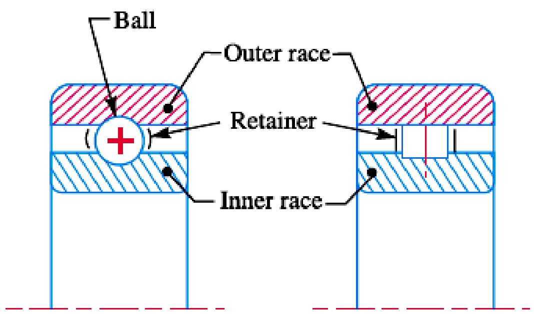
After studying this unit, you should be able to

- ***describe the types of rolling bearings***
- ***calculate the design procedure of rolling bearings***

Types of Rolling Contact Bearings

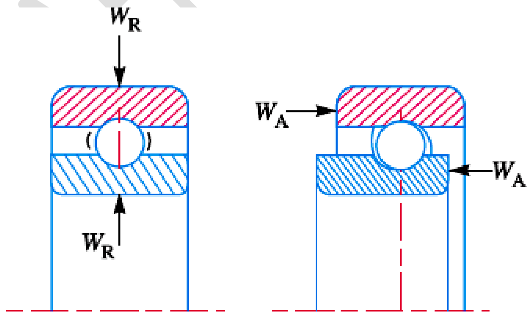
Following are the two types of rolling contact bearings:

1. Ball bearings; and 2. Roller bearings.



(*a*) Ball bearing. (*b*) Roller bearing.

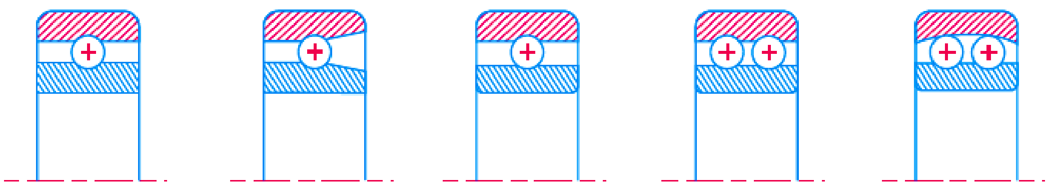
Ball and roller bearings.



(*a*) Radial ball bearing. (*b*) Thrust ball bearing.

Radial and thrust ball bearings.

Types of Radial Ball Bearings



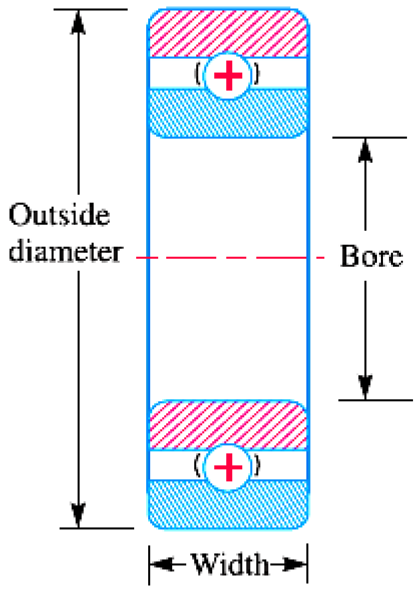
(*a*) Single row deep groove. (*b*) Filling notch. (*c*) Angular contact. (*d*) Double row. (*e*) Self-aligning.

Types of radial ball bearings.

Standard Dimensions and Designations of Ball Bearings

The dimensions that have been standardized on an international basis are shown in Fig. 27.4. These dimensions are a function of the bearing bore and the series of

bearing. The standard dimensions are given in millimeters. There is no standard for the size and number of steel balls.



Standard designations of ball bearings.

The bearings are designated by a number. In general, the number consists of atleast three digits. Additional digits or letters are used to indicate special features e.g. deep groove, filling notch etc. The last three digits give the series and the bore of the bearing. The last two digits from 04 onwards, when multiplied by 5, give the bore diameter in millimetres. The third from the last digit designates the series of the bearing. The most common ball bearings are available in four series as follows :

1. Extra light (100), 2. Light (200),

3. Medium (300), 4. Heavy (400)

Notes : 1. If a bearing is designated by the number 305, it means that the bearing is of medium series whose bore is 05 × 5, i.e., 25 mm.

2. The extra light and light series are used where the loads are moderate and shaft sizes are comparatively large and also where available space is limited.

3. The medium series has a capacity 30 to 40 per cent over the light series.

4. The heavy series has 20 to 30 per cent capacity over the medium series. This series is not used extensively in industrial applications.

Static Equivalent Load for Rolling Contact Bearings

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that which occurs under the actual conditions of loading.

1. $W_{0R} = X_0 . W_R + Y_0 . W_A$; and 2. $W_{0R} = W_R$

where :
W_R = Radial load,
W_A = Axial or thrust load,
X_0 = Radial load factor, and
Y_0 = Axial or thrust load factor.
According to IS : 3824 – 1984, the values of X_0 and Y_0 for different bearings are given in the
following table :

Table Values of X_0 and Y_0 for radial bearings.

S.No.	Type of bearing	Single row bearing		Double row bearing	
		X_0	Y_0	X_0	Y_0
1.	Radial contact groove ball bearings	0.60	0.50	0.60	0.50
2.	Self aligning ball or roller bearings and tapered roller bearing	0.50	0.22 cot θ	1	0.44 cot θ
3.	Angular contact groove bearings :				
	α = 15°	0.50	0.46	1	0.92
	α = 20°	0.50	0.42	1	0.84
	α = 25°	0.50	0.38	1	0.76
	α = 30°	0.50	0.33	1	0.66
	α = 35°	0.50	0.29	1	0.58
	α = 40°	0.50	0.26	1	0.52
	α = 45°	0.50	0.22	1	0.44

Notes : 1. The static equivalent radial load (W_{0R}) is always greater than or equal to the radial load (W_R).
2. For two similar single row angular contact ball bearings, mounted 'face-to-face' or 'back-to-back', use
the values of X_0 and Y_0 which apply to a double row angular contact ball bearings. For two or more similar single
row angular contact ball bearings mounted 'in tandem', use the values of X_0 and Y0 which apply to a single row
angular contact ball bearings.
3. The static equivalent radial load (W_{0R}) for all cylindrical roller bearings is equal to the radial load (W_R).
4. The static equivalent axial or thrust load (W_{0A}) for thrust ball or roller bearings with angle of contact
〈 90°, under combined radial and axial loads is given by
W_{0A} = 2.3 WR.tan 〈 + W_A
This formula is valid for all ratios of radial to axial load in the case of direction bearings. For single
direction bearings, it is valid where W_R / W_A δ 0.44 cot 〈.
5. The thrust ball or roller bearings with 〈 = 90° can support axial loads only. The static equivalent axial
load for this type of bearing is given by
W_{0A} = W_A

Dynamic Equivalent Load for Rolling Contact Bearings

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$W = X \cdot V \cdot W_R + Y \cdot W_A$

where V = A rotation factor,

= 1, for all types of bearings when the inner race is rotating,

= 1, for self-aligning bearings when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary. The values of radial load factor (X) and axial or thrust load factor (Y) for the dynamically loaded bearings may be taken from the following table:

Table Values of *X* and *Y* for dynamically loaded bearings.

Type of bearing	*Specifications*	$\frac{W_A}{W_R} \le e$		$\frac{W_A}{W_R} > e$		*e*
		X	*Y*	*X*	*Y*	
Deep groove ball bearing	$\frac{W_A}{C_0} = 0.025$				2.0	0.22
	= 0.04				1.8	0.24
	= 0.07				1.6	0.27
	= 0.13	1	0	0.56	1.4	0.31
	= 0.25				1.2	0.37
	= 0.50				1.0	0.44
Angular contact ball bearings	Single row		0	0.35	0.57	1.14
	Two rows in tandem		0	0.35	0.57	1.14
	Two rows back to back	1	0.55	0.57	0.93	1.14
	Double row		0.73	0.62	1.17	0.86
Self-aligning bearings	Light series : for bores					
	10 – 20 mm		1.3		2.0	0.50
	25 – 35	1	1.7	6.5	2.6	0.37
	40 – 45		2.0		3.1	0.31
	50 – 65		2.3		3.5	0.28
	70 – 100		2.4		3.8	0.26
	105 – 110		2.3		3.5	0.28
	Medium series : for bores					
	12 mm		1.0	0.65	1.6	0.63
	15 – 20		1.2		1.9	0.52
	25 – 50		1.5		2.3	0.43
	55 – 90		1.6		2.5	0.39
Spherical roller bearings	For bores :					
	25 – 35 mm		2.1		3.1	0.32
	40 – 45	1	2.5	0.67	3.7	0.27
	50 – 100		2.9		4.4	0.23
	100 – 200		2.6		3.9	0.26
Taper roller bearings	For bores :					
	30 – 40 mm				1.60	0.37
	45 – 110	1	0	0.4	1.45	0.44
	120 – 150				1.35	0.41

Dynamic Load Rating for Rolling Contact Bearings under Variable Loads

The approximate rating (or service) life of ball or roller bearings is based on the fundamental equation,

$$L = \left(\frac{C}{W}\right)^{K} x10^{6} \text{ revolutions}$$

$$C = W\left(\frac{L}{10^{6}}\right)^{\frac{1}{K}} \quad kN$$

where L = Rating life,
C = Basic dynamic load rating,
W = Equivalent dynamic load,
and

k = 3, for ball bearings,
= 10/3, for roller bearings.
The relationship between the life in revolutions (L) and the life in working hours (L_H) is given by
$L = 60\ N\ .\ L_H$ revolutions
where N is the speed in r.p.m.
Now consider a rolling contact bearing subjected to variable loads. Let W_1, W_2, W_3 etc., be the loads on the bearing for successive n_1, n_2, n_3 etc., number of revolutions respectively.

$$W = \left[\frac{L_1(W_1)^3 + L_2(W_2)^3 + L_3(W_3)^3}{L_1 + L_2 + L_3 + \cdots}\right]^{\frac{1}{3}}$$

Reliability of a Bearing

The reliability (R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. According to Wiebull, the relation between the bearing life and the reliability is given as

$$\frac{L}{L_{90}} = \left[\frac{log_e(1/_R)}{log_e(1/_{R_O})}\right]^{\frac{1}{b}}$$

where ,b=1.117

This expression is used for selecting the bearing when the reliability is other than 90%.

Example.1. *A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to stationary equivalent radial load of 3 kN for 10 per cent of time, 2 kN for 20 per cent of time, 1 kN for 30 per cent of time and no load for remaining time of cycle. If the total life expected for the bearing is 20×10^6 revolutions at 95 per cent reliability, calculate dynamic load rating of the ball bearing.*

Solution. *Given : $W_1 = 3$ kN ; $n_1 = 0.1\ n$; $W_2 = 2$ kN ; $n_2 = 0.2\ n$; $W_3 = 1$ kN ;*
$n_3 = 0.3\ n$;
$W_4 = 0$; $n_4 = (1 – 0.1 – 0.2 – 0.3)\ n = 0.4\ n$; $L95 = 20 \times 10^6$ rev
Let L_{90} = Life of the bearing corresponding to reliability of 90 per cent,
L_{95} = Life of the bearing corresponding to reliability of 95 per cent
$= 20 \times 10^6$ revolutions ... (Given)
We know that:

$$\frac{L}{L_{90}} = \left[\frac{log_e(1/_R)}{log_e(1/_{R_O})}\right]^{\frac{1}{b}}$$

$$\frac{L_{95}}{L_{90}} = \left[\frac{log_e(1/R_{95})}{log_e(1/R_{90})}\right]^{\frac{1}{b}} == \left[\frac{log_e(1/0.95)}{log_e(1/0.90)}\right]^{\frac{1}{1.17}} = (\frac{0.0513}{0.1054})^{0.8547} = 0.54$$

$$L_{90} = \frac{L_{95}}{0.54} = \frac{20x10^6}{0.54} = 37x10^6 \ rev.$$

We know that equivalent radial load,

$$W = \left[\frac{n_1(W_1)^3 + n_2(W_2)^3 + n_3(W_3)^3 + n_4(W_4)^3}{n_1 + n_2 + n_3 + n_4}\right]^{\frac{1}{3}}$$

$$= \left[\frac{n_1(W_1)^3 + n_2(W_2)^3 + n_3(W_3)^3 + n_4(W_4)^3}{n_1 + n_2 + n_3 + n_4}\right]^{\frac{1}{3}}$$

$$= (\frac{0.1\,nx3^3 + 0.2nx2^3 + 0.3nx1^3 + 0.4nx0^3}{0.1n + 0.2n + 0.3n + 0.4n})^{\frac{1}{3}}$$

=(2.7+1.6+0.3+0)$^{1/3}$=1.663 kN

We also know that dynamic load rating,

$$C = W(\frac{L}{10^6})^{\frac{1}{K}}$$

$$C = W(\frac{L_{90}}{10^6})^{\frac{1}{K}}$$

=1.663(37x10^6/10)$^{1/3}$=5.54 kN ***Ans.***

Example.2. ***The rolling contact ball bearing are to be selected to support the overhung countershaft. The shaft speed is 720 r.p.m. The bearings are to have 99% reliability corresponding to a life of 24 000 hours. The bearing is subjected to an equivalent radial load of 1 kN. Consider life adjustment factors for operating condition and material as 0.9 and 0.85 respectively. Find the basic dynamic load rating of the bearing from manufacturer's catalogue, specified at 90% reliability.***

Solution. ***Given : N = 720 r.p.m. ;***
L_H = 24 000 hours ; W = 1 kN
We know that life of the bearing corresponding to 99% reliability,
L_{99} = 60 N. L_H = 60 × 720 × 24 000 = 1036.8 × 10^6 rev
Let L_{90} = Life of the bearing corresponding to 90% reliability.
Considering life adjustment factors for operating condition and material as 0.9 and 0.85 respectively, we have

$$\frac{L}{L_{90}} = \left[\frac{log_e(1/_R)}{log_e(1/_{R_O})}\right]^{\frac{1}{b}} x0.9x0.85$$

$\frac{L_{99}}{L_{90}} = \left[\frac{log_e(1/_{R_{99}})}{log_e(1/_{R_{90}})}\right]^{\frac{1}{1.17}} x0.9x0.85$*=(0.01005/0.1054)*$^{0.8547}$*x0.9x0.85=0.1026*

L_{90}= L_{99}/0.1026=1036x10^6/0.1026=10 105x10^6 rev.

We know that dynamic load rating

$$C = W(\frac{L}{10^6})^{\frac{1}{K}}$$

$C = W(\frac{L_{90}}{10^6})^{1/3}$ ***k = 3, for ball bearing***

=1(10 105x10^6/10^6)$^{1/3}$=21.62 kN Ans.

Selection of Radial Ball Bearings

In order to select a most suitable ball bearing, first of all, the basic dynamic radial load is calculated. It is then multiplied by the service factor (K_S) to get the design basic dynamic radial load capacity. The service factor for the ball bearings is shown in the following table.

Table Values of *X* and *Y* for dynamically loaded bearings.

Type of bearing	*Specifications*	$\frac{W_A}{W_R} \le e$		$\frac{W_A}{W_R} > e$		*e*
		X	*Y*	*X*	*Y*	
Deep groove ball bearing	$\frac{W_A}{C_0}$ = 0.025				2.0	0.22
	= 0.04				1.8	0.24
	= 0.07				1.6	0.27
	= 0.13	1	0	0.56	1.4	0.31
	= 0.25				1.2	0.37
	= 0.50				1.0	0.44
Angular contact ball bearings	Single row		0	0.35	0.57	1.14
	Two rows in tandem		0	0.35	0.57	1.14
	Two rows back to back	1	0.55	0.57	0.93	1.14
	Double row		0.73	0.62	1.17	0.86
Self-aligning bearings	Light series : for bores					
	10 – 20 mm		1.3		2.0	0.50
	25 – 35	1	1.7	6.5	2.6	0.37
	40 – 45		2.0		3.1	0.31
	50 – 65		2.3		3.5	0.28
	70 – 100		2.4		3.8	0.26
	105 – 110		2.3		3.5	0.28
	Medium series : for bores					
	12 mm		1.0	0.65	1.6	0.63
	15 – 20		1.2		1.9	0.52
	25 – 50		1.5		2.3	0.43
	55 – 90		1.6		2.5	0.39
Spherical roller bearings	For bores :					
	25 – 35 mm		2.1		3.1	0.32
	40 – 45	1	2.5	0.67	3.7	0.27
	50 – 100		2.9		4.4	0.23
	100 – 200		2.6		3.9	0.26
Taper roller bearings	For bores :					
	30 – 40 mm				1.60	0.37
	45 – 110	1	0	0.4	1.45	0.44
	120 – 150				1.35	0.41

Table Values of service factor (K_S).

S.No.	*Type of service*	*Service factor (K_S) for radial ball bearings*
1.	Uniform and steady load	1.0
2.	Light shock load	1.5
3.	Moderate shock load	2.0
4.	Heavy shock load	2.5
5.	Extreme shock load	3.0

After finding the design basic dynamic radial load capacity, the selection of bearing is made from the catalogue of a manufacturer. The following table shows the basic static and dynamic capacities for various types of ball bearings.

Table Basic static and dynamic capacities of various types of radial ball bearings.

Bearing No.	Basic capacities in kN							
	Single row deep groove ball bearing		Single row angular contact ball bearing		Double row angular contact ball bearing		Self-aligning ball bearing	
	Static (C_0)	Dynamic (C)	Static (C_0)	Dynamic (C)	Static (C_0)	Dynamic (C)	Static (C_0)	Dynamic (C)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
200	2.24	4	—	—	4.55	7.35	1.80	5.70
300	3.60	6.3	—	—	—	—	—	—
201	3	5.4	—	—	5.6	8.3	2.0	5.85
301	4.3	7.65	—	—	—	—	3.0	9.15
202	3.55	6.10	3.75	6.30	5.6	8.3	2.16	6
302	5.20	8.80	—	—	9.3	14	3.35	9.3
203	4.4	7.5	4.75	7.8	8.15	11.6	2.8	7.65
303	6.3	10.6	7.2	11.6	12.9	19.3	4.15	11.2
403	11	18	—	—	—	—	—	—
204	6.55	10	6.55	10.4	11	16	3.9	9.8
304	7.65	12.5	8.3	13.7	14	19.3	5.5	14
404	15.6	24	—	—	—	—	—	—
205	7.1	11	7.8	11.6	13.7	17.3	4.25	9.8
305	10.4	16.6	12.5	19.3	20	26.5	7.65	19
405	19	28	—	—	—	—	—	—
206	10	15.3	11.2	16	20.4	25	5.6	12
306	14.6	22	17	24.5	27.5	35.5	10.2	24.5
406	23.2	33.5	—	—	—	—	—	—
207	13.7	20	15.3	21.2	28	34	8	17
307	17.6	26	20.4	28.5	36	45	13.2	30.5
407	30.5	43	—	—	—	—	—	—
208	16	22.8	19	25	32.5	39	9.15	17.6
308	22	32	25.5	35.5	45.5	55	16	35.5
408	37.5	50	—	—	—	—	—	—
209	18.3	25.5	21.6	28	37.5	41.5	10.2	18
309	30	41.5	34	45.5	56	67	19.6	42.5
409	44	60	—	—	—	—	—	—
210	21.2	27.5	23.6	29	43	47.5	10.8	18
310	35.5	48	40.5	53	73.5	81.5	24	50
410	50	68	—	—	—	—	—	—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
211 311 411	26 42.5 60	34 56 78	30 47.5 —	36.5 62 —	49 80 —	53 88 —	12.7 28.5 —	20.8 58.5 —
212 312 412	32 48 67	40.5 64 85	36.5 55 —	44 71 —	63 96.5 —	65.5 102 —	16 33.5 —	26.5 68 —
213 313 413	35.5 55 76.5	44 72 93	43 63 —	50 80 —	69.5 112 —	69.5 118 —	20.4 39 —	34 75 —
214 314 414	39 63 102	48 81.5 112	47.5 73.5 —	54 90 —	71 129 —	69.5 137 —	21.6 45 —	34.5 85 —
215 315 415	42.5 72 110	52 90 120	50 81.5 —	56 98 —	80 140 —	76.5 143 —	22.4 52 —	34.5 95 —
216 316 416	45.5 80 120	57 96.5 127	57 91.5 —	63 106 —	96.5 160 —	93 163 —	25 58.5 —	38 106 —
217 317 417	55 88 132	65.5 104 134	65.5 102 —	71 114 —	100 180 —	106 180 —	30 62 —	45.5 110 —
218 318 418	63 98 146	75 112 146	76.5 114 —	83 122 —	127 — —	118 — —	36 69.5 —	55 118 —
219 319	72 112	85 120	88 125	95 132	150 —	137 —	43 —	65.5 —
220 320	81.5 132	96.5 137	93 153	102 150	160 —	146 —	51 —	76.5 —
221 321	93 143	104 143	104 166	110 160	— —	— —	56 —	85 —
222 322	104 166	112 160	116 193	120 176	— —	— —	64 —	98 —

Example 27.3. ***Select a single row deep groove ball bearing for a radial load of 4000 N and an axial load of 5000 N, operating at a speed of 1600 r.p.m. for an average life of 5 years at 10 hours per day. Assume uniform and steady load.***

Solution. ***Given :*** $W_R = 4000$ N ; $W_A = 5000$ N ; $N = 1600$ r.p.m.

Since the average life of the bearing is 5 years at 10 hours per day, therefore life of the bearing in hours,

$L_H = 5 \times 300 \times 10 = 15\,000$ hours ... (Assuming 300 working days per year)

and life of the bearing in revolutions,

$L = 60\, N \times L_H = 60 \times 1600 \times 15\,000 = 1440 \times 10^6$ rev

We know that the basic dynamic equivalent radial load,

$W = X.V.W_R + Y.W_A$ ...(i)

In order to determine the radial load factor (X) and axial load factor (Y), we require W_A / W_R and
W_A / C_0. Since the value of basic static load capacity (C_0) is not known, therefore let us take
$W_A / C_0 = 0.5$. Now from Table 27.4, we find that the values of X and Y corresponding to W_A / C_0
= 0.5 and W_A/W_R = 5000 / 4000 = 1.25 (which is greater than e = 0.44) are
X = 0.56 and Y = 1
Since the rotational factor (V) for most of the bearings is 1, therefore TT_T,
W = 0.56 × 1 × 4000 + 1 × 5000 = 7240 N
From Table 27.5, we find that for uniform and steady load, the service factor (K_S) for ball bearings is 1. Therefore the bearing should be selected for W = 7240 N.
We know that basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{\frac{1}{K}}$$

$$C = 7240\left(\frac{1440x10^6}{10^6}\right)^{1/3} = 81760\ N$$

$$= 81.76\ kN$$

From Table 27.6, let us select the bearing No. 315 which has the following basic capacities,
C_0 = 72 kN = 72 000 N and C = 90 kN = 90 000 N
Now W_A / C0 = 5000 / 72 000 = 0.07
∴ From Table 27.4, the values of X and Y are
X = 0.56 and Y = 1.6
Substituting these values in equation (i), we have dynamic equivalent load,
W = 0.56 × 1 × 4000 + 1.6 × 5000 = 10 240 N
∴ Basic dynamic load rating,

$C = 10\ 240\ (1440x10^6/10^6)^{1/3}$ ***k=3 for ball bearings***

C=115 635 N =115.635 kN

From Table 27.6, the bearing number 319 having C = 120 kN, may be selected. Ans.

Example 27.4. ***A single row angular contact ball bearing number 310 is used for an axial flow compressor. The bearing is to carry a radial load of 2500 N and an axial or thrust load of 1500 N. Assuming light shock load, determine the rating life of the bearing.***

Solution. ***Given : W_R = 2500 N ; W_A = 1500 N***
From Table 27.4, we find that for single row angular contact ball bearing, the values of radial factor (X) and thrust factor (Y) for W_A / W_R = 1500 / 2500 = 0.6 are
X = 1 and Y = 0
Since the rotational factor (V) for most of the bearings is 1, therefore dynamic equivalent load,

$W = X.V.W_R + Y.W_A = 1 \times 1 \times 2500 + 0 \times 1500 = 2500$ N

From Table 27.5, we find that for light shock load, the service factor (K_S) is 1.5. Therefore the design dynamic equivalent load should be taken as
$W = 2500 \times 1.5 = 3750$ N
From Table 27.6, we find that for a single row angular contact ball bearing number 310, the basic dynamic capacity,
C = 53 kN = 53 000 N
We know that rating life of the bearing in revolutions,

$$L = \left(\frac{C}{W}\right)^K x10^6 \;\; = \left(\frac{53000}{3750}\right)^3 x10^6 = 2823x10^6 \text{ rev.}$$

Example 27.5. *Design a self-aligning ball bearing for a radial load of 7000 N and a thrust load of 2100 N. The desired life of the bearing is 160 millions of revolutions at 300 r.p.m. Assume uniform and steady load,*
Solution. Given : $W_R = 7000$ N ; $W_A = 2100$ N ; L = 160 × 106 rev ; N = 300 r.p.m.
From Table 27.4, we find that for a self-aligning ball bearing, the values of radial factor (X) and thrust factor (Y) for $W_A / W_R = 2100 / 7000 = 0.3$, are as follows :
X = 0.65 and Y = 3.5
Since the rotational factor (V) for most of the bearings is 1, therefore dynamic equivalent load,
$W = X.V.W_R + Y.W_A = 0.65 \times 1 \times 7000 + 3.5 \times 2100 = 11\,900$ N
From Table 27.5, we find that for uniform and steady load, the service factor K_S for ball bearings is 1. Therefore the bearing should be selected for W = 11 900 N.
We know that the basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{\frac{1}{K}} = 11900\left(\frac{160x10^6}{10^6}\right)^{1/3} = 6400 \text{ N} = 64.6 \text{ kN}$$

... (k = 3, for ball bearings)
From Table 27.6, let us select bearing number 219 having C = 65.5 kN **Ans.**

Example 27.6. *Select a single row deep groove ball bearing with the operating cycle listed below, which will have a life of 15 000 hours.*

Fraction of cycle	Type of load	Radial (N)	Thrust (N)	Speed (R.P.M.)	Service factor
1/10	Heavy shocks	2000	1200	400	3.0
1/10	Light shocks	1500	1000	500	1.5
1/5	Moderate shocks	1000	1500	600	2.0
3/5	No shock	1200	2000	800	1.0

Assume radial and axial load factors to be 1.0 and 1.5 respectively and inner race rotates.
Solution. Given : $L_H = 15\,000$ hours ; $W_{R1} = 2000$ N ; $W_{A1} = 1200$ N ; $N_1 = 400$ r.p.m. ; $K_{S1} = 3$;

W_{R2} = 1500 N ; WA2 = 1000 N ; N_2 = 500 r.p.m. ; K_{S2} = 1.5 ; W_{R3} = 1000 N ;
W_{A3} = 1500 N ;
N_3 = 600 r.p.m. ; K_{S3} = 2 ; W_{R4} = 1200 N ; W_{A4} = 2000 N ; N_4 = 800 r.p.m. ; K_{S4} = 1 ;
X = 1 ; Y = 1.5

We know that basic dynamic equivalent radial load considering service factor is
$W = [X.V.W_R + Y.W_A]\ K_S$ ...(i)
It is given that radial load factor (X) = 1 and axial load factor (Y) = 1.5. Since the rotational factor (V) for most of the bearings is 1, therefore equation (i) may be written as
$W = (W_R + 1.5\ W_A)\ K_S$
Now, substituting the values of W_R, W_A and K_S for different operating cycle, we have
$W_1 = (W_{R1} + 1.5\ W_{A1})\ K_{S1} = (2000 + 1.5 \times 1200)\ 3 = 11\ 400$ N
$W_2 = (W_{R2} + 1.5\ W_{A2})\ K_{S2} = (1500 + 1.5 \times 1000)\ 1.5 = 4500$ N
$W_3 = (W_{R3} + 1.5\ W_{A3})\ K_{S3} = (1000 + 1.5 \times 1500)\ 2 = 6500$ N
and $W_4 = (WR4 + 1.5\ W_{A4})\ K_{S4} = (1200 + 1.5 \times 2000)\ 1 = 4200$ N
We know that life of the bearing in revolutions
$L = 60\ N.L_H = 60\ N \times 15\ 000 = 0.9 \times 10^6\ N$ rev
∴ Life of the bearing for 1/10 of a cycle,

$L_1=1/10x0.9x10^6N_1=1/10x0.9x10^6x400=36x10^6$ rev

Similarly, life of the bearing for the next 1/10 of a cycle,

$L_2=1/10x0.9x10^6N_2=1/10x0.9x10^6x500=45x10^6$ rev

Similarly, life of the bearing for the next 1/5 of a cycle

$L_3=1/5x0.9x10^6N_3=1/5x0.9x10^6x600=108x10^6$ rev

and life of the bearing for the next 3/5 of a cycle,

$L_4=3/5x0.9x10^6N_4=3/5x0.9x10^6x800=432x10^6$ rev

$$W = \left[\frac{L_1(W_1)^3 + L_2(W_2)^3 + L_3(W_3)^3 + L_4(W_4)^3}{L_1 + L_2 + L_3 + L_4}\right]^{\frac{1}{3}}$$

$$= \left[\frac{36x10^6(11400)^3 + 45x10^6(4500)^3 + 108x10^6(6500)^3 + 432x10^6(4200)^3}{36x10^6 + 45x10^6 + 108x10^6 + 432x10^6}\right]^{\frac{1}{3}}$$

$$=\left[\frac{1.191x10^8x10^{12}}{621x10^6}\right]^{\frac{1}{3}}=5279\ N$$

and L = L1 + L2 + L3 + L4
= 36 × 106 + 45 × 106 +108 × 106 + 432 × 106 = 621 × 106 rev

We know that dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{\frac{1}{K}} = 5279\left(\frac{621x10^6}{10^6}\right)^{1/3} = 44.08\ kN$$

From Table 27.6, the single row deep groove ball bearing number 215 having C = 44 kN may be selected. C_o=35.5 Ans.

Lecture 13

Springs

Introduction

A spring is defined as an elastic body, whose functions to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and spring loaded valves.

3. To control motion by maintaining contact between two elements as in cams and followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.

Objectives

After studying this unit, you should be able to

- describe the types of springs
- calculate the strength of springs

Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig.

(*a*) Compression helical spring. (*b*) Tension helical spring.

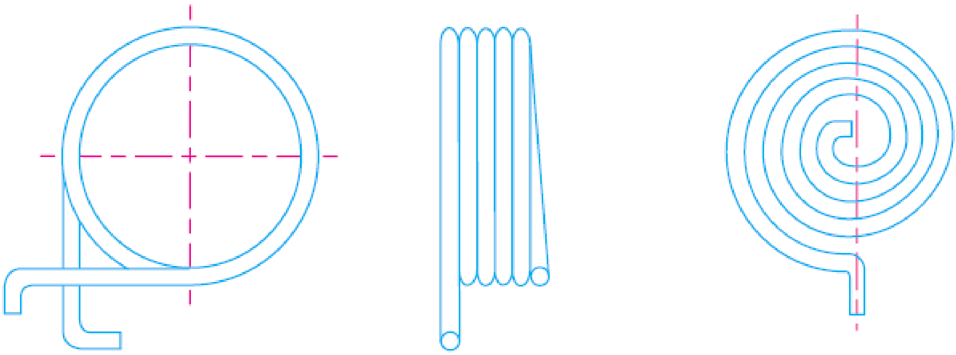
Fig. Helical springs.

2. Conical and volute springs. The conical and volute springs, as shown in Fig., are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig.

(*a*) Conical spring. (*b*) Volute spring.

Fig. Conical and volute springs.

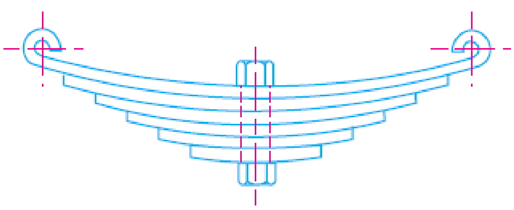
3. Torsion springs. ***These springs may be of helical or spiral type as shown in Fig.. The helical type may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks***



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

. **Fig.** . Torsion springs.

4. Laminated or leaf springs. ***The laminated or leaf spring (also known as flat spring or carriage spring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. These are mostly used in automobiles.***



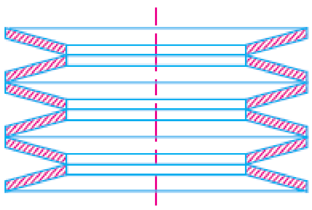


Fig. . Laminated or leaf springs. **Fig.** Disc or bellevile springs.

Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. Solid length. *When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically, Solid length of the spring,*

$L_S = n'.d$

where n' = *Total number of coils, and*

d = *Diameter of the wire.*

2. Free length. *The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,*

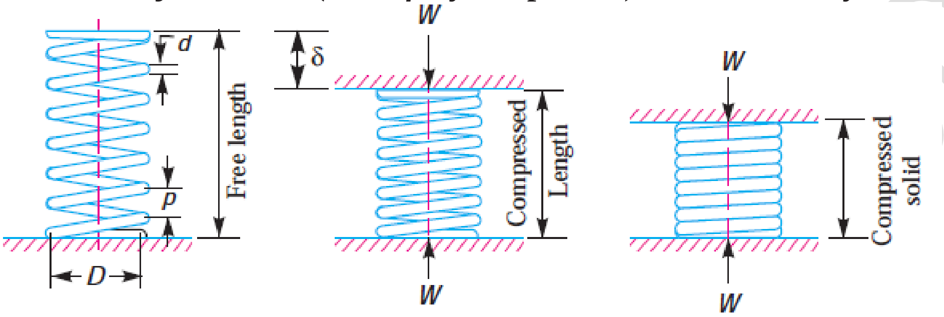


Fig. [illegible]. Compression spring nomenclature.

Free length of the spring,

L_F = *Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)*

$= n'.d + \delta_{max} + 0.15\ \delta_{max}$

The following relation may also be used to find the free length of the spring, i.e.

$L_F = n'.d + \delta_{max} + (n' - 1) \times 0.1$ mm

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. Spring index. *The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,*

Spring index, $C = D / d$

where D = *Mean diameter of the coil, and*

d = *Diameter of the wire.*

4. Spring rate. *The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,*

Spring rate, $k = W / \delta$

where W = *Load, and*

δ = *Deflection of the spring.*

5. Pitch. *The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically, Pitch of the coil,*

$$P = \frac{free\ length}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, i.e. Pitch of the coil,

$$P = \frac{L_F - L_S}{n'} + d$$

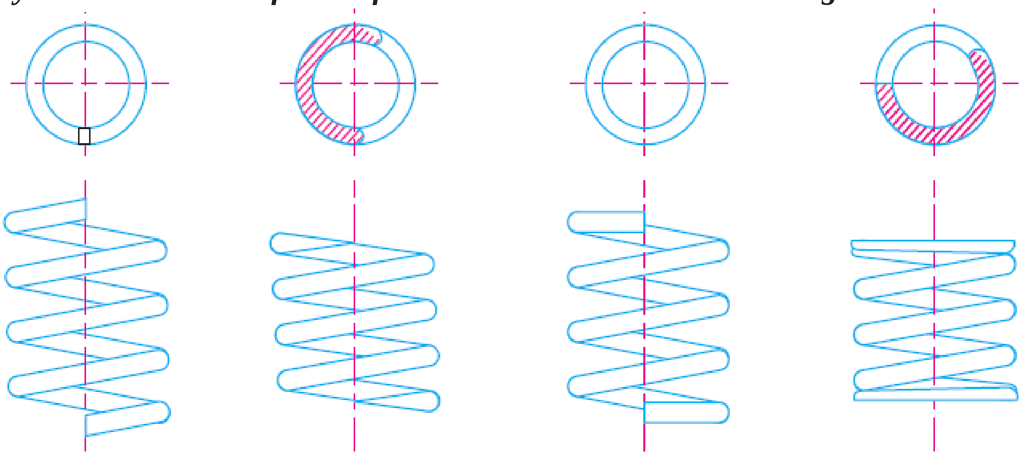
where L_F = *Free length of the spring,*

L_S = *Solid length of the spring,*

n' = Total number of coils, and
d = Diameter of the wire.

End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig.



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

Fig End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as inactive coils. The turns which impart spring action are known as active turns. As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.

Table Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\,d$	$p \times n + 2d$

where *n = Number of active turns,*
p = Pitch of the coils, and
d = Diameter of the spring wire.

Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig.

Let

D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

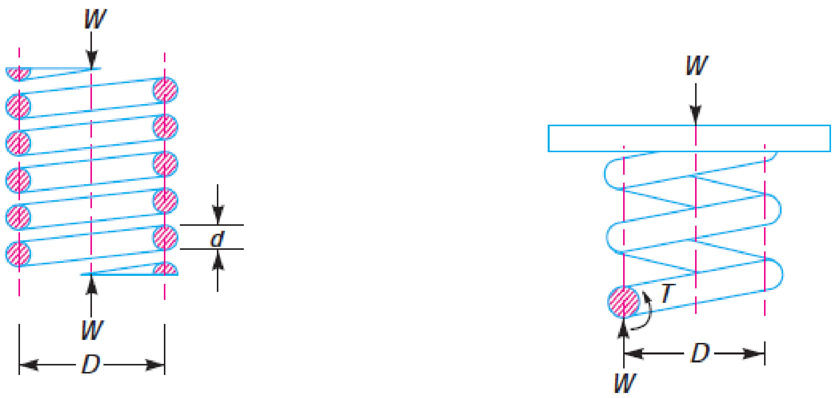
W = Axial load on the spring,

τ= Maximum shear stress induced in the wire,

C = Spring index = D/d,

p = Pitch of the coils, and

δ= Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig.

Now consider a part of the compression spring as shown in Fig. 23.10 (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W x \frac{D}{2} = \frac{\pi}{16} x \tau_1 x d^3$$

$$\tau_1 = \frac{8WD}{\pi d^3}$$

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. *Direct shear stress due to the load W, and*
2. *Stress due to curvature of wire.*

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{Load}{cross\ sectional\ area\ in\ the\ wire}$$

$$= \frac{W}{\frac{\pi}{4} d^2} = \frac{4W}{\pi d^2}$$

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \mp \tau_2 = \frac{8WD}{\pi d^3} \mp \frac{4W}{\pi d^2}$$

The positive sign is used for the inner edge of the wire and negative sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2}$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_s \frac{8WD}{\pi d^3}$$ *substitutingD/d=C*

where K_S = Shear stress factor $=1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\frac{8WD}{\pi d^3} x \frac{1}{2C}$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation (iii). It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses. In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (d).

∴Maximum shear stress induced in the wire,

$$\tau = K x \frac{8WD}{\pi d^3} = K \frac{8WC}{\pi d^2}$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig.

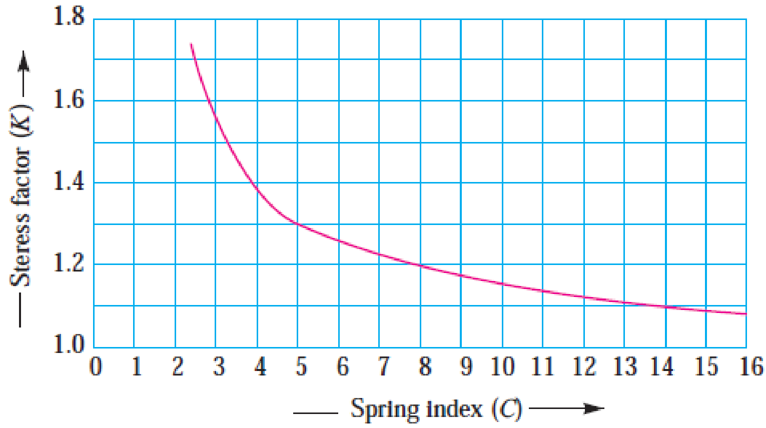


Fig. Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: *The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that*

$K = K_S \times K_C$

where K_S = *Stress factor due to shear, and*
K_C = *Stress concentration factor due to curvature.*

Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that:
Total active length of the wire,
l = Length of one coil × No. of active coils = π D × n
Let *θ = Angular deflection of the wire when acted upon by the torque T.*
∴ Axial deflection of the spring,
δ = θ × D/2 .. ..(i)
We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G\theta}{l}$$

$$\theta = \frac{T.l}{G.J} \qquad \text{considering } \frac{T}{J} = \frac{G\theta}{l}$$

where J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} d^4$$

and G = Modulus of rigidity for the material of the spring wire.
Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{G.J} = \frac{W x \frac{D}{2}.\pi.D.n}{\frac{\pi}{32} d^4.J} = \frac{16.WD^2}{Gd^4} =$$

Substituting this value of ϑin equation (i), we have

$$\delta = \frac{16\,WD^2 n}{Gd^4} x \frac{D}{2}$$

$$\delta = \frac{8\,WD^3 n}{Gd^4} = \frac{8WC^3 n}{G.d} \qquad \text{... (} C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{Gd^4}{8D^3 n} = \frac{Gd}{8C^3 n} = Constant$$

Example 1 Design a helical compression spring for a maximum load of 1000 N for a deflection of 25mm using the value of spring index as 5.(take τ_{max}*=420N/mm*^{2}

and modulus of rigidity, for spring material G=84KN/mm^2

solution: given data:

N_{max}=1000 N

δ=25 mm

C=5

τ_{max}=420N/mm^2

G=84KN/mm^2

$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4x5-1}{4x5-4} + \frac{0.615}{5}$=1.31

Diameter of the spring wire

let D= mean diameter of the spring coil, and

d= diameter of spring wire

Using the relation

$$\tau = K\frac{8WC}{\pi d^2} \qquad d^2 = \frac{K.8.WC}{\pi\tau} = \frac{1.31x8x1\,000x5}{\pi x420} = 40$$

d=6.3 mm

or

We shall take a standard wire size SWG3 having diameter 6.401 mm answer

D=5d=5x6.401=32.005 mm

number of active turns of the coils

let n= number of active turns of the coils

$$\delta = \frac{8WC^3n}{G.d}$$

$n = \frac{\delta\, Gd}{8WC^3} = \frac{25x6.3x84x10^3}{8x1\,000x5^3} = 13.23 = 14$ answer

for square and ground ends, the total number of turns

n'=n+2=14+2=16

taking 1 mm clearance between adjacent coils, free length of the spring

L_F = n'.d + δ_{max} + (n' – 1) × 0.1 mm

= 16x6.3+25+(16-1)0.1=126.5 mm

we know that the pitch of the coil

$P = \frac{free\ length}{n'-1} = \frac{140.8}{16-1} = 9.4\ mm$ answer

***Example 2** Design a close coiled helical compression spring for a service factor load ranging from 225 kg to 275 kg. The axial deflection of spring the spring for the load range is 6 mm. Assume a spring index of 5.*

Permissible shear stress intensity 4200 kg/cm^2. Modulus of rigidity G=0.84x10^6 kg/cm^2.

Neglect the effect of stress concentration. Draw a fully dimension sketch of the spring, showing details of the finish of the end coils.

Solution:

given min. load W_1=225 kg

max. load W_2= 275kg

Axial deflection for the load range from 225kg to 275kg for a load of 50 kg

δ=6 mm= 0.6 cm

spring index, C=5

Permissible shear stress τ= 4200 kg/cm^2

modulus of rigidity G= 0.84x10^6 kg/cm^2

first of all, let us find the mean diameter of the spring coil for the maximum load of W_1= 275 kg

let D= Mean diameter of the spring

d= diameter of the spring wire

$$T = Wx\frac{D}{2} = \frac{\pi}{16}x\tau_1 xd^3$$

$$275x\frac{5d}{2} = \frac{\pi}{16}x4200xd^3 \qquad c=D/d=5$$

d=0.8839 cm

we shall use a wire of SWG 2/0 whose diameter is 0.8839 cm

$\therefore$ mean diameter of the spring coil

D=5d= 5x0.8839=4.4195 cm

we know the outer diameter of the spring coil =D+d=4.4195+0.8839=5.3034cm

and the inner diameter of the spring coil =D-d= 4.4195-0.8839= 3.5356 cm

now let us find out the active numbers of turns of the coil

using the relation

$$\delta = \frac{8WC^3n}{G.d}$$

$$n = \frac{\delta\, Gd}{8WC^3} = \frac{0.6x0.84x10^6x0.8839}{8x50x5^3}$$

= 8.3=9 coils

total number of turns using square and ground end

n'=n+2=9+2=11 *answer*

since the compression produced under 50 kg load is 0.6 cm therefore maximum compression produced under the maximum load of 275 kg

δ_{max}*=0.6/50x275= 3.3 cm*

we know that the free length of the coil

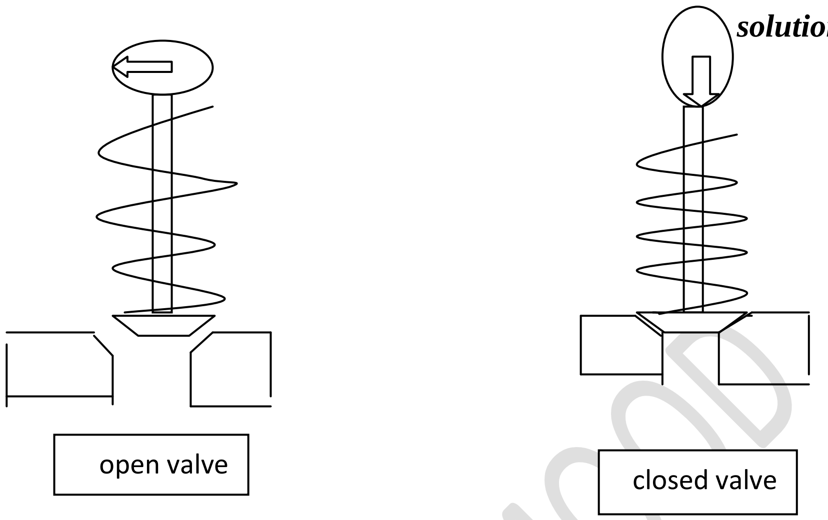
$L_F = n'.d + \delta_{max} + (n' - 1) \times 0.1$ *mm*
=11x0.8839+3.3+(11-1)x0.1=14.023 or 14.2 cm

and the pitch of the coil

$$P = \frac{free\ length}{n'-1} = \frac{14.2}{11-1} = 1.42\ cm$$ **answer**

Example 3
The following are the data for a helical spring used for an engine:
Length of the spring when valve is open= 4 cm
Length of the spring when valve is close= 5 cm
spring load when valve is open = 40 kg
spring load when valve is open= 20kg
maximum inside diameter of spring= 2.8 cm
maximum permissible shear stress for the material of the spring= 400kg/cm^2 and modulus of rigidity= 8x10^5 kg/cm^2. Design the spring

solution:



Given

Length of the spring when valve is open=l_1= 4 cm
Length of the spring when valve is close=l_2= 5 cm
spring load when valve is open =W_1= 40 kg
spring load when valve is open= W_2= 20kg
maximum inside diameter of spring=d_1= 2.8 cm
maximum permissible shear stress for the material of the spring=τ= 400kg/cm^2 and modulus of rigidity= G= 8x10^5 kg/cm^2. Design the spring
Mean diameter of the spring coil= inside diameter of spring +dia. of wire spring
=(2.8+d) cm
since the diameter of the spring wire is obtained by maximum spring load therefore the maximum twisting moment on the spring

$$T = Wx\frac{D}{2} = \frac{\pi}{16}x\tau_1 xd^3$$

$$=40x\left(\frac{2.8+d}{2}\right) = \frac{\pi}{16}x4000xd^3$$

d=0.43 or 0.45 cm
mean diameter of the spring coil= D=2.8+d=2.8+0.45=3.25 cm
we know that the spring index
C=D/d=3.25/0.45=7.2

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4x7.2-1}{4x7.2-4} + \frac{0.615}{7.2} = 1.2$$

Now taking Whals factor into account we may determine the diameter of the wire

$$\tau == K\frac{8WC}{\pi d^2}$$

$$d^2 = \frac{K.8.W_1 C}{\pi\tau} = \frac{1.2x8x40x7.2}{\pi x4000}$$

d=0.47 cm

we shall use wire of SWG 6 whose diameter is 0.4877 cm

and mean diameter of the spring coil=D=2.8+d=2.8+0.4877=3.2877 cm

number of turns of coils

let n=number of active turns of the coil

we are given that compression of the spring caused by load of (W_1-W_2) 40-20=20kg

is (l_2-l_1) 5-4=1 cm

now using the relation

$$\delta = \frac{8\,WD^3n}{Gd^4}$$

$$n = \frac{\delta d^4 G}{8wD^3} = \frac{1x0.4877^4x8x10^5}{8x20x3.2877^3} = 9.25 = 10$$

taking the ends of the spring as square and ground total numbers of turns of the spring

n'=10+2=12

since the deflection for 20 kg is 1 cm therefore maximum deflection for 40 kg load

δ_{max}=1/20x40=2 cm

we know that the free length of the coil

L_F = n'.d + δ_{max} + (n' – 1) × 1
=12x0.4877+2+(12-1)x0.1=8.9524cm

and the pitch of the coil

$P = \frac{free\ length}{n'-1}$=8.9524/12-1=0.8138 cm answer

Example 4

A close coiled helical compression spring of 12 active coils has a spring stiffness of k. It is cut into two springs having 5 and 7 turns. Determine the spring stiffness of resulting springs.

solution

number of active coils=12

stiffness of spring=k=W/δ

$$\delta = \frac{8WD^3n}{Gd^4}$$

$$\frac{W}{\delta} = \frac{Gd^4}{8D^3n}$$

since G, D and d are constant therefore substiuting

$$\frac{Gd^4}{8D^3} = X$$

$$\frac{W}{\delta} = k = \frac{X}{n}$$

X=kn

The spring is cut into two springs with n_1=5 and n_2=7

k_1=stiffness of spring having 5 turns

k_2= stiffness of spring having 7 turns

$$k_1 = \frac{X}{n_1} = \frac{12k}{5} = 2.4\ k$$

$$k_2 = \frac{X}{n_2} = \frac{12k}{7} = 1.7\ k$$ ***answer***

Example 5

Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions;

diameter of valve seat=100 mm

operating pressure=1 N/mm^2

maximum pressure when valve blows off freely=1.075 N/mm^2

maximum lift of the valve when the pressure is 1.075 N/mm^2

maximum allowable stress= 400 N/mm^2 modulus of rigidity= 84x10^3 N/mm^2

spring index=5.5 Draw a neat sketch of the free spring showing the main dimensions

solution

since the valve is a Rams bottom safety valve therefore the spring will be under tension, we know that the initial tensile force acting on the spring (before the valve lifts.

$$W_1 = \frac{\pi}{4} D_1^2 P_1 = \frac{\pi}{4} 100^2 x 1 = 7855\ N.$$

$$W_2 = \frac{\pi}{4} D_1^2 P_2 = \frac{\pi}{4} 100^2 x 1.075 = 8445\ N$$

Force which produces the deflection of 6 mm

W=W$_2$-W$_1$=8445-7855=590 N

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4x5.5-1}{4x5.5-4} + \frac{0.615}{5.5} = 1.28$$

$$W_2 x \frac{D}{2} = \frac{\pi}{16} \tau d^3$$

$$8445 x \frac{5.5d}{2} = \frac{\pi}{16} x 400 x d^3$$

d=17.2 or 18 mm

∴ mean diameter D=5.5d=5.5x18=99 mm

outside diameter= D+d=99+18=117 mm

inside diameter= D-d= 99-18=81 mm

number of active turns

$$\delta = \frac{8WC^3 n}{d.G}$$

$$6 = \frac{8x590x5.5^3 n}{18x84x10^3}$$

n=11.38 or 12 ans.

Lecture 14

Gear Design

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another. Several types of gears are in common use. This chapter introduces various types of gears and details the design, specification and selection of spur gears in particular.

LEARNING OBJECTIVES

At the end of this section you should be:

- ***familiar with gear nomenclature;***
- ***able to select a suitable gear type for different applications;***
- ***able to determine gear train ratios;***
- ***able to determine the bending stress for a spur gear using the Lewis formula;***
- ***able to select appropriate gears for a compound gearbox using spur gears;***

Gears can be divided into several broad classifications.

1. Parallel axis gears:
(a) spur gears (see Figure 6.5),
(b) helical gears (see Figures 6.6 and 6.7),
(c) internal gears.
2. Non-parallel, coplanar gears (intersecting axes):
(a) bevel gears (see Figure 6.8),
(b) face gears,
(c) conical involute gearing.
3. Non-parallel, noncoplanar gears (nonintersecting axes):
(a) crossed axis helicals (see Figure 6.9),
(b) cylindrical worm gearing (see Figure 6.10),
(c) single enveloping worm gearing,
(d) double enveloping worm gearing,
(e) hypoid gears,
(f) spiroid and helicon gearing,
(g) face gears (off centre).
4. Special gear types:
(a) square and rectangular gears,
(b) elliptical gears,

Figure 6.5 Spur gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

Figure 6.6 Helical gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

Figure 6.7 Double helical gears.

Figure 6.8 Bevel gears.

Figure 6.9 Crossed axis helical gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

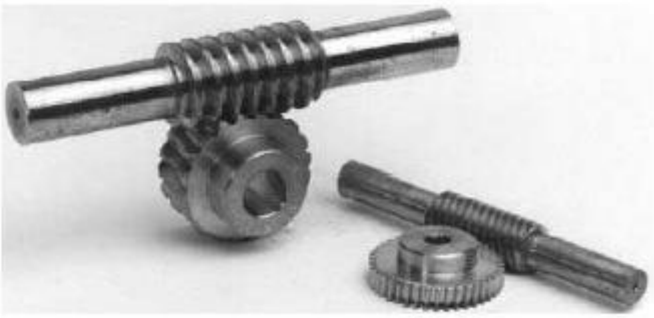


Figure 6.10 Worm gears. Photograph courtesy of Hinchliffe Precision Components Ltd.

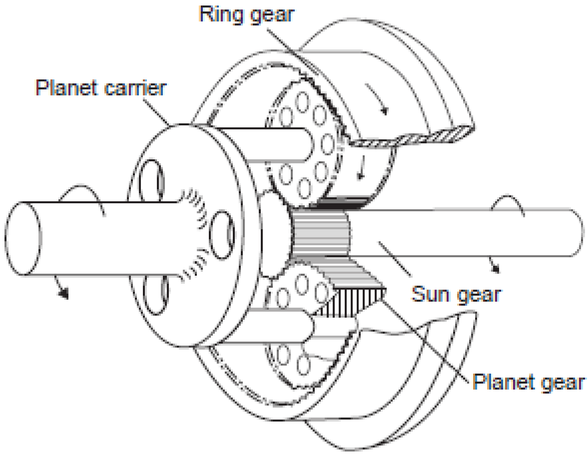


Figure 6.11 Epicyclic gears (reproduced from Townsend, 1992).

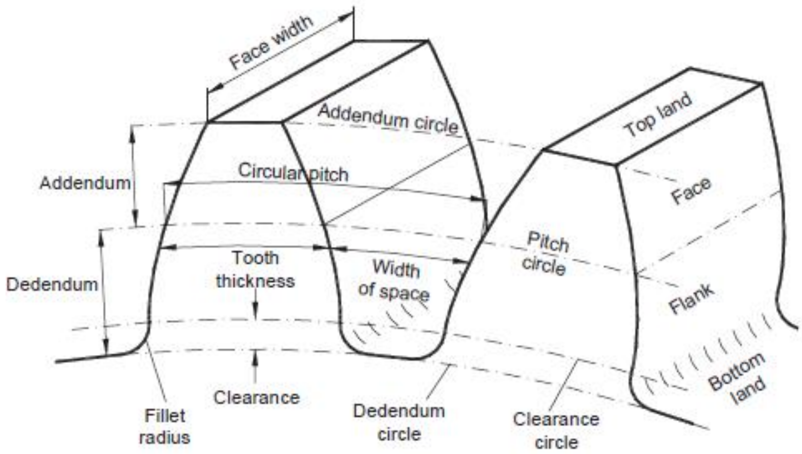


Figure 6.12 Spur gear schem showing principle terminology.

Terms used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage.

***1. Pitch circle.** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.*

***2. Pitch circle diameter.** It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as pitch diameter.*

***3. Pitch point.** It is a common point of contact between two pitch circles.*

***4. Pitch surface.** It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.*

***5. Pressure angle or angle of obliquity.** It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ The standard pressure angle is 20°.*

***6. Addendum.* It is the radial distance of a tooth from the pitch circle to the top of the tooth.**
***7. Dedendum.* It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.**
***8. Addendum circle.* It is the circle drawn through the top of the teeth and is concentric with the pitch circle.**
***9. Dedendum circle.* It is the circle drawn through the bottom of the teeth. It is also called root circle.**
***Note :* Root circle diameter = Pitch circle diameter × cos ϕ where ϕ is the pressure angle.**
***10. Circular pitch.* It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by Pc. Mathematically.**

Gear trains

A gear train is one or more pairs of gears operating together to transmit power. Figure below shows examples of a simple gear train, a reverted gear train and a non-reverted gear train.

$$\frac{n_P}{n_G} = \frac{N_G}{N_P} = \frac{d_G}{d_P} = \frac{w_G}{w_p}$$

Example .1
Consider the gear train shown in Figure below. Calculate the speed of gear five.
Solution

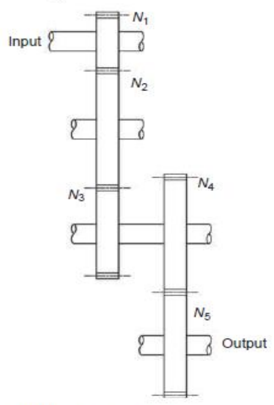


Figure 6.20 Example gear train.

$$n_2 = -\frac{N_1}{N_2} n_1$$
$$n_3 = -\frac{N_2}{N_3} n_2$$
$$n_4 = n_3$$
$$n_5 = -\frac{N_4}{N_5} n_4$$
$$n_5 = \frac{N_4 N_2 N_1}{N_5 N_3 N_2} n_1$$

Beam Strength of Gear Teeth – Lewis Equation

The beam strength of gear teeth is determined from an equation (known as *Lewis equation) and the load carrying ability of the toothed gears as determined by this

equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the end of the driving teeth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several teeth. But it is almost certain that at some time during the contact of teeth, the proper distribution of load does not exist and that one tooth must transmit the full load. In any pair of gears having unlike number of teeth, the gear which have the fewer teeth (i.e. pinion) will be the weaker, because the tendency toward undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller.

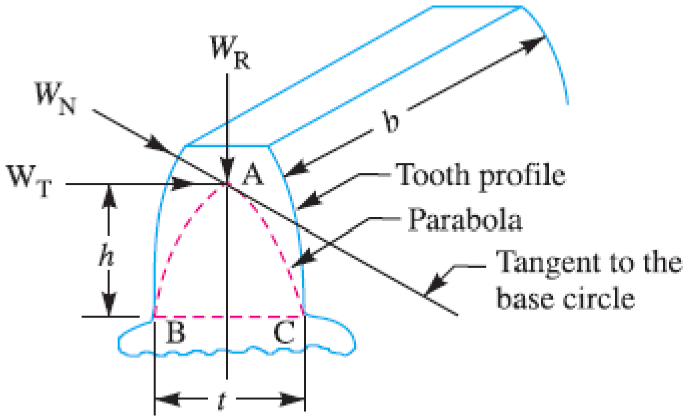


Fig. Tooth of a gear.

$$\sigma_b = \frac{W_t}{FmY}$$

where Wt is transmitted load (N); F is face width(m or mm); m is module (m or mm); and Y is the Lewis form factor and can be found from Table 27

Table 6.6 Values for the Lewis form factor *Y* defined for two different tooth standards (Mitchener and Mabie, 1982)

N, number of teeth	Y ($\phi = 20°$; $a = 0.8$ m; $b =$ m)	Y ($\phi = 20°$; $a =$ m; $b = 1.25$ m)
12	0.33512	0.22960
13	0.34827	0.24317
14	0.35985	0.25530
15	0.37013	0.26622
16	0.37931	0.27610
17	0.38757	0.28508
18	0.39502	0.29327
19	0.40179	0.30078
20	0.40797	0.30769
21	0.41363	0.31406
22	0.41883	0.31997
24	0.42806	0.33056
26	0.43601	0.33979
28	0.44294	0.34790
30	0.44902	0.35510
34	0.45920	0.36731
38	0.46740	0.37727
45	0.47846	0.39093
50	0.48458	0.39860
60	0.49391	0.41047
75	0.50345	0.42283
100	0.51321	0.43574
150	0.52321	0.44930
300	0.53348	0.46364
Rack	0.54406	0.47897

a, addendum; *b*, dedendum; ϕ, pressure angle; *m*, module.

Simple gear selection procedure

The procedure is outlined below.

1. Select the number of teeth for the pinion and the gear to give the required gear ratio

2. Select a material. This will be limited to those listed in the stock gear catalogues.

Table 6.11 Permissible bending stresses for various commonly used gear materials

Material	Treatment	σ_{uts} (MPa)	Permissible bending stress σ_P (MPa)
Nylon		65 (20°C)	27
Tufnol		110	31
080M40		540	131
080M40	Induction hardened	540	117
817M40		772	221
817M40	Induction hardened	772	183
045M10		494	117
045M10	Case hardened	494	276
655M13	Case hardened	849	345

3. Select a module, m

4. Calculate the pitch diameter, d = mN.

5. Calculate the pitch line velocity, V = (d/2) . n . (2π/60).
6. Calculate the dynamic factor, Kv = 6/(6 + V).

7. Calculate the transmitted load, Wt = Power/V.

8. Calculate an acceptable face width using the Lewis formula in the form,

$$F = \frac{W_t}{K_v m Y \sigma_p}$$

Example 1
A 20° full depth spur pinion is to transmit 1.25 kW at 850 rpm.The pinion has 18 teeth. Determine the Lewis bending stress if the module is 2 and the face width is 25mm.
Solution
Calculating the pinion pitch diameter
$d_P = m N_P = 2x\ 18 = 36$ mm
Calculating the pitch line velocity

$$V = \frac{d}{2} * 10^{-3} * n * \frac{2\pi}{60}$$

$$= \frac{0.036}{2} * 10^{-3} * 850 * \frac{2\pi}{60} = 1.602\ m/s$$

Calculating the velocity factor

$$K_v = \frac{6.1}{6.1 + V}$$

$$= \frac{6.1}{6.1+V} = \frac{6.1}{6.1+1.602} = 0.7920$$

Calculating the transmitted load

$$W_t = \frac{power}{V} = \frac{1250}{1.602} = 780.2\ N$$

From Table 27 for N_P = 18, the Lewis form factor Y = 0.29327. The Lewis Equation for bending stress gives:

$$\sigma_b = \frac{W_t}{K_v F m Y}$$

$$= \frac{780.2}{0.792*0.025*0.002*0.29327} = 67.18x10^6 N/mm^2$$

=67.18 Mpa answer

Example 2:

A gearbox is required to transmit 18 kW from a shaft rotating at 2650 rpm.The desired output speed is approximately 12 000 rpm. For space limitation and standardization reasons a double step-up gearbox is requested with equal ratios.

Solution

Overall ratio =12 000/2650 = 4.528.

First stage ratio =$\sqrt{4.528}$ = 2.128.

This could be achieved using a gear with

38 teeth and pinion with 18 teeth (ratio = 38/18 = 2.11).

The gear materials listed in Tables 6.7 to 6.10 are 817M40 and 655M13 steels.

from Table 6.11 Permissible bending stresses for various commonly used gear materials

Table 6.11 Permissible bending stresses for various commonly used gear materials

Material	Treatment	σ_{uts} (MPa)	Permissible bending stress σ_P (MPa)
Nylon		65 (20°C)	27
Tufnol		110	31
080M40		540	131
080M40	Induction hardened	540	117
817M40		772	221
817M40	Induction hardened	772	183
045M10		494	117
045M10	Case hardened	494	276
655M13	Case hardened	849	345

Calculations for gear 1: $Y_{38} = 0.37727$, $n = 2650$ rpm.

m	1.5	2.0
$d = mN$ (mm)	57	76
$V = \frac{d}{2} \times 10^{-3} \times n \frac{2\pi}{60}$ (m/s)	7.9	10.5
$W_t = \frac{Power}{V}$ (N)	2276	1707
$K_v = \frac{6.1}{6.1 + V}$	0.4357	0.3675
$F = \frac{W_t}{K_v m Y \sigma_p}$ (m)	0.027	0.018

$m = 1.5$ gives a face width value greater than the catalogue value of 20 mm, so try $m = 2$. $m = 2$ gives a face width value less than the catalogue value of 25 mm, so design is OK.

Calculations for pinion 1: Y18 = 0.29327, n = 5594 rpm. (No need to do calculations for m = 1.5, because it has now been rejected.)

m	***2.0***
d (mm)	***36***
V (m/s)	***10.5***
Wt (N)	***1707***
Kv	***0.3676***
F (m)	***0.023***

m = 1.5 gives a face width value greater than the catalogue value of 20mm, so try m = 2.

m = 2 gives a face width value less than the catalogue value of 25mm, so design is OK.

Calculations for gear 2: Y38 = 0.37727, n = 5594 rpm.

m	***2.0***
d (mm)	***76***
V (m/s)	***22.26***
Wt (N)	***808.6***
Kv	***0.215***
F (m)	***0.0144***

m = 2 gives a face width value lower than catalogue specification, so the design is OK. Calculations for pinion 2: Y18 = 0.29327, n = 11 810 rpm.

m	*2.0*
d (mm)	*36*
V (m/s)	*22.26*
Wt (N)	*808.6*
Kv	*0.215*
F (m)	*0.0186*

m = 2 gives a face width value lower than catalogue specification, so the design is OK.

Lecture15

Pressure Vessels

The pressure vessels (i.e. cylinders or tanks) are used to store fluids under pressure. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property.
The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

Classification of Pressure Vessels

The pressure vessels may be classified as follows:

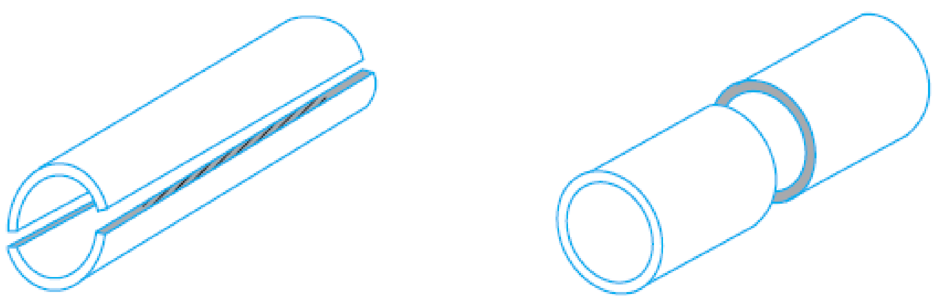
1. According to the dimensions. ***The pressure vessels, according to their dimensions, may be classified as thin shell or thick shell. If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a thin shell. On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a thick shell. Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.***

2. According to the end construction.
The pressure vessels, according to the end construction, may be classified as open end or closed end. A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, whereas a tank is an example of a closed end vessel. In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure, whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

Stresses in a Thin Cylindrical Shell due to an Internal Pressure

The analysis of stresses induced in a thin cylindrical shell are made on the following
assumptions:

1. ***The effect of curvature of the cylinder wall is neglected.***
2. ***The tensile stresses are uniformly distributed over the section of the walls.***
3. ***The effect of the restraining action of the heads at the end of the pressure vessel is neglected.***

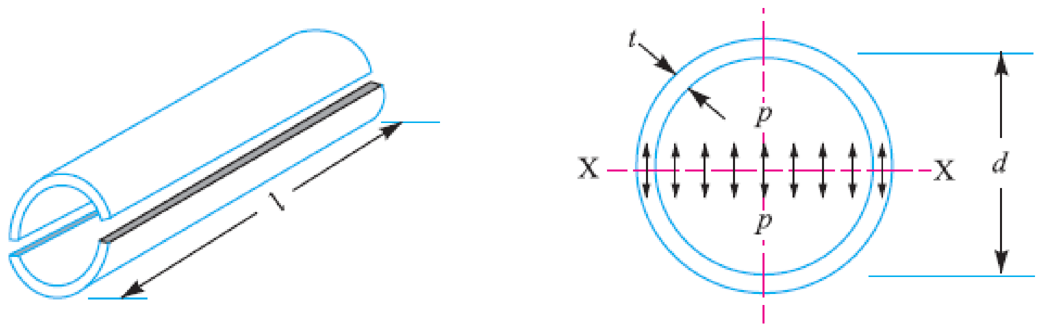


(*a*) Failure of a cylindrical shell along the longitudinal section.

(*b*) Failure of a cylindrical shell along the transverse section.

Fig. 7.1. Failure of a cylindrical shell.

When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following
two ways:
1. It may fail along the longitudinal section (i.e. circumferentially) splitting the cylinder into two troughs, as shown in Fig. 7.1 (a).
2. It may fail across the transverse section (i.e. longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. 7.1 (b). Thus the wall of a cylindrical shell subjected to an internal pressure has to withstand tensile stresses of the following two types:
(a) Circumferential or hoop stress, and (b) Longitudinal stress.
These stresses are discussed, in detail, in the following articles.



(a) View of shell. (b) Cross-section of shell.

Fig. 7.2. Circumferential or hoop stress.

Let p = Intensity of internal pressure,
d = Internal diameter of the cylindrical shell,
l = Length of the cylindrical shell,
t = Thickness of the cylindrical shell, and
σt_1 = Circumferential or hoop stress for the material of the cylindrical shell.
We know that the total force acting on a longitudinal section (i.e. along the diameter X-X) of the shell
= Intensity of pressure × Projected area = p × d × l ...(i)
and the total resisting force acting on the cylinder walls
= σt_1 × 2t × l ...(... of two sections) ...(ii)
From equations (i) and (ii), we have

$\sigma_{t1} \times 2t \times l = p \times d \times l$ *or*

$$\sigma_{t1} = \frac{p.d}{2t} \quad \text{or} \quad t = \frac{p.d}{2\sigma_{tl}}$$

The following points may be noted:
1. In the design of engine cylinders, a value of 6 mm to 12 mm is added in equation (iii) to permit reboring after wear has taken place. Therefore

$t = \frac{p.d}{2\sigma_{tl}} + 6 - 12\ mm$

2. *In constructing large pressure vessels like steam boilers, riveted joints or welded joints are used in joining together the ends of steel plates. In case of riveted joints, the wall thickness of the cylinder,*

$$t = \frac{p.d}{2\sigma_{tl}\gamma}$$

where ζ = Efficiency of the longitudinal riveted joint.

3. *In case of cylinders of ductile material, the value of circumferential stress (σ_{t1}) may be taken 0.8 times the yield point stress (σy) and for brittle materials, σ_{t1} may be taken as 0.125 times the ultimate tensile stress (σu).*

4. *In designing steam boilers, the wall thickness calculated by the above equation may be compared with the minimum plate thickness as provided in boiler code as given in the following table.*

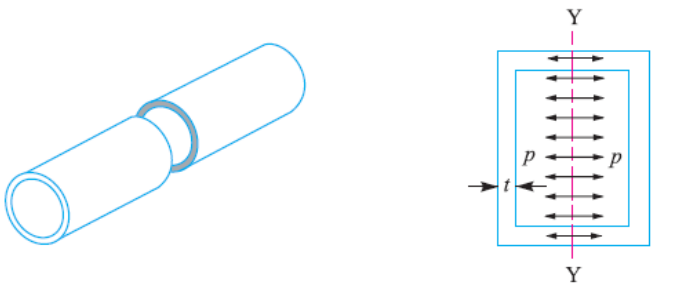
Table 7.1. Minimum plate thickness for steam boilers.

Boiler diameter	Minimum plate thickness (t)
0.9 m or less	6 mm
Above 0.9 m and upto 1.35 m	7.5 mm
Above 1.35 m and upto 1.8 m	9 mm
Over 1.8 m	12 mm

Note: *If the calculated value of t is less than the code requirement, then the latter should be taken, otherwise the calculated value may be used.*
The boiler code also provides that the factor of safety shall be at least 5 and the steel of the plates and rivets shall have as a minimum the following ultimate stresses.
Tensile stress, σ_t = 385 MPa
Compressive stress, σ_c = 665 MPa
Shear stress, τ = 308 MPa

Longitudinal Stress

*Consider a closed thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.3 (a) and (b). A tensile stress acting in the direction of the axis is called longitudinal stress. In other words, it is a tensile stress acting on the *transverse or circumferential section Y-Y (or on the ends of the vessel).*



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.3. Longitudinal stress.

Let σ_{t2} = Longitudinal stress.
In this case, the total force acting on the transverse section (i.e. along Y-Y)
= Intensity of pressure × Cross-sectional area

$$= p x \frac{\pi}{4} d^2 \quad \text{..........(i)}$$

and total resisting force = $\sigma_{t2} \times \pi d.t$ ...(ii)

From equations (i) and (ii), we have

$$p x \frac{\pi}{4} d^2 = \sigma_{t2} \times \pi d.t$$

$$t = \frac{p.d}{4\sigma_{t2}}$$

From above we see that the longitudinal stress is half of the circumferential or hoop stress. Therefore, the design of a pressure vessel must be based on the maximum stress i.e. hoop stress.

Example 7.2. ***A thin cylindrical pressure vessel of 500 mm diameter is subjected to an internal pressure of 2 N/mm^2. If the thickness of the vessel is 20 mm, find the hoop stress, longitudinal stress and the maximum shear stress.***

Solution. ***Given : d = 500 mm ; p = 2 N/mm^2 ; t = 20 mm***
We know that hoop stress,

$$\sigma_{t1} = \frac{p.d}{2t} = \frac{2x500}{2x20} = 25\ {}^{N}/_{mm^2} = 25 \quad Mpa$$

Longitudinal stress
We know that longitudinal stress,

$$\sigma_{t2} = \frac{p.d}{4t} = \frac{2x500}{4x20} = 12.5\ N/mm^2 = 12.5\ Mpa$$

Maximum shear stress
We know that according to maximum shear stress theory, the maximum shear stress is one-half the algebraic difference of the maximum and minimum principal

stress. Since the maximum principal stress is the hoop stress (σ_{t1}) and minimum principal stress is the longitudinal stress (σt_2), therefore maximum shear stress,

$\tau_{mX} = \frac{\sigma_{t1}-\sigma_{t2}}{2} = \frac{25-12.5}{2} = 6.25\ N/mm^2$*=6.25 Mpa* ***ANSWER.***

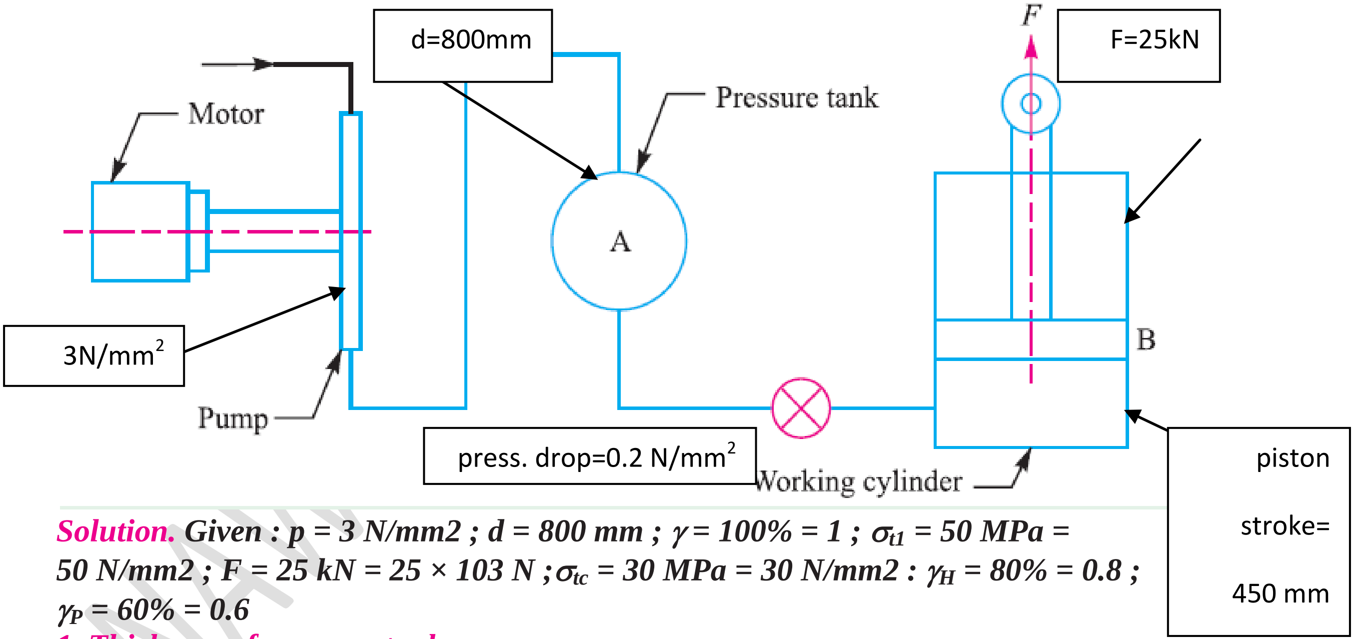
Example 7.3. ***An hydraulic control for a straight line motion, as shown in Fig. below utilises a spherical pressure tank 'A' connected to a working cylinder B. The pump maintains a pressure of 3 N/mm² in the tank.***

1. If the diameter of pressure tank is 800 mm, determine its thickness for 100% efficiency of the joint. Assume the allowable tensile stress as 50 MPa.

2. Determine the diameter of a cast iron cylinder and its thickness to produce an operating force F = 25 kN. Assume (i) an allowance of 10 per cent of operating force F for friction in the cylinder and packing, and (ii) a pressure drop of 0.2 N/mm2 between the tank and cylinder. Take safe stress for cast iron as 30 MPa.

3. Determine the power output of the cylinder, if the stroke of the piston is 450 mm and the time required for the working stroke is 5 seconds.

4. Find the power of the motor, if the working cycle repeats after every 30 seconds and the efficiency of the hydraulic control is 80 percent and that of pump 60 percent.



Solution. *Given : p = 3 N/mm2 ; d = 800 mm ; γ = 100% = 1 ; σ_{t1} = 50 MPa = 50 N/mm2 ; F = 25 kN = 25 × 103 N ; σ_{tc} = 30 MPa = 30 N/mm2 : γ_H = 80% = 0.8 ; γ_P = 60% = 0.6*

1. Thickness of pressure tank

We know that thickness of pressure tank,

$t = \frac{p.d}{2\sigma_{tl}\gamma} = \frac{3x800}{2x50x1}$*=24 mm*

2. Diameter and thickness of cylinder

Let D = Diameter of cylinder, and

t_1 = Thickness of cylinder.

Since an allowance of 10 per cent of operating force F is provided for friction in the cylinder and packing, therefore total force to be produced by friction,

$F_1 = F + \frac{10}{100}$*=1.1F=1.1x25x10³=27 500 N*

We know that there is a pressure drop of 0.2 N/mm² between the tank and cylinder, therefore pressure in the cylinder,

p_1 = Pressure in tank – Pressure drop = 3 – 0.2 = 2.8 N/mm^2
and total force produced by friction (F_1),

$$27500 = \frac{\pi}{4}.D^2.2.8 = 2.2D^2$$

D=112 mm

We know that thickness of cylinder,

$$t = \frac{p.d}{2\sigma_{tl}} = \frac{2.8x112}{2x30} = 5.2 \text{ mm} \qquad Ans.$$

3. Power output of the cylinder

We know that stroke of the piston
= 450 mm = 0.45 m ...(Given)
and time required for working stroke
= 5 s ...(Given)
∴Distance moved by the piston per second
=0.45/5=0.09 m
We know that work done per second
= Force × Distance moved per second
= 27 500 × 0.09 = 2475 N-m
∴ Power output of the cylinder
= 2475 W = 2.475 kW Ans. ...(1 N-m/s = 1 W)

4. Power of the motor

Since the working cycle repeats after every 30 seconds, therefore the power which is to be produced by the cylinder in 5 seconds is to be provided by the motor in 30 seconds.
∴ Power of the motor

$$\frac{\text{power of the cyl.}}{\gamma_H \gamma_p} x \frac{5}{30} = 0.86 \, kw \qquad Ans.$$

Lecture 16

Seals

Seals are devices used to prevent or limit leakage of fluids or particulates. The aims of this chapter are to introduce the variety of seal configurations, give guidelines for the selection of seals and introduce calculation methods for the quantification of some seal leakage rates.

Learning Objectives

At the end of this section you should:

- ***be able to identify a number of the different types of sealing devices;***
- ***be able to select a seal type for rotating, reciprocating or static conditions;***
- ***be able to determine the groove dimensions for a standard O ring;***
- ***be able to estimate the leakage flow through a labyrinth seal.***

Introduction

The purpose of a seal is to prevent or limit flow between components. Seals are an important aspect of machine design where pressurized fluids must be contained within an area of a machine such as a hydraulic cylinder, contaminants excluded or lubricants retained. Seals fall into two general categories.

1 Static seals, where sealing takes place between two surfaces that do not move relative to each other.

2 Dynamic seals, where sealing takes place between two surfaces that move relative to each other by, for example, rotary or reciprocating motion. Any clearance between the two components will permit the passage of fluid molecules in either direction, the direction depending on the pressures and momentum associated with the fluid. The basic sealing problem is illustrated in Figure 16.1 where either boundary shown may be stationary or moving. Fluid can move between different regions of space by means of diffusion, free convection or forced convection. The size of a typical gas or vapor molecule is of the order of 10^{-9}m in diameter. They can therefore diffuse through very small gaps, such as pores in a machine casing or seal component. Convection involves the mixing of one portion of fluid with another by means of gross movements of the mass of fluid. The fluid motion may be caused by external mechanical means, such as by a fan or pump in which case the process is called forced convection. Alternatively if the fluid motion is caused by density differences, due to for instance temperature differences, the process is called natural or free convection.

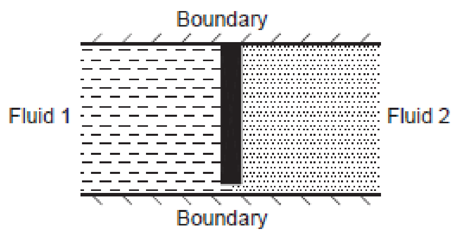


Figure 16-1 The basic sealing geometry.

Static seals

Static seals aim at providing a complete physical barrier to leakage flow. To achieve this the seal material must be resilient enough to flow into and fill any irregularities in the surfaces being sealed and at the same time remain rigid enough to resist extrusion into clearances.

Elastomeric seal rings

The 'O' ring, Figure 16-2, is a simple and versatile type of seal with a wide range of applications for both static and dynamic sealing. An 'O' ring seal is a molded elastomeric ring 'nipped' in a cavity in which the seal is located. Elastomeric seal rings require the seal material to have an interference fit with one of the mating

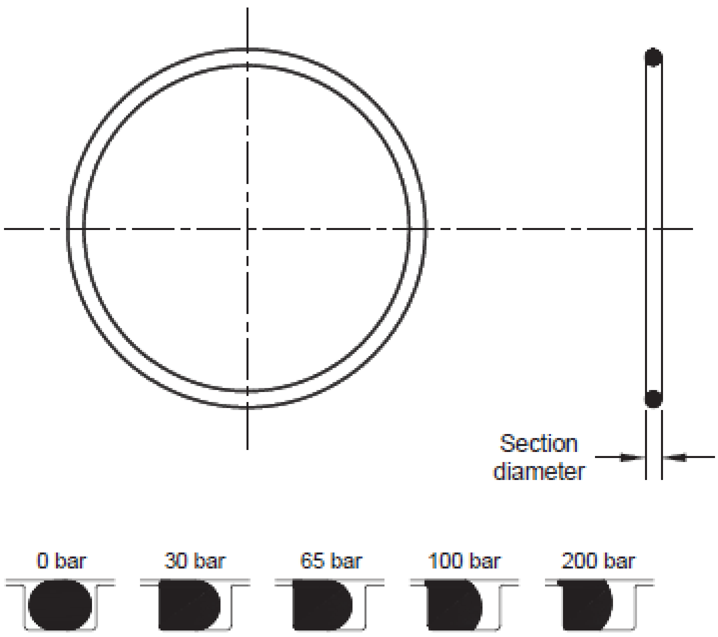


Figure 16-2 Principle of operation of 'O' rings sealing a fluid against pressure.

parts of the assembly. 'O' rings are available in a wide range of sizes with internal diameters from 3.1 to 249.1 mm and section diameters of 1.6, 2.4, 3.0, 4.1, 5.7 and 8.4 mm as defined in British Standard BS 4518. Table 32 shows the dimensions for a small number of the seal sizes available. A more extensive table is normally available in the form of a sales catalogue from manufacturers or in BS 4518.Figure 9.6 shows the groove dimensions which must be specified to house the 'O' ring seal and ensure the seal is nipped or compressed sufficiently to enable effective sealing.

Table 32 'O' ring seal dimensions (mm) (limited range tabulated only for illustration)

Reference number	Internal diameter	Section diameter	$d_{nominal}$ (Fig. 9.6a)	$D_{nominal}$ (Fig. 9.6a)	$D_{nominal}$ (Fig. 9.6b)	$d_{nominal}$ (Fig. 9.6b)	B	R
0031-16	3.1	1.6	3.5	5.8	6	3.7	2.3	0.5
0041-16	4.1	1.6	4.5	6.8	7	4.7	2.3	0.5
0051-16	5.1	1.6	5.5	7.8	8	5.7	2.3	0.5
0061-16	6.1	1.6	6.5	8.8	9	6.7	2.3	0.5
0071-16	7.1	1.6	7.5	9.8	10	7.7	2.3	0.5
0081-16	8.1	1.6	8.5	10.8	11	8.7	2.3	0.5
0091-16	9.1	1.6	9.5	11.8	12	9.7	2.3	0.5
0101-16	10.1	1.6	10.5	12.8	13	10.7	2.3	0.5
0111-16	11.1	1.6	11.5	13.8	14	11.7	2.3	0.5
0036-24	3.6	2.4	4	7.7	8	4.3	3.1	0.5
0046-24	4.6	2.4	5	8.7	9	5.3	3.1	0.5
0195-30	19.5	3.0	20	24.8	25	20.2	3.7	1.0
0443-57	44.3	5.7	45	54.7	55	45.3	6.4	1.0
1441-84	144.1	8.4	145	160	160	145	9.0	1.0
2491-84	249.1	8.4	250	265	265	250	9.0	1.0

Source: British Standard BS4518.

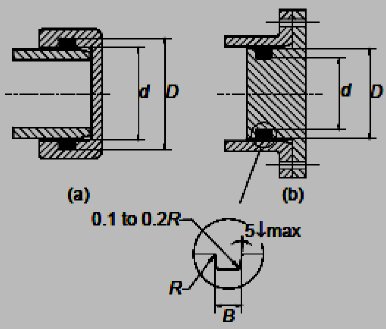


Figure 16.3 O' ring groove dimensions.

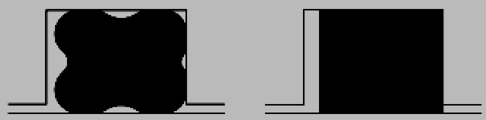


Figure 9.7 X ring and rectangular seal.

Example 16.1
Specify suitable groove dimensions for an 0195-30 'O' ring to seal against a solid cylinder
Solution
From Table 32 and with reference to Figure 9.6, B=3.7mm, R=1mm, groove fillet radius=0.2mm.

Gaskets

A gasket is a material or composite of materials clamped between two components with the purpose of preventing fluid flow. Gaskets are typically made up of spacer rings, a sealing element, internal reinforcement, a compliant surface layer and possibly some form of surface antistick treatment. Figure 16.4 shows a typical application for a gasket seal. When first closed a gasket seal is subject to compressive stresses produced by the assembly. Under working conditions, however, the compressive load may be relieved by the pressures generated within the assembly or machine. This must be accounted for in the detailed design or by use of a factor to allow for the relaxation of gasket compression. Typical gasket designs are illustrated in Table 33The choice of material depends on the temperature of operation, the type of fluid being contained and the leakage rate that can be tolerated.

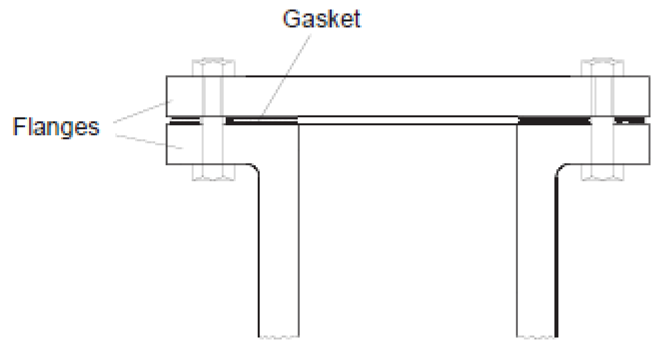
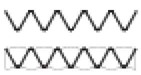


Figure 16.4 Typical gasket application.

Table 33 Typical gasket designs

Type	Cross-section	Comment
Flat		Available in a wide variety of materials. Easily formed into other shapes
Reinforced		Fabric or metal reinforced. Improves torque retention and blowout resistance in comparison with flat types
Flat with rubber beads		Rubber beads located on flat or reinforced material. Gives high unit sealing pressure
Flat with metal grommit		The metal grommit gives protection to the base material
Plain metal jacket		The metal jacket gives protection to the filler on one edge and across the surface
Corrugated or embossed		Corrugations provide increased sealing pressure capability
Profile		Multiple sealing surfaces
Spiral wound		Interleaving pattern of metal and filler

Reproduced with alterations from Czernik, 1996.

Dynamic seals

The term 'dynamic seal' is used to designate a device used to limit the flow of fluid between surfaces that move relative to each other. The range of dynamic seals is extensive with devices for both rotary and reciprocating motion. The requirements of dynamic seals are often conflicting and require compromise. Effective sealing may require high contact pressure between a stationary component and a rotating component, but minimal wear is also desired for long seal life.

Seals for rotating machinery

The functions of seals on rotating shafts include retaining working fluids, retaining lubricants and excluding contaminants, such as dirt and dust. The selection of seal type depends on the shaft speed ,working pressure and desired sealing effectiveness.
.Their application to rotating shafts is generally limited to use when the shaft speed is below 3.8 m/s and seal pressures below 14 bar. The typical geometry for a radial lip seal, commonly known as an oil ring, is shown in Figure 16.5.

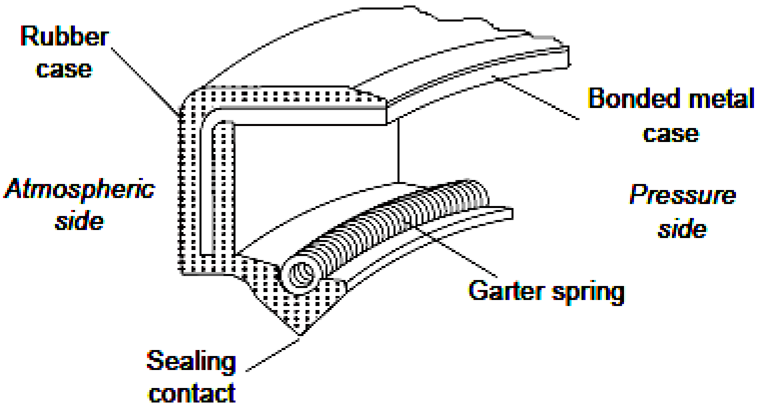


Figure 16.5 **Radial lip seal.**

A labyrinth seal in its simplest form consists of a series of radial fins forming a restriction to an annular flow of fluid as shown in Figure 16.6. In order for the fluid to pass through the annular restriction, it must accelerate. Just after the restriction the fluid will expand and decelerate with the formation of separation eddies in the cavity downstream of the fin as illustrated schematically in Figure 16.7.These turbulent eddies dissipate some of the energy of the flow reducing the pressure. This process will be repeated in subsequent cavities until the pressure reaches downstream conditions. Labyrinth seals are essentially a controlled clearance seal without rubbing contact. As there is no surface-to-surface contact, very high relative speeds are possible and the geometry can be arranged to limit leakage to tolerable levels.

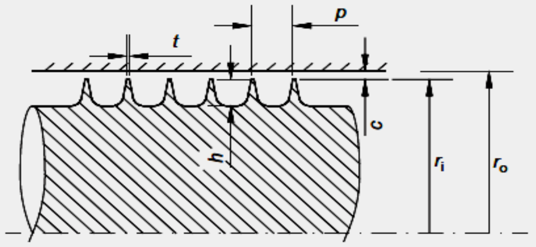


Figure 16.6 Labyrinth seal geometry.

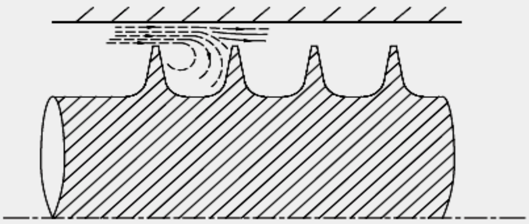


Figure 16.7 Labyrinth seal flow.

Flow through a labyrinth can be estimated using Eq.

$$\dot{m} = A\alpha\gamma\varphi\sqrt{\rho_o P_0}$$

$\dot{m}$: mass flow rate (kg/s)

$A = \pi(r_0^2 - r_i^2)$ ***:area of the angular gap between the fin tips and the casing (m^2)***

α: flow coefficient

γ :carry over correction factor

φ: expansion ratio

P_0: upstream pressure (Pa)

ρ_0: density at the upstream conditions(kg/cm^3)

The carry over correction factor, γ, varies as a function of the clearance to pitch ratio.As a crude approximation,   can be taken as varying linearly for c/p values of 0 to 0.11 as listed in Table 34.

Table 34 Relationships for the carry over correction factor as a function of fin number

Carry over correction factor	Number of fins
$\gamma = 1 + 3.27(c/p)$	2
$\gamma = 1 + 5(c/p)$	3
$\gamma = 1 + 6.73(c/p)$	4
$\gamma = 1 + 8.82(c/p)$	6
$\gamma = 1 + 10.2(c/p)$	8
$\gamma = 1 + 11.2(c/p)$	12

c, radial clearance (m); p, pitch (m)

The expansion ratio, φ , is given by:

$$\varphi = \sqrt{\frac{1-(\frac{P_n}{P_0})}{n+ln(\frac{P_0}{P_n})}}$$

where:

P_n:downstream pressure(Pa)

n: number of fins

Example
Determine the mass flow rate through a labyrinth seal on a 100 mm diameter shaft.The labyrinth consists of six fins, height 3.2mm, pitch 4.5mm, radial clearance 0.4 mm and tip width 0.3mm.The pressure is being dropped from 4 bar absolute, 353 K, to atmospheric conditions (1.01 bar).Take the gas constant R as 287 J/(kg K).
Solution
The outer radius of the annular gap is
(100/2)+3.2+0.4=53.6 mm. The inner radius of the annular gap is
(100/2)+3.2=53.2mm.
The annulus gap area is:
$A = \pi(r_0^2 - r_i^2)$ $=\pi[(53.6x10^{-3})^2-(53.2x10^{-3})]=1.342x10^{-4}\ m^2$

α=0.71 , γ=1+8.82(c/p)for n=6 , c=0.4 mm and p=4.5 mm so

γ=1+8.82(0.4/4.5)=1.784

$$\varphi = \sqrt{\frac{1-(\frac{P_n}{P_0})}{n+ln(\frac{P_0}{P_n})}} = \sqrt{\frac{1-(\frac{1.01x10^5}{4x10^5})}{n+ln(\frac{4x10^5}{1.01x10^5})}} = 0.3183$$

p=ρRT

$\rho_0=P_0/RT_0$ 1st law of thermodynamic

$\rho_0= 4x10^5/ 287x353)= 3.948\ kg/m^3$

$m^{\cdot}= 1.342x10^{-4}x0.71x1.784x0.3183\sqrt{3.948x4x10^5}=0.0680\ kg/s$ ans.