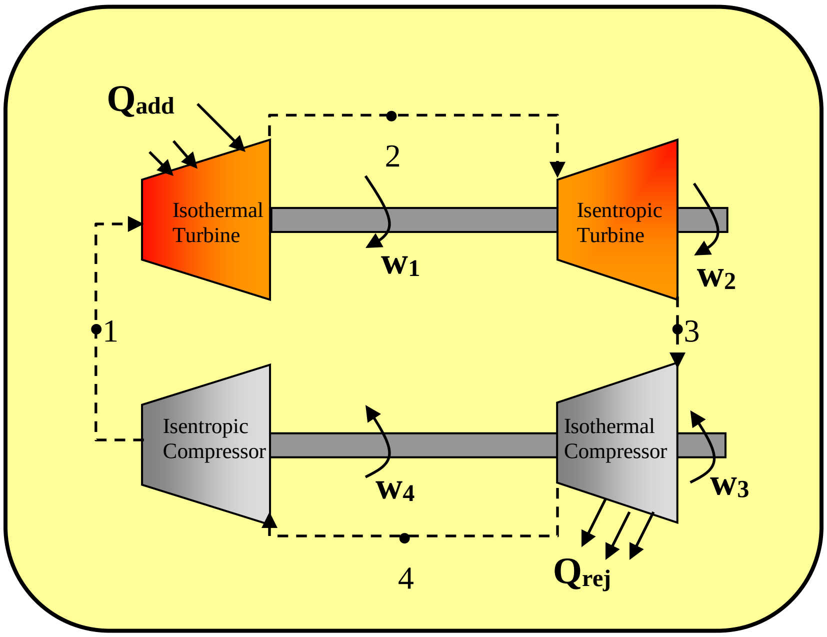
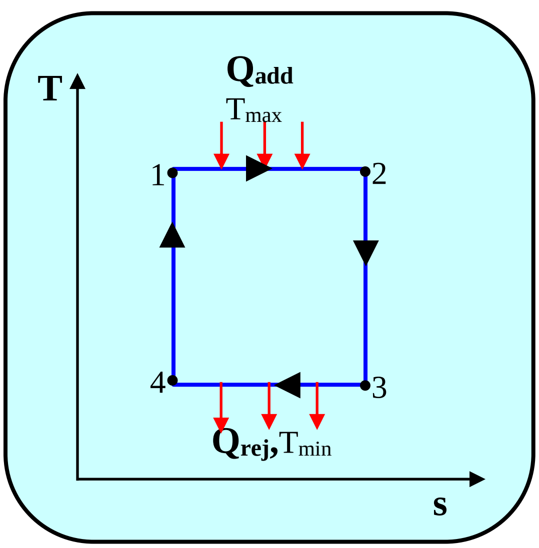
The Ministry of Higher Education
& Scientific Research
Northern Technical University
Technical College / Kirkuk
Mechanical Power Technique Eng. Dept.

Thermodynamic

For first year students





Subject	Year of study	Hours in week			Units
		Theory	Practical	Total	
Thermodynamic	First year students	2	2	4	6

Aim of the subject: - Define the students the foundation of thermo- dynamic which A/C equipment working unit, also studying the relation & rules of 1^{st} &2^{nd} law of thermodynamic.

Course Weekly out line

Week No.	Syllabus
1	Introduction – Reference – Units
2	Important definition – force – pressure - system
3	Atmospheric , gauge & absolute pressure – unit of pressure
4	Temperature, its units & transformation, zero law.
5	Definition of energy – kinetic & potential energies – work – power flow & internal energy – enthalpy- energy diagram
6	Definition of state – property, process – property diagrams -1^{st} law of thermo dynamic, (P- V) diagram.
7	General equation of an ideal gasses – energy equation cyclic process –work done for closed system
8	Ideal gases – ideal gasses laws (boyle , Charles , Gaylosic)
9	Gas constant – Avogadro law specific heat at constant volume &pressure
10	Particular closed system processes – constant volume &constant pressure processes
11	Constant temperature
12	– adiabatic &polytrophic processes
13	Open flow system application of open flow system
14	Steam ,steam formation, the (p,v) phase diagram
15	Dryness fraction ,liquid line ,steam line ,wet steam
16	Calculation of steam ,steam table
17	Superheated steam ,super heated steam table
18	Steam process with drawing each processes on (P-

	V)diagram
19	**2nd law of thermodynamic –heat engine heat pump**
20	**Statement of2nd law of thermodynamic (Kelvin, Planck &Clausius statement**
21	**Carnot cycle - reversed Carnot cycle**
22	**Reversible &i Reversible processes**
23	**Entropy – calculation – (T-S) diagram**
24	**Entropy equation for an ideal gasses.**
25	**representation on (T-S) diagram all system processes**
26	**Entropy change in irreversible process**
27	**Air standard cycles – Otto cycles**
28	**Diesel cycle -dual cycle**
29	**Steam cycles – simple Rankine cycle**
30	**Rankine cycle with super heated**

References:-

1- Foundation of thermodynamics .5th. Edition by sonntay
2- Engineering thermodynamics by Yunus A. Çengel, 4th. Edition
3- Applied thermodynamic for engineering tech. by Eastop, 3rd. Edition
4- Thermodynamic and transport properties of fluid SI units, arranged by Y .R. Mayhew & G.F, C. Rogers

Unit one

Introduction of thermodynamic

Mechanical engineering thermodynamic:-
Energy transferred between heat and work.
Heat transferred to work by heat engine.
Work transferred to heat by heat pump or ref.

Force (F): - the force acting 1 kg mass moving with acceleration of m/sec^2.

Force S.I unit (N).

$N = (kg. m/sec^{2)}$.

Mass S.I unit (kg), symbol (m).

Acceleration S.I unit (m/sec^2), symbol (a).

Gravitational acceleration, symbol (g). $g = 9.81\ m/sec^2$.

$F = m*a$

$= kg * m/sec^2 = N$

For weight (w) = m* g = N

Pressure (p): - defined as the force per unit area, symbol.

$P = F/A = N/m^2$, where A =area (m^2).

Absolute pressure (p_a):- weight of liquid column per unit area.

$$Pa = \frac{F}{A} = \frac{m * g}{A} = \frac{\rho * V * g}{A} = \frac{\rho * A * h * g}{A} = \rho * h * g$$

$Pa = \rho * h * g$

Where:-
ρ = density of liquid (kg/m^3).

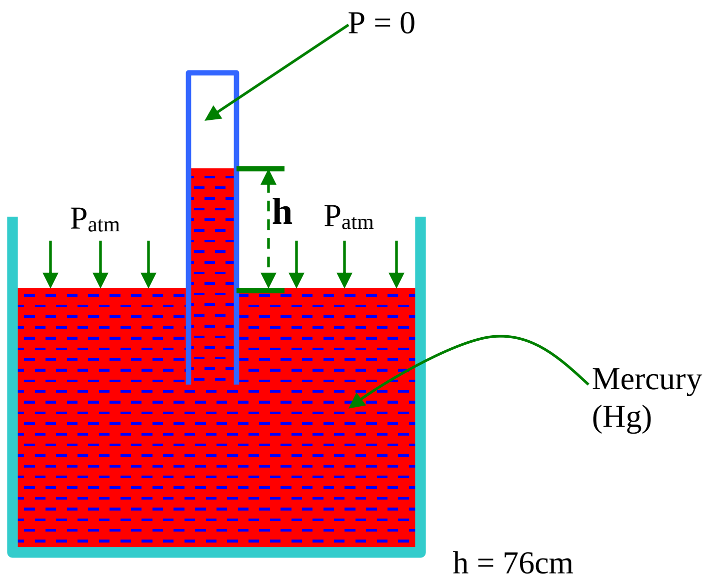
V = volume of liquid (m^3).

h = height of liquid (m).

$Pa = (kg/m^{3)} * m * (m/sec^2) = (kg) * (m/sec^2)/m^2 = N/m^2$.

Atmospheric pressure (p_{atm}) :- weight of atmospheric per unit area of earth's surface.

The standard instrument for recording of (p_{atm}) is the barometer as shown in fig .



The density of mercury (Hg) = 13616 kg/m^3.

P_{atm} is measure in meters of mercury (m.Hg).

The conversion from (m.Hg) to (N/m^2) is given by: - for example,

1 (m.Hg) = ρ * h * g

= 13616 * 9.81*1

= 133660 N/m^2

= 1.3366 * 10^5 N/m^2 = 1.3366 bar = 133.66 KN/m^2

1 bar = 10^5 N/m^2 =10^2 KN/m^2

P_{atm} = 1 atmosphere = 1.01325 bar

=101.325 KN/m^2 = 76 cm Hg = 0.76 m. Hg

= 13616 * 0.76 * 9.81

= 101325 N/m^2

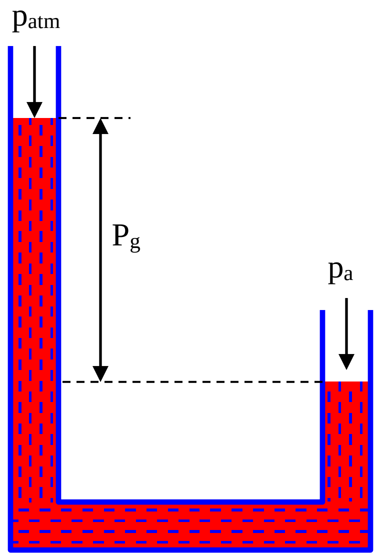
Gauge pressure (p_g):- it is defined as:-

$$p_g = Pa - P_{atm}$$

Where:-

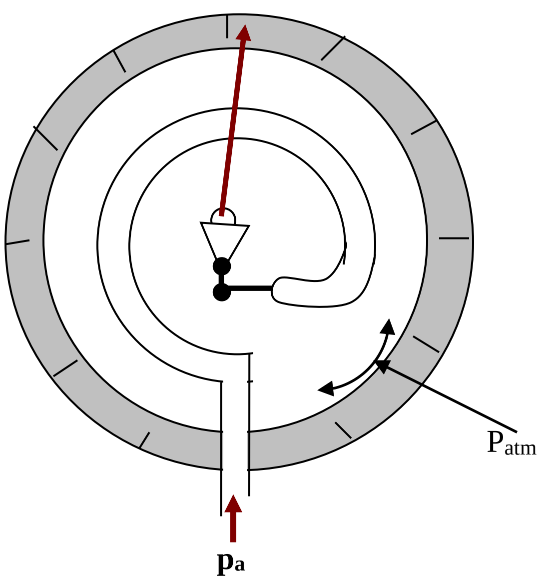
P_{atm} = 1.01325 bar

1- p_g is measured by differential manometer in which the P to be measured applied to one line of U tube containing barometer fluid to the other side as shown in fig.



(Differentional manometer)

2- p_g is measured with bourdon gauge :-



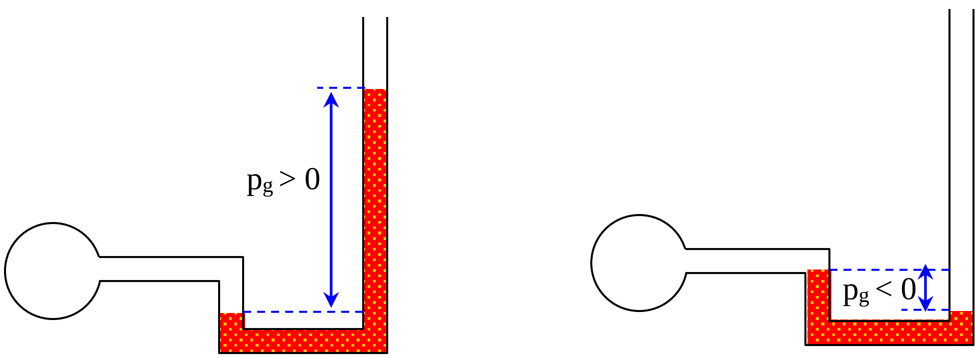
To convert the pressure reading from a bourdon gauge to a (p) measured we used,

$p_g = p_a - p_{atm}$

p_g = + vie when $p_a - p_{atm} > 0$

p_g = - vie when $p_a - p_{atm} < 0$ or $p_{atm} - p_a > 0$

Both bourdon gauge & manometers can be read either positive or negative (p_g) as shown in fig.



The (-ve) value of p_g is called vacuum which given by,

$p_g = p_{atm} - p_a$

Vacuum = $p_{atm} - p_a$

EX: - A steam turbine exhausted in to a condenser the gauge on which read (0.65 m.Hg) vacuum. Express this in absolute pressure, assuming the atmosphere pressure is 1.013bar.

Sol:-
Vacuum = $p_{atm} - p_a$
Vacuum = 0.65 * 13616 * 9.81

= 86822.4N/m^2

= 0.868224 bar

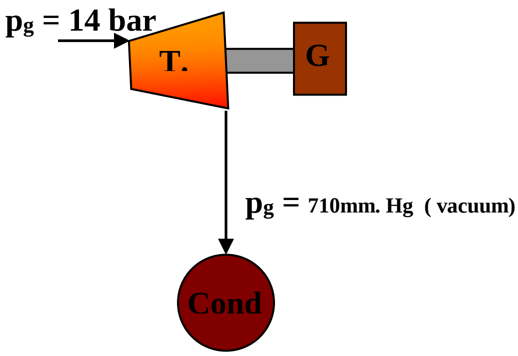
$p_a = p_{atm}$ – Vacuum

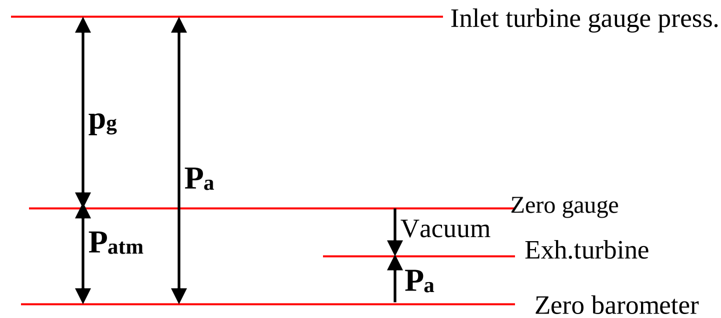
= 1.013 -0.868224

= 0.145 bar

EX: - A turbine is supplying with steam at a gauge pressure of 14 bar, after expansion in the turbine the steam passed to a condenser which is maintained at a vacuum of 710mm.Hg by means of pumps the barometer pressure is 770 mm.Hg. Express the inlet & exhaust steam absolute pressure in N/m^2.

Sol:-





<u>Inlet turbine</u>

$p_g = p_a - p_{atm}$

$p_{atm} = 770$ mm.Hg = 0.77 m.Hg * 13616 * 9.81

= 102851 N/m^2

= 1.02851* 10^5 N/m^2 =1.02851 bar

$p_a = p_g + p_{atm}$

= 14 + 1.02851 = 15.0285 bar = 15.0285*10^5 N/m^2

<u>Exhausted from turbine</u>

Vacuum = 710 mm.Hg = 0.71 m.Hg * 13616 * 9.81

= 94836N/m^2 = 0. 94836bar

$p_a = p_{atm}$ – vacuum

=1.02851-0. 94836= 0.08015 bar =0.08015*10^5 N/m^2=8.015*10^3 N/m^2

EX:- If the vacuum in the condenser is 720mm.Hg and barometer reading is 763mm.Hg find the condenser absolute pressure and atmosphere pressure in bar.

Sol:-

Vacuum = 720 mm.Hg = 0.72 m.Hg * 13616 * 9.81

= 0. 961725312bar

$p_{atm} = 763$ mm.Hg = 0.763 m.Hg * 13616 * 9.81

= 102851 N/m^2

= 1.019161685* 10^5 N/m^2 =1.019161685 bar

$p_a = p_{atm}$ – vacuum =1.019161685 -0. 961725312 = 0.0574336373 bar

EX: - Find the pressure in bar for an oil of density is 0.8kg/m^{3}and height of 30cm.

Sol:

$P = \rho * h * g$

$= 0.8 * 0.3 * 9.81 = 2.3544\ N/m^2 = 0.000023544$ bar

EX:- A vessel is connected to tube manometer the difference in the liquid level is 20 mm find the pressure in the vessel if the liquid is: - (a) water, (b) Hg

Sol:-

(a) Water
$P = \rho * h * g$

$= 1000 * 0.02 * 9.81 = 196.2 N/m^2 = 0.001962$ bar

(b) Hg.

$P = \rho * h * g$

$= 13616 * 0.02 * 9.81 = 2671.4592 N/m^2 = 0.026714592$bar

Temperature

Temperature: - Is the property which determines the ability of the system to transfer heat.

Temperature scale: - Is an arbitrary set of number and method for assigning each number to a definition level of temperature for example the melting point of ice and boiling point of water at standard atmosphere pressure

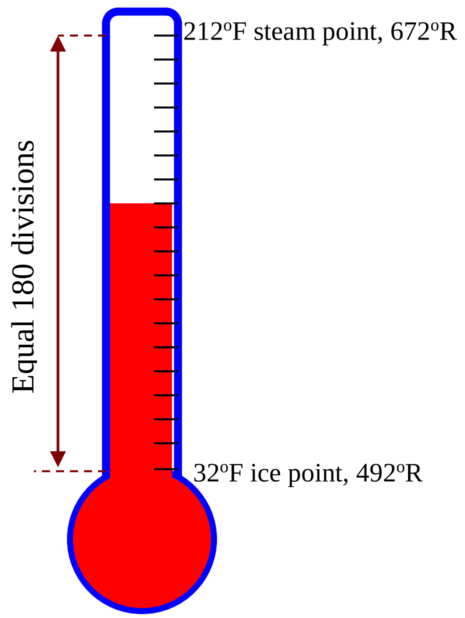
Thermometer: - is a measuring device yielding a number at each temperature level the number is functionally related to the temperature.

Actual thermometer is based on changes of certain property with temperature. For example:-

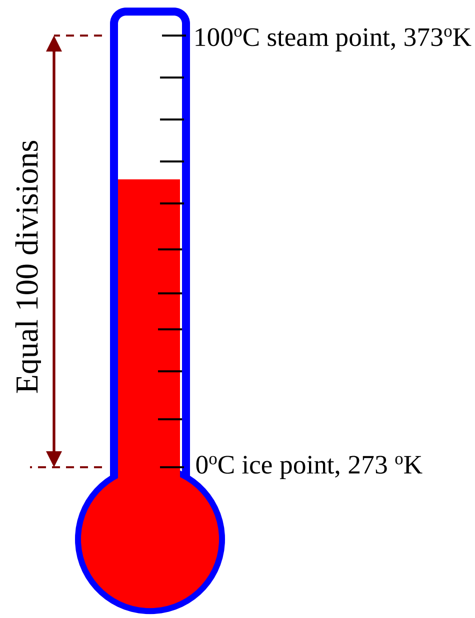
a- Volumetric expansion of gases, liquid, and solids.
b- Pressure exerted by gases.
c- Electrical resistance of solids.
d- Vapor pressure of liquids.
e- Thermo electricity.

- The mercury in glass thermometer provides an example of the observed quantity and height of the column of mercury dependent not only on temperature but also the external pressure acting on the bulb and stem.
- The ice point or freezing point of water occurs at 273.15 $^oK = 0\ ^oC$ and the steam point of boiling point of water occurs at 373.15 $^oK= 100\ ^oC$. The interval is100 ^oC and this basis of Celsius scale whose determine ice point 0 ^oC.

- On the Fahrenheit scale interval between the ice and steam point is divided in to 180^o while the datum is the freezing point at 32^oF and steam point at212^oF.
- The absolute scale corresponding to the Fahrenheit scale is the Rankine scale.



Fahrenheit thermometer



Celsius thermometer

$$\frac{{}^oC - 0}{100} = \frac{{}^oF - 32}{180} = \frac{{}^oK - 273}{100} = \frac{R - 492}{180}$$

$${}^oF = \frac{9}{5}\,{}^oC + 32 \longrightarrow {}^oF = {}^oR - 460$$

$${}^oC = \frac{5}{9}({}^oF - 32) \longrightarrow {}^oC = {}^oK - 273$$

EX: - At p_{atm} the boiling point of water is 100^oC and for helium $15\ ^oK$. Express these in^oF.

Sol:-

For water

$^oF = 9/5\ ^oC + 32 = 9/5 * 100 + 32 = 212\ ^oF$ boiling point of water.

For helium

$^oC = {}^oK - 273$

$= 15 - 273 = -258\,^oC$

$^oF = 9/5\,^oC + 32$

$= 9/5 * (-258) + 32 = -432.4\,^oF$ boiling point of helium

EX: - The following fixed point at p_{atm} are given in ^oF & ^oR. Express them to degree Celsius. Oxygen point 297.32 ^oF, gold point 2400^oR, sulfur point 832.28 ^oF , Antimony 1620 ^oR.

Sol:-
Oxygen point

$$\frac{^oC - 0}{100} = \frac{^oF - 32}{180}$$

$$\frac{^oC}{100} = \frac{297.32 - 32}{180}$$

$^oC = 147.4^o$

gold point

$$\frac{^oC - 0}{100} = \frac{^oR - 492}{180}$$

$$\frac{^oC}{100} = \frac{2400 - 492}{180}$$

$^oC = 1060^o$

Sulfur point

$$\frac{^oC - 0}{100} = \frac{^oF - 32}{180}$$

$$\frac{^oC}{100} = \frac{832.28 - 32}{180}$$

$^oC = 444.6^o$

For Antimony

$$\frac{{}^{o}C - 0}{100} = \frac{{}^{o}R - 492}{180}$$

$$\frac{{}^{o}C}{100} = \frac{1620 - 32}{180}$$

${}^{o}C = 882.222^{o}$

Thermal Equilibrium: - If a temperature of two bodies is equal when heat flows between them, two bodies is called bodies at thermal equilibrium.

Zero law of thermodynamics: - when there are two bodies in thermal equilibrium with third body, the two bodies is become in state of thermal equilibrium and this defined as (Zero Law of thermodynamic).

Energy

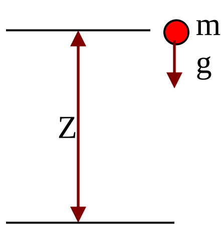
Energy: - Is the capacity either latent or apparent to exert a force through the distance.

$\boxed{E = F * S}$ …….. Joule (N.m)

Where:-
E = Energy
F = Force
S = distance

Potential Energy "pot. E.":- It is energy stored in a mass by virtue of its position in gravitational.

$\boxed{Pot.\ E = m.\ g\ .Z}$ ... Joule (N.m)



Where:-
Z = height

Kinetic energy "K.E":- it is energy wanted to move a mass with velocity.

$\boxed{K.E = \frac{1}{2}\ m\ V^2}$ …….. Joule (N.m)

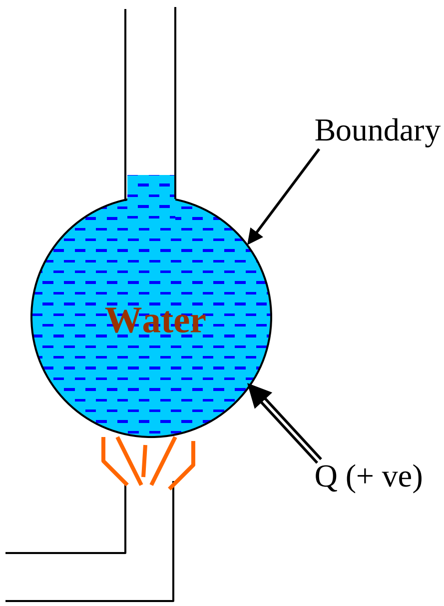
Where:-

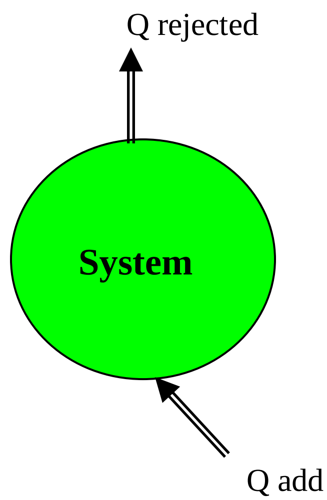
V= velocity m/sec

Heat "Q":- is regarded as energy in thermo. If it crossing the boundary of the system by virtue of temperature difference.

Q = KJ or Kcal

q = KJ /kg or Kcal / kg





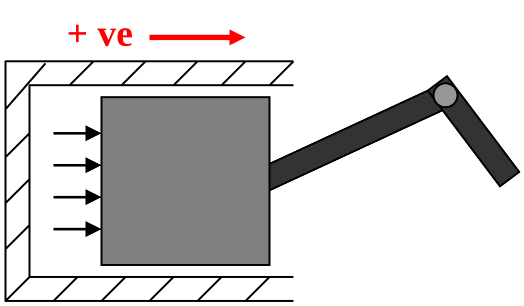
Q = (+ ve) when added to the system.
Q = (- ve) when rejected from system.

System: - Is a region neither necessary of constant volume nor fixed in space, where transfer of mass & energy are to be studied.

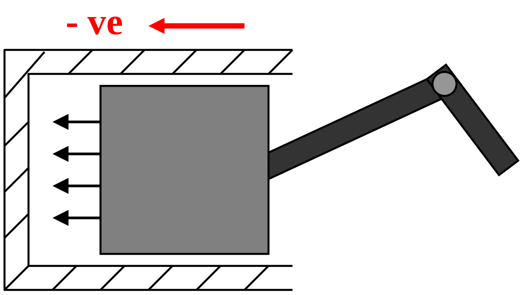
Boundary: - The actual envelope in closing the system is defined as boundary.

Surrounding: - The region out side the system is defined as surrounding.

Work: - It is the mechanical energy crossing the boundary of system and the only effect of which is rising of a weight out side or inside the system.



W = kJ
w = kJ /kg



Work is (+ ve) if it is done by the system on the surrounding.
Work is (- ve) if it is done on the system from the surrounding.

Internal energy (U):- It is energy stored in the mass of substance in the system.

U = kJ
u = kJ/kg

$$\frac{du}{dT} = Cv$$

du = Cv . dT

$$\int_{u_1}^{u_2} du = \int_{T_1}^{T_2} Cv \, . \, dT$$

$u_2 - u_1 = Cv\,(T_2 - T_1)$ or

$$\boxed{\Delta u = Cv.\,\Delta T}$$

$U_2 - U_1 = m.\,Cv\,(T_2 - T_1)$ or

$$\boxed{\Delta u = m.Cv.\,\Delta T}$$

Flow energy (F.E):- It is energy necessary to flow or to move a fluid at steady rate without changing its state.
The force necessary to move the piston without changing P, V, T of the gas.

F= p * A

The energy necessary to move the gas without changing its state = the energy necessary to move piston a distance "L".

F.E = p*A* L = p* V = m * p* ν (Joule)

Where (ν) = specific volume m^3 /kg = 1/ρ, where (ρ) = density kg/m^3.

Flow energy per unit mass of the gas (F.E /m) = p.ν (J /kg)

$$\boxed{\textbf{F.E/mass} = 10^2 * p * \nu \ (kJ / kg)}$$

Where:-
(p)= pressure (bar)
(ν) = specific volume m^3 /kg

Power: - is the rate of doing work, and is measured in (J/sec) or (N.m / sec) = watt,

1metric (h.P) = 75 kg

F.m = 75 * 9.81 = 735.75 watt ≈ 740 watt

1 h.P = 0.74 kW.

1 British h.P = 0.745 kW = 745 watt.

1Cal = 4.1868 joule or 1kCal = 4.1868 kJ
Enthalpy (H):- is the sum of internal energy plus flow energy of a moving gas.

$$\mathbf{H = U + p.V \longrightarrow kJ}$$

$$\mathbf{h = u + p.\nu \longrightarrow kJ/kg}$$

As Uis property and p,V are properties of the gas therefore H is a property.
EX: - Express the following quantities in term of kJ and kCal.
a- h.P. hour (h.P.h). b- kW .hour (kW .h).
Sol:-
a-
h.p = 0.7457 kW = 0.7457 kJ /sec

h.p.h = 0.7457 kJ /sec * 3600 sec = <u>2685 kJ</u>

1kCal = 4.1868 kJ

2685 / 4.186 = <u>640.2 kCal</u>

b-
kW.h = 1 kJ /sec *3600 sec = <u>3600 kJ</u>

1kCal = 4.1868 kJ

3600/ 4.186 = <u>859.845 kCal</u>

EX: - Nitrous Oxide of density 31.084 kg/m^3 is carried in a pipe line 60.96m above sea level the gas is at temperature 148.9°C. It flows along a pipe at the rate of 0.096m/sec and its specific at constant volume (Cv) is 0.161kCal / kg.K, taking sea level as datum for heights

and 0°C as datum for energy involving temperature evaluate the potential energy, kinetic energy, internal energy and total energy of the gas.
Sol:-

$\rho = 31.084\ kg/m^3$

$h\ (Z) = 60.96m$

$T = 148.9^oC$

$C = 0.096\ m/sec$

$Cv = 0.161\ kCal\ /kg\ .K$

$Cv = 0.161 * 4.186 = 0.673946\ kJ\ /kg.K$

$P.E = m.g.Z(J)$

$= g.Z\ (J/kg)$

$= 9.81 * 60.96 = 598.0176\ kJ/kg$

$K.E = ½\ m * C^2\ (J)$

$= ½\ C^2\ (J/kg)$

$= ½\ (0.096)^2 = 0.004608\ kJ/kg$

$U = m.\ Cv\ .T\ (J)$

$u = Cv.T(J/kg)$

$= 0.673946 * (148.9 +273) = 284.337\ kJ/kg$

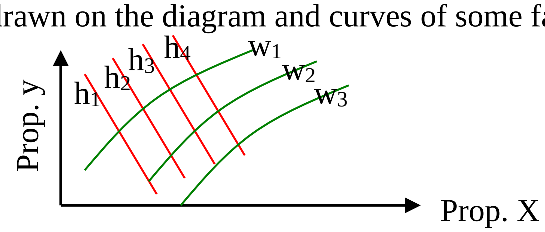
$T.E = Pot\ .E + K.E +u$

$= 598.0176 + 0.004608 + 284.337 = 882.3592\ kJ/kg$

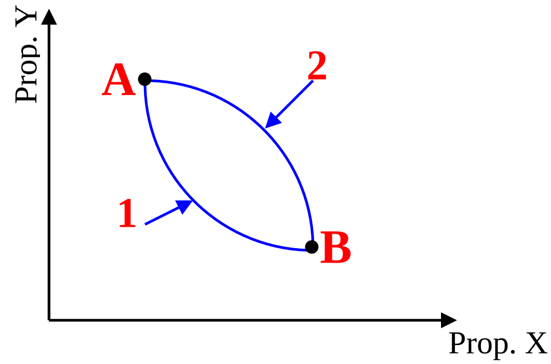
Property: - It is an observable characteristic of a system or part of a system.

State: - It is the condition of the system or part of system, at any constant of time, described by its properties.

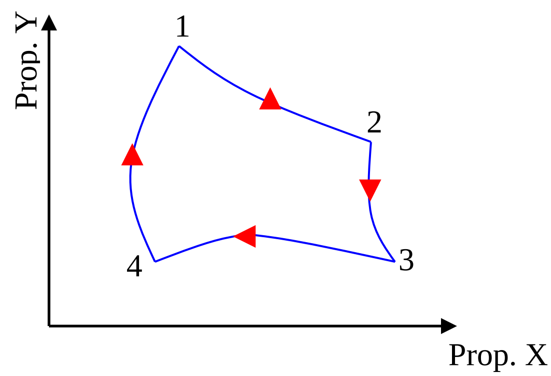
Property diagram: - It is a plane described by two coordinates representing two in depended properties; a point in the plane represents state families of curves of other properties such as T, W and H can be drawn on the diagram and curves of some families could not intersect.



Process: - It is locus of state represented on property diagram at which gas take to the same point from primary state or inlet state to the end of final state either through the process A (1) B or A (2) B.



Cycle: - It is sum of different processes such that the final process brings the substance to the original state.



Problems

1-1 A turbine is supplied with steam at a gauge pressure of 14bar. After expansion in the turbine the steam passes to a condenser which is maintained at a vacuum of 710mm Hg by means of pumps. The barometric pressure 772mm Hg. Express the inlet and exhaust steam (absolute) pressures N/m^2. Take the density of mercury as 13.6*10^{3}kg/m^3

(1502999, 8271.8N/m^2)

1-2 A mercury manometer measuring the pressure in a steam plant condenser reads 63cm vacuum. The barometer in the plant room reads 76cm of mercury. What is the absolute pressure in the condenser in N/m^2?

(17344N/m^2)

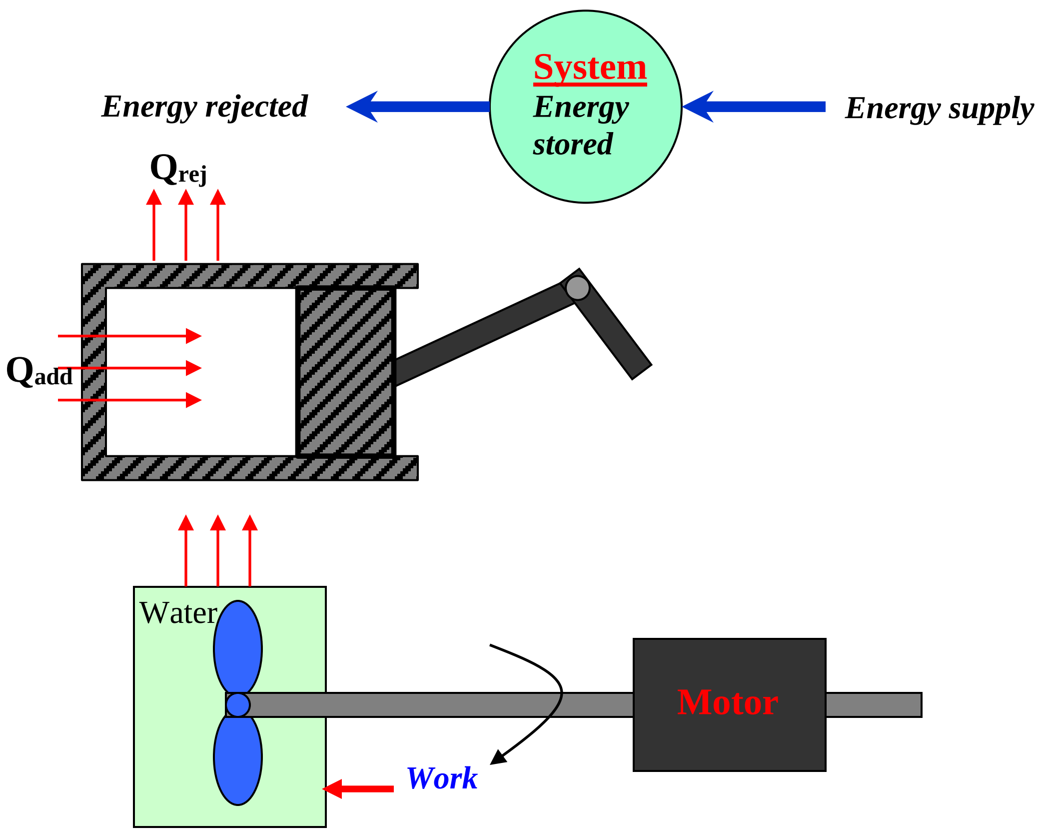
Unit two

The first law of thermodynamics

The first law of thermodynamics

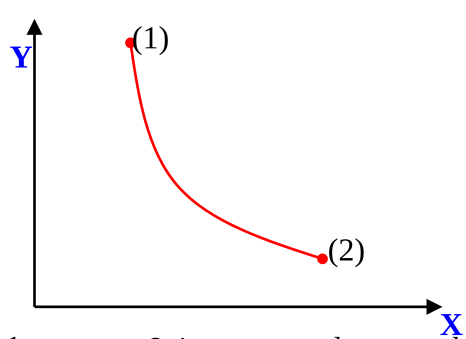
The first law of thermodynamics means that Energy can neither be created nor dissipated it can be transferred from a type to another.

Energy supplied = energy rejected + stored energy



Application to close system process

For the process shown on the property diagram the system changes its state from (1) to (2) through the shown process while heat energy "Q_{12}" is supplied to it,



Work "W_{12}" done by the system & increase or decrease the stored energy will appear as a change in pot. E, K.E, and U.

$Q_{12} = W_{12}$ + stored energy

$$\mathbf{Q_{12} = W_{12} + \Delta\ Pot.E_{12} + \Delta\ K.E_{12} + \Delta\ U_{12}}$$

$$\mathbf{Q_{12} = W_{12} + \Delta\ E_{12}} \text{ ——— } \textbf{(1) energy equation}$$

Where:-

$$\mathbf{\Delta\ E_{12} = \Delta\ Pot.E_{12} + \Delta\ K.E_{12} + \Delta\ U_{12}}$$

Usually in closed system processes the change in Pot.E and change in K.E is negligible. Thus the Equ. (1) Reduced to $Q_{12} = W_{12} + \Delta\ U_{12}$

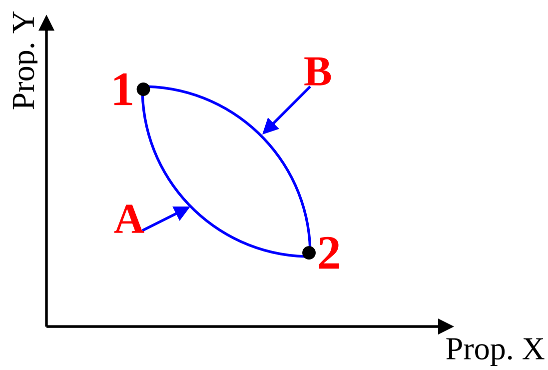
$\boxed{\mathbf{Q_{12} = W_{12} + \Delta\ U_{12}}}$ ———— for closed system process

Cyclic process:-

In cyclic process the total change in stored energy should equal at sum of changes of stored energy during the different processes constituting the cycle as substance is returned back to the original state then there is no change in the stored energy and thus :-

$$\boxed{\oint dQ = \oint dw}$$

The symbol $\oint$, which is called the cyclic integral

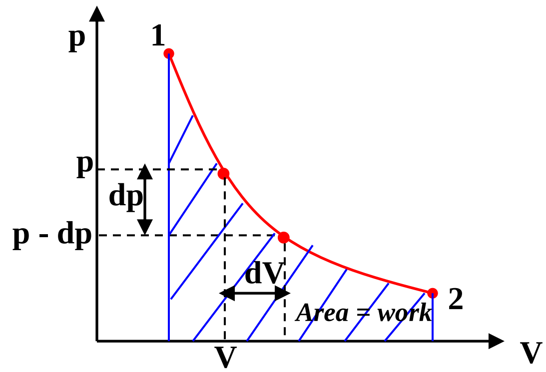


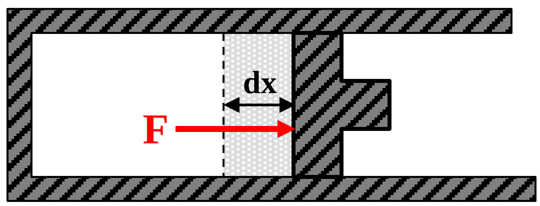
Working done at the moving boundary a closed system:-

For the shown system in diagram the cylinder is filled with a certain gas and fitted with a friction less piston against the certain resistance. The force necessary to move the piston is:-

F = P * area of the piston = P* A

Assume that the piston moves every small distance dx, such that the pressure between V & V + dV will be assumed not to change.





The energy – work done to move the piston the infinitesimal distance should equal to:-

Work done = F * dx = p*A*dx

dw = p* dV

The total work done developed by the gas on the piston a sit moves from (1) to (2),

$\int_1^2 dw = \int_1^2 p.dV$, there fore,

$$\boxed{w_{12} = \int_1^2 p.dV}$$

w_{12} can be calculated by measuring the area under the process represented on the p.V diagram.

EX: - The energy of system increases 120 kJ while 150 kJ work is transferred to surrounding. Is heat added or taken a way from the system.

Sol:-

$\Delta E_{12} = + 120$ kJ

$w_{12} = + 150$ kJ

$Q_{12} = w_{12} + \Delta E_{12}$

$= 150 + 120 = 270$ kJ added

EX: - During an expansion process. The work done and the heat received by the system are respectively 20 kJ and 50000 J. Calculate the change of energy for the system.

Sol:-

$w_{12} = + 20$ kJ

$Q_{12} = + 50000$ J $= 50$ kJ

$Q_{12} = w_{12} + \Delta E_{12}$

$\Delta E_{12} = Q_{12} - w_{12} = 50 - 20 = +30$ kJ

EX: - A closed system consists of a cylinder of water stirred by a paddle wheel. For the process the work was 34 kJ the initial internal energy was 120 kJ and the final internal energy after (1) hour of stirring was 144 kJ. Find the heat transferred in kJ /hour. Is the temperature of the system raising or falling.

Sol:-

$w_{12} = - 34$ kJ

$U_1 = 120$ kJ

$U_2 = 144$ kJ

$\Delta U_{12} = 144 - 120 = 24$ kJ

$Q_{12} = w_{12} + \Delta U_{12} = - 34 + 24 = - 10$ kJ / h the temperature is rising

Problems

2-1 During the expansion of 1kg of a fluid the internal energy changes from an initial value of 1162kJ/kg to a final value of 1025kJ/kg of shaft work are obtained, calculate the quantity and direction of exchange between the fluid and surroundings. Assume expansion is frictionless. (52kJ out wards)

2-2 In a certain non – flow process with air as the working fluid, 1000kJ is removed from the system and the internal energy decreases by 200kJ. Is the process an expansion or compression? What work is performed? (800kJ compression work)

Unit three

Ideal gas

Ideal (perfect) gas

Ideal gas it is the gas which has the following properties:-

1- It follows the general gas equation.
2- Its specific heat at constant pressure and specific heat at constant volume are constant.

Ideal gas laws:-

1- Boyle's law: - The pressure of the gas is inversely proportional to the volume at constant temperature.

$$p \propto \frac{1}{V} \quad \text{at } T = \text{constant}, \quad p = \frac{C}{V} \quad \text{or} \quad pV = \text{constant},$$

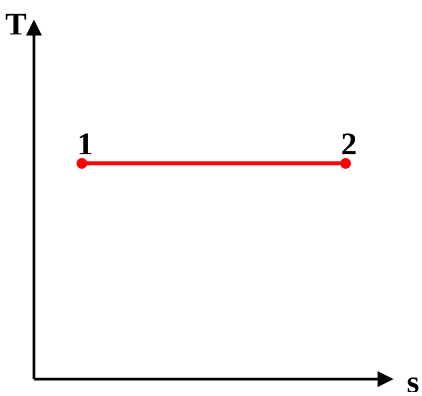
$$\boxed{p_1V_1 = p_2V_2} \quad \text{—— } \mathbf{p.V = C}$$

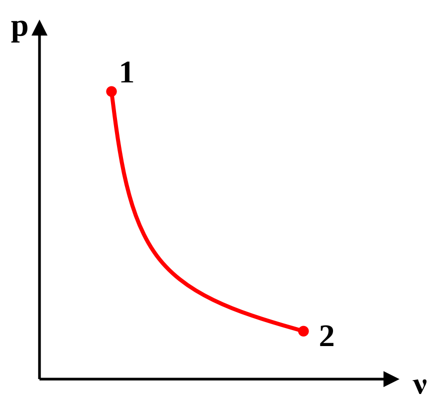
$$V = m\,.\,v \quad \text{or} \quad v = \frac{V}{m}$$

$$p_1.m\,.v_1 = p_2\,.\,m\,.v_2$$

$$p_1.v_1 = p_2\,.v_2$$

$$\boxed{\frac{p_1}{p_2} = \frac{v_2}{v_1} = \frac{V_2}{V_1}} \quad \text{—— at } T = \text{constant}$$

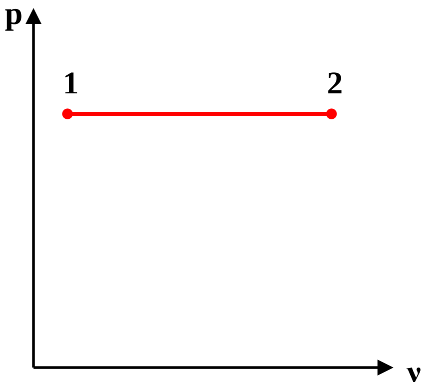




2- Charles law: - The volume of the gas is directly proportional to the temperature at constant pressure.

$V \propto T$ at p = constant, therefore $V = C.\,T$ or, V/T = const.

$$\boxed{\frac{T_2}{T_1} = \frac{v_2}{v_1} = \frac{V_2}{V_1}} \quad \text{—— at } p = \text{constant}$$

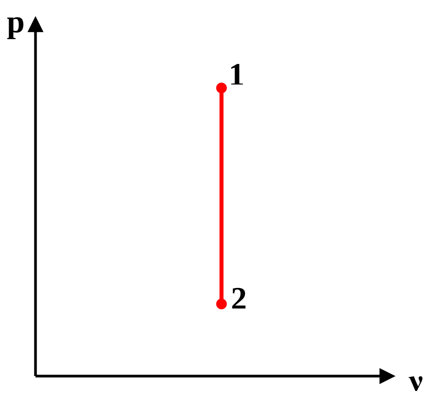


3- Gay – Lussac law: - The pressure of the gas is directly proportional to the temperature at constant volume.

$p \propto T$ at V = constant, therefore $p = C.T$ or,

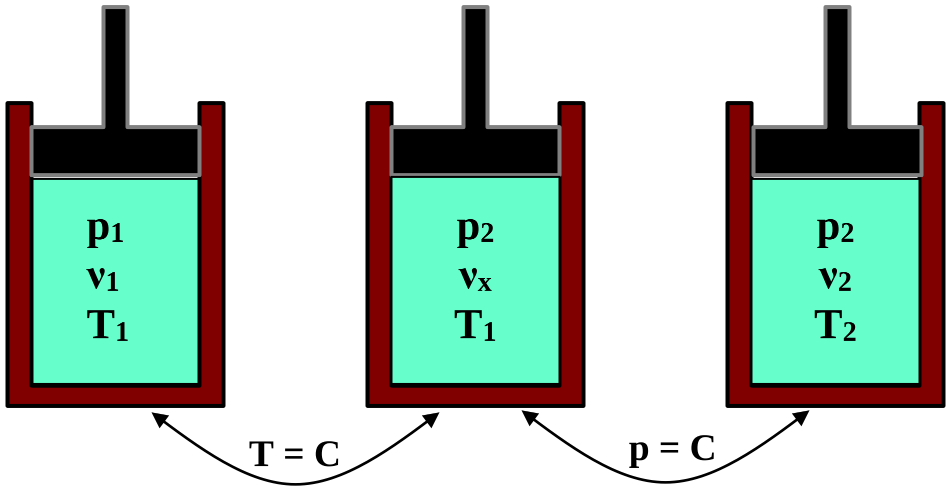
$$\frac{p}{T} = C$$

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$ ———— at V = constant



Equation of State of an Ideal Gas:-

A relationship between p, V& T



Boyle's law

$$p_1 v_1 = p_2 v_x$$

$$v_x = \frac{p_1}{p_2} * v_1$$

Charles law

$$\frac{v_x}{T_1} = \frac{v_2}{T_2}$$

$$v_x = \frac{T_1}{T_2} * v_2$$

$$\frac{p_1}{p_2} * v_1 = \frac{T_1}{T_2} * v_2$$

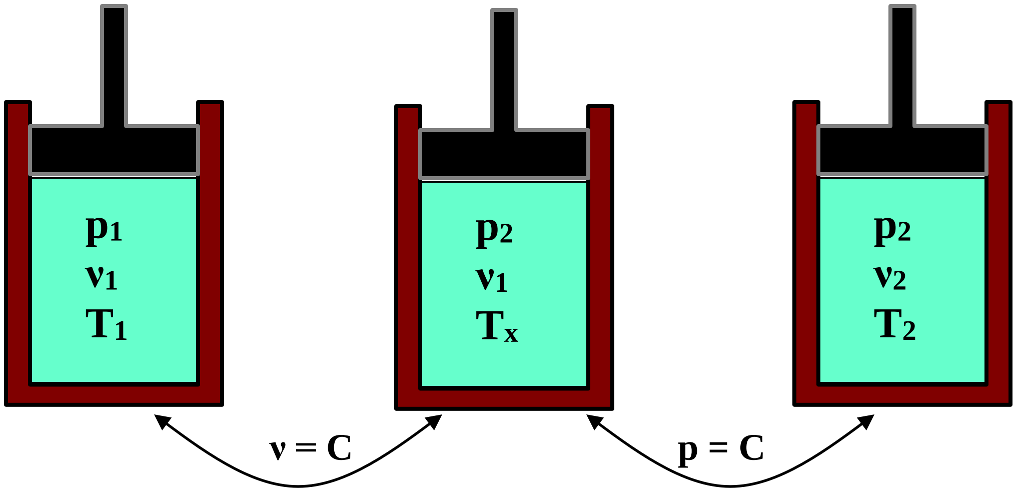
$$\boxed{\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{pv}{T} = R$$

Where (R) is the gas constant kJ/kg.K

$Pv = RT$

$\boxed{pV = mRT}$ — Is called general of state of an ideal gas (general equation of gas).



Gay – Lussac law

$$\frac{p_1}{T_1} = \frac{p_2}{T_x}$$

$$T_x = \frac{T_1}{p_1} * p_2$$

Charles law

$$\frac{v_1}{T_x} = \frac{v_2}{T_2}$$

$$T_x = \frac{T_2}{v_2} * v_1$$

$$\frac{T_1}{p_1} * p_2 = \frac{T_2}{v_2} * v_1$$

$$\boxed{\frac{v_2 * p_2}{T_2} = \frac{v_1 * p_1}{T_1}}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{pv}{T} = R$$

Where (R) is the gas constant kJ/kg.K

$Pv = RT$

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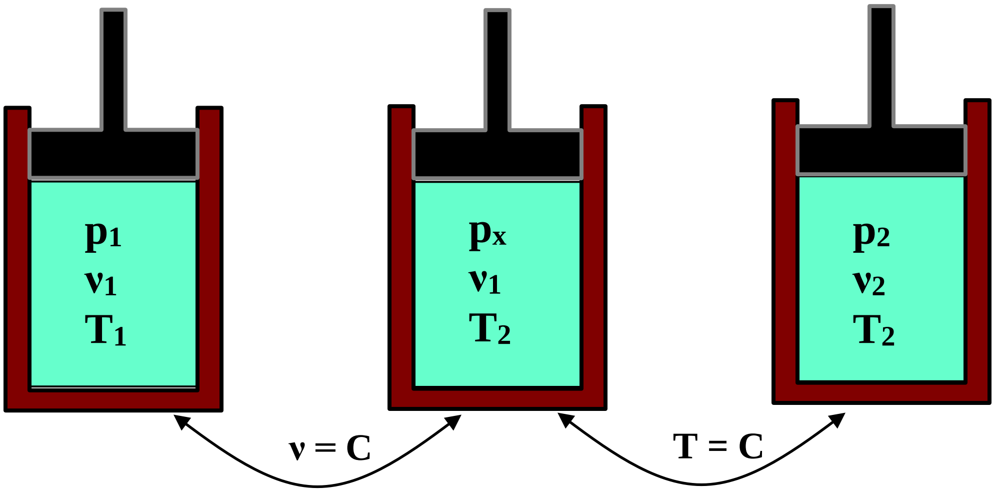


Gay – Lussac law

$$\frac{p_1}{T_1} = \frac{p_x}{T_2}$$

$$p_x = \frac{T_2}{T_1} * p_1$$

Boyle's law

$$p_x v_1 = p_2 v_2$$

$$p_x = \frac{p_2}{v_1} * v_2$$

$$\frac{T_2}{T_1} * p_1 = \frac{p_2}{v_1} * v_2$$

$$\boxed{\frac{v_2 * p_2}{T_2} = \frac{v_1 * p_1}{T_1}}$$

= R

T_1 T_2 T

Where (R) is the gas constant kJ/kg.K

$Pv = RT$

$\boxed{pV = mRT}$ — Is called general of state of an ideal gas (general equation of gas).

EX: - An ideal gas occupies a volume of 0.105 m^3 at temperature 20 C^o and the pressure of 1.5 kg f /cm^2 absolute. Find the final temperature if the gas compressed to pressure7.5 kg f / cm^2 and occupies a volume of 0.04 m^3 (note 1kg f / cm^2 = 1bar).

Sol:-

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

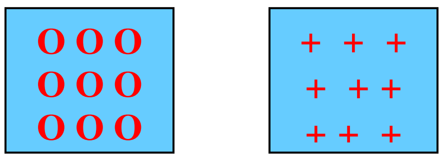
$$T2 = \frac{p_2\, v_2}{p_1\, v_1} * T_1$$

$T_1 = 20 + 273 = 293\ ^oK$

$$T2 = \frac{7.5 * 0.04}{1.5 * 0.105} * 293 = 558\ ^oK$$

Avogadro law: - All ideal gases have the same amount of molecular in equal volume at the same pressure and temperature.

Gas A Gas B



V V

M_1 M_2

Where:-
M = molecular weight.

n = is the number of moles.

$m_1 = M_1.n \longrightarrow$ gas A

$m_2 = M_2.n \longrightarrow$ gas B

$$v_1 = \frac{V}{m_1} = \frac{V}{M_1.n} \longrightarrow v_1\, M_1 = \frac{V}{n}$$

$$v_2 = \frac{V}{m_2} = \frac{V}{M_2.n} \longrightarrow v_2\, M_2 = \frac{V}{n}$$

Therefore,
$v_1\, M_1 = v_2\, M_2 = v_3\, M_3 = v\, M$

At the standard condition (i .e $p = p_o = 1$ bar, & $T = T_o = 273\ ^oK$). The const. (M. v) is obtained for 1 kg. mol of any gas and found to be, $M\, v_o = 22.4\ m^3 / mol$

1kg mol of O_2 = 32 kg

N_2 = 28 kg

H_2 = 2kg

CO = 28 kg

CO_2 = 44kg

CH_4 = 16kg

$$\mathbf{v_o = \frac{22.4}{M}\ (m^3/kg)}$$

p.V = m.R.T

10^2 p.V = m.R.T, where p = bar

$$10^2\, p * \frac{V}{m} = R.T$$

10^2 p.v = R.T

$10^2\, p_o.v_o = R_o.T_o$ …. At standard condition

$$10^2 * 1 * \frac{22.4}{M} = R * 273$$

$$M.R = \frac{10^2 * 22.4}{273} = 8.314\ kJ / mol .K$$

Therefore MR = G = 8.314 kJ / mol .^{o}K which is called the universal gas constant and from which you find the specific gas constant. (R) For each gas as following.

$$\mathbf{R = \frac{G}{M} = \frac{8.314}{M}}$$

For example:-

$$R_{O2} = \frac{8.314}{32} = 0.2598\ kJ/kg .K$$

$$R_{N2} = \frac{8.314}{28} = 0.2969\ kJ/kg .K$$

$$R_{CO2} = \frac{8.314}{44} = 0.1889 \text{ kJ/kg.K}$$

<u>Note:-</u>
The general equation of gas (10^2 p .V = m R T) where p = bar, V =m^3, m = kg, R = kJ/kg .K

EX: - A vessel of volume 0.2m^3contains nitrogen at 1.013 bar and 15°C if 0.2kg of nitrogen is now pumped in to vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molecular weight of nitrogen is 28, and it may be assumed to be a perfect gas.

Sol:-

$$\text{Gas constant, } R = \frac{8.314}{M} = \frac{8.314}{28} = 0.2969 \text{kJ/kg.K}$$

$p_1\ V_1 = m_1\ R\ T_1$

$$m_1 = \frac{p_1\ V_1}{R\ T_1} = \frac{10^2 * 1.013 * 0.2}{0.2969 * 288} = 0.237 \text{ kg}$$

(Where T_1 = 15 + 273 = 288°K).

0.2 kg of nitrogen are added, hence m_2 = 0.2 + 0.237 = 0.437kg. then for the final condition,

$10^2\ P_2V_2 = m_2\ R\ T_2$

But $V_2 = V_1$, & $T_2 = T_1$,

$$p_2 = \frac{m_2\ R\ T_2}{V_2} = \frac{0.437 * 0.2969 * 288}{10^2 * 0.2} = 1.87\text{bar}$$

Specific heat: - "C"

Specific heat it is it is the amount of heat add to unit mass of substance (1kg) to increase its temperature 1°C unit kJ /kg .K.
For solid and liquid, there is one value of "C".
But for gases, we have two value of "C" called Cp and Cv ,

Where:-
Cv is the specific heat at constant volume (kJ/kg.K).
Cp is the specific heat at constant pressure (kJ/kg.K).

Specific heat at constant volume "Cv":-

For a close system the energy equation is,

$Q_{12} = w_{12} + \Delta U_{12}$

For constant volume Cv = constant

$\Delta V = 0$,$w_{12} = 0$ therefore,

$Q_{12} = \Delta U_{12} = m\ Cv\ \Delta T$

Also,
$q_{12} = \Delta u_{12} = Cv\ \Delta T$

$$\boxed{\mathbf{Cv = \frac{\Delta u}{\Delta T}}}$$

Specific heat at constant pressure "Cp"

$q_{12} = w_{12} + \Delta u_{12}$

$= (p_2 .V_2 - p_1 .V_1) + u_2 - u_1$

$= (u_2 + p_2 .V_2) - (u_1 + p_1 .V_1)$

$q_{12} = h_2 - h_1 = \Delta h = Cp\ \Delta T$

$$\boxed{\mathbf{Cp = \frac{\Delta h}{\Delta T}}}$$

The relation between Cp & Cv:-

$H = p .V + U$

$\Delta H = \Delta\ p.V + \Delta\ U$

For a perfect gas, from equation pV = mRT .Also for a perfect gas, from Joule's law, U = m Cv T. Hence, substituting,

$\left(m .Cp .\Delta T = m . R .\Delta T + m .Cv .\Delta T \right) \div mR\Delta T$

$\boxed{\mathbf{Cp = R + Cv}}$ ….. Therefore Cp> Cv

Ratio of specific heats

The ratio of the specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma) or gas index.

$$\boxed{\gamma = \frac{Cp}{Cv}}$$

Relationships between Cp, Cv, R and γ can be derived from equation,

$$Cp = R + Cv$$

$$R = Cp – Cv$$

Dividing through by Cv

$$\frac{Cp}{Cv} - 1 = \frac{R}{Cv}$$

Therefore using equation, $\gamma = Cp / Cv$, then ,

$$\gamma - 1 = \frac{R}{Cv}$$

$$Cv = \frac{R}{(\gamma - 1)}$$

$$\left(Cv = \frac{R}{(\gamma - 1)} \right) * \gamma$$

$$Cv\,\gamma = \frac{\gamma\, R}{(\gamma - 1)}$$

Also from equation, $Cp = \gamma\, Cv$, substituting in equation ,

$$Cp = \frac{\gamma R}{(\gamma - 1)}$$

EX: - A certain perfect has specific heats as follows, Cp = 0.846kJ/kg.K, Cv = 0.657 kJ/kg.K. Find the gas constant and the molecular weight of the gas

Sol:-

$Cp - Cv = R$

$R = Cp - Cv$

$= 0.846 - 0.657 = 0.189$ kJ/kg.K

$R = 8.314 / M$

$M = 8.314 / 0.189$

$= 44$

EX:- A perfect gas has a molecular weight of 26 and a value of γ = 1.26. calculate Specific heat at constant pressure & the heat rejected per kg of gas , when the gas is contained in a riged vessel at 3 bar and 315 ºC and is then cooled until the pressure falls to 1.5 bar.

Sol:-

$R = 8.314 / M = 8.314 / 26 = 0.3198$ kJ/kg.K

$$Cv = \frac{R}{(\gamma - 1)} = \frac{0.3198}{(1.26 - 1)} = 1.229 \text{kJ/kg.K}$$

$Cp / Cv = \gamma$

$Cp = \gamma * Cv = 1.26 * 1.229 = 1.548$ kJ/kg.K

The volume remains constant for the mass of gas present, and hence the specific volume remains constant

$p_1 v_1 = R T_1$ & $p_2 v_2 = R T_2$

Therefore since $v_1 = v_2$

$T_2 = T_1 (p_2 / p_1)$

$= 588 (1.5 / 3) = 294$ K

Where $T_1 = 315 + 273 = 588$K

Heat rejected per kg of gas $= Cv (T_2 - T_1)$

$= 1.229 (588 - 294)$

$= 361$kJ/kg

Problems

3 -1 The molecular weight of carbon dioxide, CO_2, is 44. In an experiment the value γ for CO_2 was found to be 1.3. Assuming that CO_2 is a perfect gas, calculate the gas constant, R, and the specific heats at constant pressure and constant volume, Cp, Cv
(0.189 kJ/kg.K; 0.63kJ/kg.K; 0.819kJ/kg.K)

3-2 Oxygen, O_2, at 200 bar is to be stored in a steel vessel at 20°C the capacity of the vessel is 0.04m^3. Assuming that O_2 is a perfect gas, calculate the mass of oxygen that can be stored in the vessel. The vessel is protected against excessive pressure by a fusible plug which will melt if the temperature rises too high. At what temperature must the plug melt to limit the pressure in the vessel to 240bar? The molecular weight of oxygen is 32
(10.5 kg; 78.6°C)

3-3 A quantity of a certain perfect gas is compressed from an initial state of 0.085m^3, 1 bar to a final state of 0.034m^3, 3.9 bar. The specific heats at constant volume are 0.724 kJ/kg.K, and the specific heats at constant pressure are 1.02kJ/kg.K. The observed temperature rise is 146K. Calculate the gas constant, R, the mass of gas present, and the increase of internal energy of the gas. (0.296kJ/kg.K; 0.11kg; 11.63kJ)

Unit four

Particular closed system process

Particular closed system process

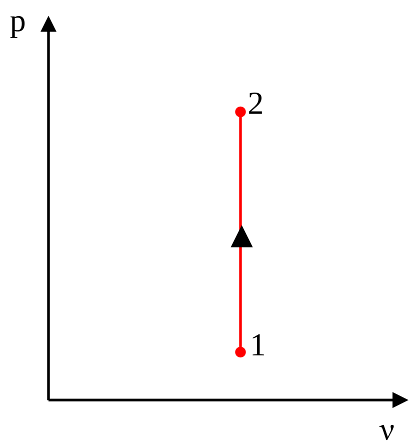
1- The isochoric process (constant volume process):- In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work can be done on or by the system, other than paddle wheel work input. It will be assumed that constant volume implies zero work unless stated otherwise.

From the non- flow energy equation,

$$Q_{12} = U_{12} + W_{12}$$

Since no work is done, we therefore have

$$Q_{12} = U_2 - U_1$$



$W_{12} = 0$ (Area under curve p.v diagram = 0)

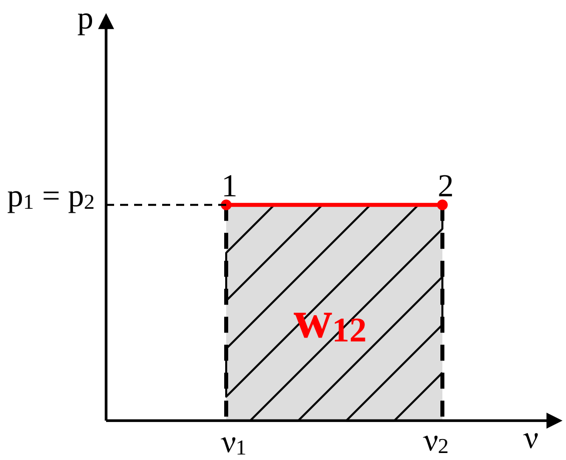
$Q_{12} = m\, Cv\, (T_2 - T_1)$

$v_1 = v_2$

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$ ------ p / T = constant

2- Isobaric process (constant pressure process):- for a constant pressure process the boundary must move against an external resistance as heat is supplied ; for instance a fluid in a cylinder behind a piston can be made to undergo a constant pressure process. Since the piston is pushed through a certain distance by the force exerted by the fluid, then work is done by the fluid on its surroundings.

$$W = \int_{v_1}^{v_2} p\, dv$$



Therefore, since p is constant,

$w = p\,(v_2 - v_1)$

From the non -flow energy equation,

$Q_{12} = W_{12} + \Delta U_{12}$

$q_{12} = w_{12} + \Delta u_{12}$

$= p\,(v_2 - v_1) + u_2 - u_1$

$= (p\,v_2 + u_2) - (p\,v_1 + u_1)$

Enthalpy, $h = p\,v_1 + u_1$, hence,

$q_{12} = h_2 - h_1$

or for mass ,m, of a fluid ,

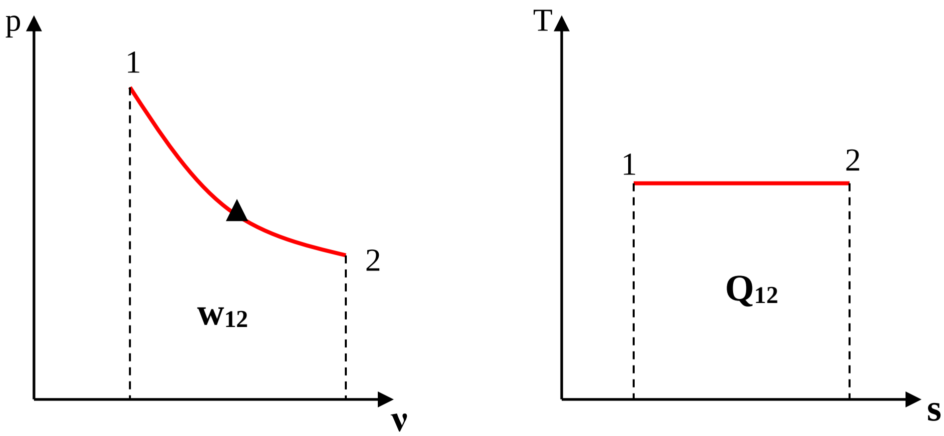
$Q_{12} = H_2 - H_1$
$Q_{12} = m\,Cp\,(T_2 - T_1)$

$q_{12} = Cp\,(T_2 - T_1)$

$$\frac{V}{T} = C = \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\boxed{\frac{\mathbf{V_1}}{\mathbf{T_1}} = \frac{\mathbf{V_2}}{\mathbf{T_2}}}$$ ------ V/ T = constant

3- The isothermal process (constant temperature process):- in an isothermal expansion heat must be added continuously in order to keep the temperature at the initial value. Similarly in an isothermal compression heat must be removed from the fluid continuously during the process.



$V_2 > V_1$ & $p_1 > p_2$

$T_2 = T_1$

When the temperature is constant as an isothermal process then we have,

$pV = mRT = \text{constant}$

Therefore for an isothermal process for a perfect gas,

$p.V = \text{constant}$ or $p.v = \text{constant}$

$\boxed{p_1.V_1 = p_2.V_2}$ ---- p.V = constant

$Q_{12} = W_{12} + \Delta U_{12}$

$\Delta U_{12} = m\,Cv(T_2 - T_1) = 0$

$Q_{12} = W_{12}$

$$W_{12} = \int_1^2 p\, dV$$

In this case, pV = constant, or p = c / V = m R T/ V (where c = constant).

$$W_{12} = \int_{V1}^{V2} (m\,R\,T/ V)\, dV$$

$$W_{12} = m\,R\,T \int_{V1}^{V2} (dV / V)$$

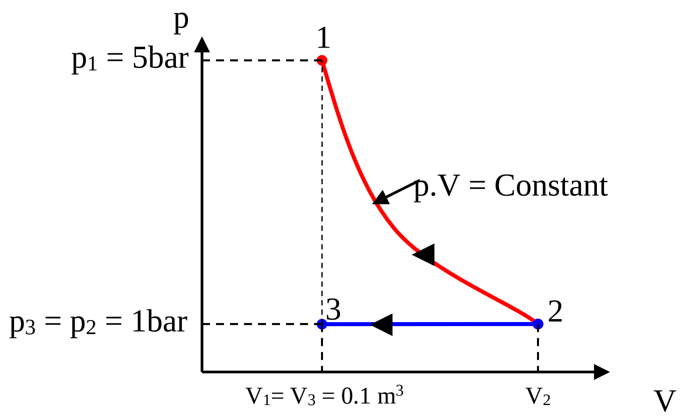
$$W_{12} = m\,R\,T\,(\ln V_2 - \ln V_1)$$

$= m\ R\ T\ \ln (V_2/V_1) = pV \ln (V_2/V_1)$

$$\boxed{W_{12} = m\ R\ T\ \ln (V_2/V_1)}$$

EX: - O_2 gas at a pressure of 5 bar, volume is (0.1) m^3 and temperature (1227C°) expand isothermally until its pressure is 1 bar it is then cooled isobarically to original volume. Calculate the change of internal energy, work done and heat transferred during each process.

Sol:-



$p_1 = 5$bar, $V_1 = 0.1\ m^3$, $T_1 = 1227 + 273 = 1500^oK$, $p_2 = 1$ bar

$T_2 = T_1 = 1500^oK$ (isothermal process)

$p_3 = p_2 = 1$bar (isobaric process)

$V_3 = V_1$ (cooled isobarically to original volume)

V_2 =? , T_3 =? , Q_{12} =? , W_{12} =? , $\Delta\ U_{12}$ =? , Q_{23} =? , W_{23} =? , $\Delta\ U_{23}$ =? ,

Process (1-2) isothermal process

$$Q12 = w_{12} = m\ .R\ .\ T \ln \frac{V_2}{V_1}$$

$$p_1.V_1 = p_2.V_2$$

$$V_2 = \frac{p_1.V_1}{p_2} = \frac{5 * 0.1}{1} = 0.5\ m^3$$

$$R = \frac{8.314}{MO_2} = \frac{8.314}{32} = 0.259\ kJ\ /kg\ .K$$

$10^2 . p_1 . V_1 = m .R .T_1$

$$m = \frac{10^2 . p_1 . V_1}{R .T_1} = \frac{10^2 * 5 * 0.1}{0.259 * 1500}$$

$Q_{12} = W_{12} = 0.128 * 0.128 * 1500 * \ln (0.5 / 0.1)$

$= 800$ kJ

Process (2-3) isobaric process

$\Delta U_{23} = m . Cv .(T_3 – T_1)$

$Cp = R + Cv$

$Cv = Cp – R = 1 – 0.259 = 0.741$ kJ /kg .K

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$T_3 = \frac{V_3}{V_2} * T_2 = \frac{0.1}{0.5} * 1500 = 300\ ^oK$$

$\Delta U_{23} = 0.128 * 0.791(300 – 1500)$

$= - 113.8$ kJ

$Q_{23} = \Delta H_{23} = m. Cp (T_3 – T_2)$

$= 0 .128 * 1 (300 – 1500)$

$= - 153.8$ kJ

$W_{23} = Q_{23} - \Delta U_{23}$

$= - 153.8 – (- 113.8) = - 40$ kJ

Or

$W_{23} = p_3 V_3 – p_2 V_2$

$= 10^2 * p_2 (V_3 – V_2)$

$=10^2 * 1 (0.1 – 0.5)$

$= -40$ kJ

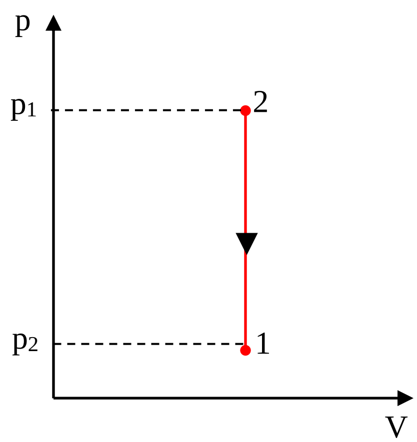
$Q_{23} = W_{23} + \Delta U_{23}$

$= -40 - 113.8$

$= -153.8$ kJ

EX: - Air in a closed vessel of fixed volume 0.15m^3, its pressure is 10 bar and at temperature 250°C. If the vessel is cooled so that the pressure falls to 2.5 bar, determine the final temperature, and the heat transferred. Take (Cv = 0.71kJ/kg .K, R = 0.286 kJ / kg.K).

Sol:-
Closed vessel = constant volume
$V_1 = 0.15m^3 = V_2$, $p_1 = 10$ bar, $T_1 = 250+273 = 523°K$, $p_2 = 2.5$bar, $T_2 = ?$, $Q_{12} = ?$



$$\frac{p_2}{T_2} = \frac{p_1}{T_1}$$

$$T_2 = \frac{p_2}{p_1} * T_1 = \frac{2.5}{10} * 523 = 130.75°K = -142.25\ °C$$

$Q_{12} = m\,Cv\,(T_2 - T_1)$

$10^2 .p_1.V_1 = m .R .T_1$

$$m = \frac{10^2 * 10 * 0.15}{0.286 * 523} = 1.003kg$$

$Q_{12} = 1.003 * 0.717\ (130.75 - 523) = -282.08$ kJ

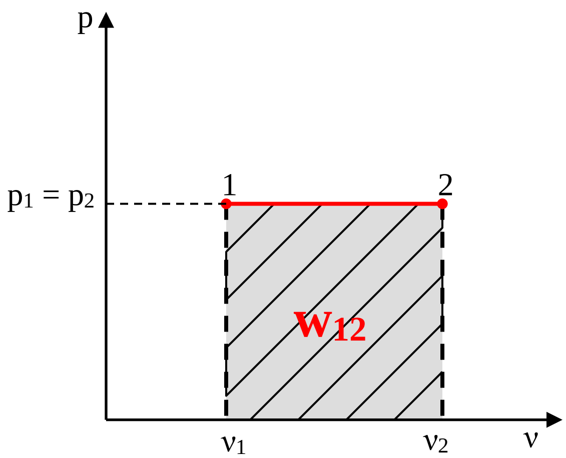
EX: - 2 kg of air initially at 150°C expand at constant pressure at 0.6bar until the volume is doubled. Take Cp = 1.005 kJ /kg.K, γ =1.4, Find,

a- the final temperature
b- the work transferred
c- the heat transferred

Sol:-

m = 2kg

$T_1 = 150+273 = 423$ K , p = 0.6bar , $V_2 = 2V_1$



$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$T_2 = \frac{V_2 * T1}{V_1}$$

$$T_2 = \frac{2V_1 * 423}{V_1} = 846 \text{ K} - 273 = 573\ ^oC$$

$$W_{12} = 10^2 * p\,(V_2 - V_1)$$

$$Cp = \frac{\gamma R}{\gamma - 1}$$

$$R = \frac{Cp\,(\gamma - 1)}{\gamma}$$

$$R = \frac{1.005\,(1.4 - 1)}{1.4} = 0.287 \text{ kJ / kg .K}$$

$$10^2 * p * V_1 = m * R * T_1$$

$$10^2 * 0.6 * V_1 = 2 * 0.287 * 423$$

$V_1 = 4.0467\ m^3$, $V_2 = 8.0934 m^3$

$W_{12} = 10^2 * 0.6\ (8.0934 - 4.0467) = +\ 242.802\ kJ$

$Q_{12} = m*Cp\ (T_2 - T_1)$

$= 2* 1.005 * (846 - 423) = +850.23\ kJ$

EX: - Air initially at 127°C and 1bar is compressed isothermally to 8bar. Find for unit mass of air,

a- change of internal energy
b- heat transferred
c- work done

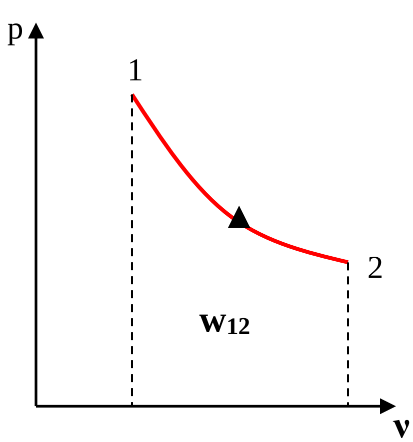
Sol: -

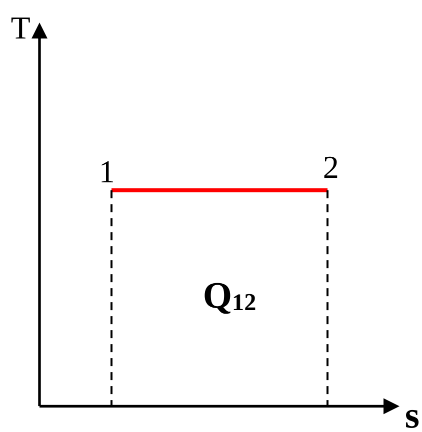
$T_1 = 127+273 = 400^oC$, $p_1 = 1bar$, isothermally T= constant, $p_2 = 8bar$

$\Delta u_{12} = 0$

$$q_{12} = w_{12} = R\ .T\ \ln\frac{v_2}{v_1} \quad \ldots\ldots\ (v_2/v_1 = p_1/p_2)$$

$$= R\ .T\ \ln\frac{p_1}{p_2} = 0.287 * 400 * \ln\frac{1}{8} = -238.719\ kJ/kg$$



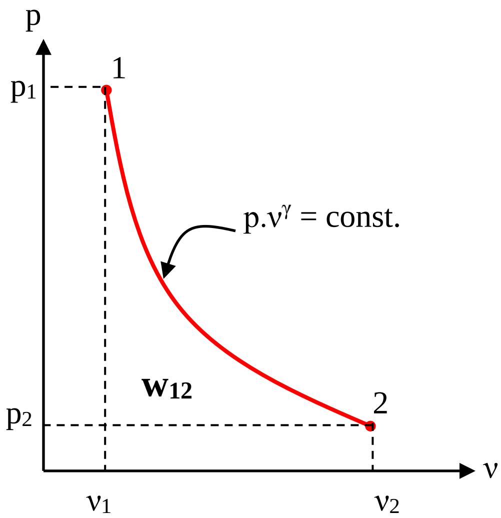


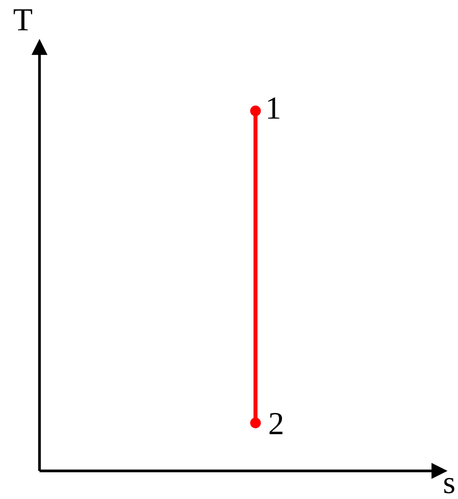
4- The adiabatic process: - is one in which no heat is transferred to or from the fluid during the process.

$Q_{12} = 0$ ……. (An adiabatic process)

$Q_{12} = \Delta\ u_{12} + w_{12}$

$w_{12} = -\ \Delta\ u_{12}$





$$\int_1^2 p.dV = -\, m\, Cv \int_1^2 dT$$

$$p.V = m\,.R.T$$

$$p = \frac{m.\,R.\,T}{V} \qquad \&\; Cv = \frac{R}{\gamma - 1}$$

$$\int_1^2 \frac{m.\,R.\,T}{V}.dV = -\, m \frac{R}{\gamma - 1} \int_1^2 dT$$

$$-\,(\gamma - 1) \int_1^2 \frac{dV}{V} = \int_1^2 \frac{dT}{T}$$

$$-\,(\gamma - 1)\,[\ln V_2 - \ln V_1] = \ln T_2 - \ln T_1$$

$$-\,(\gamma - 1) \ln \frac{V_2}{V_1} = \ln \frac{T_2}{T_1}$$

$$\ln \left(\frac{V_2}{V_1}\right)^{-(\gamma - 1)} = \ln \frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \frac{T_2}{T_1}$$

$$\boxed{\mathbf{T_1\,.V_1^{\,\gamma - 1} = T_2\,.V_2^{\,\gamma - 1} = const.}} \quad \ldots\ldots\ldots.a$$

$$\frac{p_1 . V_1}{T_1} = \frac{p_2 . V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{p_2 . V_2}{p_1 . V_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \frac{p_2 . V_2}{p_1 . V_1}$$

$$\frac{V_1^{\gamma -1} * V_1}{V_2^{\gamma -1} * V_2} = \frac{p_2}{p_1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

$$\boxed{\mathbf{p_1 . V_1^{\gamma} = p_2 . V_2^{\gamma} = const.}} \quad \text{……….b}$$

$$\frac{p_1 . V_1}{T_1} = \frac{p_2 . V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{p_2 . T_1}{p_1 . T_2}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma -1} = \left(\frac{p_2 . T_1}{p_1 . T_2}\right)^{\gamma -1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2 . T_1}{p_1 . T_2}\right)^{\gamma -1}$$

$$\frac{T_2 . T_2^{\gamma-1}}{T_1 . T_1^{\gamma-1}} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1}\right)^{\gamma} = \left(\frac{p_2}{p_1}\right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{T_2}{p_2^{\frac{\gamma-1}{\gamma}}} = \frac{T_1}{p_1^{\frac{\gamma-1}{\gamma}}} = \text{const.}} \quad \ldots\ldots\ldots\ldots\ldots\text{c}$$

$p.V^{\gamma}$ = constant

$T.\ V^{\gamma-1}$ = constant

$$\frac{T}{p^{\gamma-1/1}} = \text{constant}$$

$w_{12} = -\Delta\ U_{12}$

$$w_{12} = \int_1^2 p.dV$$

$p.V^{\gamma}$ = constant

$$p = \frac{\text{Const}}{V^{\gamma}}$$

$$w_{12} = \int_1^2 \frac{\text{const.}}{V^{\gamma}} .dV$$

$$= \text{const.} \int_1^2 V^{-\gamma} .dV$$

$$= \text{const.} \left(\frac{V^{1-\gamma}}{1-\gamma}\right)_1^2$$

$$= \text{const.}\left(\frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma}\right)$$

$$= \frac{\text{const.}V_2^{1-\gamma} - \text{const.}\ V_1^{1-\gamma}}{1-\gamma}$$

$$= \frac{p.V_2^{\gamma}.V_2^{1-\gamma} - p.V_1^{\gamma}\ V_1^{1-\gamma}}{1-\gamma}$$

$$\boxed{\mathbf{w_{12} = \frac{p_2.V_2 - p_1.V_1}{1-\gamma}}}$$

$$w_{12} = \frac{p_2.V_2 - p_1.V_1}{1-\gamma} = \frac{m..R\,.T_2 - m..R\,.T_1}{1-\gamma} = \frac{m..R\,(T_2 - T_1)}{1-\gamma}$$

$$w_{12} = \frac{m..R\,(T_1 - T_2)}{\gamma - 1} = m\,Cv\,(T_1 - T_2) = -\,m\,Cv\,(T_2 - T_1) = -\Delta\,U_{12}$$

We can call this process isentropic process because $s_1 = s_2$

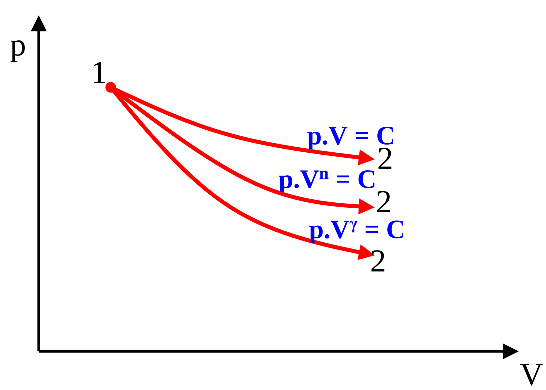
5- The polytropic process: -

$$\boxed{p.V^n = \text{constant}}$$

The index n depends only on the heat and work quantities during the process. For example,

When $n = 0$ $\quad p.V^0$ = constant, i.e. p = constant
When $n = \infty$ $\quad p.V^{\infty}$ = constant

or $p^{1/\infty}.V$ = constant , i.e. V = constant
When $n = 1$ $\quad p.V$ =constant, i.e. T = constant
When $n = \gamma$ $\quad p.V^{\gamma}$ = constant, i.e. reversible adiabatic



Generally $(1 < n < \gamma)$

As $p.V^n$ = constant

$\therefore$ From general equation of gases

$$\boxed{\mathbf{T.V^{n-1} = constant}} \quad ..... a$$

$$\boxed{\mathbf{\frac{T}{p^{n-1/n}} = constant}} \quad ..... b$$

$$\boxed{\mathbf{W_{12} = \frac{p.V_1 - p_2\, V_2}{n-1}}} \quad ....c$$

For calculation of heat,

$Q_{12} = w_{12} + \Delta U_{12}$

$$= \frac{p_1.V_1 - p_2.V_2}{n-1} + m\,.Cv\,(T_2 - T_1)$$

$$= \frac{m\,R\,(T_1 - T_2)}{n-1} - m\,\frac{R}{\gamma - 1}(T_1 - T_2)$$

$$= m\,R\,(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma-1}\right)$$

$$= m\,R\,(T_1 - T_2)\left(\frac{(\gamma-1) - (n-1)}{(n-1)(\gamma-1)}\right)$$

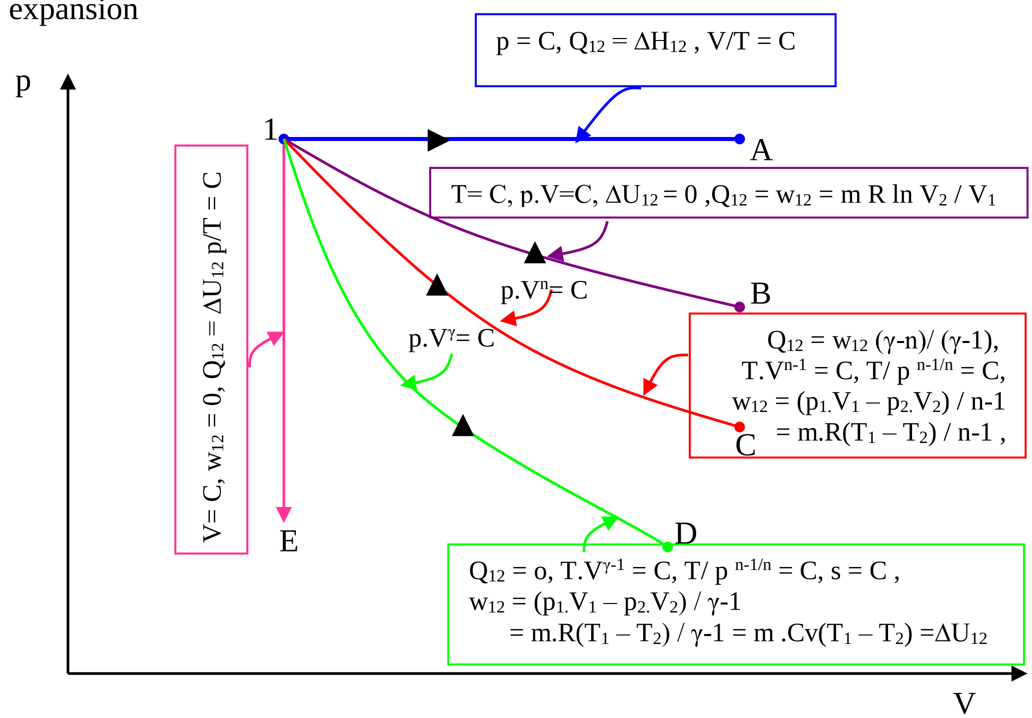
$$= m\,R\,(T_1 - T_2)\left(\frac{(\gamma - n)}{(n-1)(\gamma-1)}\right)$$

$$Q_{12} = \frac{m\,R\,(T_1 - T_2)}{n-1} - \frac{\gamma - n}{\gamma-1}$$

$$Q_{12} = w_{12} - \frac{\gamma - n}{\gamma-1}$$

$$Q_{12} = w_{12} \cdot \frac{\gamma - n}{\gamma - 1} \quad \ldots\ldots d$$

n = is called the index of expansion



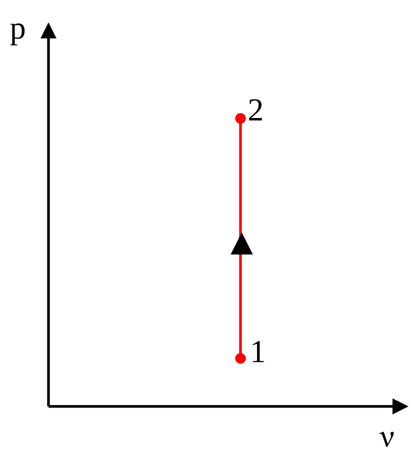
State 1 to state A is constant pressure heating,
State 1 to state B is isothermal expansion,
State 1 to state C is polytropic expansion,
State 1 to state D is reversible adiabatic expansion,
State 1 to state E is constant Volume cooling,

EX: - 2kg of gas, occupying 0.7m^3, had an original temperature of 15°C. It was then heated at constant volume until its temperature became 135°C. How much heat transferred to the gas? & what was its final pressure? [Take Cv = 0.75 kJ / kg .K, R= 0.29kJ /kg .K].

Sol:-

m = 2kg, $V_1 = 0.7m^3$, $T_1 = 15 + 273 = 288$ °K , $T_2 = 135 + 273 = 408$ °K,

Q_{12} =? , p_2 =? ,



$Q_{12} = m .Cv (T_2 – T_1)$

$= 2 * 0.75 (408 – 288)$

$= 172.8$ kJ

$p_1/ T_1 = p_2/ T_2$

$10^2 p_1.V_1 = m.R.T_1$

$$p_1 = \frac{m. R.T_1}{10^2 V_1} = \frac{2 * 0.29 * 288}{10^2 * 0.7} = 2.38 \text{ bar}$$

$p_2 = (p_1/ T_1) *T_2 = (2.38/ 288) *408 = 3.38$ bar

EX: - A gas whose pressure, volume and temperature are 275kN / m^2, 0.09 m^3 and 185°C, respectively, has its state changed at constant pressure until its becomes15°C. How much heat is transferred and how much work is done during the process? [Take R= 0.29 kJ/ kg .K, Cp = 1.005 kJ/kg.K] .

Sol:-

$p_1 = 275$ kN / m^2 = 2.75 bar = p_2 at constant pressure, $V_1 = 0.09$ m^3,

$T_1 = 185 +273 = 458$ °K,

$Q_{12} = \Delta H_{12}$

$= m. Cp. (T_2-T_1)$

$10^2 p_1.V_1 = m.R.T_1$

$$m= \frac{10^2 p_1.V_1}{R.T_1} = \frac{10^2 * 2.75* 0.09}{0.29 * 458} = 0.186 \text{ kg}$$

$Q_{12} = m. Cp. (T_2-T_1) = 0.186 * 1.005 (288- 458) = -31.7781$kJ

$W_{12} = 10^2. p_1 (V_2-V_1)$

$V_2/ T_2 = V_1/ T_1$

$V_2= (T_2 / T_1)* V_1$

$= (288 / 458) * 0.09 = 0.0565$ m^3

$W_{12} = 10^2. p_1 (V_2-V_1)$

$= 10^2 * 2.75(0.0565- 0.09) = - 9.2125$ kJ

Or

$\Delta U_{12} = m \cdot Cv\,(T_2 - T_1)$

$R = Cp - Cv$

$Cv = Cp - R = 1.005 - 0.29 = 0.715$ kJ/kg .K

$\Delta U_{12} = 0.186 * 0.715(288 - 458) = -22.6083$ kJ

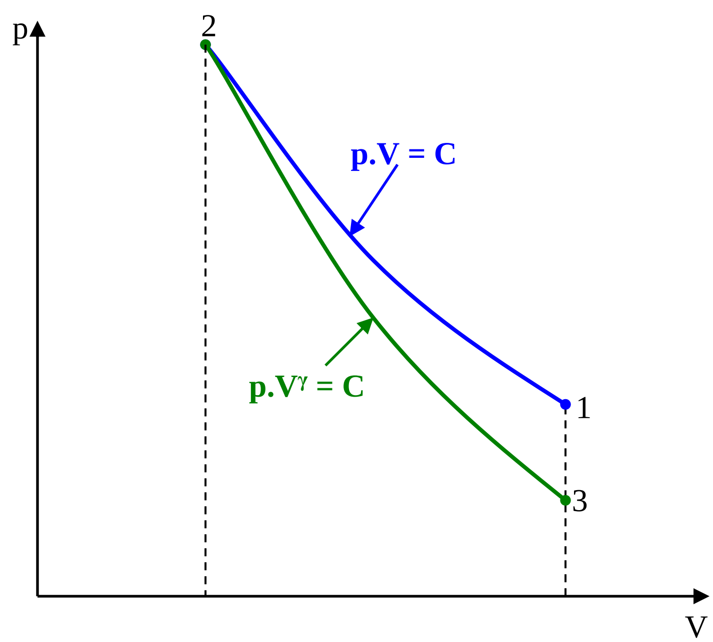
$w_{12} = Q_{12} - \Delta U_{12}$

$= -31.7781 - (-22.6083) = -9.1698$kJ

EX: - A quantity of gas occupies a volume of 0.3m^3 at a pressure of 100kN / m^2 and a temperature of 20^{o}C. The gas is compressed isothermally to a pressure of 500kN/m^2 and then expanded adiabatically to its initial volume. Determine, a) the heat received or rejected during compression, b) the change of internal energy during the expansion. [Take R= 0.29 kJ/ kg .K, Cp = 1.005 kJ/kg.K] .

Sol:-

$V_1 = 0.3m^3$, $p_1 = 100kN/m^2 = 1bar$, $T_1 = 20 + 273 = 293K$, $p_2 = 500\ kN/m^2$



Process (1-2) isothermal compression

$Q_{12} = m\,R\,T_1 \ln (V_2/V_1)$

$$m = \frac{10^2\ p_1.V_1}{R.T_1} = \frac{10^2 * 1 * 0.3}{0.29 * 293} = 0.35 \text{ kg}$$

$p_1.V_1 = p_2.V_2$

$$V_2 = \frac{p_1.V_1}{p_2} = \frac{1*0.3}{5} = 0.06\ m^3$$

$Q_{12} = 0.35*0.29*293 \ln(0.06/0.3)$

$= -47.86$ kJ

Process (2-3) adiabatic expansion

$$w_{23} = \frac{10^2(p_3.V_3 - p_2.V_2)}{1-\gamma}$$

$\gamma = Cp / Cv$

$Cv = Cp - R = 1.005 - 0.29 = 0.715$ kJ/kg.K

$\gamma = 1.005 / 0.715 = 1.4$

$$p_3 = \frac{p_2 . V_2^{\gamma}}{V_3^{\gamma}} = \frac{5*(0.06)^{1.4}}{(0.3)^{1.4}} = 0.52\ \text{bar}$$

$$w_{12} = \frac{(5*0.06 - 0.52*0.3)*10^2}{1-1.4} = 0.36*10^2\ \text{kJ}$$

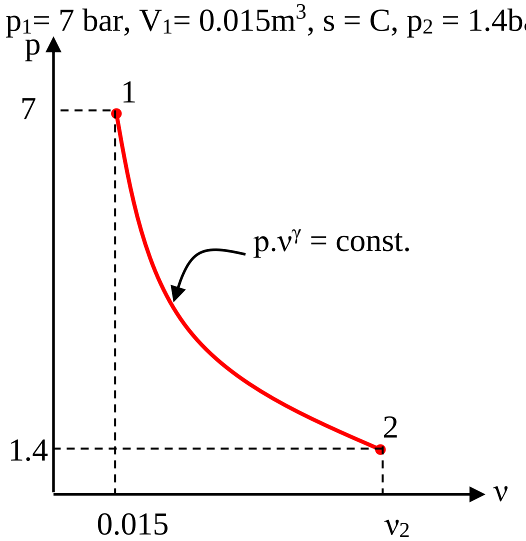
$\Delta U_{12} = -w_{12} = -0.36*10^2$ kJ

EX: - A gas expands adiabatically from a pressure and volume of 7 bars &0.015 m^3, respectively, to a pressure of 1.4 bar. Determine the final volume and the work done by the gas. What is the change of internal energy in this case?

[Take Cp= 1.046 kJ/kg.K , Cv = 0.752 kJ/kg .K].

Sol:-

p_1= 7 bar, V_1= 0.015m^3, s = C, p_2 = 1.4bar



$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$$V_2^{\gamma} = \frac{p_1.\,V_1^{\,\gamma}}{p_2}$$

γ = Cp/Cv = 1.046 / 0.752= 1.39

$$V_2^{1.39} = \frac{7*\,(0.015)^{\,1.39}}{1.4} = 0.014579$$

$V_2^{1.39} = 0.014579$

$V_2 = 0.014579^{1/1.39} = 0.0477m^3$

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2)*10^2}{\gamma - 1}$$

$$w_{12} = \frac{(7*0.015 - 1.4*0.0477)*10^2}{1.39 - 1} = 9.555kJ$$

$w_{12} = -\Delta U_{12} = -\ 9.555$ kJ

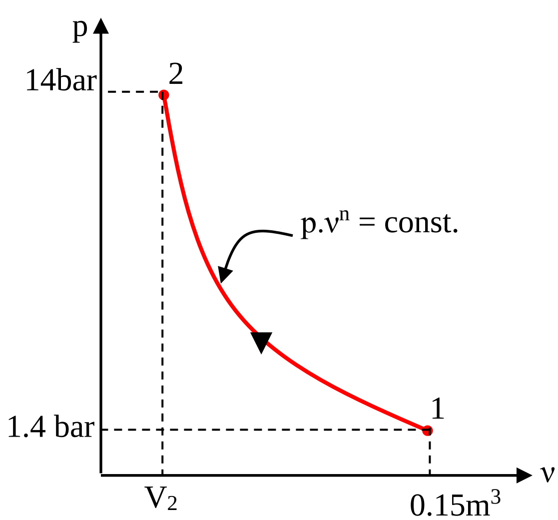
EX: - 0.25kg of air at a pressure of 1.4 bar occupies 0.15 m^3 and from this condition it is compressed to 14 bar according to the law p.V$^{1.25}$ = C. Determine,

a- the change of internal energy ,
b- the work done,
c- The heat transferred, [Take Cp= 1.005 kJ/kg.K , Cv = 0.718 kJ/kg .K].

Sol:-

m = 0.25 kg, p_1 = 1.4 bar, $V_1 = 0.15\ m^3$

p_1= 7 bar, V_1= 0.015m^3, s = C, p_2 = 1.4bar



$\Delta U_{12} = m.\ Cv\ (T_2 - T_1)$

$10^2\ p_1.V_1 = m.\ R.T_1$

$R = Cp - Cv$

$= 1.005 - 0.718 = 0.287\ kJ/kg.K$

$$T_1 = \frac{10^2\ p_1.V_1}{m\ .R} = \frac{10^2 * 1.4 * 0.15}{0.25 * 0.287} = 292.68\ K$$

$$\frac{T_1}{p_1^{n-1/n}} = \frac{T_2}{p_2^{n-1/n}}$$

$$\frac{292.68}{1.4^{(1.25-1)/1.25}} = \frac{T_2}{14^{(1.25-1)/1.25}}$$

$T_2 = 463.867\ K$

$\Delta U_{12} = 0.25 * 0.718\ (463.867 - 292.68) = 30\ .73\ kJ$

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2) * 10^2}{n-1}$$

$$w_{12} = \frac{m.\ R\ (T_1 - T_2)}{n-1} = \frac{0.25 * 0.287\ (292.68 - 463.867)}{1.25-1} = -\ 49\ .13 kJ$$

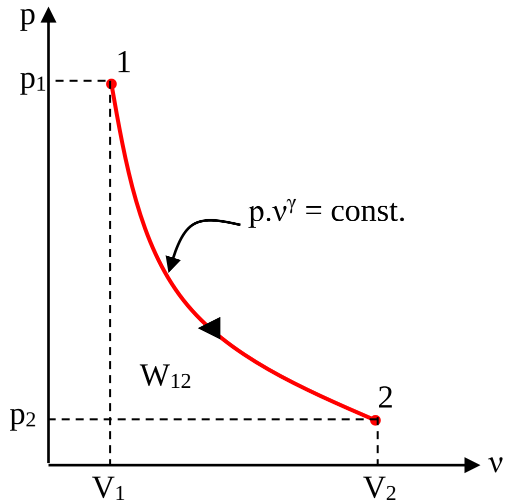
$Q_{12} = w_{12} + \Delta U_{12}$

$= -\ 49\ .13 + 30\ .73 = -18.4 kJ$

EX: - If 1 kg of gas expands adiabatically, its temperature falls from 240 ºCto116 ºC, while the volume is doubled. The gas does 19.5 kJ of work in this process. Find the values of the specific heats Cv, Cp and the molecular weight of the gas.

Sol:-

$m = 1kg$, $T_1 = 240 + 273 = 513\ K$, $T_2 = 116 + 273 = 389\ K$, $w_{12} = +\ 91.5\ kJ$



$Q_{12} = 0$

$w_{12} = -\ m\ .Cv\ (T_2 - T_1)$

$10^2\ .p_1\ .V_1 = m.\ R.T_1$

$$\left(\frac{V_2}{V_1}\right)^{\gamma -1} = \frac{T_1}{T_2}$$ the volume is doubled when it expand

$(2)^{\gamma -1} = 513/\ 389$

$(\gamma -1) \ln 2 = \ln (513/\ 389)$

$\gamma = 1.399$

$w_{12} = -\ m.\ (R/\gamma -1)\ (T_2 - T_1)$

$$R = \frac{w_{12} * (\gamma -1)}{m\ (T_1 - T_2\ )} = \frac{91.5 * (1.399 -1)}{1 * (513 - 389)} = 0.294 \text{kJ/kg.K}$$

$Cv = R/\ (\gamma -1)$

$= 0.294\ /\ (1.399 -1)$

$= 0.738$ kJ/kg.K

$Cp = R + Cv$

$= 0.294 + 0.738 = 1.032$ kJ/kg.K

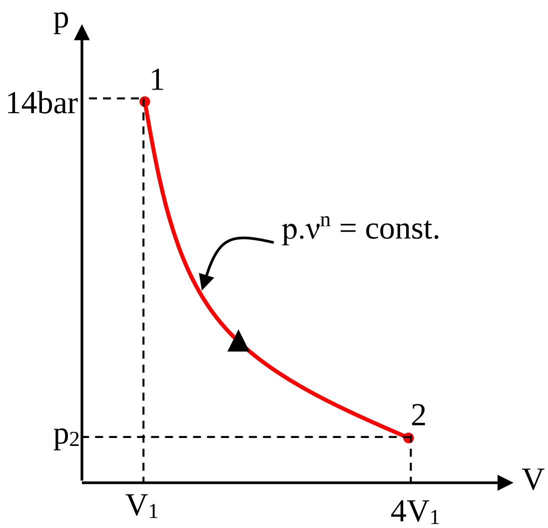
$M = 8.314\ /\ R = 8.314\ /0.294 = 28\ .27$ kg/mol.

EX: - 0.675 kg of gas at 14 bar & 280°C is expanded to four times the original volume according to the law $p.V^{1.3} = C$. Determine

- **a- the original and final volume of the gas ,**
- **b- the final pressure**
- **c- The final temperature [Take R= 0.287 kJ/kg.K].**

Sol:-

$m = 0.675$ kg, $p_1 = 14$ bar, $T_1 = 280 + 273 = 553$ °K, $n = 1.3$, $V_2 = 4\,V_1$



$$10^2 .p_1 .V_1 = m. R.T_1$$

$$V_1 = \frac{m.R.T_1}{10^2\, p_1} = \frac{0.675 * 0.287 * 553}{10^2 * 14} = 0.0765 \text{ m}^3 \text{ the original volume}$$

$V_2 = 4V_1 = 4* 0.0765 = 0.306$ m^3 the final volume

$$p_1.V_1^n = p_2.V_2^n$$

$p_2 = (V_1/ V_2)^n\, p_1 = (V_1/ 4V_1)^{1.3} * 14 = 2.309$ bar $= 0.2309$ MN /m^3 the final pressure

$$T_1 .V_1^{n-1} = T_2 .V_2^{n-1}$$

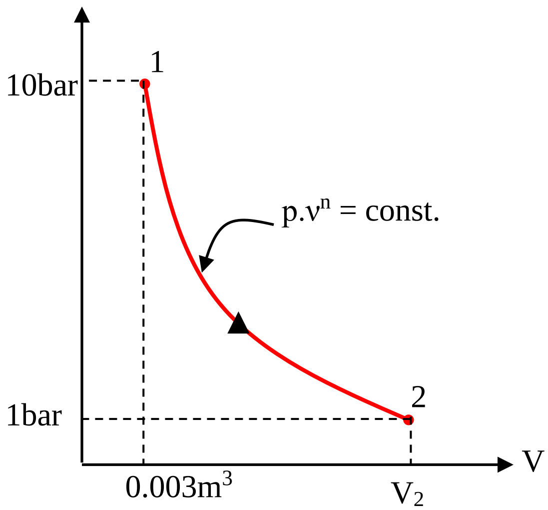
$T_2 = (V_1/ V_2)^{n-1} * T_1 =(V_1/ 4V_1)^{1.3-1} * 553 = 364 . 84$ °K $= 91.84$ °C final temperature

EX: - A gas expand according to the law $p.V^{1.3} = C$ from a pressure of 10 bar & a volume 0.003m^3 to a pressure 1 bar. How much heat was transferred?

[Take $\gamma = 1.4$, Cv = 0.718 kJ/kg .K]

Sol:-

$p_1 = 10$ bar, $V_1 = 0.003$m^3, $p_2 = 1$ bar



$$Q_{12} = w_{12} \frac{\gamma - n}{\gamma - 1}$$

$$w_{12} = \frac{(p_1.V_1 - p_2.V_2) * 10^2}{n - 1}$$

$$p_1.V_1^n = p_2.V_2^n$$

$$V_2 = (p_1 / p_2)^{1/n} V_1 = (10/1)^{1/1.3} * 0.003 = 0.0176 \text{ m}^3$$

$$w_{12} = \frac{(10*0.003 - 1*0.0176) * 10^2}{1.3 - 1} = 4.122 \text{ kJ}$$

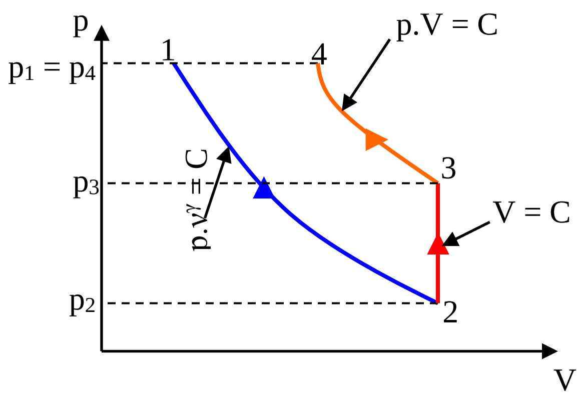
$$Q_{12} = 4.122* \frac{1.4 - 1.3}{1.4 - 1} = 1.03 \text{ kJ}$$

EX: - A gas at a pressure of 14 bar and a temperature of 360 °C is expanded adiabatically to a pressure of 1 bar the gas is then heated at constant volume until it again attains 360°C when its pressure is found to be 2.2 bar and finally it is compressed isothermally until the original pressure of 14 bar is attained. Find for until mass.

a- the value of the adiabatic index γ ,

b- The change in internal energy during each process. [Take Cp = 1.005 kJ/kg .K]

Sol:-



p_1 = 14 bar, T_1 = 360 + 273= 633 ^oK, p_2 = 1 bar, p_3 = 2.2 bar, m=1kg

$V_3 = V_2$ heated at constant volume

$T_4 = T_3$ = 633 ^oK isothermal compression

$p_4 = p_1$ = 14 bar compressed to original pressure

$p_2 / T_2 = p_3 / T_3$

$T_2 = (p_2 / p_3) * T_3 = (1 / 2.2) * 633 = 287.73\ ^oK$

$$\frac{T_1}{p_1^{\gamma-1/\gamma}} = \frac{T_2}{p_2^{\gamma-1/\gamma}}$$

$(p_2 / p_1)^{\gamma-1/\gamma} = T_2 / T_1$

$$\frac{\gamma - 1}{\gamma} \ln (p_2 / p_1) = \ln (T_2 / T_1)$$

$$\frac{\gamma - 1}{\gamma} \ln (1 / 14) = \ln (287.73 / 633)$$

$$\frac{\gamma - 1}{\gamma} (- 2.64) = -0.788$$

$[1- (1/\gamma)](- 2.64) = -0.788$

$[1- (1/\gamma)] = 0.2986$

$0.7013 = (1/\gamma)$

$\gamma = 1. 425$

$\Delta U_{12} = m.\ Cv\ (T_2 - T_1)$

$\Delta U_{12} = m.\ (Cp / \gamma)\ (T_2 - T_1)$

$= 1 * (1.005 / 1.425)\ (287.73 - 633)$

$= - 243.36$ kJ

$\Delta U_{23} = m.\ (Cp / \gamma)\ (T_3 - T_2)$

$= 1 * (1.005 / 1.425)\ (633 - 287.73)$

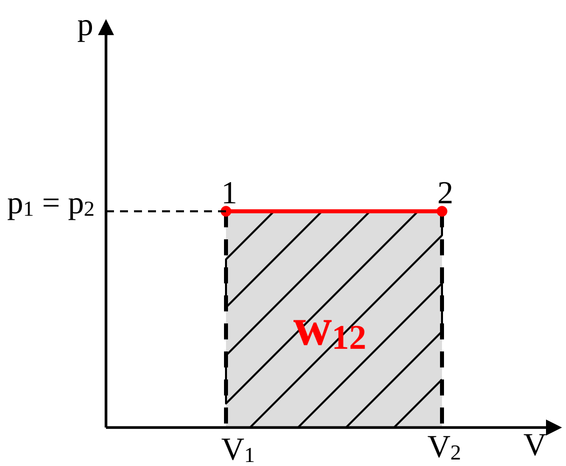
$= 243.36$ kJ

$\Delta U_{34} = 0$

EX: - A certain gas has a density of 1.875 kg /m^3 at 1bar and 15 ^oC. Calculate the characteristic gas constant. When 0.9 kg of this gas is heated from 15 ^oC to 250^oC at constant pressure, the heat required is 175 kJ. Calculate the specific heat capacity of the gas at constant pressure and the specific heat at constant volume. Calculate also, the change of internal energy and the external work done during the heating.

Sol:-

$\rho_1 = 1.875$ kg/m^3, $p_1 = 1$ bar, $T_1 = 15 + 273 = 288\ ^oK$, m = 0.9 kg, $T_2 = 250 + 273 = 523^oK$
$p_1 = p_2 = 1$ bar at constant pressure, $Q_{12} = 175$ kJ



$Q_{12} = \Delta H_{12}$

$= m.\ Cp\ (T_2 - T_1)$

$175 = 0.9 * Cp * (523 - 288)$

$Cp = 0.827$ kJ/kg.K

$\rho_1 = m / V_1$

$V_1 = m / \rho_1 = 0.9 / 1.875 = 0.48\ m^3$

$10^2 .p_1 .V_1 = m. R.T_1$

$$R = \frac{10^2 .p_1 .V_1}{m. T_1} = \frac{10^2 * 1 * 0.48}{0.9 * 288} = 0.185\ kJ/kg.K$$

$Cv = Cp - R$

$= 0.827 - 0.185 = 0.642\ kJ/kg.K$

$\Delta U_{12} = m .Cv (T_2 - T_1)$

$= 0.9 * 0.642 * (523 - 288)$

$= 135.78\ kJ$

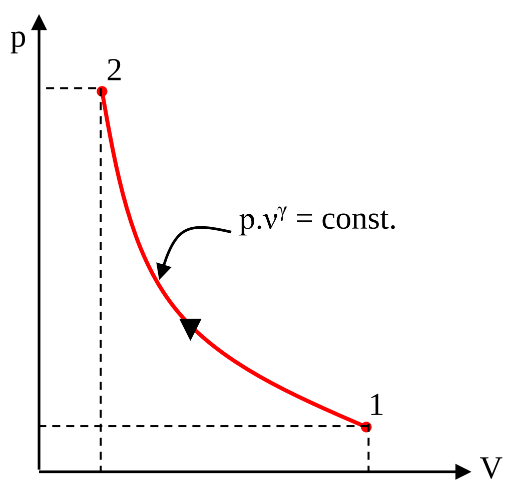
$w_{12} = Q_{12} - \Delta U_{12}$

$= 175 - 135.78 = 39 .22\ kJ /kg.K$

EX: - A gas whose original pressure and temperature were 300 kN /m^2 and 25 ^{o}C. Respectively, is compressed according to the law p.V$^{1.4}$ = C until its temperature becomes 180 ^{o}C. Determine the new pressure of the gas.

Sol:-

$p_1 = 300\ kN /m^2$, $T_1 = 25 + 273 = 298\ ^oK$, $p.V^{1.4} = C$, $T_2 = 180 + 273 = 453\ ^oK$



$$\frac{T_2}{p_2^{\gamma-1/\gamma}} = \frac{T_1}{p_1^{\gamma-1/\gamma}}$$

$$\frac{T_2^{\gamma/\gamma-1}}{p_2} = \frac{T_1^{\gamma/\gamma-1}}{p_1}$$

$$\frac{(453)^{1.4/1.4-1}}{p_2} = \frac{(298)^{1.4/1.4-1}}{300 * 10^{-2}}$$

$p_2 = 12.99$ bar

EX: - A certain mass of air initially at a pressure of 4.8 bar is expanded adiabatically to a pressure of 0.94 bar. It is then heated at constant volume until it attains its initial temperature, when the pressure is found to be 1.50 bar. State the type of compression necessary to bring the air back to its original pressure and volume. Using the information calculate the value of γ. If the initial temperature of the air is 190 ºC, determine the work done per kg of air during the adiabatic expansion [take R = 0.29 kJ /kg .K].

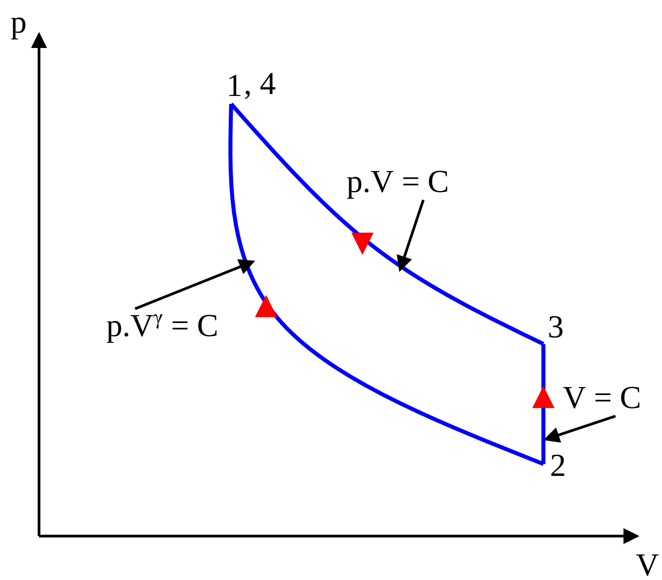
Sol:-

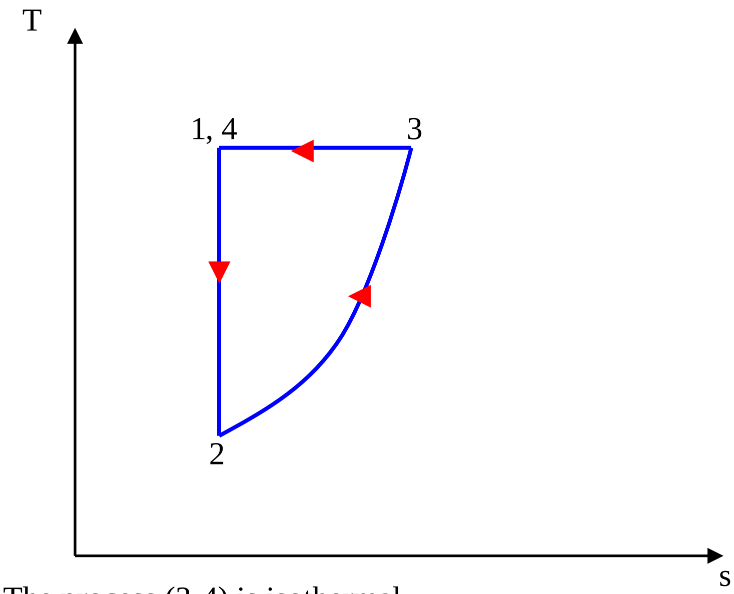
$p_1 = 4.8$ bar, $T_1 = 190 + 273 = 463$ ºK,

s = C adiabatic exp.   exp. $p_2 = 0.94$ bar,

V = C heated at constant volume to initial temp. $T_3 = T_1 = 463$ ºK, $p_3 = 1.5$ bar

$p_4 = p_1 = 4.8$ bar $V_4 = V_1$ (air back to its original pressure and volume).





The process (3-4) is isothermal

$$\frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}}$$

$$\frac{T_1}{T_2} = \frac{p_1^{(\gamma-1)/\gamma}}{p_2^{(\gamma-1)/\gamma}}$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$$\frac{0.94}{T_2} = \frac{1.5}{463}, \quad T_2 = 290\ ^oK$$

$$\ln\frac{T_1}{T_2} = \ln\left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

$$\text{Ln}\,\frac{T_1}{T_2} = [(\gamma-1)/\gamma]\ln\frac{p_1}{p_2}$$

$$\text{Ln}\,\frac{463}{290} = [(\gamma-1)/\gamma]\ln\frac{4.8}{0.94}$$

$\gamma = 1.402$

$$w_{12} = \frac{10^2 (p_1 V_1 - p_2 V_2)}{\gamma - 1} = Cv\,(T_1 - T_2)$$

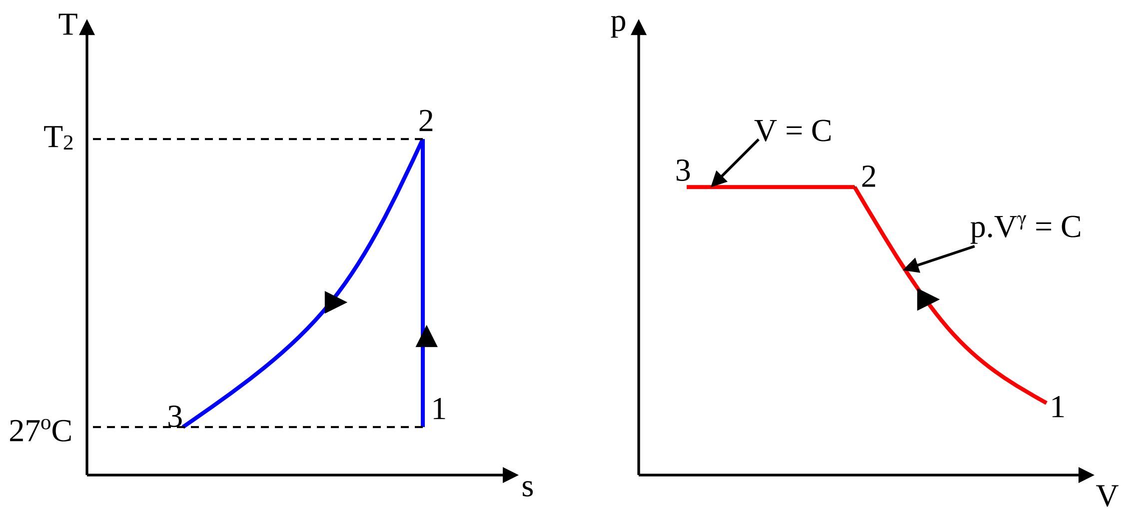
$$Cv = \frac{R}{\gamma - 1} = \frac{0.29}{1.4 - 1} = 0.725 \text{ kJ//kg.K}$$

$w_{12} = 0.725\,(463 - 290) = 125.425$ kJ /kg

EX: - 1 kg of air at 1 bar and 27 °C is compressed to 6 bar adiabatically. Find the final temperature and work done. If the air is then cooled at constant pressure to original temperature of 27°C, what amount of heat is rejected and, what further work of compression is done? The final state could have been reached by a reversible isothermal compression, instead of by the two other processes. What would be the work done and heat transferred in this case? [Take Cp = 1005 kJ/kg.K, Cv = 0.717 kJ/kg.K]

Sol:-

m=1kg, p_1 = 1 bar, T_1 = 27 + 273 = 300 °K, p_2 = 6 bar, T_2 =? , $T_3 = T_1$ =300 °K, $p_3 = p_2$, Q_{23} =? , w_{12} =?



Process (1-2) adiabatic compression
Process (2-3) cooled at constant pressure

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

$\gamma = Cp/Cv = 1.005 / 0.717 = 1.4$

$$\frac{300}{T_2} = \left(\frac{1}{6}\right)^{(1.4-1)/1.4}$$

T_2 = 500.55 °K = 227.55°C

$w_{12} = m.Cv\ (T_1 - T_2)$

$= 1 * 0.717\ (300 - 500.55) = -143.79$ kJ

$Q_{23} = m.\ Cp\ (T_3 - T_2)$

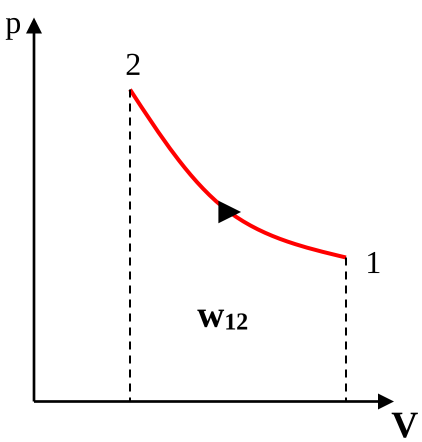
$= 1 * 1.005\ (300 - 500.55) = -201.55$ kJ

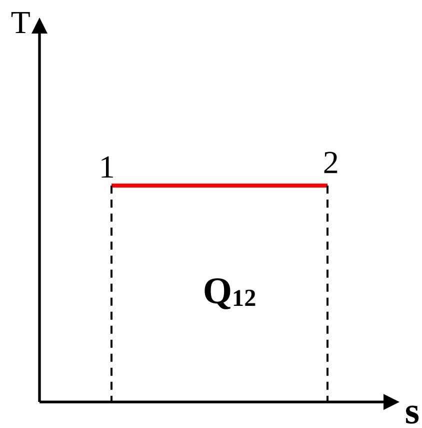
- Instead of by the two processes the final state could have been reached by reversible isothermal compression process

$$Q_{12} = w_{12} = m.\ R\ .T \ln\frac{V_2}{V_1} \qquad \ldots\ldots\ (V_2/V_1 = p_1/p_2)$$

R= Cp-Cv = 1.005 – 0.717 =0.288 kJ/kg.K

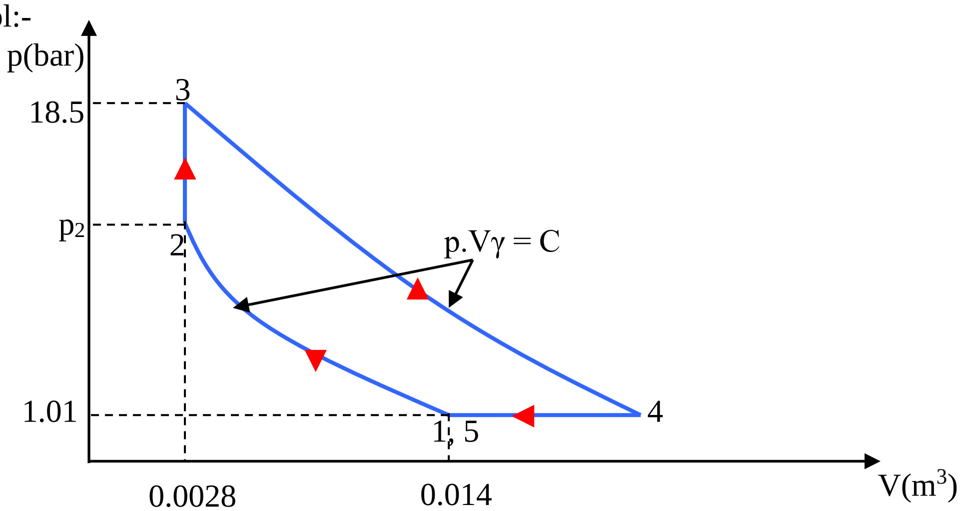
$$= m.R\ .T_1 \ln\frac{p_1}{p_2} = 1*0.288 * 300 * \ln\frac{1}{6} = -154\ .8 \text{ kJ}$$





EX: - Air occupying 0.014m³ at 1.01 bar and 15°C is compressed adiabatically to a volume of 0.0028 m³. Heat is then supplied at constant volume until the pressure becomes 18.5 bar. Adiabatic expansion returns the air to the initial pressure of 1.01 bar, and then it is cooled at constant pressure until it attains the original volume of 0.014m³. Calculate for each process, the heat transferred, the work done and change in internal energy. [Take γ = 1.4, R = 0.29 kJ/kg.K]

Sol:-



$V_1 = 0.014\ m^3$, $p_1 = 1.01$ bar, $T_1 = 15 + 273 = 288\ ^oC$, $V_2 = 0.0028\ m^3$, $p_3 = 18.5$

$V_3 = V_2 = 0.0028\ m^3$, $p_4 = p_1 = 1.01$ bar $= p_5$, $V_5 = V_1$

Process (1-2) adiabatic compression

Process (2-3) heat at constant volume

Process (3-4) adiabatic expansion

Process (4-1, 5) cooled at constant pressure

$Q_{12} = 0$

$w_{12} = -\Delta U_{12}$

$= -m\ Cv\ (T_2 - T_1)$

$10^2\ p_1.\ V_1 = m\ .R\ .T_1$

$$m = \frac{10^2\ p_1.\ V_1}{R\ .T_1} = \frac{10^2 * 1.01 * 0.014}{0.29 * 288} = 0.0169\ kg$$

$Cv = R / (\gamma-1) = 0.29 / (1.4 - 1) = 0.725$ kJ/kg.K

$T_1.\ V_1^{\gamma-1} = T_2.\ V_2^{\gamma-1}$

$288 * 0.014^{1.4-1} = T_2 * 0.0028^{1.4-1}$

$T_2 = 548.25\ ^oK$

$w_{12} = -m\ Cv\ (T_2 - T_1)$

$= -0.0169 * 0.725 * (548.25 - 288) = -3.1887$ kJ

$\Delta U_{12} = -w_{12} = 3.1887$

$Q_{23} = \Delta U_{23} = m\ Cv\ (T_3 - T_2)$

$p_2 / T_2 = p_3 / T_3$

$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$$p_2 = \frac{p_1.V_1^{\gamma}}{V_2^{\gamma}} = \frac{1.01 * (0.014)^{1.4}}{(0.0028)^{1.4}} = 9.613\ bar$$

$9.613 / 548.25 = 18.5 / T_3$

$T_3 = 1124.38\ ^oK$

$Q_{23} = 0.0169 * 0.725\ (1124.38 – 548.25) = 7.059\ kJ = \Delta U_{23}$

$Q_{34} = 0$

$W_{34} = - \Delta U_{34}$

$= - m\ Cv\ (T_4 – T_3)$

$T_3 . V_3^{\gamma-1} = T_4 . V_4^{\gamma-1}$

$p_3 . V_3^{\gamma} = p_4 . V_4^{\gamma}$

$18.5 *(0.0028)^{1.4} = 1.01 *V_4^{\gamma}$

$V_4 = 0.0223\ m^3$

$1124.38*(0.0028)^{1.4-1} = T_4*(0.0223)^{1.4-1}$

$T_4 = 490\ ^oK$

$W_{34} = - 0.0169 * 0.725\ (490 – 1124.38) = 7.773\ kJ$

$W_{34} = - \Delta U_{34} = - 7.773\ kJ$

$Q_{41} = \Delta H_{41}$

$= m .Cp\ (T1 – T_4)$

$$Cp = \frac{\gamma R}{\gamma - 1} = \frac{1.4 * 0.29}{1.4 – 1} = 1.015\ kJ/kg.K$$

$Q_{41} = 0.0169 *1.015\ (288 – 490) = - 3.342\ kJ$

$\Delta U_{41} = m\ Cv\ (T_1 – T_4)$

$= 0.0169*0.725\ (288 – 490) = - 2.475\ kJ$

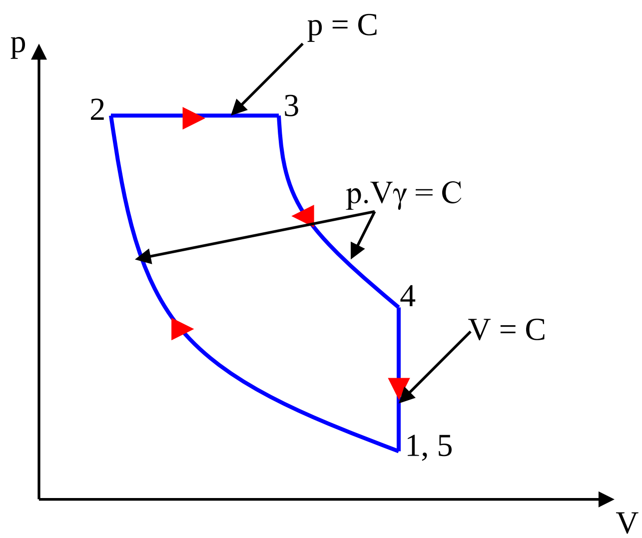
$w_{41} = Q_{41} - \Delta U_{41}$

$=- 3.342 – (- 2.475) = - 0.867 kJ$

EX: - 1 kg of air is initially at 0.9 bar and 40 ºC is compressed adiabatically till the volume is reduced to one – sixteenth of its initial value. Heat is then added at constant pressure till the temperature is 1400ºC. The air is then expanded adiabatically until the initial volume is reached and finally it is cooled at constant volume until the original state of 0.9 bar and 40ºC is Attained. Calculate the work done, internal energy and heat transferred for each process. [Take γ = 1.4, Cp = 1.004kJ/kg.K]

Sol:-

m = 1 kg, p_1 = 0.9 bar, T_1 = 40 + 273 = 313 °K, $V_2 = V_1/16$, T_3 = 1400+ 273 = 1673 °K



(1-2) Process adiabatic compression, s = C

(2 – 3) Process isobaric heat add at constant pressure process, $p_2 = p_3$

(3-4) Process adiabatic expansion, s = C

(4-1) Process isochoric cooling at constant volume process $V_4 = V_1$

$V_5 = V_1$ cooled at constant volume to the original state

$p_5 = p_1$ cooled at constant volume to the original state

$w_{12} = m\,.Cv\,(T_1 - T_2) = -\Delta U_{12}$

$\gamma = Cp/Cv$

$Cv = Cp/\gamma = 1.004 / 1.4 = 0.717$ kJ/kg.K

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{T_2}{T_1}$$

$$\left(\frac{V_1}{V_1/16}\right)^{1.4-1} = \frac{T_2}{313}$$

T_2 = 948.84 °K

w_{12} = 1*0.717 (313 - 948.84) = -455.896 kJ

$\Delta U_{12} = 455.896$ kJ

$Q_{12} = 0$

$Q_{23} = m.\ Cp\ (T_3 - T_2)$

$= 1 * 1.004\ (1673 - 948.84) = 727.1$ kJ

$\Delta U_{23} = m\ .Cv\ (T_3 - T_2)$

$= 1 * 0.717\ (1673 - 948.84) = 519\ .222$ kJ

$w_{23} = Q_{23} - \Delta U_{23}$

$= 727.1 – 519.222 = 207.88$ kJ

$w_{34} = m\ .Cv\ (T_3 – T_4)$

$10^2\ p_{1.}\ V_1 = m\ .R\ .T_1$

$R = Cp – Cv = 1.004 – 0.717 = 0.287$ kJ/kg.K

$10^2 *0.9 *V_1 = 1*0.287 *313$

$V_1 = 0.998\ m^3$

$V_2 = V_1 / 16 = 0.998 / 16 = 0.0624\ m^3$

$V_2 / T_2 = V_3 / T_3$

$0.0624 / 948.84 = V_3 / 1673 \quad , V_3 = 0.11 m^3$

$$\left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_3}{T_4}$$

$$\left(\frac{0.998}{0.11}\right)^{1.4-1} = \frac{1673}{T_4}$$

$T_4 = 692.47\ ^oK$

$w_{34} = 1*0.717 * (1673 – 692.47) = 703.04$ kJ

$\Delta U_{34} = - 703.04$ kJ/kg.K

$Q_{41} = \Delta U_{41} = m\ .Cv\ (T_1 – T_4) = 1*0.717 * (313 - 692.47) = - 272.08$ kJ

$w_{41} = 0$

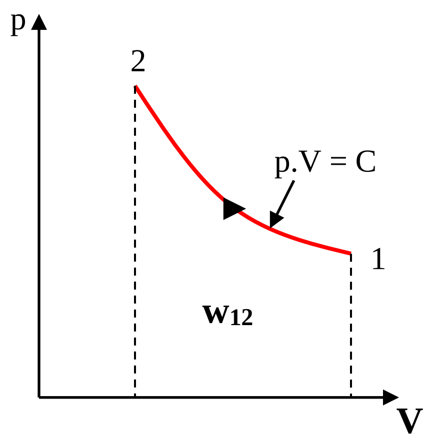
EX: - One kilogram of a gas is at 1.1 bar and 15°C. It is compressed until its volume is 0.1m³. Calculate the final pressure and temperature if the compression is a- isothermal, b- adiabatic. Calculate also the work done, change in internal energy and heat transferred in each case. [Take Cp = 0.92 kJ/kg.K, Cv = 0.66 kJ/kg.K] .

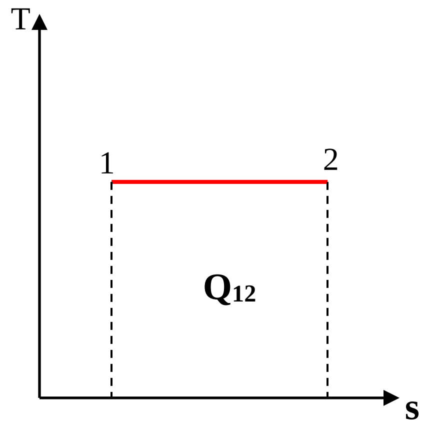
Sol:-

m = 1kg, Cp = 0.92 kJ/kg.K , p_1 = 1.1 bar, Cv = 0.66 kJ/kg.K , T_1 = 15 + 273 = 288 °K,

$V_2 = 0.1 m^3$,

p_2 =? , T_2 =? , w_{12} =? , ΔU_{12} =? , Q_{12} =?





a-

$10^2\, p_1 . V_1 = m . R . T_1$

R = Cp – Cv = 0.92 – 0.66 = 0.26 kJ/kg.K

$$V_1 = \frac{m . R . T_1}{10^2\, p_1} = \frac{1*0.26 * 288}{10^2 * 1.1} = 0.68\ m^3$$

$p_1.V_1 = p_2.V_2$

$$p_2 = \frac{p_1.V_1}{V_2} = \frac{1.1*0.68}{0.1} = 7.488 \text{ bar} = 0.7488\ MN/m^2$$

$10^2\, p_2 . V_2 = m . R . T_2$

$$T_2 = \frac{10^2\, p_2 . V_2}{m . R} = \frac{10^2*7.488*0.1}{1 * 0.26} = 288°K = 15°C$$

Or $T_1 = T_2$ isothermal compression process is at constant temperature.

$$Q_{12} = w_{12} = m.\, R\, .T \ln\frac{V_2}{V_1}$$

$$= 1 * 0.26 * 288 \ln (0.1/ 0.68) = - 143.54 \text{ kJ}$$

$\Delta U_{12} = 0$

<u>b-</u>

$10^2\, p_{1.}\, V_1 = m\, .R\, .T_1$

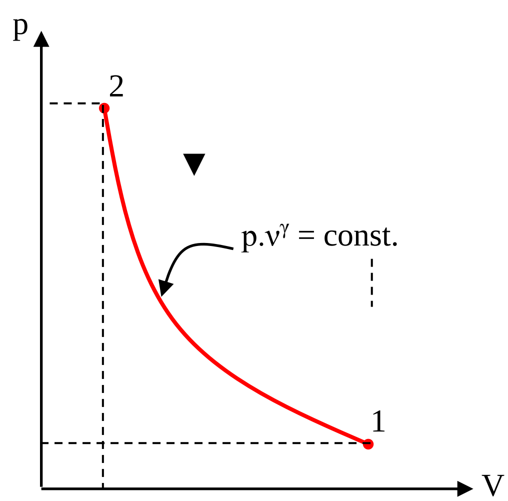
$R = Cp – Cv = 0.92 – 0.66 = 0.26$ kJ/kg.K

$$V_1 = \frac{m\, .R\, .T_1}{10^2\, p_1} = \frac{1*0.26 * 288}{10^2 * 1.1} = 0.68 \text{ m}^3$$

$\gamma = Cp / Cv = 0.92 / 0.66 = 1.393$

$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$$p_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} * p_1 = \left(\frac{0.68}{0.1}\right)^{1.393} * 1.1 = 15.89 \text{ bar} = 1.589 \text{ MN/m}^2$$



$$T_2 = \frac{10^2\, p_{2.}\, V_2}{m\, .R} = \frac{10^2*15.89*0.1}{1 * 0.26} = 611.1^oK = 338.1^oC$$

$w_{12} = m\, .Cv\, (T_1 - T_2) = - \Delta U_{12}$

$w_{12} = 1*0.66\, (288 - 611.1) = -213.25$ kJ

$\Delta U_{12} = 213.25$ kJ

$Q_{12} = 0$

Problems

4 -1 0.1m^3 of gas is compressed from a pressure of 120 kN / m^2 and temperature 25^{o}C to a pressure of 12 bar according to the law p .V$^{1.2}$ = constant. Calculate;
1- The work done on the gas
2- The change in internal energy
3- The quantity heat transferred, Assume, Cv = 0.72 kJ/kg.K , R = 0.285kJ/kg.K
(-28.2kJ, 14.2kJ, -14kJ)

4 -2 1 kg of air enclosed in a rigid container is initially at 4.8 bar and 150^{o}C. The container is heated until the temperature is 200^{o}C. Calculate the pressure of the air finally and the heat supplied during the process (5.37 bar ; 35.9kJ/kg)

4 -3 Oxygen (molecular weight 32) expands reversibly in a cylinder behind a piston at a constant pressure of 3 bar. The volume initially is 0.01m^3; the initial temperature is 17^{o}C. Calculate the work done by the oxygen and the heat flow to or from the cylinder walls during the expansion. Assume oxygen to be a perfect gas and take Cp = 0.917kJ/kg.K (6kJ; 21.16kJ)

4 -4 0.05 m^3 of a perfect gas at 6.3 bar undergoes a reversible isothermal process to a pressure of 1.05 bar. Calculate the heat flow to or from the gas (56.4kJ)

4-5 1kg of air is compressed isothermally & reversibly from 1 bar and 30^{o}C to 5 bar. Calculate the work done on the air and the heat flow to or from the air.
(140kJ/kg; -140kJ/kg)

4 -6 1kg of air at 1 bar, 15^{o}C is compressed reversibly and adiabatically to a pressure of 4 bar. Calculate the final temperature and the work done on the air. (155^{o}C; 100.5kJ/kg)

4 -7 Nitrogen (molecular weight 28) expands reversibly in perfectly thermally insulated cylinder from 3.5bar, 200^{o}C to a volume of 0.09m^3. If the initial volume occupied was 0.03m^3, calculate the work done during the expansion. Assume nitrogen to be perfect gas and take Cv = 0.741kJ/kg.K (9.31kJ)

4 -8 1kg of air at 1.02bar, 20^{o}C is compressed reversibly according to a law p .V$^{1.3}$ = const., to a pressure of 5.5bar. Calculate the work done on the air and the heat flow to or from the cylinder walls during the compression. (133.5kJ/kg; -33.38kJ/kg)

4 -9 Oxygen (molecular weight 32) is compressed reversibly and polytropically in a cylinder from 1.05 bar, 15^{o}C to 4.2bar in such a way that one- third of the work input is rejected as heat to the cylinder walls . calculate the final temperature of the oxygen. Assume oxygen to be aperfect gas and take Cv = 0.649kJ/kg.K (113^{o}C)

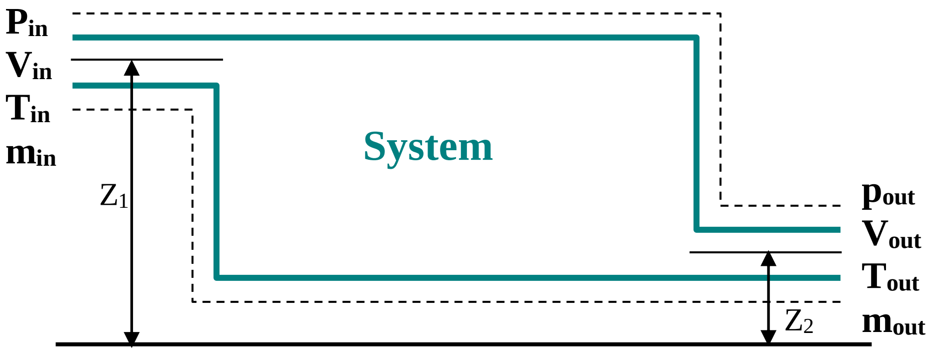
4 -10 0.05 kg of carbon dioxide (molecular weight 44), occupying a volume of 0.03m^3 at 1.025bar , is compressed reversibly until the pressure is 6.15bar. calculate the final temperature the work done on the CO_2 and the heat flow to or from the cylinder walls ,
1- when the process is according to the law p.v$^{1.4}$ = constant ,
2- when the process is isothermal ,
assume carbon dioxide to be a perfect gas , and take γ = 1.3
(270^{o}C ; 5.138kJ/kg ; 1.713kJ; 52.6^{o}C ; 5.51kJ/kg ; -5.51kJ)

Unit five

Application to open flow system

Application to open flow system

The open flow system is that one which has mass and energy crossing its boundary. As shown in fig.



$$\boxed{Q_{12} - w_{12} = \Delta E_{12} + [\Delta E_{out\ flow} - \Delta E_{in\ flow}]}$$

Where:-

ΔE_{12} = change of stored energy.

$\Delta E_{12} = [m_2(u_2 + K.E_2 + pot.E_2)] - [m_1(u_1 + K.E_1 + pot.E_1)]$

$\Delta E_{out\ flow}$: - $m_{out}\ (h_{out} + K.E_{out} + pot_{out})$

$\Delta E_{in\ flow}$: - $m_{in}\ (h_{in} + K.E_{in} + pot_{in})$

$\mathbf{Q_{12} - w_{12}} = [m_2(u_2 + K.E_2 + pot.E_2)] - [m_1(u_1 + K.E_1 + pot.E_1)] + [m_{out}\ (h_{out} + K.E_{out} + pot_{out}) - m_{in}(h_{in} + K.E_{in} + pot_{in})]$ this is called u.S.F.E.E[un steady flow energy equation]

In case where $m_{in} = m_{out}$ there will be no change in the system mass.

$\therefore$ $m_1 = m_2$ and also $\Delta E_{12} = 0$ the equation be written in the form,

$\mathbf{Q_{12} - w_{12}} = [m_{out}\ (h_{out} + K.E_{out} + pot_{out}) - m_{in}\ (h_{in} + K.E_{in} + pot_{in})]$

But $m_{out} = m_{in} = m$ & put instead of out &in (2, 1) we can write the equation,

$Q_{12} - w_{12} = m\ [(h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) + g\ (Z_2 - Z_1)]$

$$\boxed{Q_{12} - w_{12} = \Delta H_{12} + \Delta K.E_{12} + \Delta pot.E_{12}}$$

… S.F.E.E this is called steady flow energy equation

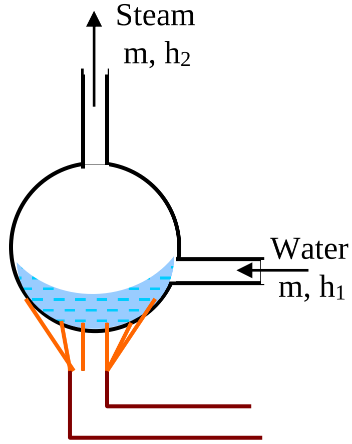
Application of open flow for S.F.E.E

1- The boiler steam

$$Q_{12} - \cancelto{0}{w_{12}} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

Work = 0, K.E = 0, pot.E = 0,

$$\boxed{Q_{12} = \Delta H_{12}}$$

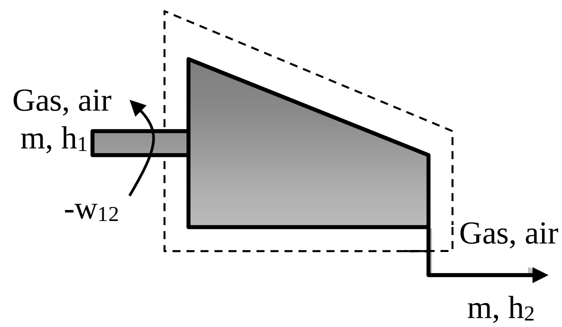


2- The compressor:-

$$Q_{12} - w_{12} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

K.E = 0, pot.E = 0,

$$\boxed{Q_{12} - w_{12} = \Delta H_{12}}$$

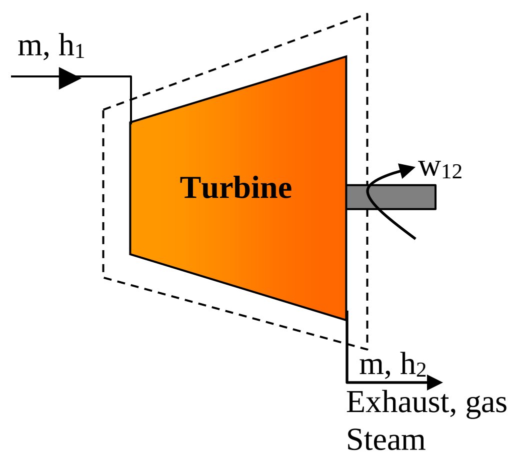


3- The turbine

$$\cancelto{0}{Q_{12}} - w_{12} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

K.E = 0, pot.E = 0, $Q_{12} = 0$

$$\boxed{w_{12} = H_1 - H_2}$$

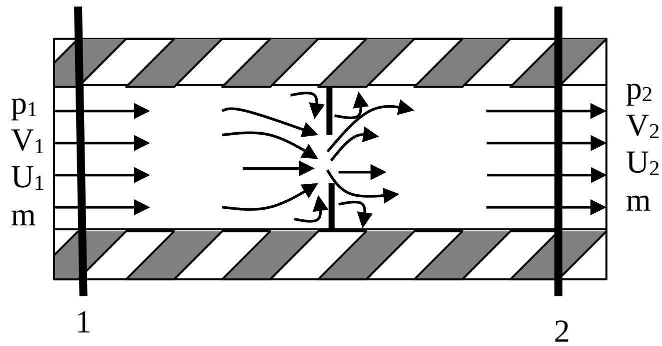


4- The throttle valve

$$\cancelto{0}{Q_{12}} - \cancelto{0}{w_{12}} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

$K.E = 0$, $pot.E = 0$, $Q_{12} = 0$, $w_{12} = 0$

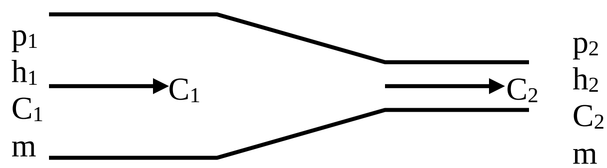
$$\boxed{H_1 = H_2}$$



5- The nozzle

$$\cancelto{0}{Q_{12}} - \cancelto{0}{w_{12}} = [\Delta H_{12} + \Delta K.E_{12} + \cancelto{0}{\Delta pot.E_{12}}]$$

$pot.E = 0$, $Q_{12} = 0$, $w_{12} = 0$



$\Delta H_{12} + \Delta K.E_{12} = 0$

$m\,[(h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2)] = 0$

$\frac{1}{2}(C_2^2 - C_1^2) = (h_1 - h_2)$

$C_2^2 = 2(h_1 - h_2) + C_1^2$

$C_2 = \sqrt{2(h_1 - h_2) + C_1^2}$

$$\boxed{C_2 = \sqrt{2(h_1 - h_2)}}$$

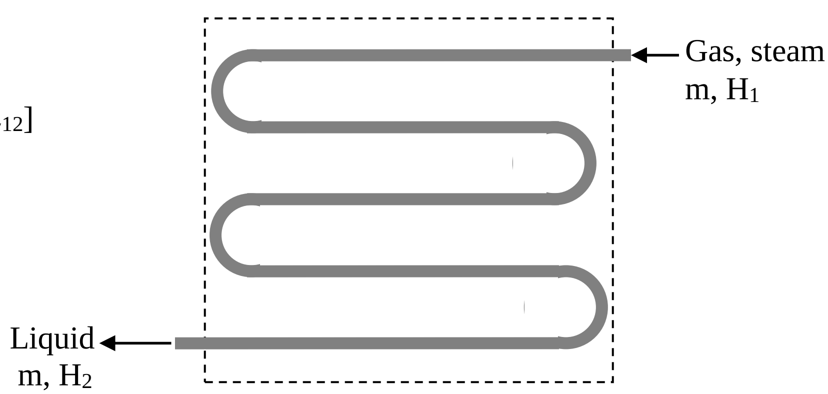
…. In case C_1 is very small

6- Condenser

$$Q_{12} - \cancelto{0}{w_{12}} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

$pot.E = 0$, $\Delta K.E_{12} = 0$, $w_{12} = 0$

$$\boxed{Q_{12} = \Delta H_{12}}$$

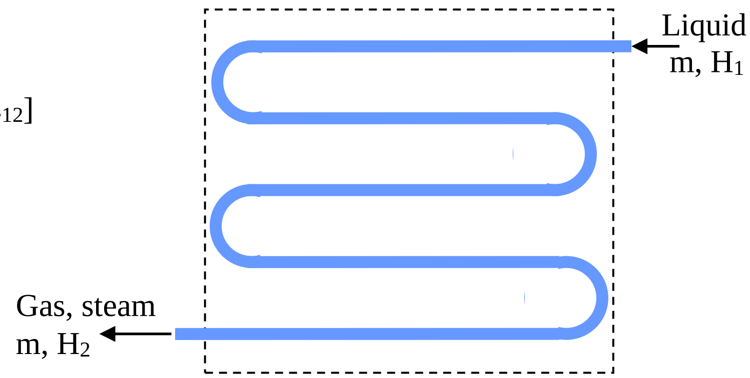


7- Evaporator

$$Q_{12} - \cancelto{0}{w_{12}} = [\Delta H_{12} + \cancelto{0}{\Delta K.E_{12}} + \cancelto{0}{\Delta pot.E_{12}}]$$

pot.E = 0, $\Delta K.E_{12} = 0$, $w_{12} = 0$

$$\boxed{\mathbf{Q_{12} = \Delta H_{12}}}$$



EX: - Fluid flow in a pipe the pressure decrease due to closed valve from 10 bar to 1 bar if the specific volume of fluid increase from 0.3m^3 /kg to 1.8 m^3/kg find the change in internal during the throttle process.

Sol:-

p_1 = 10bar, p_2 = 1bar, $v_1 = 0.3$ m^3 /kg, $v_2 = 1.8$m^3/kg, ΔU_{12} =?

The process is throttle therefore,

$H_2 = H_1$

$m\, h_2 = m\, h_1$

$u_2 + p_2 v_2 = u_1 + p_1 v_1$

$u_2 - u_1 = p_1 v_1 - p_2 v_2$

$\Delta U_{12} = 10^2 (p_1 v_1 - p_2 v_2)$

$= 10^2 (10 * 0.3 - 1 * 1.8) = 120$ kJ/kg

EX: - Fluid with 2800 kJ/kg specific enthalpy enter a nozzle with small value of velocity, the rate of flow is 14kg/sec the specific enthalpy and volume at exit equal 2250kJ/kg &1.25m^3/kg respectively find exit velocity & exit area.

Sol:-

h_1 = 2800 kJ/kg, nozzle, $C_1 = 0$, $m^{\cdot}$ = 14kg/sec, h_2 = 2250kJ/kg,

v_2 = 1.25m^3/kg, C_2 =? , A_2 =?

$C_2 = \sqrt{2\,(h1 - h_2)}$

$C_2 = \sqrt{2\,(2800 - 2250)} = 33.166$ m/sec

$$A_2 = \frac{V_2\ (m^3/sec)}{C_2\ (m/sec)}$$

$V_2 = \nu\ (m^3/kg) * \dot{m}\ (kg/sec)$

$= 1.25 * 14 = 17.5\ m^3/sec$

$A_2 = 17.5 / 33.166 = 0.528\ m^2$

EX: - Vapor enter condenser with rate 35 kg/sec and specific enthalpy 2200 kJ/kg. The liquid leaves the condenser with 255kJ/kg specific enthalpy .find the heat loss from the condenser.

Sol:-

$\dot{m} = 35 kg/sec$, $h_1 = 2200$ kJ/kg, $h_2 = 255$kJ/kg, Q_{12} =?

$Q_{12} = \Delta H_{12} = \dot{m}\ (h_2 - h_1)$

$= 35$ kg/sec $(255 - 2200)$ kJ/kg

$= - 68075$ kJ/sec $= - 68075$ kW

EX: - A fluid flow in turbine with rate 45kg/min the specific enthalpy decrease about 580kJ/kg. After leaving the turbine and the heat lost to surrounding with rate 2100kJ/min find the power reduced by the turbine?

Sol:-

$\dot{m} = 45/60 = 0.75$kg/sec, $\Delta h_{12} = - 580$ kJ/kg, $Q_{12} = - 2100$ kJ/min $= 2100 /60 = -35$ kJ/sec, w =?

$Q_{12} - w_{12} = \Delta H_{12}$

$\Delta H_{12} = \dot{m} * \Delta h_{12}$

$= 0.75 * - 580 = - 435$ kW

$- 35 - w_{12} = -435$

$w_{12} = 400$ kW

EX: - Ferion gas enter the compressor at 0.4bar and 0.1 m^3 /kg specific volume, the gas compressed to 2bar and 0.01 m^3 /kg specific volume, the increase in internal energy is 116kJ/kg. Calculate the heat quantity if the work done is 180 kJ/kg.

Sol:-

Compressor, $p_1 = 0.4$bar, $\nu_1 = 0.1 m^3/kg$, $p_2 = 2$bar, $\nu_2 = 0.01 m^3/kg$, $\Delta u_{12} = +116$ kJ/kg ,

w_{12} = - 180 kJ/kg, Q_{12} =?

$q_{12} - w_{12} = \Delta h_{12}$

$\Delta h_{12} = h_2 - h_1$

$= (u_2 + p_2 . v_2) - (u_1 + p_1 . v_1)$

$= (u_2 - u_1) + 10^2 (p_2 . v_2 - p_1 . v_1)$

$= 116 + 10^2 (2 * 0.01 - 0.4 * 0.1)$

= 114kJ/kg

q_{12} – 180 = 114

q_{12} = -66 kJ/kg

Problems

5-1 Stead flow of steam enters a condenser with an enthalpy of 2400 kJ/kg and a velocity of 366 m/sec. the condensate leaves the condenser with an enthalpy of 162kJ/sec and a velocity of 6 m/sec what is the heat transferred to the cooling water per kg steam condensed. (- 69198 kJ/kg)

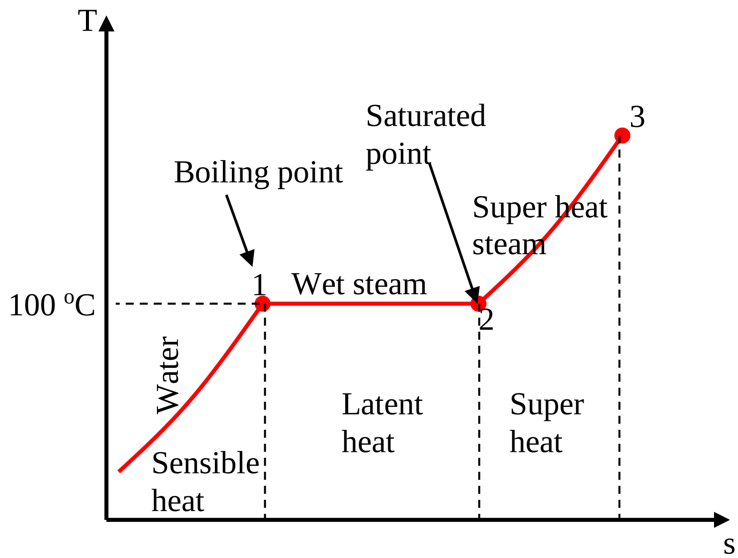
5-2 An air compressor delivers 4.5 kg of air per minute at a pressure of 7 bar and a specific volume of 0.17 m^3 /kg. Ambient conditions are pressure 1bar and specific volume 0.86 m^3/kg. The initial and final internal energy values for the air are 28 kJ/kg and 110 kJ /kg respectively. Heat rejected to the cooling jacket is 76kJ/kg of air pumped. Neglecting changes in kinetic and potential energies, what is the shaft power required driving the compressor? (14.3kW)

Unit six

Steam

Steam

Steam formation: - it is water vapor formed through water heating by the addition of the following heats:-



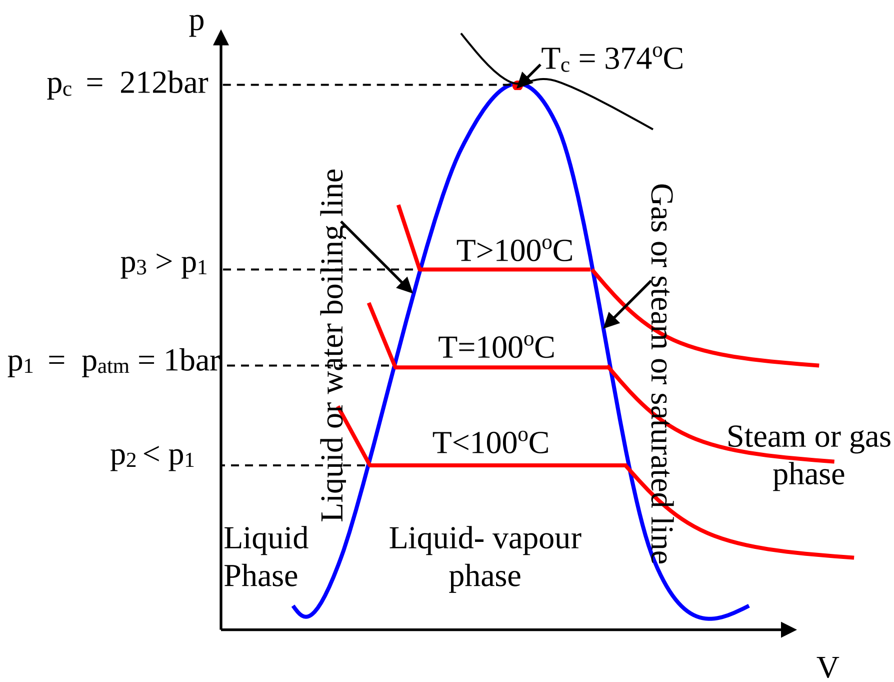
1- Sensible heat: - It is quantity of heat added to the water to raise its temperature from the room temperature to boiling temperature.

$$\boxed{Q_{sen} = m\ Cp\ .\Delta T}$$ ... Where Cp = 4.186 kJ/kg.K

2- Latent heat: - It is heat added to the water at boiling temperature (constant temperature) to transfer it from liquid phase to gases phase.

Steam formed after latent heat addition is called dry and saturated steam and temperature of which is constant and equal to saturated or boiling temperature
Steam formed between boiling point (1) & saturated point (2) is called wet steam & its temperature equal to saturation temperature or (boiling temperature) as shown in fig above.

3- Super heat: - It is quantity of heat added to the dry and saturated steam to raise its temperature above saturated temperature, in this case the steam regarded as gas.



The pressure - volume (p.V) phase diagram for steam.

- Critical point, p_c = 212bar ,T_c =374°C

Above p_c no latent heat is added as shown in fig and the water transfer suddenly to steam.

Steam tables: - it is given values of steam properties [p, v, T, h, u, and s] where:-

p = pressure (bar)

T = temperature (°C)

u = specific internal energy (kJ.kg)

v = specific volume (m^3/kg)

h = specific enthalpy (kJ/kg)

s = specific entropy (kJ/kg)

t_s = saturation temperature (°C)

f = fluid [liquid phase]

g = gas [vapor phase]

f_g = between fluid and gas phase [wet steam], **QL** = Δh_{fg}

Wet steam: - There are no values of wet steam properties in the steam table.

Dryness fraction(x): - It is quantity depends of how far it's from point (1)

It is ratio between saturated steams in the mixture of the net steam to the total mass of the mixture (water + steam)

$$x = \frac{\text{Quantity of saturated steam}}{\text{Total mass of mixture}}$$

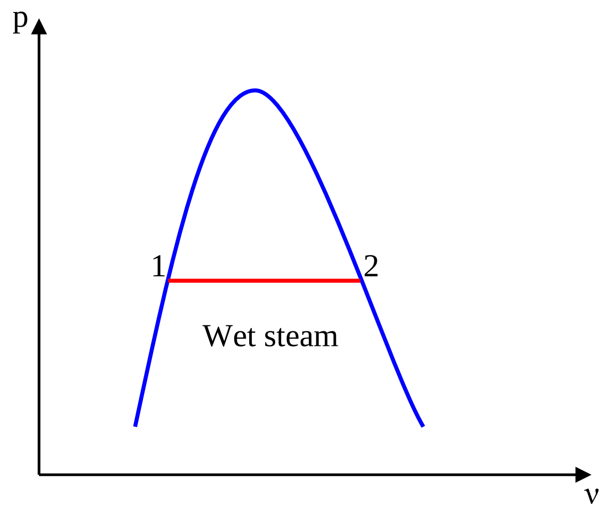


Fig (1)

x = 0 at point (1) boiling point

x = 1.0 at point (2) saturated point

Relation of wet steam

For a wet steam the total volume of the mixture is given by the volume of the liquid present plus the volume of dry steam present , therefore the specific volume is given by ,

$$\nu = \frac{\text{Volume of liquid + volume of dry steam}}{\text{Total mass of wet steam}}$$

Now for 1kg of wet steam there are x kg of dry steam and (1-x) kg of liquid, where x is the dryness fraction

$\nu = \nu_f\ (1 - x) + \nu_g\ x$

The volume of the liquid is usually negligibly small compared to the volume of dry saturated steam,

$$\boxed{\nu = x.\nu_g}$$

The enthalpy of wet steam is given by the sum of the enthalpy of the liquid plus the enthalpy of the dry steam,

$h = (1 - x)\, h_f + x\, h_g$

$h = h_f + x\,(h_g - h_f)$

$$\boxed{h = h_f + x\,(h_{fg})}$$

similary, the interanal energy of a wet steam is given by the internal energy of the liquid plus the internal energy of the dry steam,

$u = (1 - x)\, u_f + x\, u_g$

$u = u_f + x\,(u_g - u_f)$

$$\boxed{u_x = u_f + x\,(u_g - u_f)}$$

Entropy of the dry steam in the mixture. For wet steam with dryness fraction, x, we have

$s = (1 - x)\, s_f + x\, s_g$

$s = s_f + x\,(s_g - s_f)$

$$\boxed{s_x = s_f + x\,(s_{fg})}$$

Steam defines as follow:-

1- **Wet steam: -** is define by its pressure & dryness x for example ,steam at 9bar & 0.85 dryness
2- **Dry and saturated steam :-** pressure only &dry and saturated for example dry and saturated steam at 5 bar or steam at 5 bar dry and saturated [point 2] as shown in fig (1)
3- **Super heated steam :-** define by pressure and temperature for example steam at 30 bar and 350 ^oC

EX:-

Find steam properties of the following condition,

1- steam at 30 bar , dry &saturated
2- steam at 30 bar ,0.85 dryness
3- steam at 30 bar , 350 ^oC

Sol:-
1- from table at 30 bar

$t_s = 233.8\ ^oC$, $v_g = 0.06665\ m^3/kg$, $u_g = 2603$ kJ/kg, $h_g = 2803$ kJ/kg,$s_g = 6.186$ kJ/kg.K

2- Steam at 30 bar, 0.85 dryness

Wet steam,

$\nu_x = x\ \nu_g$

$= 0.85 * 0.06665$

$= 0.0566525\ m^3 / kg$

$u_x = u_f + x\ (u_g - u_f\ )$

$= 1004 + 0.85\ (2603 – 1004)$

$= 2363.15$ kJ/kg

$h_x = h_f + x\ (h_{fg})$

$= 1008 + 0.85 * 1795$

$= 2533.75$ kJ/kg

$s_x = s_f + x\ (s_{fg})$

$= 2.645 + 0.85 * 3.541$

$= 5.655$ kJ/kg.K

3-steam at 30 bar, 350 ^oC

Steam is superheated,

$\nu_g = 0.0905\ m^3/kg$, $u_g = 2845$ kJ/kg, $h_g = 3117$ kJ/kg,s_g= 6.744kJ/kg.K

EX: - Determine the state of steam at 10 bar, if the heat added to transfer the water from 0^oC is 2500kJ/kg.

Sol:-

$q_x = h_x = 2500$ kJ/kg

From table at 10 bar, $h_g = 2778$ kJ/kg > 2500 steam is wet

$h_x = h_f + x\ (h_{fg})$

From table at 10 bar $h_f = 763$ kJ/kg, $h_{fg} = 2015$ kJ/kg

$$x = \frac{h_x - h_f}{h_{fg}} = \frac{2500 - 763}{2015} = 0.86$$

EX: - vessel its volume is 85 liter contain 0.1kg of water and 0.7kg of dry steam in equilibrium state find the pressure inside the vessel.

Sol:-

$V = 85$ liter $= 85/1000 = 0.085\ m^3$,

$m_t = m_w + m_s$

$= 0.1 + 0.7$

$= 0.8$ kg

$v_x = V/m_t = 0.085 / 0.8 = 0.106\ m^3/kg$

$v_x = x.v_g$

$x = m_s/m_t = 0.7/0.8 = 0.875$

$v_g = v_x / x = 0.106 / 0.875 = 0.121\ m^3/kg$

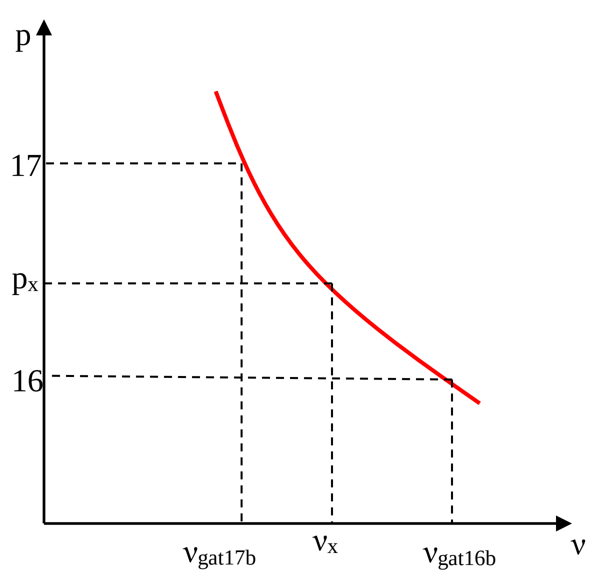
From table at p= 16 bar, $v_g = 0.1237\ m^3/kg$

at p= 17 bar, $v_g = 0.1167\ m^3/kg$

$$p_x = p_{16} + \frac{v_x - v_{g\ at\ 16bar}}{v_{g\ at\ 17bar} - v_{g\ at\ 16bar}}(p_{17} - p_{16})$$

$$p_x = 16 + \frac{0.121 - 0.1237}{0.1167 - 0.1237}(17 - 16)$$

$= 16.386$ bar



$$\frac{p_{17} - p_{16}}{v_{gat17bar} - v_{gat16bar}} = \frac{p_x - p_{16}}{v_x - v_{gat16bar}}$$

$$p_x - p_{16} = \frac{v_x - v_{gat16bar}}{v_{gat17bar} - v_{gat16bar}} * (p_{17} - p_{16})$$

$$p_x = p_{16} + \frac{v_x - v_{gat16bar}}{v_{gat17bar} - v_{gat16bar}} * (p_{17} - p_{16})$$

Degree of superheated $(\Delta t)_{SH}$: - it is the difference between the temperature of superheated steam and saturated steam at same pressure.

$\boxed{(\Delta t)_{SH} = t - t_s}$ ⟶ at p = constant

EX: - what is the degree of superheated at pressure 20bar & 300°C.

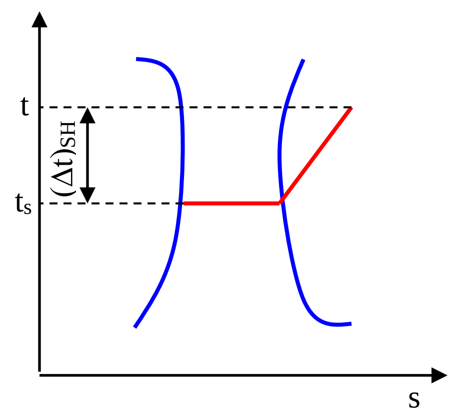
Sol:-

From table, ts = 212.4 °C,

$(\Delta t)_{SH} = t - t_s$

$= 300 - 212.4$

$= 87.6\,^{o}C$



EX: - super heated steam at 15 bar and degree of superheated 151.7 °C find its properties.

Sol:-

From table at 15 bar, $t_s = 198.3\,^{o}C$

$t = (\Delta t)_{SH} + t_s$

$= 151.7 + 198.3$

$= 350\,^{o}C$

$v_g = 0.1865\ m^3/kg$, $u_g = 2868\ kJ/kg$, $h_g = 3148\ kJ/kg$, $s_g = 7.182 kJ/kg.K$

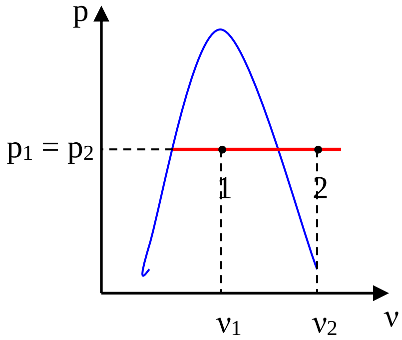
Steam process:-

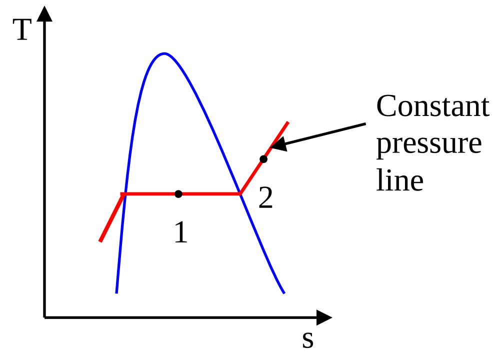
1- Isobaric process:-

$q_{12} = \Delta h_{12}$

$q_{12} - w_{12} = \Delta u_{12}$

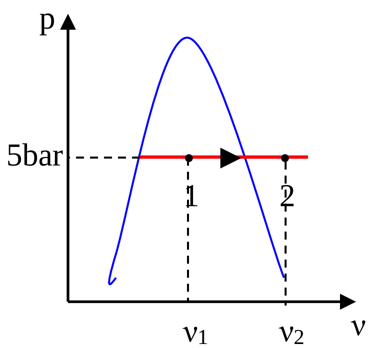
$w_{12} = 10^2\, p\, (v_2 - v_1)$





EX: - Steam is heated isobarically from 5 bar, 0.6 dry until its temperature is 300 °C find the quantity of heat add and change in internal energy.

Sol:-



Steam is heated therefore, $v_2 > v_1$

$p_1 = p_2 = 5\text{bar}$, $x = 0.6$, $T_2 = 300 + 273 = 573^oK$, $q_{12} = ?$, $\Delta u_{12} = ?$

$q_{12} = h_2 - h_1$

$\Delta u_{12} = u_2 - u_1$

Point (1) wet steam from steam table at 5 bar, $h_{f1} = 640$ kJ/kg, $h_{fg1} = 2109$ kJ/kg,

$u_{f1} = 639$kJ/kg, $u_{g1} = 2562$ kJ/kg,

$h_1 = h_{f1} + x\,.h_{fg1}$

$= 640 + 0.6 * 2109 = 1905.4$kJ /kg

$u_1 = u_{f1} + x.\,(u_{g1} - u_{f1})$

$= 639 + 0.6 *(2562 - 639) = 1792.8$ kJ/kg point (2) steam is superheated from table at p = 5bar & T = 300 ºC , u_2 = 2804 kJ/kg,

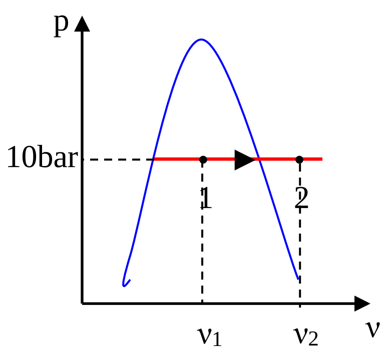
h_2 = 3065 kJ/kg ,

q_{12} = 3065– 1905.4 = 1159.6 kJ/kg (the quantity of heat added)

Δu_{12} = 2804 – 1792.8 = 1010.8 kJ/kg (change in internal energy)

EX: - Steam at 10bar, 0.95dry is heated electrically at constant pressure process. If the rate of consume power is 1kW and the rate of steam is 0.3kg/min, determine the final state of steam?

Sol:-



x = 0.95 , $p_1 = p_2$ = 10bar, Q_{12} = 1kW , $\dot{m}_s$ = 0.3 kg/min

$q_{12} = Q_{12} / \dot{m}_s$ = 1kJ/kg / (0.3 kg / 60 sec) = 200kJ/kg

$q_{12} = h_2 - h_1$

$h_2 = q_{12} + h_1$

Point (1) wet steam from steam table at 10 bar, h_{f1}= 763 kJ/kg, h_{fg1} = 2015 kJ/kg,

$h_1 = h_{f1} + x .h_{fg1}$

= 763 + 0.95 * 2015

= 2677.25kJ /kg

h_2 = 200 + 2677.25 = 2877.25 kJ/kg

From table at p = 10 bar, $h_g < h_2$ therefore the final state is superheat

From superheated table at p =10 bar,

h = 2829 at t = 200 ºC

h = 2944 at t = 250 °C

$$t_2 = 200 + \frac{2877 - 2829}{2944 - 2829}(250 - 200)$$

= 220.87 °C

$$v_{g2} = 0.2061 + \frac{2877 - 2829}{2944 - 2829}(0.2328 - 0.2061)$$

= 0.21724 m^3 /kg

$$u_{g2} = 2623 + \frac{2877 - 2829}{2944 - 2829}(2711 - 2623)$$

= 2659.73 kJ/kg

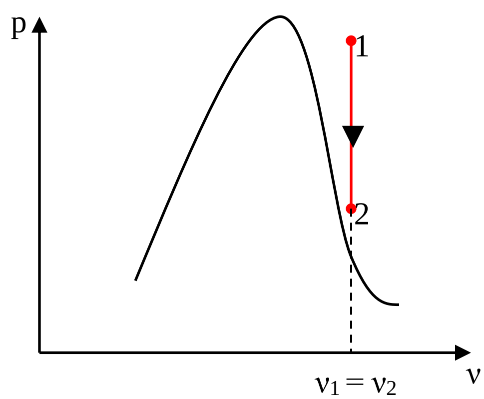
$$s_{g2} = 6.695 + \frac{2877 - 2829}{2944 - 2829}(6.926 - 6.695)$$

= 6.7914kJ/kg.K

2- Isochoric process:-

$w_{12} = 0$, $v_1 = v_2$ = constant

$$\boxed{q_{12} = \Delta u_{12}}$$

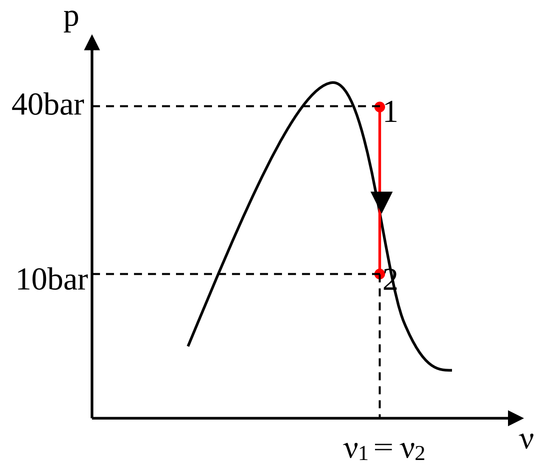


EX: - steam at 40 bar and 400°C expand isochorically until its pressure is 10 bar, find the quantity of heat add or rejected during this process.

Sol:-

$q_{12} = u_2 - u_1$
From superheated table at p = 40 bar& 400°C, u_1 = 2921kJ/kg ,v_1 = 0.0733 m^3/kg = v_2 ,

From table at p = 10 bar , v_{g2} = 0.1944 > v_2 therefore steam is wet



$v_2 = x \cdot v_{g2}$

$x = v_2 / v_{g2} = 0.0733 / 0.1944 = 0.377$

$u_2 = u_{f2} + x \cdot (u_{g2} - u_{f2})$

$= 762 + 0.377\ (2584 - 762)$

= 1449kJ/kg

q_{12} = 1449 – 2921

= - 1472 kJ/kg rejected

EX: - Radiator is using as heating system its volume is 0.054 m^3 and the steam at 1.4 bar dry & saturated when the valve of Radiator is closed the heat transfer to room & pressure reduced to 1.2 bar . Calculate the mass of steam and the amount of heat transfer to the room.

Sol:-

V = 0.054 m^3, p_1 = 1.4 bar & dry sat. , $v_1 = v_2$ closed, p_2 = 1.2 bar, m_s =?, Q_{12} = ?

$$m_s = \frac{V\ (m^3)}{v_1 (m^3 / kg)}$$

From table at p = 1.4 bar, $v_{g1} = v_1 = 1.236\ m^3/kg = v_2$, $u_1 = u_{g1} = 2517$ kJ/kg

$$m^{\cdot}_s = \frac{0.054}{1.236} = 0.044 \text{ kg}$$

From table at p = 1.2 bar, $v_{g2} = 1.428 > v_2 = 1.236\ m^3/kg$ therefore steam is wet

$v_2 = x \, . \, v_{g2}$

$x = v_2 / v_{g2} = \quad 1.236 / 1.428 = 0.866$

$u_2 = u_{f2} + x \, . \, (u_{g2} - u_{f2})$

$= 439 + 0.866\ (2512 - 439)$

$= 2234.218$kJ/kg

$q_{12} = u_2 - u_1$

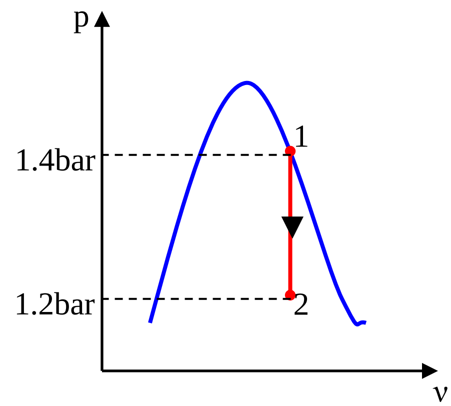
$= 2234.218 - 2517$

$= -\ 282.782$ kJ/kg

$Q_{12} = m^{\cdot}_s * q_{12}$

$= 0.044 *-\ 282.782$

$= -\ 12.442408$ kJ

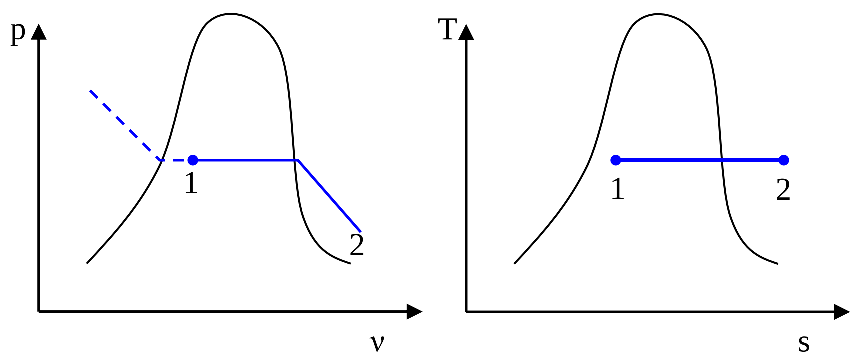


3- Isothermal process:-

$T_1 = T_2 =$ constant

$$\boxed{q_{12} - w_{12} = \Delta u_{12}}$$

$$\boxed{q_{12} = T . (s_2 - s_1)}$$



EX:- 1kg of steam expand isothermally from 40 bar dry & saturated to 5 bar find the final steam condition and the heat transfer & heat pump work during this process .

Sol:-

m = 1kg, T = const., $p_1 = 40$ bar & dry sat., $p_2 = 5$ bar, Q_{12}, w_{12},

$q_{12} = T.(s_2 - s_1)$

From table at 40 bar&dry sat., $T_1 = T_{s1} = 250.3^oC = T_2$, $s_1 = s_g = 6.070$ kJ/kg.K,

From table at p = 5 bar, $T_{s2} = 151.8^oC < T_2$ therefore steam is superheated,

From superheated table at p = 5bar &250 ^oC ,$s_2 = 7.271$ kJ/kg.K ,

$Q_{12} = m * T (s_2 - s_1)$

$= 1 * (250.3+273) (7.271 - 6.07)$

$= 300.6$kJ

$\Delta U_{12} = m * (u_2 - u_1)$

From table at 40 bar, $u_1 = 2602$kJ/kg,

From superheated table at p = 5bar &250 ^oC ,$u_2 = 2725$kJ/kg ,

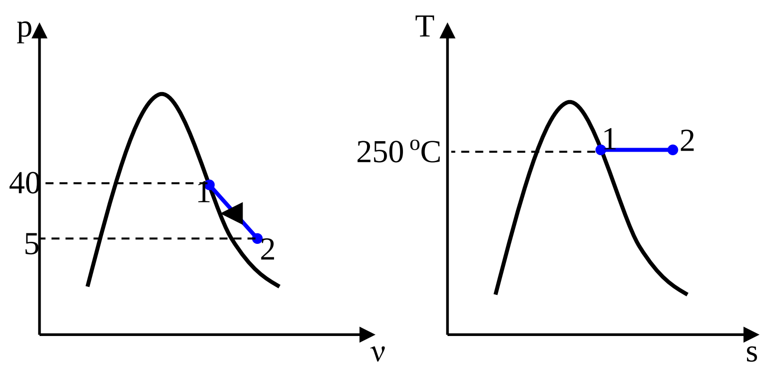
$\Delta U_{12} = 1 * (2725 - 2602)$

$= 123$ kJ

$w_{12} = Q_{12} - \Delta U_{12}$

$= 300.6 - 123$

$= 177.6$kJ



EX: - 3kg of steam expand isothermally from 30 bar and 0.9 dry to 2bar. Find the work done.

Sol:-

m = 3kg, p_1= 30bar, $x_1 = 0.9$, $p_2 = 2$bar

From table at 30 bar, $t_s = 233.8^oC = T_1 = T_2$ = const.

$s_1 = s_{f1} + x_1 * s_{fg1}$

$= 2.645 + 0.9 * 3.543 = 5.8314$ kJ/kg.K

$u_1 = u_{f1} + x_1 *(u_{g1} - u_{f1})$

$= 1004 + 0.9 (2603 - 1004)$

$= 2443.1$kJ/kg

From table at 2 bar, $t_{s2} = 120.2\,^oC < T_2$ therefore final state is superheated

From superheated table at p = 2 bar & T = 233.8°C,

$$s_2 = 7.507 + \frac{233.8 - 200}{250 - 200}(7.708 - 7.507) = 7.642876 kJ/kg.K$$

$$u_2 = 2655 + \frac{233.8 - 200}{250 - 200}(2731 - 2655)$$

$= 2706.366 kJ/kg$

$\Delta u_{12} = u_2 - u_1$

$= 2706.366 - 2443.1 = 263.276 kJ/kg$

$q_{12} = T(s_2 - s_1)$

$= (233.8 + 273)(7.642876 - 2443.1) = 918.056\ kJ/kg$

$w_{12} = m * (q_{12} - \Delta u_{12})$

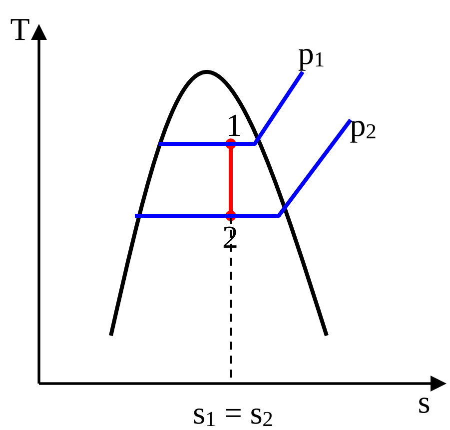
$= 3 * (918.056 - 263.276) = 1964.28\ kJ$

4- Isentropic process:-

$s_1 = s_2$, $q_{12} = w_{12} + \Delta u_{12}$, & $q_{12} = 0$ therefore $w_{12} = -\Delta u_{12}$

$$\boxed{w_{12} = u_1 - u_2}$$

$$\boxed{w_{12} = h_1 - h_2}$$ … For steam turbine [s.F.E.E]



All expand (+w), all compressed (–w)

EX: - Steam at 40 bar & 375°C expand isentropically in a turbine to 2bar find the final steam condition and work done what would the final pressure such that the steam was dry & saturated.

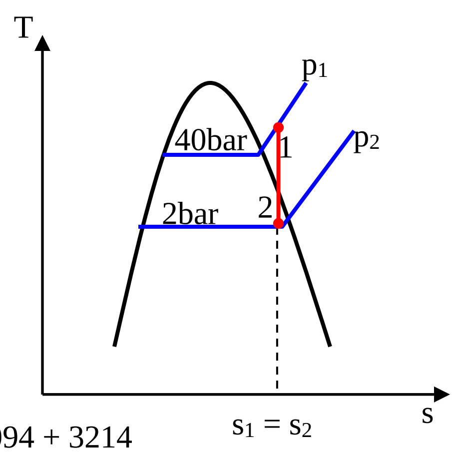
Sol:-

p_1 = 40 bar, T_1 = 375°C, p_2 = 2bar,

From table at p_1 = 40 bar& T_1 = 375°C where375°C is between 350°C &400°C

$$s_1 = \frac{6.584 + 6.764}{2}$$

= 6.675 kJ/kg.K = s_2 (isentropic process)



$$h_1 = \frac{3094 + 3214}{2}$$

= 3154 kJ/kg

From table at p_1 = 2 bar, s_{g2} = 7.127 kJ/kg.K > s_2 therefore steam is wet

$s_2 = s_{f2} + x * s_{fg2}$

$$x = \frac{s_2 - s_{f2}}{s_{fg2}}$$

$$x = \frac{6.675 - 1.53}{5.597}$$

= 0.919

$h_2 = h_{f2} + x * h_{fg2}$

= 505 + 0.919 * 2202= 2528.6 kJ/kg

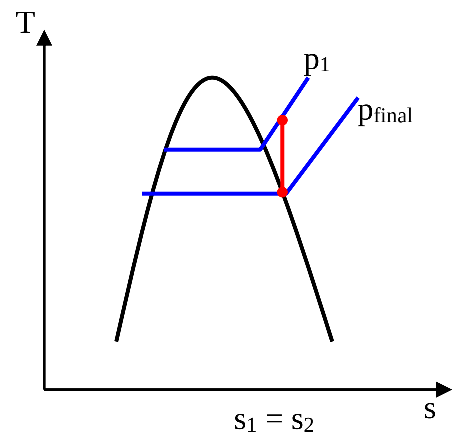
$w_{12} = h_1 - h_2$

$= 3154-2528.6$

$= 652.4$ kJ/kg

<u>When the final state is dry & saturated</u>

$s_1 = s_2 = s_{g2} = 6.675$ kJ/kg.K



From table $s_g = 6.709$ at p = 7 bar

$s_g = 6.663$ at p = 8bar

$$p_{final} = 7 + \frac{6.675 - 6.709}{6.613 - 6.709} *(8-7)$$

$= 7.717$ bar

EX: - Steam at 15bar & 330°C expand isentropically in a turbine to 0.12 bar if the flow rate of steam is 300kg/min determine the power produced in turbine.

Sol:

$p_1 = 15$bar, $t_1 = 330^oC$, $p_2 = 0.12$bar, $\dot{m}_s = 300$kg/min

Power = W = $\dot{m}_s$ ($h_1 - h_2$)

From superheated table at p = 15bar & t = 330°C, where330°C is between 300°C&350°C in superheated table

$$s_1 = 6.768 + \frac{330 - 300}{350 - 300}(6.957 - 6.768)$$

$= 6.8814$kJ/kg.K $= s_2$ (isentropic process)

$$h_1 = 3025 + \frac{330 - 300}{350 - 300}(3138 - 3025)$$

$= 3092.8$ kJ/kg

From table at p = 0.12bar, $s_{g2} = 8.085 > s_2$ therefore steam is wet

$s_2 = s_{f2} + x * s_{fg2}$

$$x = \frac{s_2 - s_{f2}}{s_{fg2}}$$

$$x = \frac{6.8814 - 0.696}{7.389}$$

$= 0.837$

$h_2 = h_{f2} + x * h_{fg2}$

$= 207 + 0.837 * 2383$

$= 2201.831 kJ/kg$

Power = (300/60) (3092 – 2201)

= 4454.845 kW

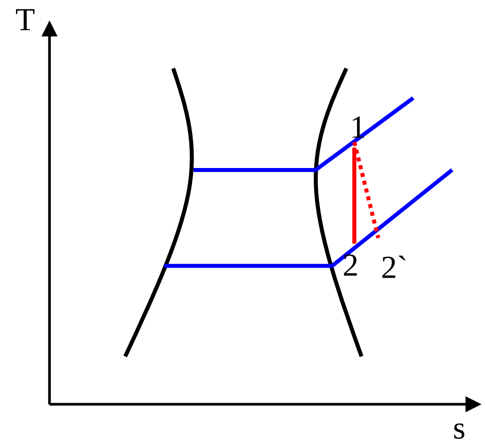
5- Adiabatic process:-

It is isentropic process with friction

η_i = isentropic efficiency or internal efficiency

$$\eta_i = \frac{\text{Adiabatic actual work}}{\text{Isentropic work}}$$

$$\boxed{\eta_i = \frac{h_1 - h_{2`}}{h_1 - h_2} = \frac{w_{12`}}{w_{12}}}$$



EX: - Steam turbine at 10 bar and 350°C is expanded adiabatically with an internal efficiency of 85% to 0.08 bar. If the flow rate of steam is 250 kg/min, determine the work produced in turbine?

Sol:-

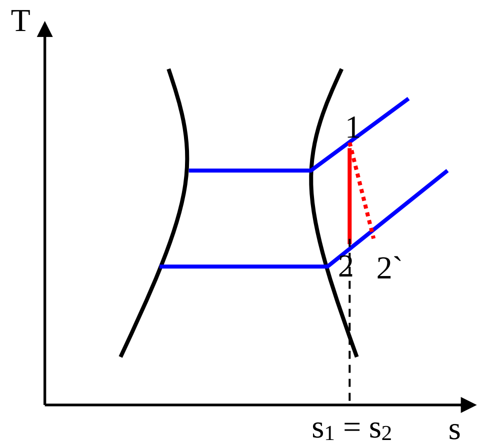
$T_1 = 350^oC$, $p_1 = 10bar$, $\eta_i = 85\%$, $p_2 = 0.08bar$, $m^{\cdot}_s = 250kg/min = 250/60 = 4.16$ kg/sec,

From superheated table at 10 bar & 350°C, $h_1 = 3158$ kJ/kg, $s_1 = 7.301$ kJ/kg.K $= s_2$

From table at 0.08 bar, $s_{g2} = 8.227$ kJ/kg.K $> s_2$ therefore steam is wet,

$s_2 = s_{f2} + x * s_{fg2}$

$$x = \frac{s_2 - s_{f2}}{s_{fg2}}$$



$$x = \frac{7.301 - 0.593}{7.634}$$

$= 0.87$

$h_2 = h_{f2} + x * h_{fg2}$

$= 179 + 0.87 * 2402$

$= 2263.7$ kJ/kg

$w_{12} = (h_1 - h_2)$

$= (3158 - 2263.7) = 894.3$ kJ/kg

Adiabatic actual work = η_i * Isentropic work

$w_{12`} = \eta_i * w_{12}$

$= 0.85 * 894.3$

$= 760.155$ kJ/kg* 4.16kg/sec = 3162.2 kJ/sec=3162.2 kW

Throttling process:-

The aim of this process is to reduce the steam pressure without any losses.

$$\boxed{h_1 = h_2}$$

This concludes that the quantity of steam after throttling is better than before throttling.

EX: - Dry and saturated steam at 50 bar is throttling to 1bar find the final condition of steam.

Sol:-

From steam table at 50 bar dry & sat. $h_1 = 2794$ kJ/kg $= h_2$,

From steam table at $p_2 = 1$bar, $h_{g2} = 2675$ kJ/kg $< h_2$ therefore steam is superheated

From superheated table at 1bar & $h_2 = 2794$ kJ/kg where 2794 is between 2777 at T = 150^{o}C& 2876 at T = 200^{o}C

$$T_2 = 150 + \frac{2794 - 2777}{2876 - 2777}(200 - 150) = 159^oC$$

EX: - Steam at 7 bar & relative humidity 0.08 throttle to 0.05 bar, determine the steam condition after throttle.

Sol:-

p_2 = 7 bar, R.H = 0.08, p_2 = 0.05

$$x = \frac{m_s}{m_T} = \frac{(1-0.08)}{(1-0.08)+0.08} = 0.92$$

$h_1 = h_{f1} + x * h_{fg1}$

$= 697 + 0.92 * 2067$

$= 2598.64 kJ/kg = h_2$

From table at p = 0.05 bar, $h_{g2} = 2423 < h_2$ the condition after throttling is superheated,

$$T_2 = 50 + \frac{2598.64 - 2594}{2688 - 2594}(100-50) \quad = 52.468^oC$$

Problems:-

6 – 1 A rigid vessel of volume 1 m^3 contains steam at 20bar and 400^{o}C. The vessel is cooled until the steam is just dry & saturated. Calculate the mass of the steam in the vessel, the final pressure of the steam, and the heat removed during the process. (6.62kg; 13.01bar; 2355kJ)

6 – 2 Steam at 7 bar, dryness fraction 0.9, expands reversibly at constant pressure until the temperature is 200^{o}C. Calculate the work done and heat supplied per kg of steam during the process. (38.2kJ/kg; 288.7kJ/kg)

6 – 3 Dry saturated steam at 7 bar expands reversibly in a cylinder behind a piston until the pressure is 0.1 bar. If heat supplied continuously during the process in order to keep the temperature constant, calculate the change of internal energy per kg of steam. (37.2kJ/kg)

6 – 4 1kg of steam in a cylinder expands reversibly behind a piston according to a law pv = constant, from 7 bar to 0.75 bar if the steam is initially dry saturated, find the temperature finally, the work done by the steam, and the heat flow to or from the cylinder walls (144^{o}C ; 427kJ/kg ; 430kJ/kg)

6 – 5 In a steam jacket cylinder, steam expands from 5 bar to 1.2 bar according to a law pv$^{1..05}$ = constant. Assuming that the initial dryness fraction is 0.9, calculate the work done and the heat supplied per kg of steam during the expansion. (221.8kJ/kg; 197.5kJ/kg)

6 – 6 Steam at 17 bar, dryness fraction 0.95, expands slowly in a cylinder behind a piston until the pressure is 4 bar. Calculate the final specific volume and the final temperature of the steam when the expansion follows the law pv = constant. (0.471m^3/kg; 150^{o}C)

6 – 7 the pressure in a steam main is 12 bar. A sample of steam is drawn off and passed through a throttling calorimeter, the pressure and temperature at exit from the calorimeter being 1 bar & 140^{o}C respectively. Calculate the dryness fraction of the steam in the main, stating any assumption made in the throttling process. (0.986)

Unit seven

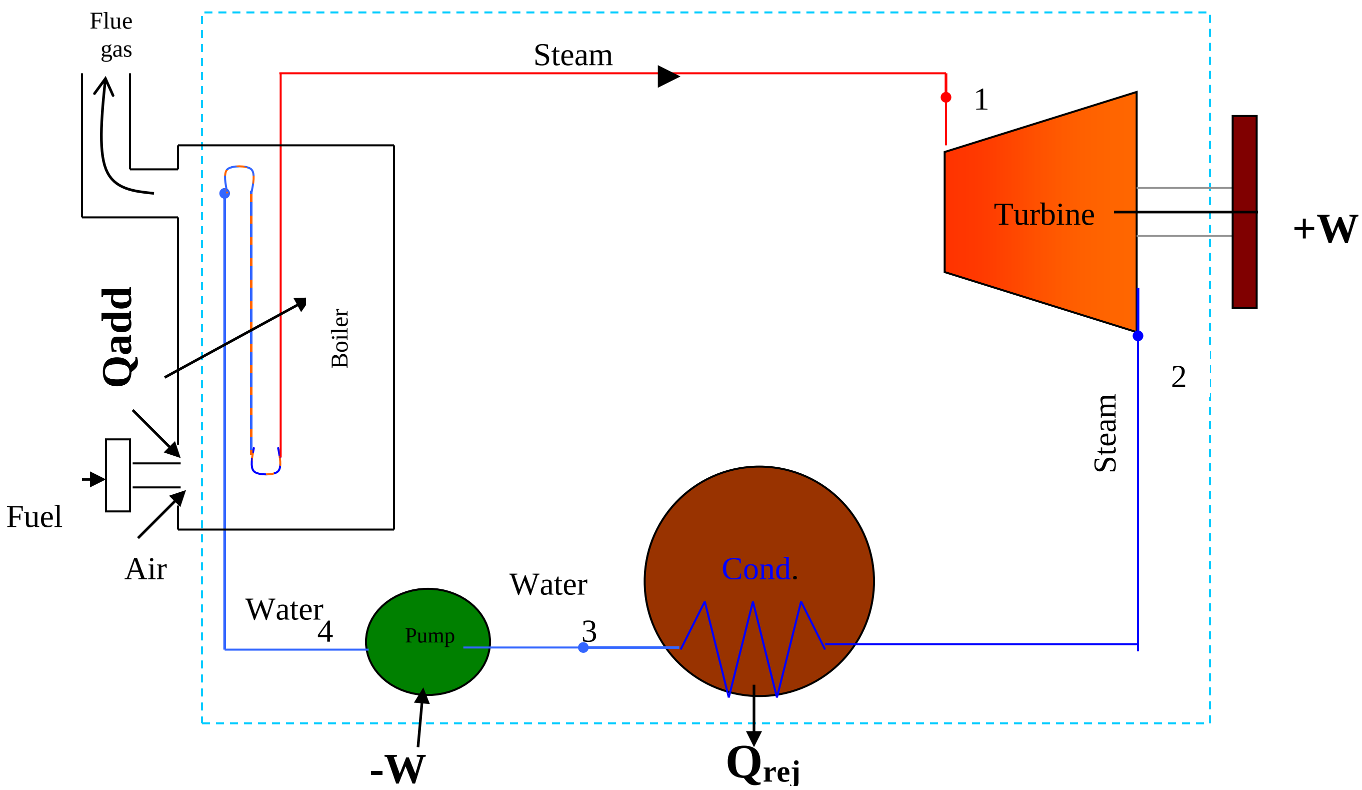
The second law of thermodynamics

The second law of thermodynamics

The first law of thermodynamic deals with process either reversible or irreversible and also either closed or open flow system.

The 2^{nd} law of thermodynamic deals with cyclic processes, these cyclic have only heat and work transfer its boundary no mass transfer

In figure shown the cycle of 2^{nd} law for steam power plant (heat engine).

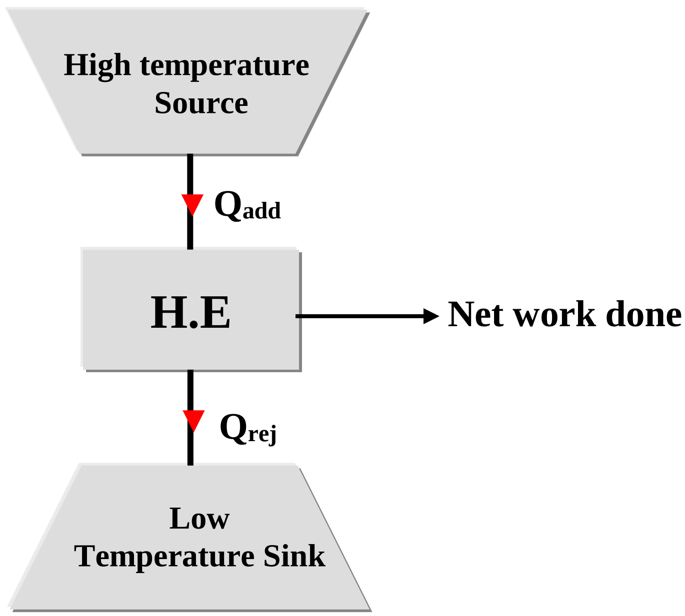


The element of steam power cycle is :-

1- boiler -open flow system – heat transfer (Qadd)
2- turbine – open flow system – energy transferd (+ve w.D)
3- condensor – open flow system – heat transferd (Qrej)
4- pump - open flow system – energy transferd (–vew.D)

In general the cycle can be classified to two main kind :-

1- Heat Engine [H.E]:-Its main object to take heat from high temperature source , transfered some of it to work and rejected the rest .
A diagrammatic representation of a heat engine is shown in fig below

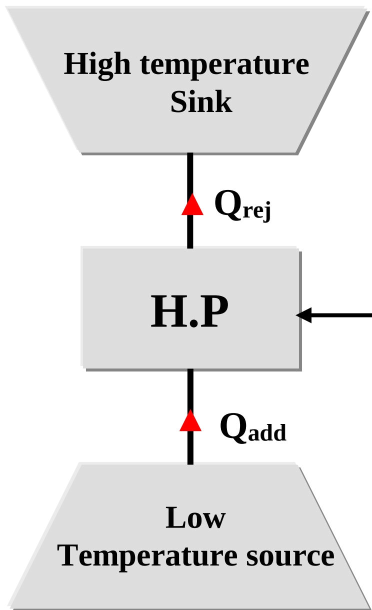


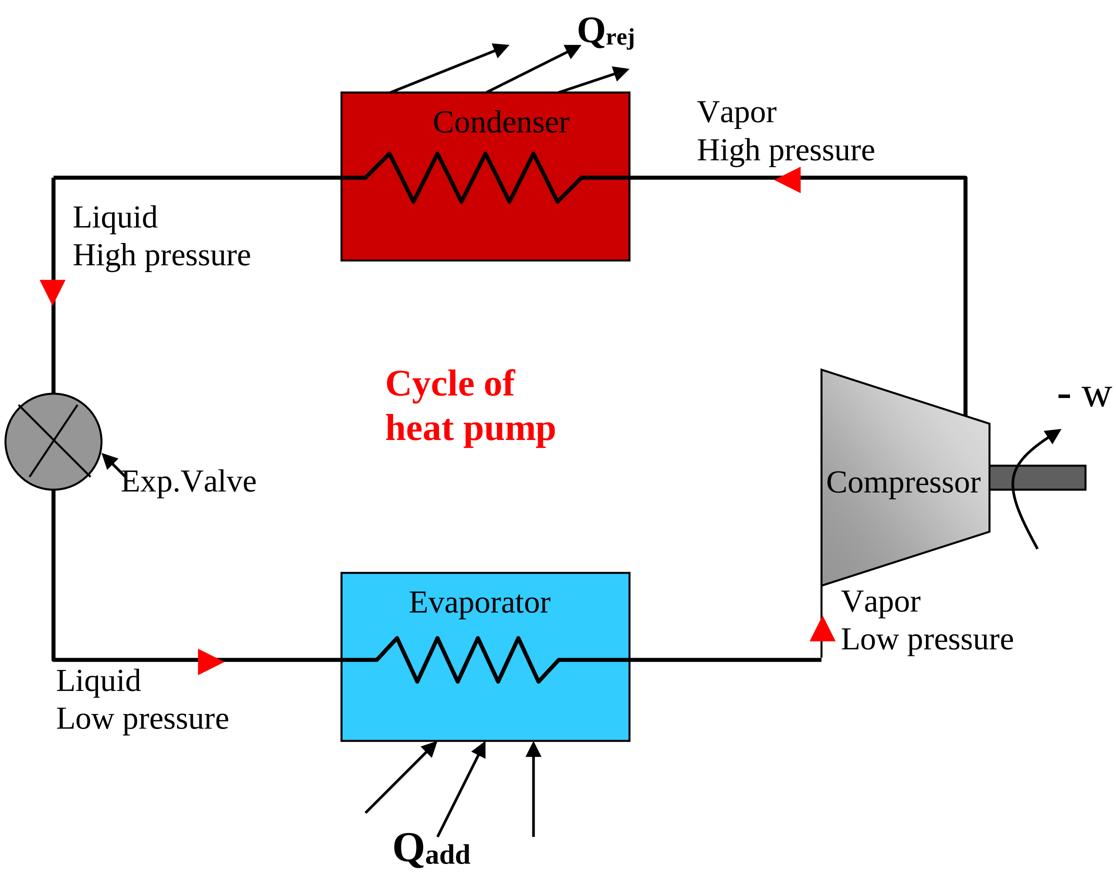
Net work done = $Q_{add} - Q_{rej}$

$$\eta_{H.E} = \frac{\text{Net w.D.}}{Q_{add}} = \frac{Q_{add} - Q_{rej}}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}}$$

2- Heat pump [H.P]:- It is reverse of heat engine it transfer heat from low temperature source to high temperature sink and consume work external.

A diagrammatic representation of a heat pump is shown in fig below





The cofficient of performance [C.O.P]of H.P:- it is ratio between heat rejected or heat added devided by work done.

$$(C\,.\,O\,.\,P)_{H.P} = \frac{Q_{rej}}{w.D} = \frac{Q_{rej}}{Q_{rej} - Q_{add}}$$

$$(C\,.\,O\,.\,P)_{ref.} = \frac{Q_{add}}{w.D} = \frac{Q_{add}}{Q_{rej} - Q_{add}}$$

$$\boxed{(C\,.\,O\,.P)_{H.P} - (C\,.\,O\,.P)_{ref.} = 1}$$

where$(C\,.\,O\,.\,P)_{H.P} > (C\,.\,O\,.\,P)_{ref.}$

The diffrence between heat engine & heat pump :-

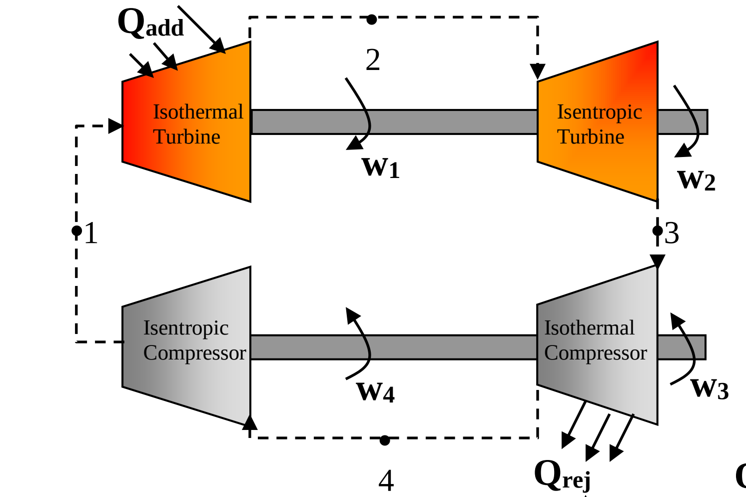
Heat engine	Heat pump
+ w	- w
High temperature source	Low temperature source
Low temperature sink	High temperature sink
$Q_{add} > Q_{rej}$	$Q_{rej} > Q_{add}$

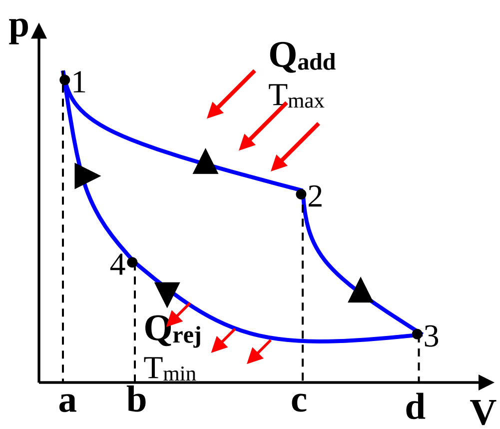
Statement of 2nd law of thermodynmic :-

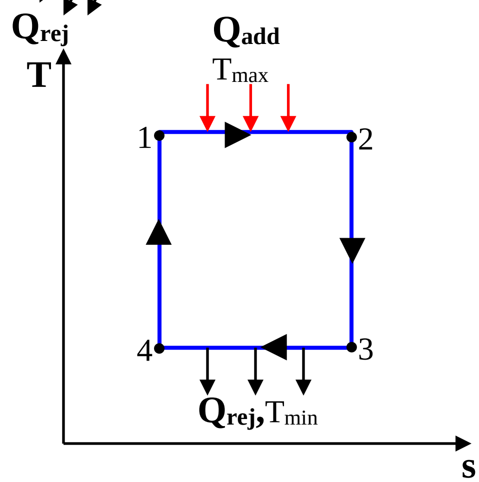
1- Kelvin – planck statement [used for heat engine] :- It is impossible to construct aheat engine which operate in a cycle and receives a given of heat from high temperature body and does an amount of works , as some of that heat must be rejected to a low temperature sink.

2- Clausius – statement [used for heat pump& ref.] :- heat can not passes by it self from a lower temperature body to higher temperature body without any equipment.

The Carnot cycle :-







Process (1-2) is isothermal expansion, T= const. = T_{max} = T_2 =T_1 & p.V = const.,

$Q_{add} = w_{12} = m.R.T_{max} \ln(V_2/V_1)= w_1$

Process(2- 3) is isentropic expansion,Q_{23} = 0& $p.V^{\gamma}$ = const., $w_{23} = \dfrac{p_2.V_2 - p_3.V_3}{\gamma - 1} = w_2$

process (4 – 3) is isothermal compresion , T= const. = T_{min}= T_3 =T_4 & p.V = const.

$Q_{rej} = w_{34} = -m.R.T_{min} \ln V_3/V_4 = m.R.T_{min} \ln V_4/V_3 = w_3$

Process(4- 1)is isentropic compresion,Q_{41}= 0& $p.V^{\gamma}$ = const.,$w_{41} = \dfrac{p_4.V_4 - p_1.V_1}{\gamma - 1} = w_4$

$Q_{add} = m.R.T_{max} \ln (V_2/V_1)$

$Q_{rej} = - m.R.T_{min} \ln (V_3/V_4)$

For process (2 – 3) isentropic exp.

$T_2.V_2^{\gamma-1} = T_3.V_3^{\gamma-1}$

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} = \frac{T_{max}}{T_{min}} \qquad \text{———————} \qquad (1)$$

For process (4 – 1) isentropic comp.

$$T_1.V_1^{\gamma-1} = T_4.V_4^{\gamma-1}$$

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = \frac{T_{max}}{T_{min}} \qquad \text{———————} \qquad (2)$$

$$\left(\frac{V_3}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\boxed{\frac{V_3}{V_2} = \frac{V_4}{V_1}} \quad \text{Or} \quad \boxed{\frac{V_3}{V_4} = \frac{V_2}{V_1}}$$

The efficiency of Carnot cycle is

$$\eta_{car} = 1 - \frac{Q_{rej}}{Q_{add}} = 1 - \frac{m.R.T_{min} \ln (V_3/V_4)}{m.R.T_{max} \ln (V_2/V_1)}$$

$$\boxed{\eta_{car} = 1 - \frac{T_{min}}{T_{max}}}$$

w_1 = area (1 2 c a 1) + ve

w_2 = area (2 3 d c 2) + ve

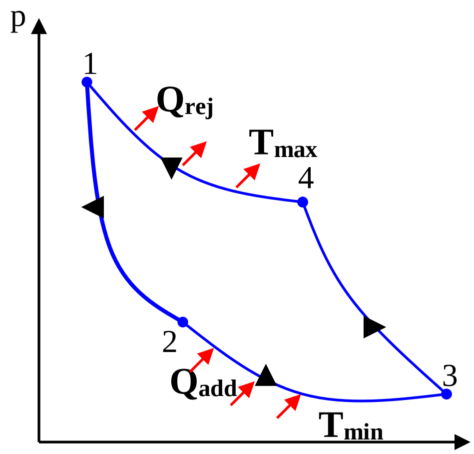
w_3 = area (4 3 d b 4) – ve

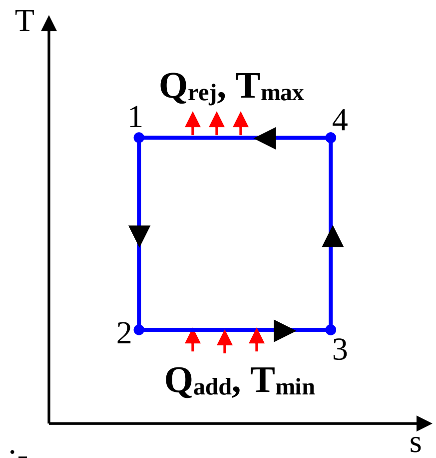
w_4 = area (1 4 b a 1) – ve

Net woork done = $w_1 + w_2 - w_3 - w_4$

= area (1 2 3 4 1) + ve

For Heat Pump working on Carnot principle we reversed carnot cycle:-





For reversed Carnot cycle as shown in figure :-

$$(C . O . P)_{H.P} = \frac{Q_{rej}}{Q_{rej} - Q_{add}}$$

$$\boxed{\mathbf{(C . O . P)_{H.P} = \frac{T_{max}}{T_{max} - T_{min}}}}$$

$$(C . O . P)_{Ref.} = \frac{Q_{add}}{Q_{rej} - Q_{add}}$$

$$\boxed{\mathbf{(C . O . P)_{Ref} = \frac{T_{min}}{T_{max} - T_{min}}}}$$

$(C . O . P)_{H.P} - (C . O . P)_{Ref.} = 1$

$(C . O . P)_{H.P} > (C . O . P)_{Ref.}$

EX:- The heat rejected from steam power plant is 1600kJ/kg. while the work done by the machine is 800kJ/kg.and work wanted to operate water pump is 20kJ/kg.Calculate the efficiency of power plant ?

Sol:-

q_{rej} = 1600 kJ/kg, w_{out} = 800kJ/kg , w_{in}= - 20kJ/kg,

$$\eta_{power\ plant} = 1 - \frac{Q_{rej}}{Q_{add}} = \frac{N.W.D}{Q_{add}}$$

$N.W.D = Q_{add} - Q_{rej}$

$N.W.D = w_{out} - w_{in} = 800 - 20 = 780$ kJ/kg

$780 = Q_{add} - 1600$

$Q_{add} = 2380$ kJ/kg

$$\eta_{power\ plant} = \frac{N.W.D}{Q_{add}}$$

$$= \frac{780}{2380} = 0.328 = 32.8\%$$

EX:- The maximum temperature of Carnot cycle forH.E is 1000°C & the minimum temperature is 200°C The heat produced Max. temperature is 6000kJ/min .Find the power of the engine ?

Sol:-

$T_{max} = 1000^oC$, $Q_{add} = 6000/60 = 100$kJ/sec $= 100$ kW

Power = w.D = $Q_{add} - Q_{rej}$

$$\frac{T_{min}}{T_{max}} = \frac{Q_{rej}}{Q_{add}}$$

$$Q_{rej} = \frac{T_{min}}{T_{max}} * Q_{add}$$

$$= \frac{200 + 273}{1000 + 273} * 100 = 37.15 \text{ kW}$$

Power = w.D = 100 - 37.15 = 62.81kW

EX:- Through the revese Carnot cycle, the work wanted to system is 400kJ/min and work produced by the system is 200kJ/min.If the haet rejected from the system is 600kJ/min how much heat is added and what power wanted to work this system also calculate C.O.P for the state a- used as ref.b- used as H.P.

Sol:-

$(w.D)_{in}$ = 400kJ/kg , $(w.D)_{out}$ = 200kJ/min , Q_{rej} = 600kJ/min , Q_{add} = ? , N.W.D =?

$(C.O.P)_{Ref}$ =? , $(C.O.P)_{H.P}$ =?

N.W.D $=(w.D)_{in} - (w.D)_{out}$

$= 400 – 200 = 200$kJ/min

N.W.D $=Q_{rej} - Q_{add}$

$200 = 600 – Q_{add}$

Q_{add} = 400 kJ/min

$$(C.O.P)_{Ref} = \frac{Q_{add}}{N.W.D} = \frac{400}{200} = 2$$

$$(C.O.P)_{H.P} = \frac{Q_{rej}}{N.W.D} = \frac{600}{200} = 3$$

$(C.O.P)_{Ref} - (C.O.P)_{H.P} = 1$

$3 – 2=1$

EX:- A ref. operates on reversed Carnot cycle the higher temperature is 40°C & lower temperature is -20°C the capacity of ref.is 10 ton Determine a- C.O.P b- power required c- Q_{rej} [Take 1 ton = 3.5kW].

Sol:-

T_{max} = 40+273 = 313°K ,T_{min} = -20 +273 = 253°K, Q_{add} = 10ton *3.5 = 35 kW

$$(C.O.P)_{Ref} = \frac{T_{min}}{T_{max} - T_{min}} = \frac{253}{313 – 253} = 4.217$$

$$N.W.D = \frac{Q_{add}}{(C.O.P)} = \frac{35}{4.217} = 8.31 \text{ kW}$$

N.W.D $=Q_{rej} - Q_{add}$

Q_{rej} = N.W.D + Q_{add}

$= 8.31 + 35 = 43.3$kW

EX:- What is more effective for increase eff. of engine working with Carnot cycle between 1100ºK & 500 ºK.

a- Increase temp. of hot vessel about 100ºC & remain the temp. of cold vessel const.

b- Decrease the temp. of cold vessel about 100ºC & remain the temp.of hot vessel const.

Sol:-

a-

$T_{max} = 1100^oK$, $T_{min} = 500^oK$

$$\eta_{car} = 1 - \frac{T_{min}}{T_{max}}$$

$$\eta_{car} = 1 - \frac{500}{1100}$$

$= 0.545 = 54.5\%$

$T_{max} = 1100 + (100 +273)$

$= 1473^oK$

$$\eta_{car} = 1 - \frac{T_{min}}{T_{max}}$$

$$\eta_{car} = 1 - \frac{500}{1473}$$

$= 0.6 = 66\%$

b-

$T_{max} = 1100\ ^oK$

$T_{max} = 500 - (100 +273)$

$= 127^oK$

$$\eta_{car} = 1 - \frac{T_{min}}{T_{max}}$$

$$\eta_{car} = 1 - \frac{127}{1100}$$

$= 0.88 = 88\%$ the case b it give more effective

EX:- Carnot engine is 25% we reversed the motor to working with cooling load of 800kJ/min at 5°C ,Determine :-

a- net work done wanted to operate in cooling machine
b- C.O.P

Sol:

η_{car} = 25% Q_{add} = 800kJ/min, T_{min} = 5+ 273 = 278°K,

N.W.D = Q_{rej} - Q_{add}

$$\eta_{car} = 1 - \frac{T_{min}}{T_{max}}$$

$$0.25 = 1 - \frac{278}{T_{max}}$$

T_{max} = 370.66°K

$$(C.O.P)_{Ref} = \frac{T_{min}}{T_{max} - T_{min}} = \frac{278}{370.66 - 278} = 3$$

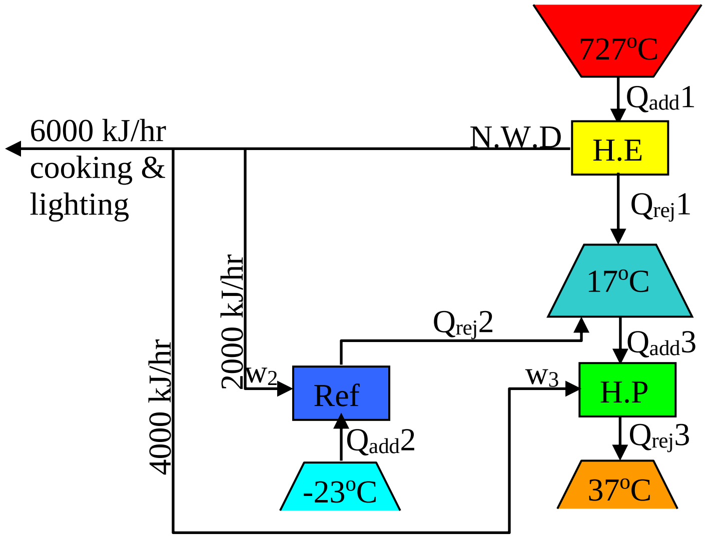
$$(C.O.P)_{Ref.} = \frac{Q_{add}}{N.W.D}$$

$$3 = \frac{800}{N.W.D}$$

N.W.D = 266.66 kJ/min

EX:- In an isolated house a heat engine (H.E) working between 727°C &17°C is installed to give power to house utilities which are composed of :-
1- cooking & lighting appliances that needs 6000kJ/hr.
2- Ref. which work between -23°C &17°C & consume 2000kJ/hr work.
3- A H.P which work between 17°C &37°C & consume 4000kJ/hr work .
-The H .E working on thermal eff. of 35.2 % of the max.possible eff.
- the ref.& H.P C.O.P are 50% of the max possible C.O.P for each machine .
Calculate :-
a- the net power of H.E (kW)
b- the heat added to H.E per hr
c- the heat absorbed by ref
d- the heat given by H.P
e- the heat added or rejected from 17°C datom.

Sol:-



Net power = $w_1 + w_2 + w_3$ = 6000+ 2000+4000

= 12000kJ/hr = 12000/3600 = 3.34kW

$Q_{add}1$= ?

$$\eta_{H.E} = \frac{N.W.D}{Q_{add}1}$$

0.71

$$\eta_{H.E\ T} = 1 - \frac{T_{min}}{T_{max}}$$

$$= 1 - \frac{17+273}{727+273}$$

= 0.71 = 71%

$\eta_{H.E}$ = 0.352 *0.71

= 0.25= 25%

$$0.25 = \frac{12000}{Q_{add}1}$$

$Q_{add}1$ = 48000 kJ/hr

$Q_{add}2$= ?

$$(C.O.P)_{Ref} = \frac{Q_{add}2}{w_2}$$

$$(C.O.P)_{Ref.}T = \frac{T_{min}}{T_{max} - T_{min}}$$

$$= \frac{-23 + 273}{(17 + 273) - (-23 + 273)}$$

$$= 6.25$$

$$(C.O.P)_{Ref} = 6.25 * 0.5 = 3.125$$

$$3.125 = \frac{Q_{add}2}{2000}$$

$Q_{add}2 = 6250$ kJ/hr

$Q_{rej}3 = ?$

$$(C.O.P)_{H.P} = \frac{Q_{rej}3}{w_3}$$

$$Q_{rej}3 = w_3 * (C.O.P)_{H.P}$$

$$(C.O.P)_{H.P.T} = \frac{T_{max}}{T_{max} - T_{min}}$$

$$= \frac{37 + 273}{(37+273) - (17-273)}$$

$$= 15.5$$

$$(C.O.P)_{H.P} = 15.5 * 0.5$$

$$= 7.75$$

$$Q_{rej}3 = 4000 * 7.75$$

$= 31000$kJ/hr

$$Q_{rej}2 = w_2 + Q_{add}2$$

$$= 2000 + 6250$$

$= 8250$ kJ/hr

$$Q_{rej}1 = Q_{add}1 - N.W.D$$

$= 48000 - 12000$

$= 36000 \text{kJ/hr}$

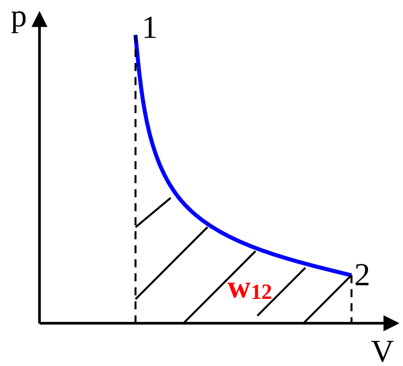
$Q_{add}3 = Q_{rej}3 - w_3$

$= 31000 - 4000$

$= 27000 \text{ kJ/hr}$

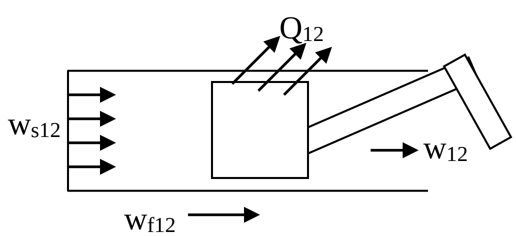
Reversible& irreversible processes :-

Reversible process :-the process in which the working substance and all elements of its surrounding after complete the process it return back to initial condition without any external effect.



There are two source of irreversible process :-

a- External source of irreversible :- Mechanical friction



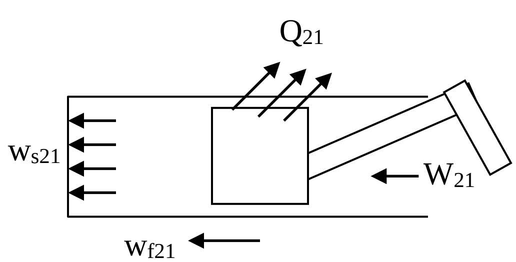
Expansion

$w_{s12} = w_{12} + w_{f12}$

$Q_{12} = w_{f12}$

$\boxed{w_{s12} = w_{12} + Q_{12}}$ …………. In case of expansion

- Compression process:-



Compression

$w_{s21} = w_{21} - w_{f21}$

$w_{21} = ws_{21} + w_{f21}$

$Q_{21} = w_{f21}$

$\boxed{w_{s21} = w_{21} - Q_{21}}$ ………… In case of compression

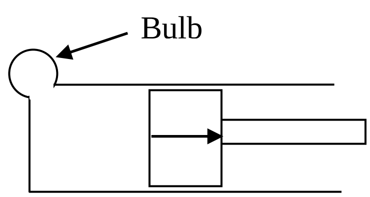
$w_{s21} = w_{s21}$

$w_{21} - Q_{21} = w_{12} + Q_{12}$

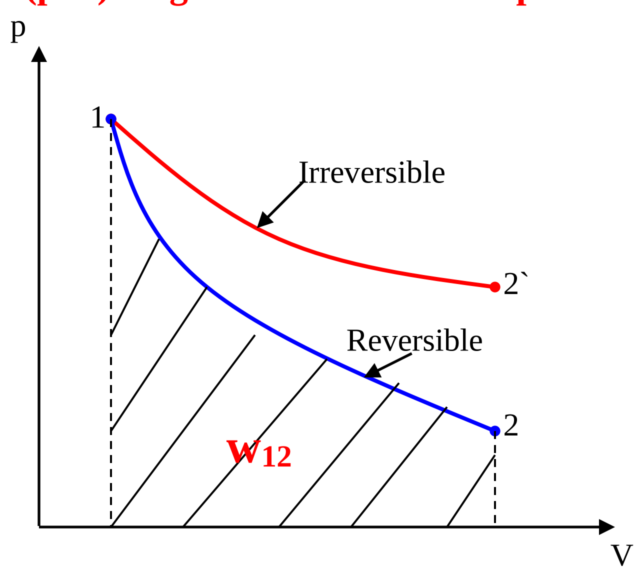
$w_{21} - w_{12} = Q_{12} + Q_{21}$

$Q_{12} + Q_{21} > 0$

b- Internal sources: -
-It is the friction between molecular of the system it self.
-If the piston moves slowly the gas fills the cylinder.
-If the piston moves rapidly the gas fills the cylinder.
-The pressure in bulb will be higher than that in cylinder.
- The net results are less work done and the gas finally will be better state.



The meaning of(p-V) diagram for irreversible process :-



Area under curve 1-2 for p.V diagram is represent work done reversible.
But Area under curve 1-2` for p.V diagram is not represent work done irreversible.

$(w.D)_{irrev} < (w.D)_{rev}$

$(w.D)_{irrev} = (w.D)_{rev}$ – friction loss

$$= \int_1^2 p.dV - \text{friction loss}$$

Friction loss :- is a friction (percentage) of $(w.D)_{rev}$

Problems :-

7 – 1 Explain why it is impossible for any engine cycle to have athermal efficiency of 100% .

7 – 2 The overall volume expansion ratio of a Carnot cycle is 15. The temperature limits of the cycle are 260°C and 21°C. Determine :
1- the volume ratios of the isothermal and adiabatic process
2 – the thermal efficiecy of the cycle Take $\gamma = 1.4$ (4.42, 3.39, 44.8%)

7 – 3 Two reversible heat engines A and B are arraged in series, A rejecting heat directly to B. Engine A recieves 200 kJ at atemperature of 421°C from hot source, while engine B is in communication with the cold sink at a temperature of 4.4°C. If the work output of A is twice that of B find :
1- the intermediat temperature between A & B
2- the efficiency of each engine
3- the heat rejected to the cold sink (143°C , 40% , 33.5% ,80kJ)

7– 4 A heat engine operates between max. & min temperature of 871°C and 60°C respectively. With an efficiency of 50% of the Carnot efficiency. It drives a heat pump whicj uses river water at 4.4°C to heat block of flats in which the temperature is to be mentained at 21.1°C. assuming that atemperature diffrence of 11.1 exists between the working fluid and the river water, on the one hand, and required room temperature on the other, and assuming a heat pump to operate on the reversed Carnot cycle, but with a coefficient of performance of 50% of the ideal value for the same temperature limits of the working fluid, find the heat input to the engine per unit heat output from the heat pump.(0.82kJ/kJheat input)

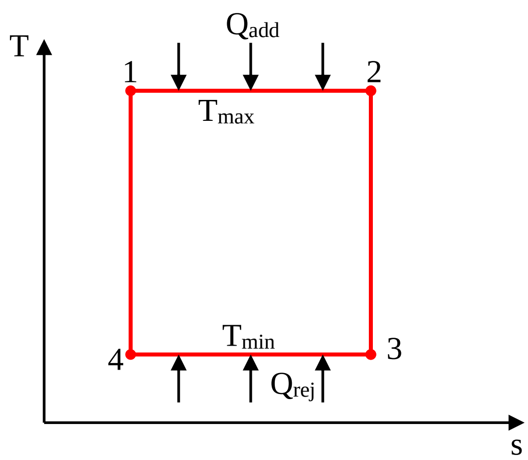
Unit eight

Entropy (s)

Entropy (s)

$$\eta_{carnot} = 1 - \frac{T_{min}}{T_{max}}$$

$$= 1 - \frac{Q_{rej}}{Q_{add}}$$



$$\frac{T_{min}}{T_{max}} = \frac{Q_{rej}}{Q_{add}}$$

$$\frac{Q_{add}}{T_{max}} = \frac{Q_{rej}}{T_{min}}$$

Thus integrating around the cycle and putting + ve for Q_{add} and –ve for Q_{rej}

$$\frac{Q_{add}}{T_{max}} + \frac{Q_{rej}}{T_{min}} = 0$$

Or $\oint (dQ/T) = 0$

The same result can be obtained to any reversible
Cycle process and the isothermal cancel each other and no heat is added or rejected in the adiagrame

$\oint (dQ/T)_{rev} = 0$

Thus the dQ/T behavies like any properties (p,v,u,h ,T)
Where :-
$\oint dp = 0$, $\oint dT = 0$, $\oint du = 0$

It exists a property of the system

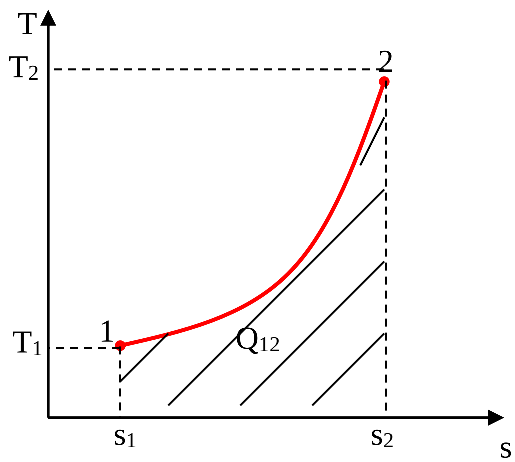
dQ/T =ds

such that

$$s_2 - s_1 = \int_1^2 ds = \int_1^2 dQ/T$$

this property is called entropy

Temperature – entropy diagram (T – s)diagram :-



$s_2 - s_1 = \int_1^2 ds = \int_1^2 dQ/T$

$ds = dQ/ T$

$dQ = T.ds$

$\int_1^2 dQ = \int_1^2 T.ds$

$$\boxed{\mathbf{Q_{12} = \int_1^2 T.ds}}$$

- plotting a property diagram between T&s it can be a range if

$s_2 - s_1 = \int_1^2 ds = \int_1^2 dQ/T$

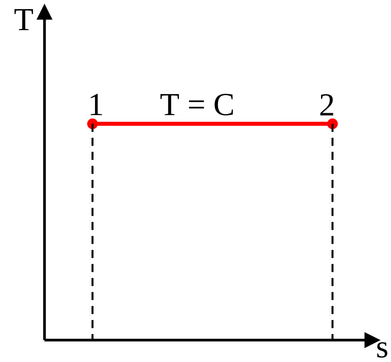
which is analoges to $w_{12} \int_1^2 p.dV$

$\int_1^2 T.ds$ is the area under the process represented on T.s diagram

T & s are two independed property

-This area calculated the reversible exchanged heat Q_{12} through this process

$Q_{isothermal} = T\ (s_2 - s_1)$

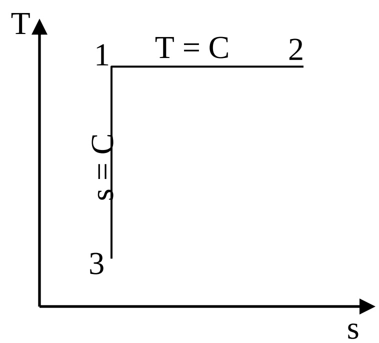


- For reversible adiabatic process is represented on T – s diagrame by a vertical line

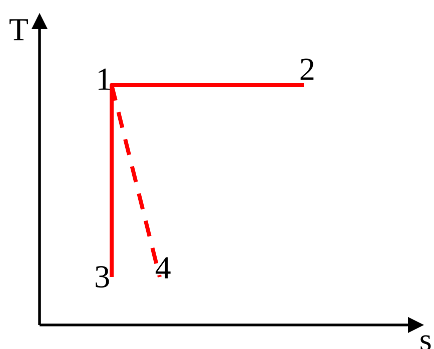
$Q_{12} = 0$

$s_1 = s_3$

is called isentropic process(s = const.)



- For irreversible adiabatic process $Q = \cancel{0}$ represented by curve 1 – 4 which is not isentropic



Entropy :- It is a property which is directly connected to the exchange heat& the change in entropy is :-

$$s_2 - s_1 = \int_1^2 dQ/T$$

Entropy Equation for an ideal gas :-

First low for closed system,

$Q_{12} = w_{12} + \Delta U_{12}$ (kJ)

$q_{12} = w_{12} + \Delta u_{12}$ (kJ/kg)
differentiate

$dq = dw + du$

$dq = p.dv + du \longrightarrow ds = dq/T$

$$\boxed{T.ds = p.dv + du} \quad \text{.... a}$$

$h = u + p.v$

$dh = du + p.dv + v.dp$

$dh = T.ds + v.dp$

$$\boxed{T.ds = dh - v.dp} \quad \text{.....b}$$

For ideal gas

$p.v = R.T$,$p/T = R/v$

$du = Cv\ .dT$, $dh = Cp\ .dT$

from equation a

$\left(T.ds = p.dv + du\right) / T$

$ds = (p/T).dv + du/T$

$ds = (R/v)\ .dv + (Cv/T).dT$

$ds = R(dv/v) + Cv\ (dT/T)$

$$\int_1^2 ds = R\int_1^2 (dv/v) + Cv \int_1^2 (dT/T)$$

$$\boxed{s_2 - s_1 = R \ln(v_2/v_1) + Cv \ln(T_2/T_1)}$$

from equation b

$\left[T.ds = dh - v.dp \right] / T$

$ds = (dh / T) - (v / T)dp$

$ds = Cp\,(dT/T) - R\,(dp/p)$

$\int_1^2 ds = Cp\int_1^2 (dT/T) - R\int_1^2 (dp/p)$

$$\boxed{s_2 - s_1 = Cp\ln(T_2/T_1) - R\ln(p_2/p_1)}$$

put $R = Cp - Cv$

$s_2 - s_1 = Cp\ln(T_2/T_1) - (Cp - Cv)\ln(p_2/p_1)$

$s_2 - s_1 = Cp\ln(T_2/T_1) - Cp\ln(p_2/p_1) + Cv\ln(p_2/p_1)$

$s_2 - s_1 = Cp\left[\ln(T_2/T_1) - \ln(p_2/p_1)\right] + Cv\ln(p_2/p_1)$

$s_2 - s_1 = Cp\ln\left[(T_2/T_1) - (p_2/p_1)\right] + Cv\ln(p_2/p_1)$

$$s_2 - s_1 = Cp\ln\frac{(T_2/T_1)}{(p_2/p_1)} + Cv\ln(p_2/p_1)$$

$$s_2 - s_1 = Cp\ln\frac{T_2.\,p_1}{T_1.p_2} + Cv\ln(p_2/p_1)$$

$$\frac{p_1.v_1}{T_1} = \frac{p_2.v_2}{T_2}$$

$$\frac{T_2\,.p_1}{T_1.p_2} = \frac{v_2}{v_1}$$

$$\boxed{s_2 - s_1 = Cp\ln(v_2 / v_1) + Cv\ln(p_2/p_1)}$$

Representation of particular process on (T- s) diagram.

a- constant volume process (isochoric)

$s_2 - s_1 = R\ln(v_2/v_1) + Cv\ln(T_2/T_1)$

$v_2 = v_1$

$R \ln (v_2/v_1) = 0$

$$\boxed{s_2 - s_1 = C_v \ln (T_2/T_1)}$$

For certain arbitrary datum the entropy is assumed = 0 at $t = t_o = 0^oC$ i.e. $T = T_o = 273\ ^oK$

And $p = p_o = 0.006112$bar and thus:-

$v_o = R\ (T_o / p_o)$ m^3/kg

$s_2 - s_1 = R \ln(v_2/v_o) + C_v \ln(T_2/T_o)$

This reduced to

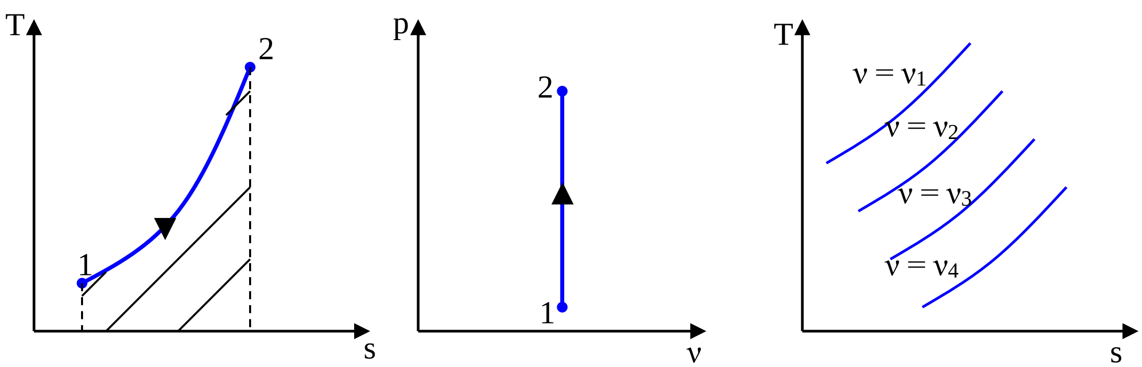
$$\boxed{s = R \ln (v/v_o) + C_v \ln(T/T_o)}$$

Appling isochoric process put v = const.

$$\boxed{s = const + C_v \ln T}$$

Which it are family curves each of which as a const. volume value a horizontal distance between any two constant volume lines is always const. as it equal :-

$\Delta s = R \ln (v_2/v_1)$



b- Isobaric process:-

$s_2 - s_1 = C_p \ln(T_2/T_1) - \overset{0}{R \ln(p_2/p_1)}$

$p_2 = p_1$

$$\boxed{s_2 - s_1 = C_p \ln(T_2/T_1)}$$

Or

$$\boxed{s_2 - s_1 = C_p \ln (v_2/v_1)}$$

Using the same datum is $s_o = 0$, $T = T_o = 273\ ^oK$ And $p = p_o = 0.006112$bar,

$s = Cp \ln(T/T_o) - R \ln(p/p_o)$

As p = const for isobaric

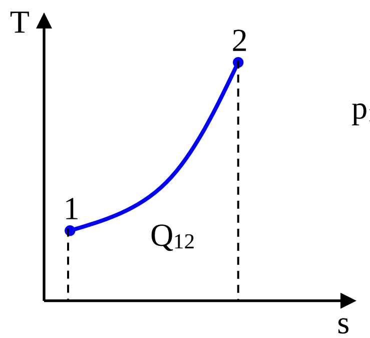
$$\boxed{\mathbf{s = const + Cp \ln T}}$$

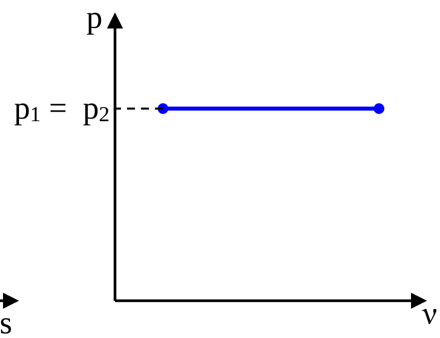
Where:-

$Const = -Cp\ln T_o - R \ln (p/p_o)$

This represented a family of curve each of which has a constant pressure Value and the horizontal distance is always constant at which equal:-

$\Delta s = R\ln (p_2 /p_1)$





C- Isothermal process (T =constant):-

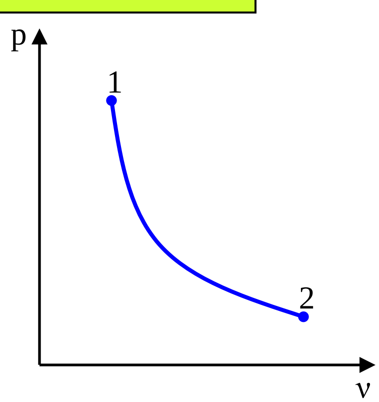
$$s_2 - s_1 = R \ln(v_2/v_1) + Cv \ln\cancelto{0}{(T_2/T_1)}$$

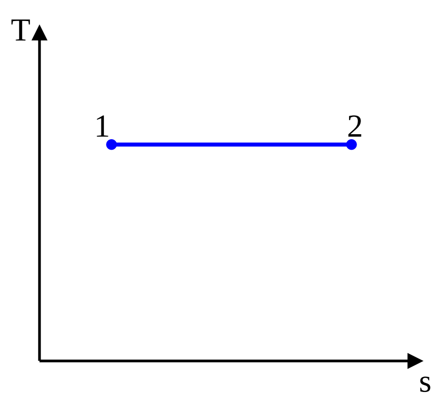
$$\boxed{\mathbf{s_2 - s_1 = R \ln(v_2/v_1)}}$$

Or

$$s_2 - s_1 = Cp \ln\cancelto{0}{(T_2/T_1)} - R \ln(p_2/p_1)$$

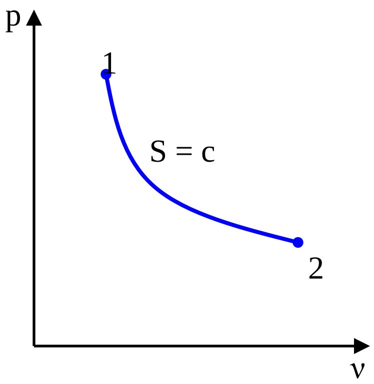
$$\boxed{\mathbf{s_2 - s_1 = - R \ln (p_2 /p_1)}}$$

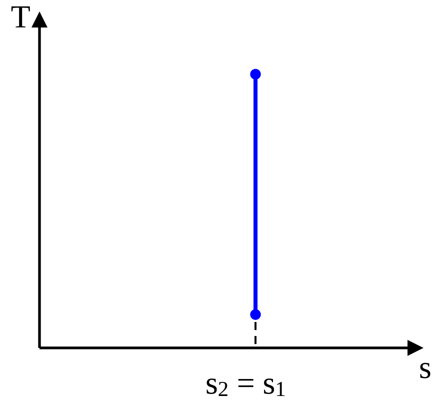




D – Isentropic process (Reversible adiabatic process) s = constant :-

$s_2 - s_1 = 0$





E- polytropic process :-

$s_2 - s_1 = R \ln (\nu_2/\nu_1) + Cv \ln(T_2/T_1)$
$T_1 . \nu_1^{n-1} = T_2 . \nu_2^{n-1}$

$(\nu_2/\nu_1)^{n-1} = T_1/ T_2$

$(\nu_2/\nu_1) = (T_1/ T_2)^{1/ n-1}$

$(\nu_2/\nu_1) = (T_2/ T_1)^{-1/ n-1}$

$Cv = R / \gamma - 1$

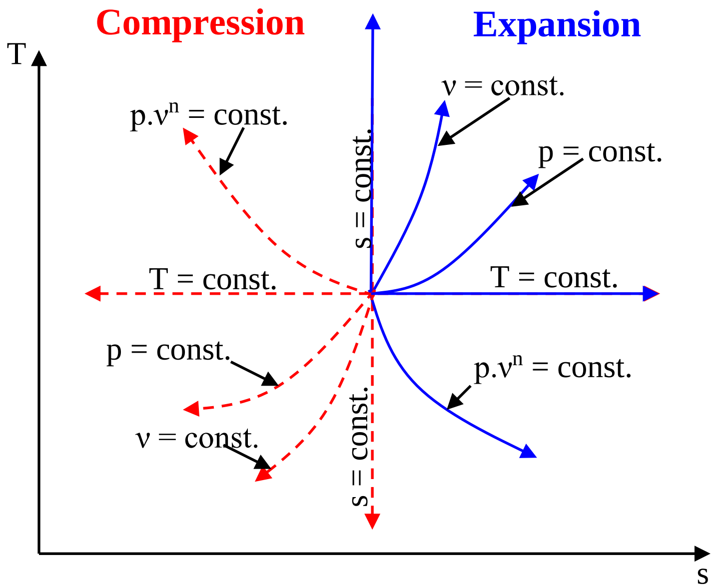
$s_2 - s_1 = R \ln (T_2/ T_1)^{-1/ n-1} + \left\{R /(\gamma - 1)\right\} \ln(T_2/T_1)$

$$= R * \frac{1}{n-1} \ln \frac{T_2}{T_1} + \frac{R}{\gamma-1} \ln \frac{T_2}{T_1}$$

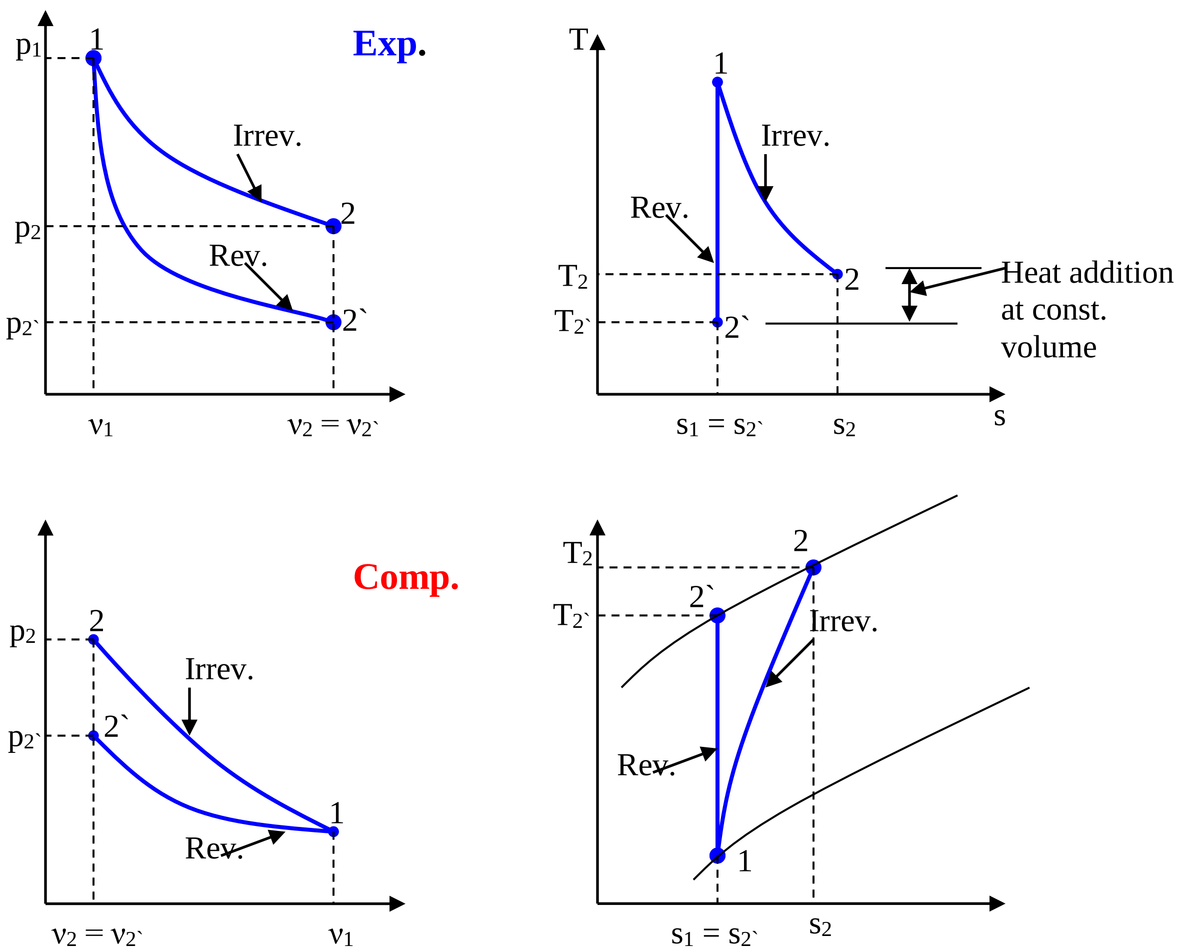
$$= R * \frac{1}{n-1} \ln \frac{T_2}{T_1} + \frac{R}{\gamma-1} \ln \frac{T_2}{T_1}$$

$$= \left(\frac{-R}{n-1} + \frac{R}{\gamma-1}\right) \ln \frac{T_2}{T_1}$$

$$\boxed{s_2 - s_1 = \frac{R\,(n-\gamma)}{(n-1)\,(\gamma-1)} \ln \frac{T_2}{T_1}}$$



Entropy change in irreversible process:-



1 - 2` Isentropic reversible adiabatic, s = const. Q =0, $\Delta s = 0$

1 - 2 Irreversible adiabatic, $s \neq$ const. $Q \neq 0$, $\Delta s \neq 0$

$$\left.\begin{matrix} p_{irrev} > p_{rev} \\ T_{irrev} > T_{rev} \end{matrix}\right\} \text{Exp \& comp., } V = \text{const.}$$

From diagram we have constant volume, heat addition from (2` - 2)

$T_{2`}.\ V_{2`} = T_1.V_1^{\gamma-1}$

$T_2.\ V_2 = T_1.V_1^{n-1}$

$$T_{2`} = T_1 \left(\frac{V_1}{V_{2`}}\right)^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\frac{T_2}{T_{2`}} = \frac{(V_1/\ V_2)^{n-1}}{(V_1/\ V_{2`})^{\gamma-1}}$$

$V_2 = V_{2`}$

$$\frac{T_2}{T_{2`}} = \frac{(V_1/\ V_2)^{n-1}}{(V_1/\ V_2)^{\gamma-1}}$$

$\boxed{\mathbf{T_2/\ T_{2`} = (V_1/V_2)^{n-\gamma}}}$ In case of compression where $n>\gamma$ & $V_1 > V_2$

$\boxed{\mathbf{s_2 - s_1 = s_2 - s_{2`} = Cv\ ln\ T_2/\ T_{2`}}}$ Where $s_1 = s_{2`}$

For expansion

$\boxed{\mathbf{T_2/\ T_{2`} = (V_1/V_2)^{n-\gamma}}}$ In case of expansion where $n<\gamma$ & $V_1 < V_2$

$\boxed{\mathbf{s_2 - s_1 = s_2 - s_{2`} = Cv\ ln\ T_2/\ T_{2`}}}$

EX: - Air in a piston – cylinder is at 1bar and 15°C. The piston is move & volume reduced to one quarter of the original size. The compressor is adiabatic and:

a- Reversible the law of comp. being $p.V^{1.4}$ = const.

b- Irreversible and final temperature being 6.6°C higher than in case (a)

Compare the w.D &Δs in the two cases

Sol:-

a-

$$w` = \frac{p_1.V_1 - p_2.V_2}{\gamma-1}$$

$$T_{2`} = T_1(V_1/V_2)^{\gamma-1}$$

$$T_{2`} = (15+273)(V_1/0.25V_1)^{1.4-1}$$

$$T_{2`} = 501.1^oK$$

$$w` = \frac{R(T_1 - T_{2`})}{\gamma-1}$$

$$w` = \frac{0.287(288 - 501.1)}{1.4-1}$$

$$= -152.9 \text{ kJ/kg}$$

$\Delta s = 0$

b-

$T_2 = T_{2`} + 6.6$

$T_{2`} = 501.1 - 273 = 228.1^oC$

$T_2 = 228.1 + 6.6$

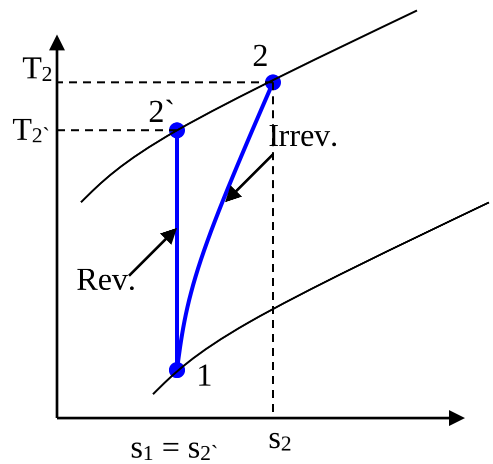
$= 234.7\ ^oC = 507.7^oK$

$w_{12} = -\Delta u_{12} = -Cv\,(T_2 - T_1)$

$$w = -\frac{R}{\gamma-1}*(T_2 - T_1)$$

$$= -\frac{0.287}{1.4-1}*(507.7 - 288)$$

$= -157.6$ kJ/kg



$\Delta s = Cv \ln (T_2/T_{2`})$

$$\Delta s = \frac{R}{\gamma - 1} \ln (T_2/T_{2`})$$

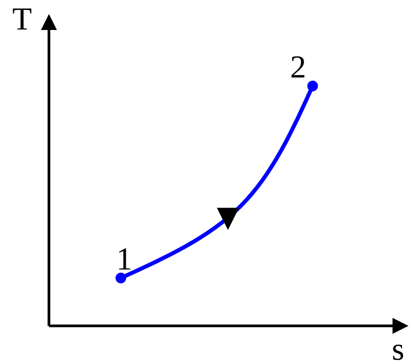
$$\Delta s = \frac{0.287}{1.4-1} \ln (507.7/501.1) = 0.009259 kJ/kg.K$$

EX: - 1kg of fluid at 1.5bar & 111.4°C is heated at constant volume to 300 °C. Calculates the final pressure & change in entropy when fluid a- air b- steam.

Sol:-

m = 1kg, p_1 = 1.5bar, T_1 = 111.4+ 273 = 384.4°K, V=const.,T_2 = 300+273 = 573 °K,p_2 =?

a- air



$p_2 / T_2 = p_1/T_1$

$p_2 = T_2 (p_1/T_1)$

$= 573 (1.5/384.4)$

$= 2.235 bar$

$\Delta s_{12} = Cv \ln (T_2/T_1)$

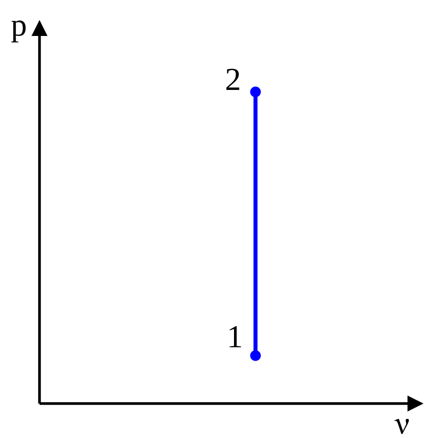
$= 0.717 \ln (573/384.4)$

$= 0.286 kJ/kg.K$

b-steam

From steam table at p_1= 1.5bar&t_s = 111.4°C,$v_1 = v_g$ = 1.159m^3/kg, $s_1 = s_g$ = 7.223kJ/kg.K

From steam table at T= 300°C & $v_1 = v_2$ = 1.159m^3/kg



p = 2bar at v = 1.316 m^3/kg

p = 3bar at v = 0.8754 m^3/kg

$$p_2 = 2 + \frac{1.159 - 1.316}{0.8754 - 1.316} (3 - 2)$$

= 2.357 bar

s = 7.892 at v = 1.316 kJ/kg.K

s = 7.702 at v = 0.8754 kJ/kg.K

$$s_2 = 7.892 + \frac{1.159 - 1.316}{0.8754 - 1.316}(7.702 - 7.892)$$

$= 7.824$ kJ/kg.K

$\Delta s_{12} = s_2 - s_1$

$= 7.824 - 7.223$

$= 0.601$kJ/kg.K

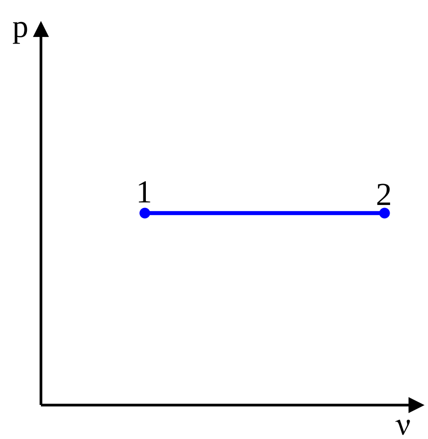
EX: - 1kg of air at 1bar &15°C is heated at constant pressure to 149°C. Calculate the Δs_{12}, ΔV_{12}, w_{12}, ΔU_{12} & ΔH_{12}.

Sol:-

$m = 1$kg, $p_1 = 1$bar, $T_1 = 15+273 = 288°K$, $p_1 = p_2 = 1$bar, $T_2 = 149 + 273 = 422°K$,

$10^2 . p_1.V_1 = m.R.T_1$

$$V_1 = \frac{m.R.T_1}{10^2 . p_1}$$



$$V_1 = \frac{1*0.287*288}{10^2 * 1}$$

$= 0.827m^3$

$V_1/T_1 = V_2/T_2$

$V_2 = T_2 (V_1/T_1)$

$= 422 (0.827 / 288)$

$= 1.212m^3$

$\Delta V_{12} = V_2 - V_1$

$= 1.212 - 0.827$

$= 0.385\ m^3$

$w_{12} = 10^2\ p_1 (V_2 - V_1)$

$=10^2 * 1 * (0.385)$

$= 38.5kJ$

$\Delta U_{12} = m.Cv. (T_2 - T_1)$

$= 1 * 0.717 (422 - 288)$

$= 94.4kJ$

$Q_{12} = \Delta H_{12} = m.Cp . (T_2 - T_1)$

$= w_{12} + \Delta U_{12} = 38.5 + 94.4 = 132.9 \text{ kJ}$

$\Delta s_{12} = Cp\ln(T_2 - T_1)$

$= 1.005 \ln(422 / 288) = 0.378 \text{kJ/kg.K}$

EX: - Consider a compressor with air at 1bar &15°C compressing
a- At constant temperature to 27.59bar.
b- At $p.V^n$ = const. & n=1.3, to same pressure, Calculate, T_2 ,w_{12} , q_{12} , Δu_{12} , Δs_{12} for two cases .

Sol:-

$p_1 = 1bar, T_1 = 15+273 = 288\ ^oK$

a-

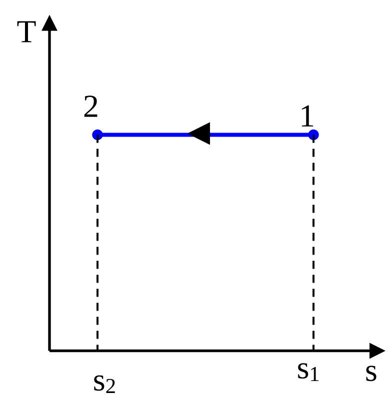
T = const. Comp.,$p_2 = 27.59bar$,

$T_1 = T_2 = 288\ ^oK$

$w_{12} = R.T_1\ln (p_1/p_2)$

$= 0.287 * 288 \ln (1/ 27.59)$

$= - 263.47 \text{ kJ/kg} = q_{12}$



$\Delta u_{12} = 0$

$\Delta s_{12} = R\ln (p_1/p_2)$

$= 0.287 \ln (1/ 27.59)$

$= - 0.915 \text{ kJ/kg.K}$

b- $p.V^n$ = const.

$$\frac{T_1}{p_1^{n-1/n}} = \frac{T_2}{p_2^{n-1/n}}$$

$T_2 = T_1 (p_2 / p_1)^{n-1/n}$

$= 288 (27.59 / 1)^{(1.3-1)/1..3}$

$= 619.2^oK$

$$w_{12} = \frac{10^2 (p_1.V_1 - p_2.V_2)}{n-1}$$

$$w_{12} = \frac{R (T_1 - T_2)}{n-1}$$

$$w_{12} = \frac{0.287 (288 - 619.2)}{1.3-1}$$

$= -316.84$ kJ/kg

$q_{12} = w_{12} + \Delta u_{12}$

$\Delta u_{12} = Cv (T_2 - T_1)$

$= 0.717 (619.2 - 288)$

$= 237.61$kJ/kg

$q_{12} = -316.84 + 237.61$

$= -79.23$ kJ/kg

$$\Delta s_{12} = \frac{(n-\gamma) R}{(\gamma-1) (n-1)} \ln \frac{T_2}{T_1}$$

$$= \frac{(1.3-1.4)\ 0.287}{(1.4-1)\ (1.3-1)} \ln \frac{619.2}{288}$$

$= -0.18594$ kJ/kgK

Problems:-

8 – 1 1m^3 of air is heated reversibly at constant pressure from 15ºC to 300ºC, and is then cooled reversibly at constant volume back to the initial temperature. The initial pressure is 1.03bar. Calculate the net heat flow and the overall change of entropy, and sketch the process on a T – s diagram. (101.5kJ; 0.246kJ/ K)

8 – 2 1kg of air is allowed to expand reversibly in a cylinder behind a piston in such a way that the temperature remains constant at 260ºC while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the

overall change of entropy. Sketch the processes on a T – s diagram.

(-161.9kJ/kg; -0.497 kJ/kg.K)

8 – 3 1kg of steam at 20 bar, dryness fraction 0.9, is heated reversibly at constant pressure to a temperature of 300ºC. Calculate the heat supplied, and change of entropy and show the process on T-s diagram, indicating the area which represented the heat flow. (415kJ/kg; 0.8173kJ/kg.K)

8 – 4 steam at 0.05 bar, 100ºC is to be condensed completely by a reversible constant pressure process. Calculate the heat flow to be removed per kg of steam, and the change of entropy. Sketch the process on a T – s diagram and shade in the area which represented heat flow. (2550kJ/kg; 8.292kJ/kg.K)

8 – 5 1kg of air at 1.013bar, 17ºC is compressed according to the law $pv^{1.3}$ = constant, until the pressure is 5 bar. Calculate the change of entropy and sketch the process on a T –s diagram, indicating the area which represents the heat flow. (-0.0885kJ/kg.K)

8 – 6 steam at 15 bar is throttled to 1bar and a temperature of 150ºC. Calculate the the initial dryness fraction and the change of entropy . Sketch the process on a T – s diagram. (0.992;1.202kJ/kg.K)

Unit nine

Gas power cycle

Gas power cycle

Air standard cycles:-

1- Otto cycles
2- Diesel cycle — comp& exp., s = constant, +ve work done
3- Dual cycle

Heat added & heat rejected:-

1- Otto cycle heat added & heat rejected at constant volume.
2- Diesel cycle heat added at constant pressure, & heat rejected at constant volume.
3- Dual cycle heat added at constant pressure & constant volume, heat rejected at constant volume.

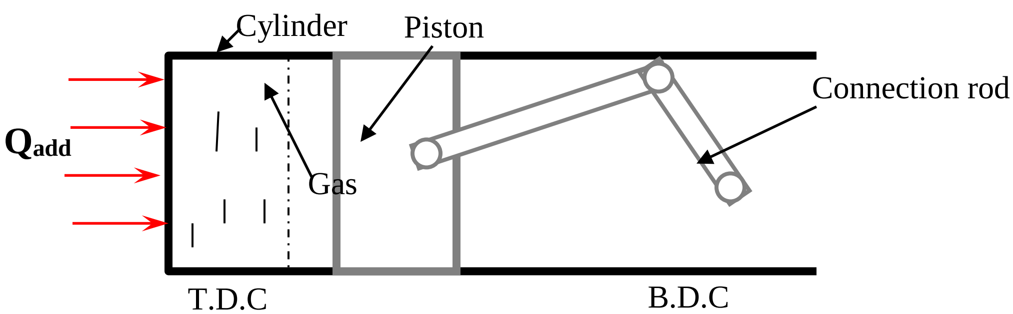
-In all these cycles the main purpose is to produce mechanical work.

The following assumptions are made:-

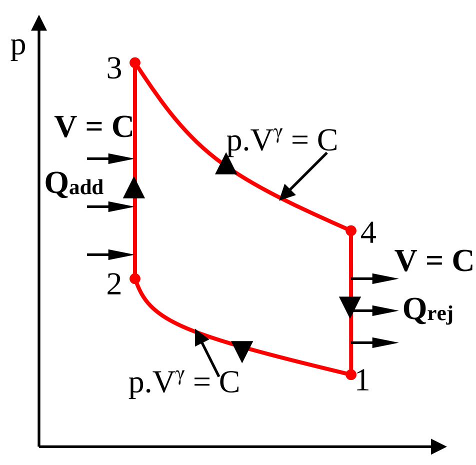
1- Air is the working substance
2- All processes are reversible at constant entropy
3- Cp, Cv , γ, e , for air are constant .

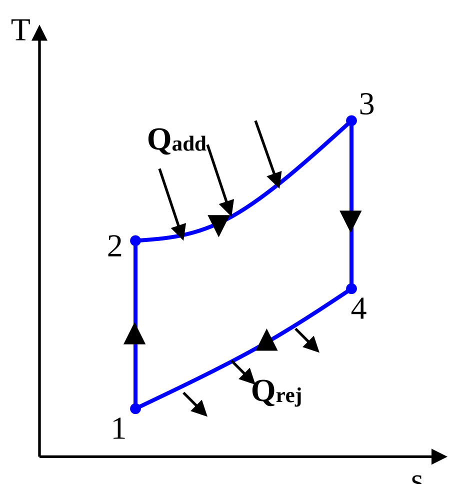
1- Otto cycle:-

The Otto cycle is the ideal air standard cycle for petrol engine, the gas engine, and the high – speed oil engine.



T.D.C = top dead center
B.D.C = bottom dead center





Process (1 - 2) reversible adiabatic comp. (s = const.) (–ve w.D) on the gas

Process (2 – 3) heat added at constant volume

Process (3 – 4) reversible adiabatic exp. (+ve w.D)

Process (4 – 1) heat rejected at constant volume & return gas to original condition.

$p_{max} = p_3$, $p_{min} = p_1$

$T_{max} = T_3$, $T_{min} = T_1$

$V_{max} = V_1 = V_4$, $V_{min} = V_2 = V_3$

+ve w.D = $-\Delta u_{34}$

$$= \frac{10^2 (p_3.V_3 - p_4.V_4)}{\gamma - 1}$$

$$= \frac{m.R (T_3 - T_4)}{\gamma - 1} = m.Cv (T_3 - T_4)$$

-ve w.D = $-\Delta u_{12}$

$$= \frac{10^2 (p_1.V_1 - p_2.V_2)}{\gamma - 1}$$

$$= \frac{m.R (T_1 - T_2)}{\gamma - 1} = m.Cv (T_1 - T_2)$$

$Q_{add} = \Delta u_{23} = m.Cv (T_3 - T_2)$

$Q_{rej} = \Delta u_{41} = m.Cv (T_4 - T_1)$

N.W.D = $Q_{add} - Q_{rej}$

Compression ratio(r):- is defined as the ratio between maximum volume in the cycle (V_1=V_4) to the minimum volume in the cycle (V_2=V_3).

$$\boxed{\mathbf{r = \frac{V_1}{V_2} = \frac{V_4}{V_3}}}$$

Process (1 – 2):-

$T_1.V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$$

$$T_2 = T_1 * r^{\gamma-1}$$

Process (3 – 4):-

$$T_3.V_3^{\gamma-1} = T_4.V_4^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1}$$

$$T_3 = T_4 * r^{\gamma-1}$$

$$\boxed{\frac{\mathbf{T_2}}{\mathbf{T_1}} = \frac{\mathbf{T_3}}{\mathbf{T_4}}}$$

$$\eta_{Otto} = 1 - \frac{Q_{rej}}{Q_{add}}$$

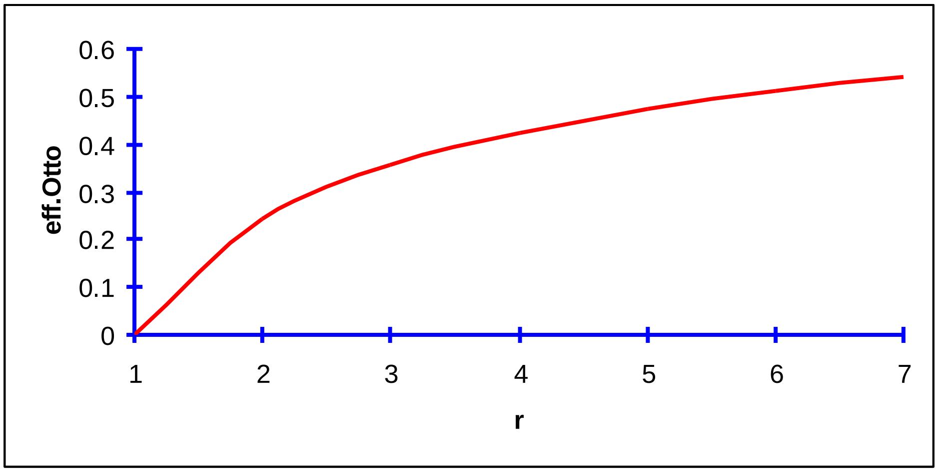
$$=1- \frac{m.Cv\,(T_4 - T_1)}{m.Cv\,(T_3 - T_2)}$$

$$=1- \frac{(T_4 - T_1)}{(T_4 * r^{\gamma-1}) - (T_1 * r^{\gamma-1})}$$

$$=1- \frac{(T_4 - T_1)}{(T_4 - T_1) * r^{\gamma-1}}$$

$$= 1- \frac{1}{r^{\gamma-1}}$$

$$\boxed{\boldsymbol{\eta_{Otto} = 1- \frac{1}{r^{\gamma-1}}}}$$



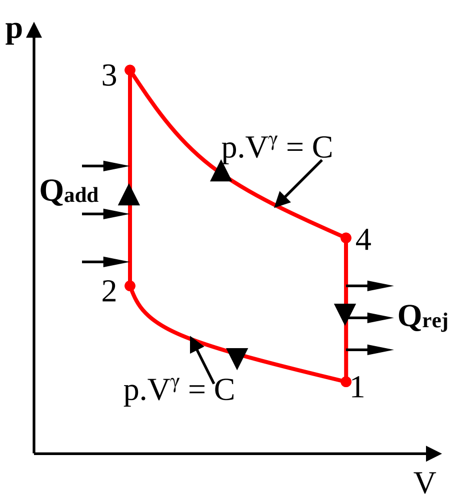
-As (r) increased eff. also increased.
-As (r) increased p_2 & T_2 increase.
- Petrol octane no, which limit the value of (r).

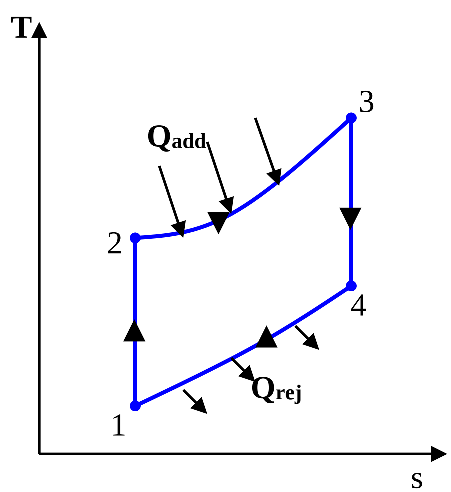
EX: - In an Otto air standard cycle the minimum pressure and temperature are 1bar &15°C, the maximum volume is 0.8m³, the minimum volume is 0.1m³, the maximum temperature in the cycle is 800°C find:-a- cycle efficiency b- N.W.D c- Q_{rej} d- p,V.T. at all cycle.[take γ = 1.4 , R = 0.287kJ/kg.K]

Sol:-

Otto cycle, p_1 = 1bar, T_1 = 15+273 = 288°K, $V_1 = V_4$ = 0.8m³, $V_2 = V_3$ = 0.1m³,

T_3 = 800+273 = 1073°K





a –

$$\eta_{Otto} = \frac{N.W.D}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}} = 1 - \frac{1}{r^{\gamma-1}}$$

$r = (V_1 / V_2)$

$= 0.8/0.1 = 8$

$$\eta_{Otto} = 1 - \frac{1}{8^{1.4-1}}$$

$= 0.565 = 56.5\%$

b-

$N.W.D = Q_{add} * \eta_{Otto}$

$Q_{add} = m.Cv(T_3 - T_2)$

$$m = \frac{10^2\ p_1 * V_1}{R * T_1} = \frac{10^2 * 1 * 0.8}{0.287 * 288} = 0.96\ kg$$

$$Cv = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.717 kJ/kg.K$$

$T_2 = T_1 * r^{\gamma-1} = 288 * 8^{1.4-1} = 662^oK$

$Q_{add} = 0.96*0.717(1073 - 662)$

$= 282kJ$

$N.W.D = 282 * 0.565$

$= 160kJ$

c –

$Q_{rej} = m.Cv(T_4 - T_1)$

$= Qadd - N.W.D$

$= 282 - 160 = 122\ kJ$

$T_3 = T_4 .r^{\gamma-1}$

$T_4 = T_3 / r^{\gamma-1}$

$= 1073 / 8^{1.4-1} = 467.05\ ^oK$

$p_1 .V_1^{\gamma} = p_2 .V_2^{\gamma}$

$p_2 = p_1(V_1/V_2)^{\gamma} = p_{1.}\ r^{\gamma}$

$= 1 * 8^{1.4} = 18.379bar$

Or

$10^2 .p_2.V_2 = m.R.T_2$

$p_2/T_2 = p_3/T_3$

$$p_3 = \frac{p_2 . T_3}{T_2} = \frac{18.2 * 1073}{662} = 29.618 \text{bar}$$

$p_3 . V_3^{\gamma} = p_4 . V_4^{\gamma}$

$p_3 = p_4 (V_4/V_3)^{\gamma} = p_4 . r^{\gamma}$

$p_4 = p_3 / r^{\gamma}$

$= 29.618 / 8^{1.4} = 1.6115 \text{bar}$

2- The Diesel cycle:-

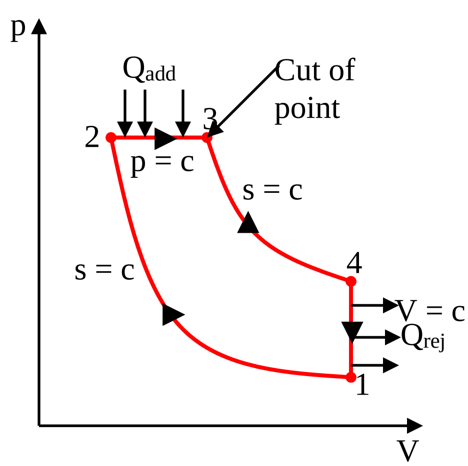
The engines in use today which are called diesel engines are far removed from the original engine invented by Diesel in 1892. Diesel worked on the idea of spontaneous ignition of powdered coal, which was blasted into the cylinder by compressed air .oil became the accepted fuel used in the compression – ignition engines, and the oil was originally blasted into the cylinder in the same way that Diesel had intended to inject the powdered coal this gave a cycle of operation which has as its ideal counterpart the ideal air standard Diesel cycle shown in fig below.

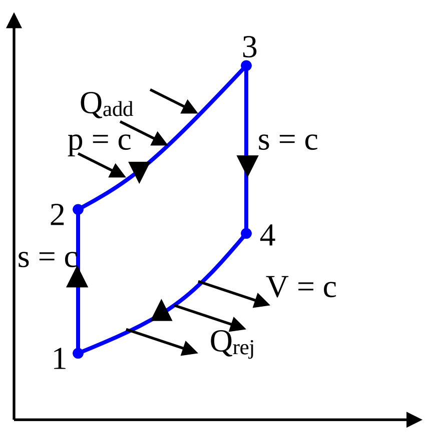
Process (1- 2) reversible adiabatic, isentropic comp., (-ve w.D)

Process (2 – 3) isobaric heat added at constant pressure which is called (cut of point) at which heat addition is stop

Process (3 - 4) isentropic exp. From cut of point to initial volume at 4 and (+ve w.D) is produce

Process (4 – 1) isochoric heat rejected at constant volume, that brings gas back to original condition.





1- Compression ratio (r):-

$$\mathbf{r = \frac{Max\ .vol.}{min.vol.} = \frac{V_1\ or\ V_4}{V_2}}$$

2- Expansion ratio (re):-

$$\mathbf{re = \frac{Max\ .vol.}{Cut\ of\ point} = \frac{V_1\ or\ V_4}{V_3}}$$

3- Cut of ratio (e):-

$$\mathbf{e = \frac{Cut\ of\ point}{min.vol.} = \frac{V_3}{V_2}}$$

$r = V_1/V_2$, $e = V_3/V_2$

$$\frac{r}{e} = \frac{V_1/V_2}{V_3/V_2} = \frac{V_1}{V_3} = \frac{V_4}{V_3} = re$$

Calculation:-

$Q_{add} = m.Cp\ (T_3 - T_2)$

$Q_{rej} = m.Cv\ (T_4 - T_1)$

$N.W.D = Q_{add} - Q_{rej}$

$$\mathbf{\eta_D = \frac{N.W.D}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}}}$$

$$\eta_D = 1 - \frac{m.Cv\ (T_4 - T_1)}{m.Cp\ (T_3 - T_2)}$$

$T_1.V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$

$\boxed{\mathbf{T_2 = T_1.r^{\gamma-1}}}$ …….1

$T_3.V_3^{\gamma-1} = T_4.V_4^{\gamma-1}$

$T_4 = T_3\ (V_3/V_4)^{\gamma-1}$

$= T_3\ (1/re)^{\gamma-1}$

$\boxed{\mathbf{T_4 = T_3\ (e/r)^{\gamma-1}}}$ ….. 2

$V_2/T_2 = V_3/T_3$ at p=const.

$T_3 = T_2*(V_3/V_2)$

$$\boxed{\mathbf{T_3 = T_2 .e}} \quad \ldots.\ 3$$

Sub 3 in 2

$T_4 = T_2 .e\ (e/r)^{\gamma-1}$

Sub 1 in this equation

$T_4 = T_1.r^{\gamma-1}.e\ (e/r)^{\gamma-1}$

$$\boxed{\mathbf{T_4 = T_1 .e^{\gamma}}} \quad \ldots..4$$

$$\eta_D = 1- \frac{Cv\ (T_4 - T_1)}{Cp\ (T_3 - T_2)}$$

$$\eta_D = 1- \frac{1(T_1 .e^{\gamma} - T_1)}{\gamma\ (T_2 .e - T_2)}$$

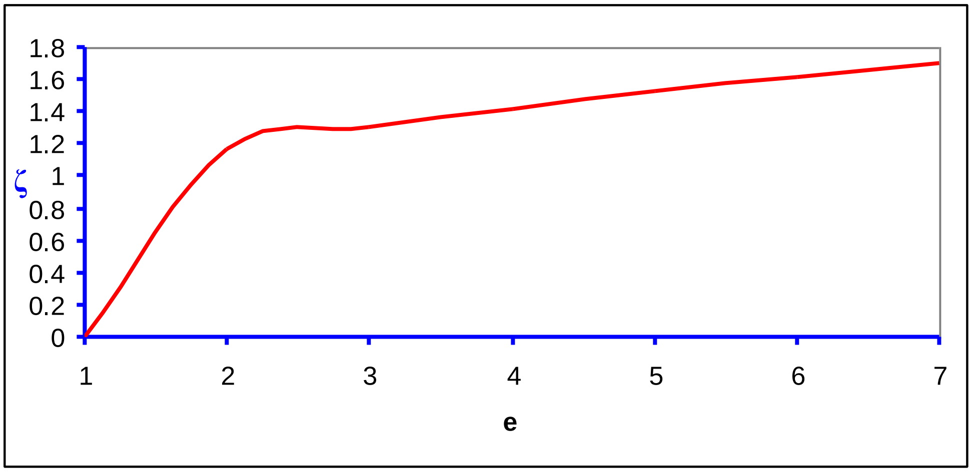
$$= 1- \frac{T_1\ (e^{\gamma} - 1)}{\gamma\ .T_2\ (e - 1)}$$

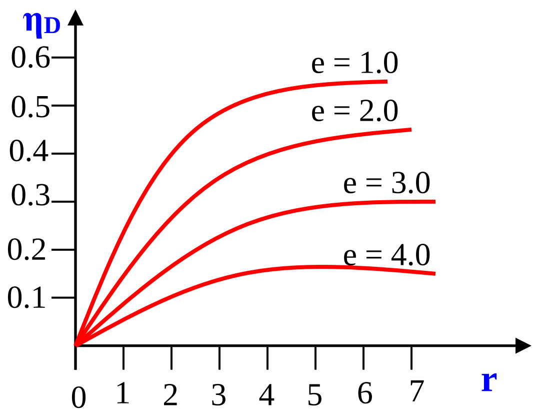
$$\eta_D = 1- \frac{T_1\ (e^{\gamma} - 1)}{\gamma.\ T_1.r^{\gamma-1}\ (e - 1)}$$

$$\eta_D = 1- \frac{1}{r^{\gamma-1}} * \frac{(e^{\gamma} - 1)}{\gamma(e - 1)}$$

Put $\zeta = \dfrac{(e^{\gamma} - 1)}{\gamma(e - 1)}$

$$\boxed{\boldsymbol{\eta_D = 1- \frac{1}{r^{\gamma-1}} * \zeta}}$$





EX: - The diesel cycle has the minimum temperature of 27°C and maximum temperature of 1327°C the quantity of heat added is 700kJ find:

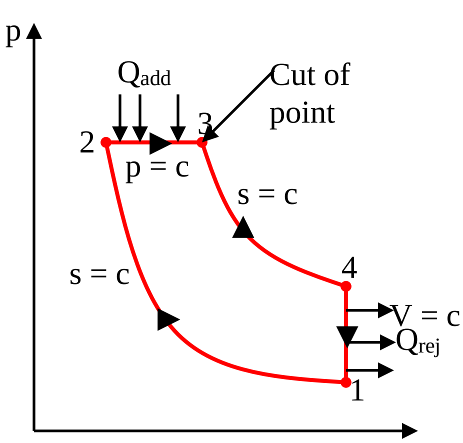
1- Efficiency of the cycle. 2- N.W.D.

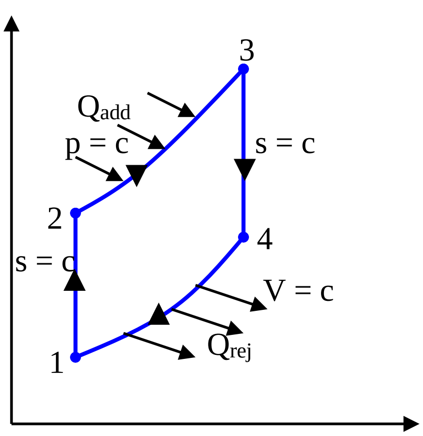
3- Compare this cycle with the Otto cycle working between the same limits of temperature and consuming the same amount of heat added.

4- The compression ratio for each cycle.

5- Efficiency of the corresponding Carnot cycle increase in net w.D between these cycle and other two cycle two cycle above.[take Cp = 1.0 kJ/kg.K , γ = 1.4, m= 1kg]

Sol:-





$T_1 = 27+273 = 300^oK$, $T_3 = 1327+273 = 1600^oK$, 1-

$$\eta_D = 1 - \frac{1}{r^{\gamma-1}} * \zeta$$

$$\zeta = \frac{(e^{\gamma} - 1)}{\gamma(e - 1)}$$

$e = V_3/V_2$

$r = V_1/V_2$

$Q_{add} = m.Cp.\ (T_3 - T_2)$

$700 = 1*1*\ (1600 - T_2)$

$T_2 = 900^oK$

$T_1.V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$

$T_2 = T_1.r^{\gamma-1}$

$900 = 300*r^{1.4-1}$

$900/300 = r^{0.4}$

$(900/300)^{1/0.4} = r$

$r = 15.5$

$V_2 / T_2 = V_3 / T_3$

$V_3 / V_2 = T_3 / T_2 = e$

$e = 1600 /900 = 1.78$

$$\zeta = \frac{(1.78^{1.4} - 1)}{1.4(1.78 - 1)}$$

$= 1.135$

$$\eta_D = 1 - \frac{1}{15.5^{1.4-1}} * 1.135$$

$= 62.3\%$

2-

$N.W.D = \eta_D * Q_{add}$

$= 0.623 * 700$

$= 436\ kJ$

3-

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$$

$T_2 = T_1 .\ r^{\gamma-1}$

$r^{\gamma-1} = T_2/T_1$

$Q_{add} = m.Cv(T_3 – T_2)$
$Cv = Cp/\gamma = 1/1.4 = 0.717 kJ/kg.K$

$700 = 1*0.717(1600 – T_2)$

$T_2 = 620^oK$

$T_2 = r^{\gamma-1} *T_1$

$620= r^{1.4-1} *300$

$r = 6.1$

$$\eta_{Otto} = 1 - \frac{1}{6.1^{1.4-1}}$$

$= 0.515 = 51.1\%$

$N.W.D = \eta_{Otto} * Q_{add}$

$= 0.515 * 700$

$= 360 kJ$

4-

$r_{diesel} = 15.5$

$r_{Otto} = 6.1$

5-

$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}}$$

$$\eta_{Carnot} = 1 - \frac{300}{1600}$$

$= 0.82 = 82\%$

$(N.W.D)_{Carnot} = \eta_{Carnot} * Q_{add}$

$= 0.82 * 700$

$= 574kJ$

Increase $(N.W.D)_{Diesel} = 574 – 436$

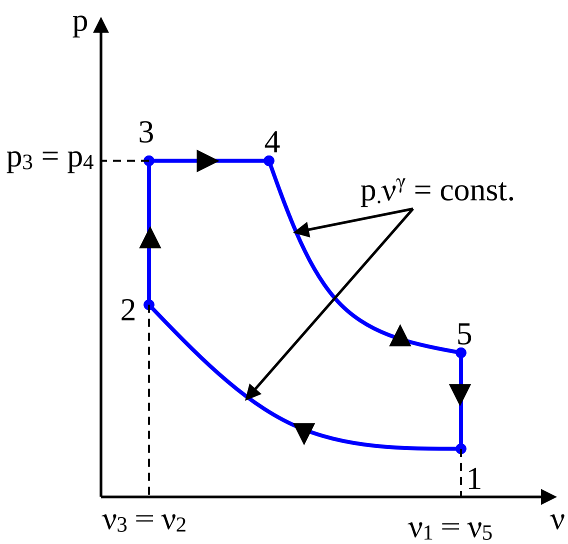
$= 138$ kJ

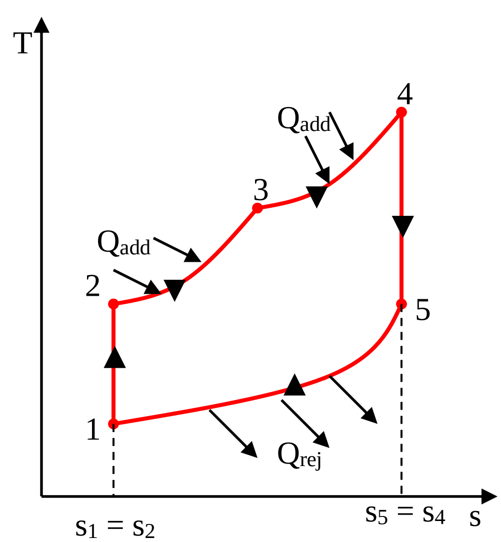
Increase $(N.W.D)_{Otto} = 574 – 360$

$= 214$ kJ

3- Dual cycle:-

Modern oil engines although still called diesel engines are more closely derived from an engine invented by Ackroyd- Stuart in 1888. all oil engines today use solid injection of the fuel ; the fuel injected by a spring –loaded injector the fuel pump being operated by a cam driven from the engine crankshaft.the ideal cycle used as a basis for comparison is called the Dual combustion cycle or the mixed cycle, as shown on a (p.v) diagram below





Process 1to 2 is isentropic compression,(-ve W.D)

$$\boxed{r = V_1/V_2}$$

Process 2 to 3 is reversible constant volume heating, $Q_{add\ 23}$ with pressure ratio,

$$\boxed{r_p = p_3/p_2}$$

Process 3 to 4 is reversible constant pressure heating, $Q_{add\ 34}$,

$$\boxed{e = V_4/V_3}$$

Process 4 to 5 is isentropic expansion, (+ve W.D)

$$\boxed{re = V_5/V_4}$$

Process 1 to 5 is reversible constant volume cooling, Q_{rej},

$Q_{add} = Q_{add\ 23} + Q_{add\ 34}$

$= m.Cv(T_3 – T_2) + m.Cp(T_4 – T_3)$

$Q_{rej} = m.Cv(T_5 – T_1)$

N.W.D = $Q_{add} - Q_{rej}$

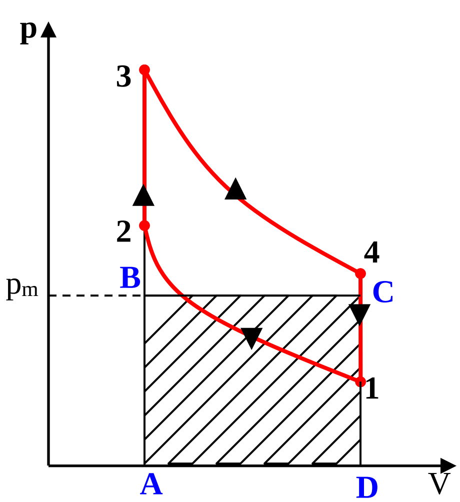
$$\eta_{Dual} = \frac{N.W.D}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}}$$

$$\eta_{Dual} = 1 - \frac{1}{r^{\gamma-1}} * D$$

$$D = \frac{r_p * e^{\gamma-1}}{(r_p - 1) + \gamma . r_p (e - 1)}$$

Mean effective pressure

Is defined as the height of a rectangle having the same length and area as the cycle plotted on (p.v) diagram the rectangle ABCDA is the same length as the cycle 12341, and area ABCDA is equal to area 12341 then the mean effective pressure,p_m, is the height AB of the rectangle.

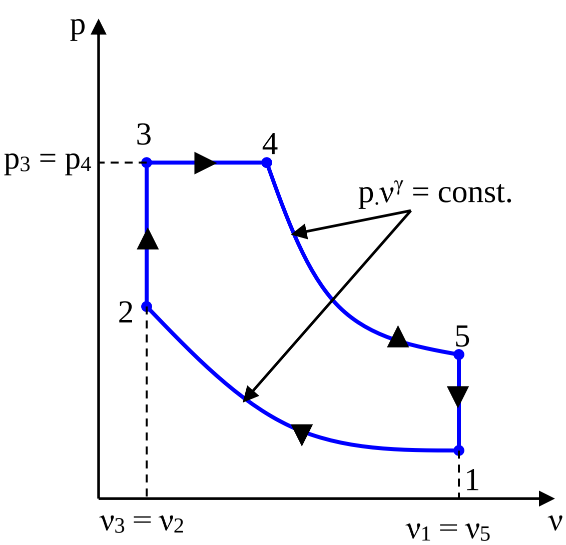


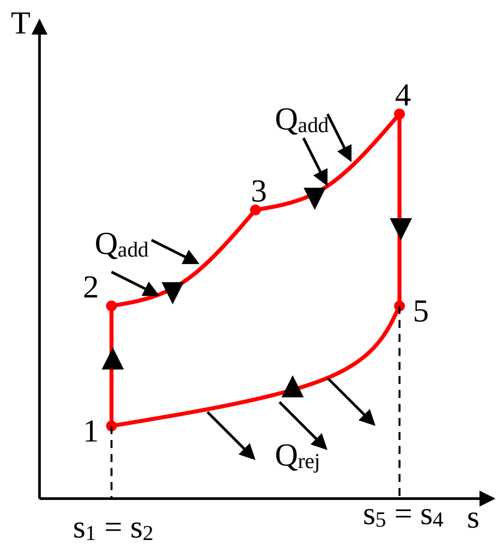
$$p_m = \frac{N.W.D}{10^2\,(v_1 - v_2)}$$

-The term $(v_1 - v_2)$ is proportional to the swept volume of the cylinder.
-The law p_m is used for three cycles (Otto, Diesel, Dual)

EX: - A Diesel engine operates on the Dual cycle with volumetric compression ratio 10:1 ambient temperature & pressure are 27°C & 2bar, maximum pressure 100bar & cut off point ratio 1.5:1 find η_{Dual} , N.W.D&p_m [Take $\gamma = 1.4$ & Cp = 1.005 kJ/kg.K].

Sol:-





r = 10:1

$10 = V_1/V_2$, $T_1 = 273 + 27 = 300^oK$, $p_1 = 2bar$, $p_{max} = p_3 = p_4 = 100bar$, $e = V_4/V_3 = 1.5$

$$\eta_{Dual} = 1 - \frac{1}{r^{\gamma-1}} * D$$

$$D = \frac{r_p * e^{\gamma-1}}{(r_p - 1) + \gamma . r_p (e - 1)}$$

$r_p = p_3/p_2$

$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$p_2 = p_1 (V_1/V_2)^{\gamma}$

$p_2 = p_1.r^{\gamma}$

$= 2 * 10^{1.4}$

$= 50bar$

$r_p = p_3/p_2$

$= 100/50$

$= 2:1$

$$D = \frac{2 * 1.5^{1.4-1}}{(2 -1) +[1.4 *2 (1.5 – 1)]}$$

$D = 0.979$

$$\eta_{Dual} = 1 - \frac{1}{10^{1.4-1}} *0.979$$

$= 0.61 = 61\%$

$$\eta_{Dual} = \frac{N.W.D}{q_{add}}$$

$N.W.D = q_{add} * \eta_{Dual}$

$q_{add} = q_{add23} + q_{add34}$

$q_{add23} = Cv (T_3 - T_2)$

$Cv = Cp/\gamma$

$= 1.005 /1.4$

$= 0.718$ kJ/kg.K

$T_2 = T_1.r^{\gamma-1}$

$= 300 * 10^{1.4-1}$

$= 753.566\ ^oK$

$T_3/ T_2 = p_3/p_2$

$T_3 = r_p * T_2$

$= 2 * 753.566 = 1507.132^oK$

$q_{add23} = 0.718\ (1507.132 - 753.566)$

$= 540.306\ kJ/kg$

$q_{add34} = Cp\ (T_4 - T_3)$

$V_4 / V_3 = T_4 / T_3$

$T_4 = T_3\ .e$

$T_4 = 1507.132 * 1.5$

$= 2260.698^oK$

$q_{add34} = 1.005\ (2260.698 - 1507.132)$

$= 753.56 kJ/kg$

$q_{add} = 540.306 + 753.56$

$= 1293.866 kJ/kg$

N.W.D = 1293.866 * 0.61

= 789.258 kJ/kg

$$p_m = \frac{N.W.D}{10^2\ (v_1 - v_2)}$$

$(v_1 - v_2) = v_1[\ 1- (v_2/v_1)]$

$= v_1\ [1- (1/r)]$

$$(v_1 - v_2) = \frac{R.T_1}{10^2.\ p_1} * \left(1-(1/r)\right)$$

$R = Cp - Cv$

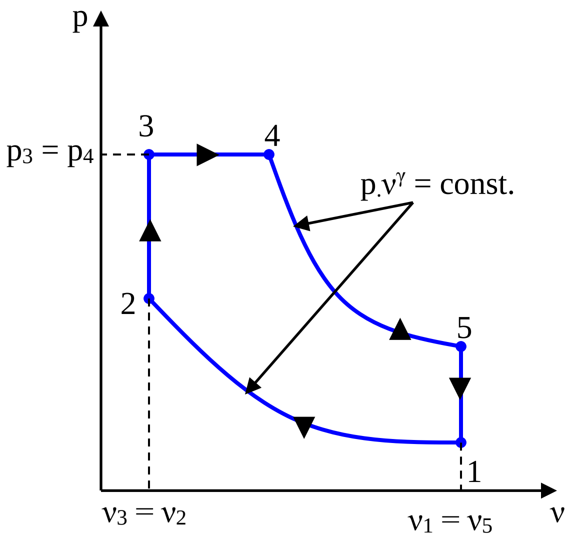
$= 1.005-0.718 = 0.287\ kJ/kg.K$

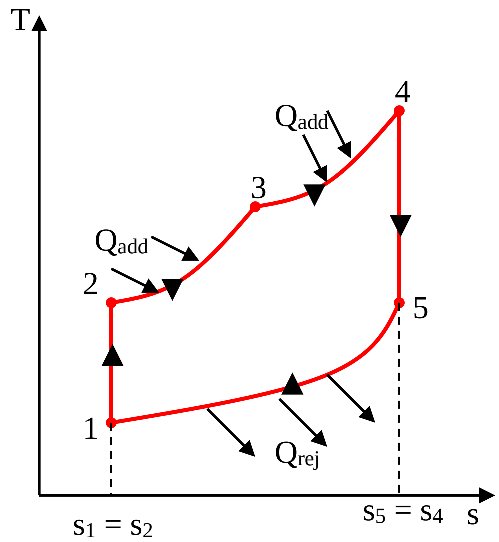
$$(v_1 - v_2) = \frac{0.287*300}{10^2 * 2} * \left(1-(1/10)\right) = 0.387 m^3/kg$$

$$p_m = \frac{789.258}{10^2 * 0.387} = 20.39 bar$$

EX: - An oil engine take air at 1.01 bar & 20°C the maximum cycle pressure is 69bar, the Compression ratio is 18:1.calculate the air standard thermal efficiency based on the Dual combustion cycle. Assume that the heat added at constant volume is equal to the heat added at constant pressure. Also calculate the mean effective pressure for the cycle. [Take γ = 1.4, Cv = 0.718kJ/kg.K]

Sol:-





$T_2 / T_1 = (v_1/v_2)^{\gamma-1} = 18^{0.4} = 3.18$

$T_2 = 3.18 * T1$

$= 3.18 * 293 = 931°K$

(Where $T_1 = 20 + 273 = 293°K$)

$p_3 / p_2 = T_3/T_2$

$T_3 = (p_3/p_2) * T_2$

$= (69 / p_2) * 931$

$p_2 / p_1 = (v_1/v_2)^{\gamma}$

$p_2 / 1.01 = (18)^{1.4}$
$p_2 = 57.8$bar

$T_3 = (p_3/p_2) * T_2$

$= (69 / 57.8) * 931 = 1112°K$

Now the heat added at constant volume is equal to the heat added at constant pressure, therefore,

$$Cv\ (T_3 - T_2) = Cp\ (T_4 - T_3)$$

$$0.718\ (1112 - 931) = 1.005\ (T_4 - 1112)$$

$T_4 = 1241.4^oK$

To find T_5 it is necessary to know the value of the volume ratio, $\nu_5 = \nu_4$. At constant pressure from 3 to 4,

$\nu_4 / \nu_3 = T_4/T_3$

$= 1241.4 / 1112 = 1.116$

Therefore,

$\nu_5 / \nu_3 = \nu_1 / \nu_4$

$$\nu_5 / \nu_3 = \frac{\nu_1}{\nu_2} * \frac{\nu_3}{\nu_4} = 18 * (1/1.116\ ) = 16.14$$

$T_4 / T_5 = (\nu_5 / \nu_4)^{\gamma-1} = 16.14^{1.4-1} = 3.04$

$T_5 = 1241.4 / 3.04 = 408\ ^oK$

$Q_{add} = Cv(T_3 - T_2) + Cp\ (T_4 - T_3)$

$= 0.718(1112 - 931) + 1.005\ (1241.4 - 1112)$

$= 260 kJ/kg$

$Q_{rej} = Cv(T_5 - T_1)$

$= 0.718\ (408 - 293) = 82.6\ kJ/kg$

$$\eta_{Dual} = 1 - \frac{Q_{rej}}{Q_{add}}$$

$$= 1 - \frac{82.6}{260}$$

$= 68.2\%$

$\eta_{Dual} = N.W.D / Q_{add}$

$N.W.D = \eta_{Dual} * Q_{add}$

$= 0.682 * 260 = 177 kJ/kg$

$N.W.D = p_m\,(v_1 - v_2)$

$(v_1 - v_2) = v_1[\,1 - (v_2/v_1)]$

$\qquad\qquad = v_1\,[1 - (1/r)]$

$$(v_1 - v_2) = \frac{R.T_1}{10^2.\,p_1} * \left(1 - (1/r)\right)$$

$R = Cp - Cv$

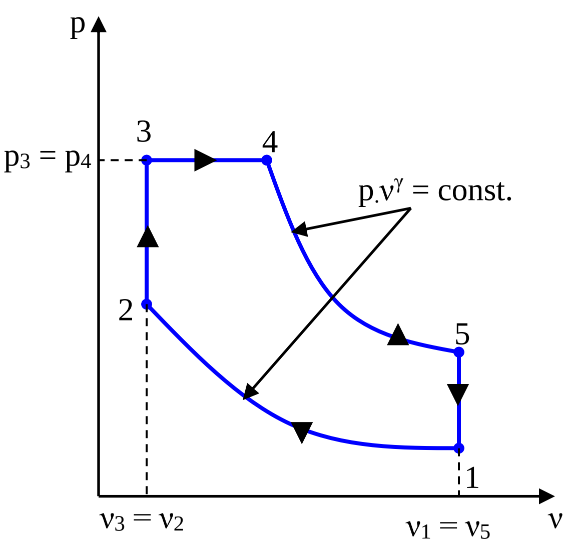
$\quad = 1.005 - 0.718 = 0.287$ kJ/kg.K

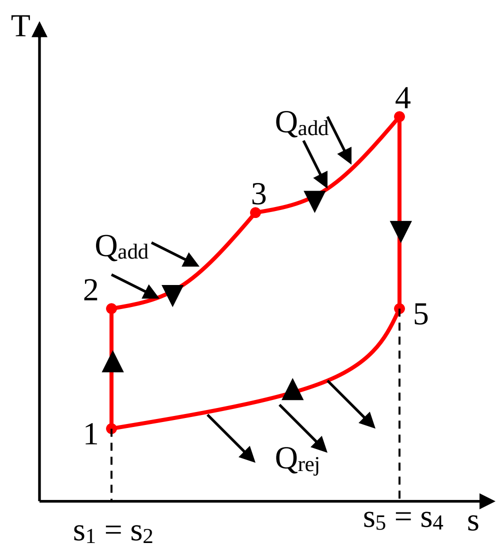
$$(v_1 - v_2) = \frac{0.287*293}{10^2 * 1.01} * \left(1 - (1/18)\right) = 0.786\,m^3/kg$$

$$p_m = \frac{177}{10^2 * 0.786} = 2.25\text{bar (mean effective pressure)}$$

EX: - In a Dual combustion cycle the maximum temperature is 2000°C and the maximum pressure is 70bar. Calculate the thermal efficiency when the pressure & temperature at the start of compression are 1bar & 17°C respectively. The compression ratio is 18:1 [Take Cp = 1.005kJ/kg.K, γ = 1.4].

Sol:-





$T_{max} = T_4 = 2000 + 273 = 2273\,^oK$, $p_{max} = p_3 = p_4 = 70$bar ,$p_1 = 1$bar, $T_1 = 17+273 = 290^oK$

$r = 18$

$$\eta = 1 - \frac{1}{r^{\gamma-1}} * D$$

$$D = \frac{r_p * e^{\gamma-1}}{(r_p - 1) + \gamma . r_p (e - 1)}$$

$r_p = p_3/p_2$

$p_1.V_1^{\gamma} = p_2.V_2^{\gamma}$

$p_2 = p_1 (V_1/V_2)^{\gamma}$

$p_2 = p_1.r^{\gamma}$

$= 1 * 18^{1.4}$

$= 57.198 bar$

$r_p = p_3/p_2$

$= 70/57.198$

$= 1.2238$

$T_2 = T_1.r^{\gamma-1}$

$= 290 * 18^{1.4-1}$

$= 921.524\ ^oK$

$T_3/ T_2 = p_3/p_2$

$T_3 = r_p * T_2$

$= 1.2238 * 921.524 = 1127.95^oK$

$e = V_4/ V_3 = T_4/T_3 = 2273 / 1127.95 = 2.0152$

$$D = \frac{1.224 * 2.0152^{1.4-1}}{(1.224 - 1) + [1.4 * 1.224 (2.0152 - 1)]}$$

$= 0.8249$

$$\eta = 1 - \frac{1}{18^{1.4-1}} * 0.8249$$

$= 0.74 = 74\%$

Cycle	Difference	Fluid used In the cycle	Similar
Carnot	Q_{add} & Q_{rej} at T = constant	Air or steam	$\eta = \frac{N.W.D}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}}$ $N.W.D = Q_{add} - Q_{rej}$ $Q_{add} > Q_{rej}$
Otto	Q_{add} & Q_{rej} at V = constant	Air	
Diesel	Q_{add} at p= constant , Q_{rej}at V = constant	Air	
Dual	Q_{add} at p&V = constant, Q_{rej}at V = constant	Air	
Rankine	Q_{add} & Q_{rej} at p= constant	steam	

Carnot	Reverse Carnot
$\eta = \frac{N.W.D}{Q_{add}} = 1 - \frac{Q_{rej}}{Q_{add}} < 1$	$C.O.P = \frac{Q_{add} \text{ or } Q_{rej}}{w.D} > 1$
W.D = $Q_{add} - Q_{rej}$	**W.D = $Q_{rej} - Q_{add}$**
$Q_{add} > Q_{rej}$	**$Q_{rej} > Q_{add}$**
H.E	**H.P&ref.**
From high temperature	**From low temperature**
+ve w.D	**-ve w.D**

Problems

9 – 1 In an air standard Otto cycle the maximum and minimum temperatures are 1400°C and 15°C. The heat supplied per kg of air is 800kJ. Calculate the compression ratio and the thermal efficiency. Calculate also the ratio of maximum and minimum pressures in the cycle. (5.26/1;48.6%;30.5/1)

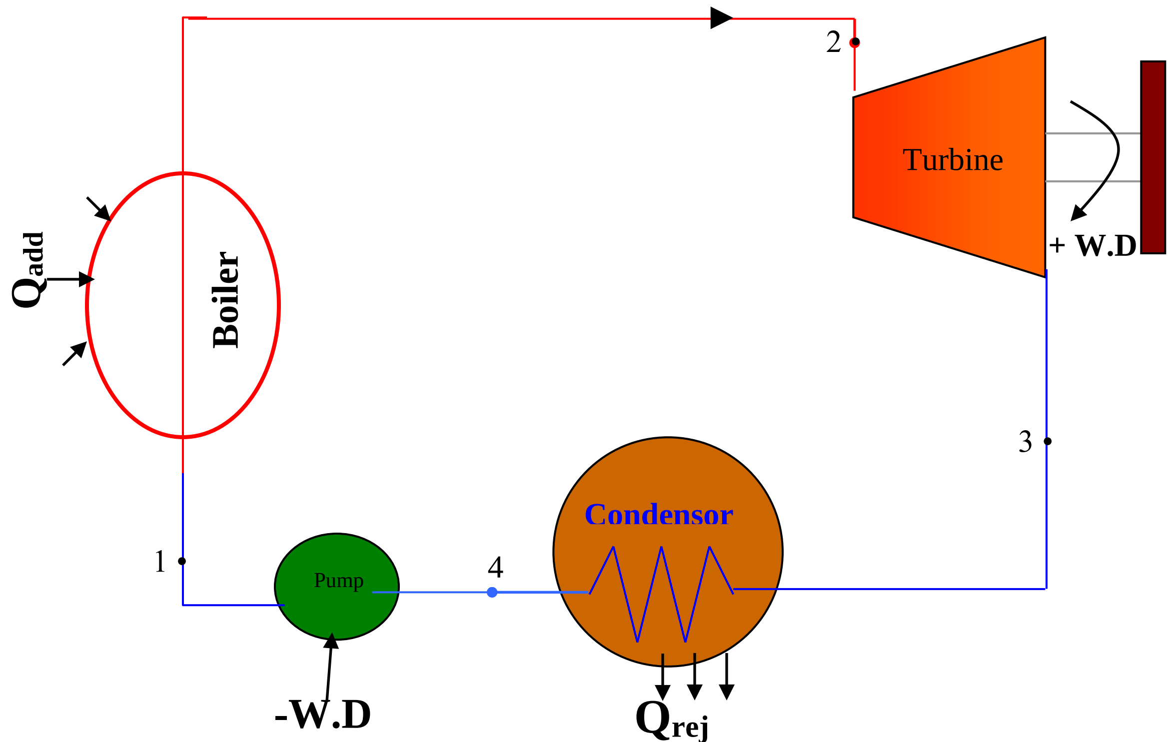
9 – 2 Calculate the thermal efficiency and mean effective pressure of an air standard diesel cycle with a compression ratio of 15/1, and maximum and minimum cycle temperatures of 1650ºC and 15ºC respectively. The maximum cycle pressure is 45bar.
(59.1%; 8.39bar)

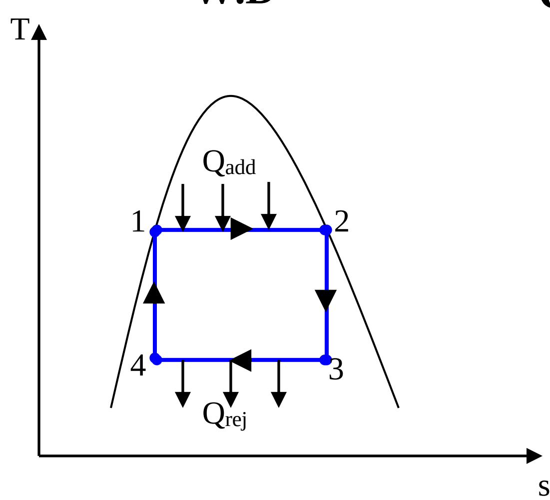
9 – 3 An air standard dual combustion cycle has a mean effective pressure of 10bar. The minimum pressure and temperature are 1bar and 17ºC respectively, and the compression ratio is 16/1. Calculate the maximum cycle temperature when the thermal efficiency is 60%. The maximum cycle pressure is 60bar. (1959ºC)

Unit ten

Steam cycles

The Carnot cycle for steam:-





Process (1 – 2) isothermal at constant temperature, $T_1 = T_2$, Q_{add}

Process (2 - 3) isentropic expansion $s_3 = s_2$ (+ Ve .W.D)

Process (3 – 4) isothermal at constant temperature, $T_3 = T_4$, Q_{rej}

Process (4 – 1) isentropic compression, $s_1 = s_4$, (- Ve .W.D)

$Q_{add} = h_2 - h_1 = h_g - h_f$ at p_1

$= T_1 (s_2 - s_1)$

$Q_{rej} = h_3 - h_4$

$= T_3(s_3 - s_4)$

$= T_3(s_2 - s_1)$

$T_{max} = T_1 = T_2$

$T_{min} = T_3 = T_4$

$+ \text{ve } W.D = h_2 - h_3$

$- \text{ve } W.D = h_1 - h_4$

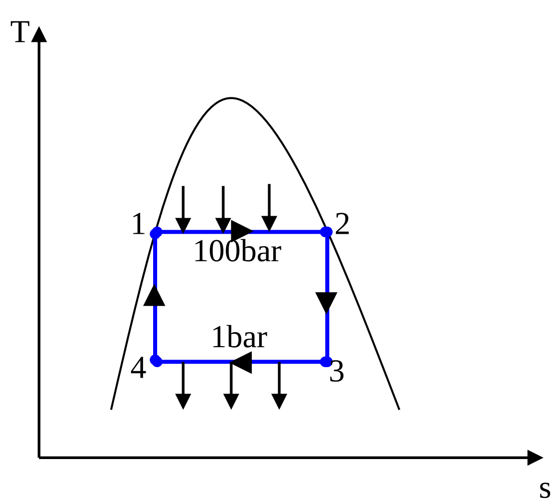
$$\eta_{Car} = 1 - \frac{Q_{rej}}{Q_{add}}$$

$$\eta_{Car} = 1 - \frac{T_{min}(s_2 - s_1)}{T_{max}(s_2 - s_1)}$$

$$\boxed{\eta_{Car} = 1 - \frac{T_{min}}{T_{max}}}$$

EX:-Find the Carnot efficiency for steam working between the pressure 100bar & 1bar.

Sol:-



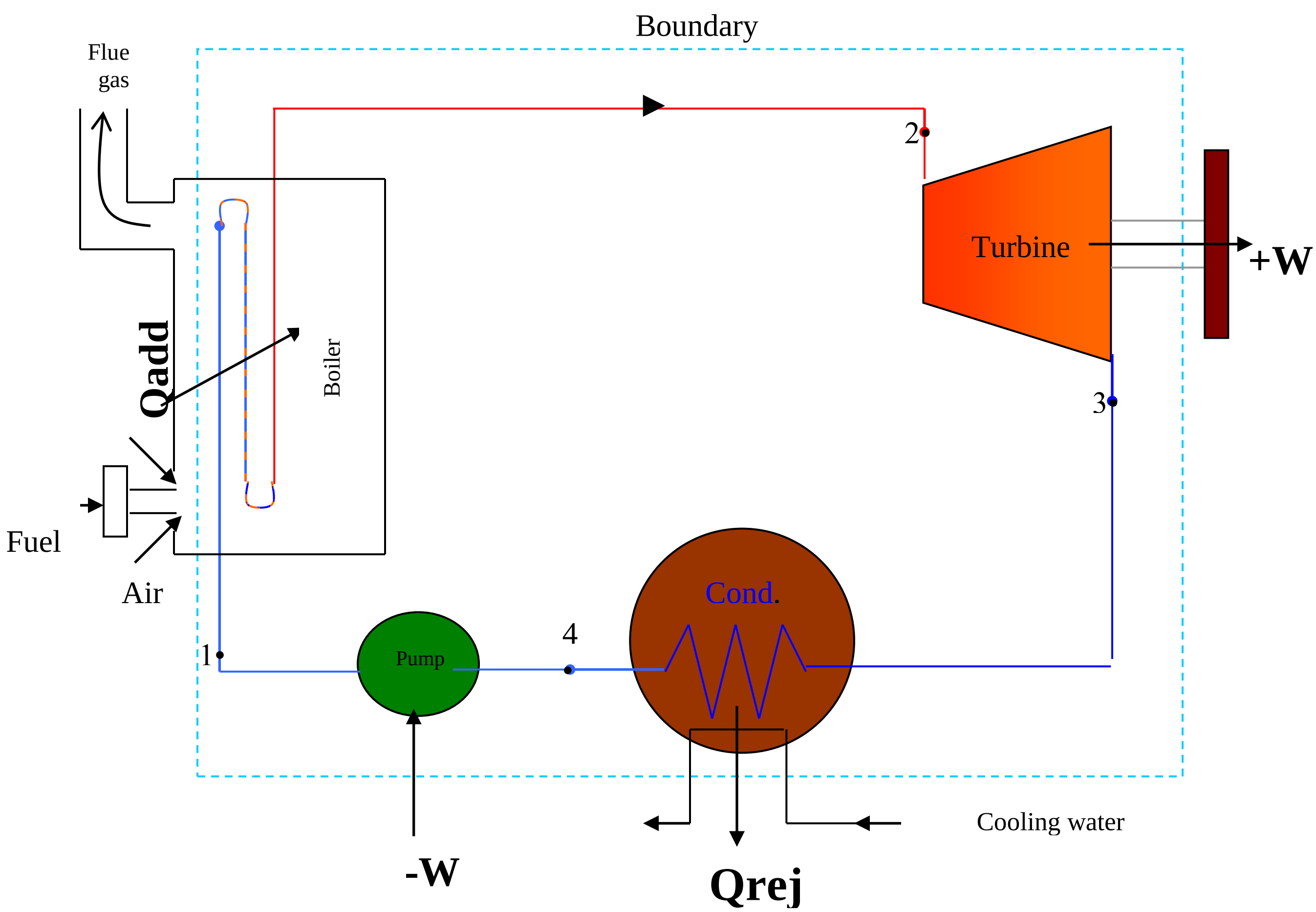
$$\eta_{Car} = 1 - \frac{T_{min}}{T_{max}}$$

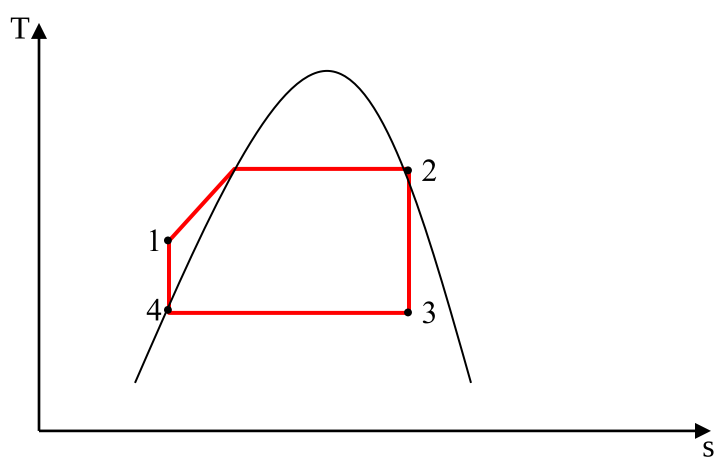
At p = 100bar from steam table t_s = 311°C = T_{max}

At p = 1bar from steam table t_s = 99.6°C = T_{min}

$$\eta_{Car} = 1 - \frac{(99.6 + 273)}{(311 + 273)} = 0.36 = 36\%$$

Simple Rankine cycle:-





Process (1 – 2) isobaric heat addition in the boiler where the water is given sensible & latent.

Process (2 – 3) isentropic expansion in turbine where (+ve w.D) is developed and expansion take place from high pressure p_1 (boiler pressure) tp the lower pressure p_2 (condenser pressure or water pressure).

Process (3 – 4) isobaric heat rejected in condenser where steam at point 3 is cond. To saturated water at 4.

Process (4 – 1) isentropic compression from low pressure (p_2) to the boiling pressre (p_1) through (-veW.D)

Calculation:-

$Q_{add} = h_2 - h_1$

$Q_{rej} = h_3 - h_4$

$= h_3 - h_f$ (at p_2)

+ve .W.D $= h_2 - h_3$

- ve .W.D $= h_1 - h_4$

$= h_1 - h_f$ (at p_2)

w.D $= 10^2\, V(p_1 - p_4)$

$= 10^2 v_{fat\ p4}\,(p_1 - p_4)$

$$\eta_{Rankine} = \frac{N.W.D}{Q_{add}}$$

$$= \frac{+ve\ .W.D - (-\ ve\ .W.D)}{Q_{add}}$$

$$= \frac{\text{Turbine work} - \text{pump work}}{Q_{add}}$$

$$= \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1}$$

Or

$$\eta_{Rankine} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_4) - (h_1 - h_4)}$$

If the feed pump neglected

$$\eta_{Rankine} = \frac{(h_2 - h_3)}{(h_2 - h_4)}$$

$$\eta_{Rankine} = \frac{(h_2 - h_3)}{(h_2 - h_f\ (\text{at } p_2))}$$

$$\textbf{Work ratio} = \frac{\textbf{N.W.D}}{\textbf{Gross work (work turbine)}}$$

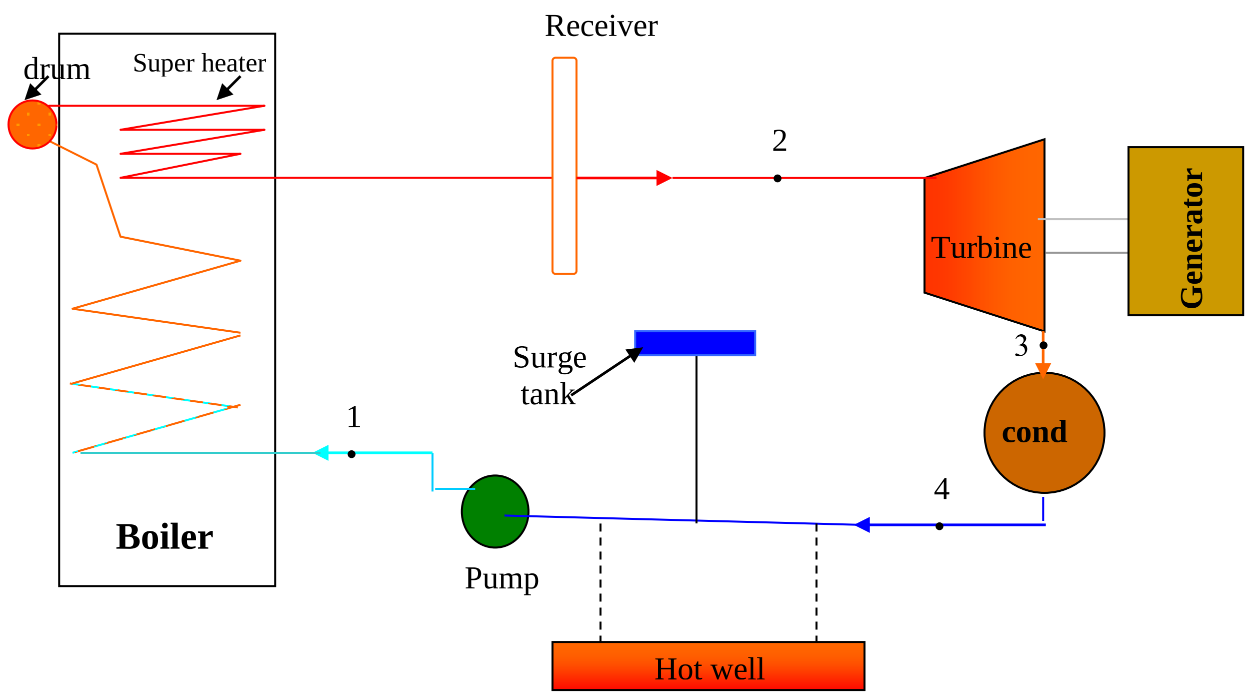
Specific steam consumption, (S.S.C):- is the steam flow in Kg /h required to develop 1KW,
Wnet KJ/Kg * S.S.C = 1KW*3600 KJ/h

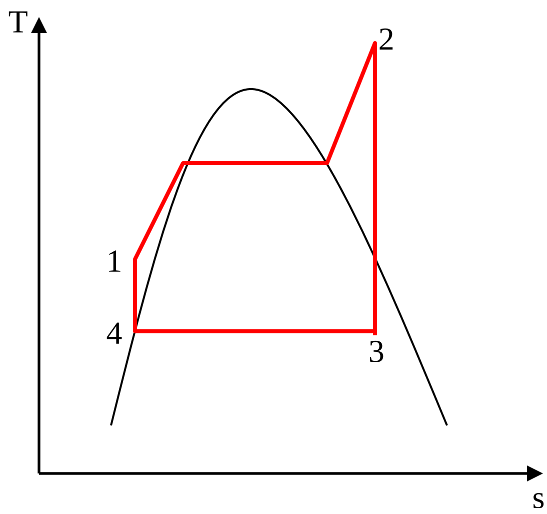
$$\text{S.S.C} = \frac{3600}{\text{Wnet}}\ (\text{Kg / kW.h})$$

$$\text{S.S.C} = \frac{3600}{(h_2 - h_3) - (h_1 - h_4)}$$

Rankine cycle with super heat:-

The purpose of super heat is to raise the efficiency by increasing h_2 which enter to the turbine.

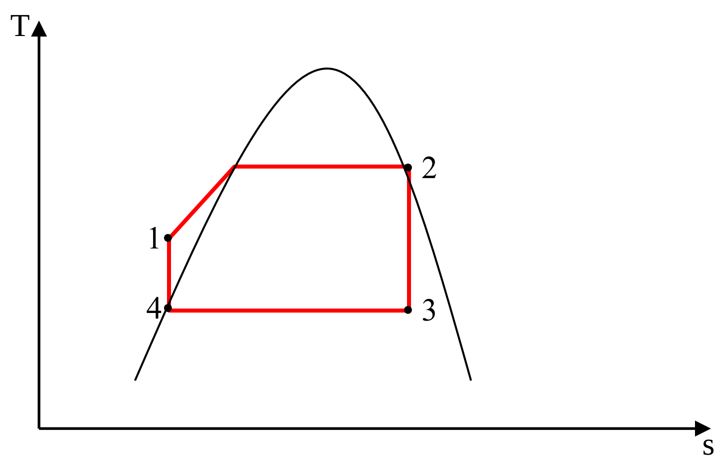




EX: - steam plant works on the Rankine cycle has the boiler pressure of 100bar and condensed pressure of 1bar. The initial steam temperature started at saturated condition then 400°C the expansion in the turbine is isentropic neglect pump work find the cycle efficiency at each condition & compare it with Carnot efficiency.

Sol:-

$P_{H.P} = 100bar$, $p_{L.P} = 1bar$,



$$\eta_{Rankine} = \frac{h_2 - h_3}{h_2 - h_{f3}}$$

$p_1 = 100bar = p_2$ dry & sat from steam table

$h_2 = 2725$ kJ/kg.K $= h_g$, $s_2 = s_g = 5.615 = s_3$

$$x = \frac{s_3 - sf_3}{s_{fg3}}$$

From steam table at p = 1bar, sf_3 = 1.303 kJ/kg.K, s_{fg3} = 6.056kJ/kg.K

$$x = \frac{5.615 - 1.303}{6.056} = 0.711$$

$h_3 = h_{f3} + x\,.h_{fg3}$

From steam table at p = 1bar, hf_3 = 417 kJ/kg, s_{fg3} = 2258kJ/kg

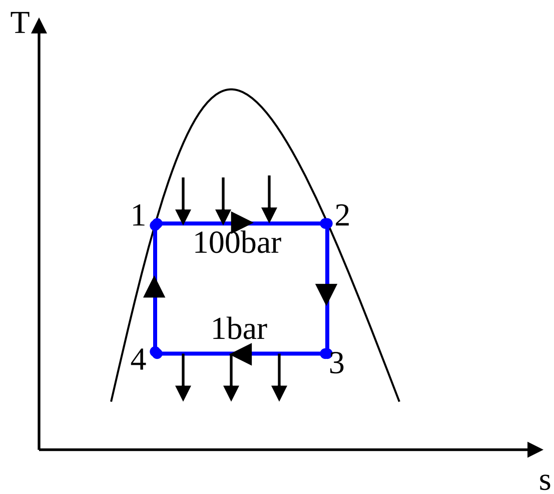
h_3 = 417 + 0.711*2258

= 2023 kJ/kg

$h_4 = h_{f3}$ = 417 kJ/kg

$$\eta_{Rankine} = \frac{2725 - 2023}{2725 - 417} = 0.304 = 30.4\ \%$$

Carnot



$$\eta_{Car} = 1 - \frac{T_{min}}{T_{max}}$$

$T_{min} = T_s$ at p = 1bar, T_{min} = 99.6 ^oC

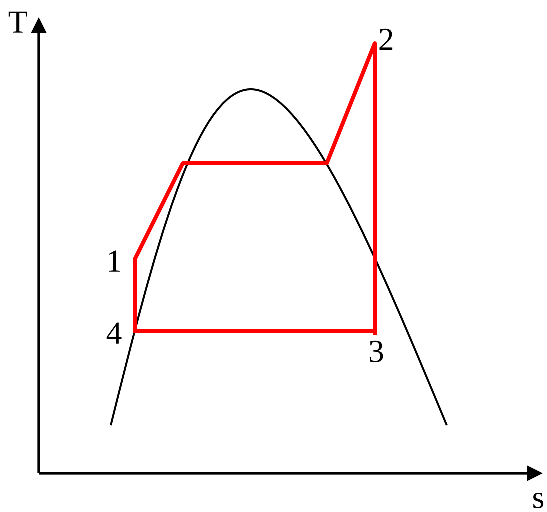
$T_{max} = T_s$ at p = 100bar, T_{max}= 311 ^oC

$$\eta_{Car} = 1 - \frac{99.6 + 273}{311 + 273}$$

= 0.361 = 36.1%

Superheated cycle

p = 100 bar & T = 400^{o}C from superheated table, h_2 = 3097kJ/kg.K, s_2 = 6.213kJ/kg.K



$s_2 = 6.213 kJ/kg.K = s_3$

From steam table at p = 1bar, sf_3 = 1.303 kJ/kg.K, s_{fg3} = 6.056kJ/kg.K

$$x = \frac{s_3 - sf_3}{s_{fg3}}$$

$$x = \frac{6.213 - 1.303}{6.056} = 0.811$$

$h_3 = h_{f3} + x .h_{fg3}$

From steam table at p = 1bar, hf_3 = 417 kJ/kg, h_{fg3} = 2258kJ/kg

$h_3 = 417 + 0.811*2258$

$= 2247.7$ kJ/kg

$h_4 = h_{f3} = 417$ kJ/kg

$$\eta_{Rankine} = \frac{h_2 - h_3}{h_2 - h_{f3}}$$

$$\eta_{Rankine} = \frac{3097 - 2247.7}{3097 - 417} = 0.317 = 31.7\ \%$$

$$\eta_{Car} = 1 - \frac{T_{min}}{T_{max}}$$

$T_{min} = T_s$ at p = 1bar, $T_{min} = 99.6\ ^oC$

$T_{max} = 400\ ^oC$

$$\eta_{Car} = 1 - \frac{99.6 + 273}{400 + 273}$$

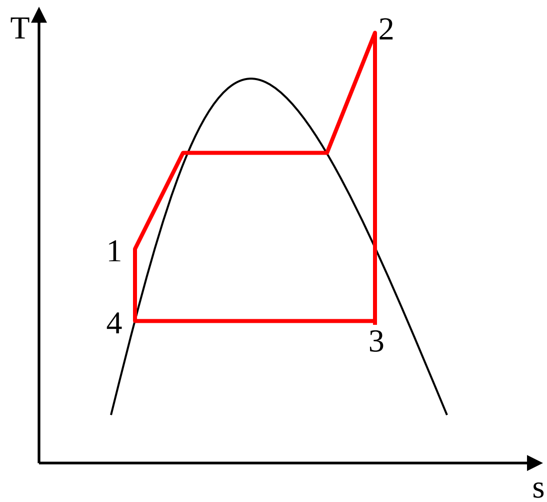
$= 0.446 = 44.6\%$

EX: - Inlet pressure to turbine is 60bar, inlet temperature to turbine is 500°C, exit pressure from turbine is 2bar, efficiency isentropic is 80%. Find Rankine efficiency Carnot efficiency & net work done.

Sol:-

$p_1 = 60$bar, $T_1 = 500^oC$

$p_3 = p_4 = 2$bar



$$\eta_{Rankine} = \frac{\eta_{isentropic}\,(h_2 - h_3)}{h_2 - h_{f3}}$$

$h_2 = 3421$kJ/kg, $s_2 = 6.874$kJ/kg.K $= s_3$

$h_3 = h_{f3} + x.h_{fg3}$

$$x = \frac{s_3 - s_{f3}}{s_{fg3}}$$

From table at p = 2bar, $s_{f3} = 2.53$kJ/kg.K,$s_{fg3} = 5.597$ kJ/kg.K,$h_{f3} = 505$kJ/kg.K

$h_{fg3} = 2202$ kJ/kg

$$x = \frac{6.874 - 1.53}{5.597} = 0.955$$

h_3 = 505 +0.955 *2202 =2607.91kJ/kg

$$\eta_{Rankine} = \frac{0.8\,(3421 - 2607.91)}{3421 - 505}$$

$= 0.223 = 22.3\%$

$$\eta_{Car} = 1 - \frac{T_{min}}{T_{max}}$$

$T_{min} = T_s$ at p = 2bar, $T_{min} = 120\ ^oC$

$T_{max} = 500\ ^oC$

$$\eta_{Car} = 1 - \frac{120 + 273}{500 + 273}$$

$= 0.49 = 49\%$

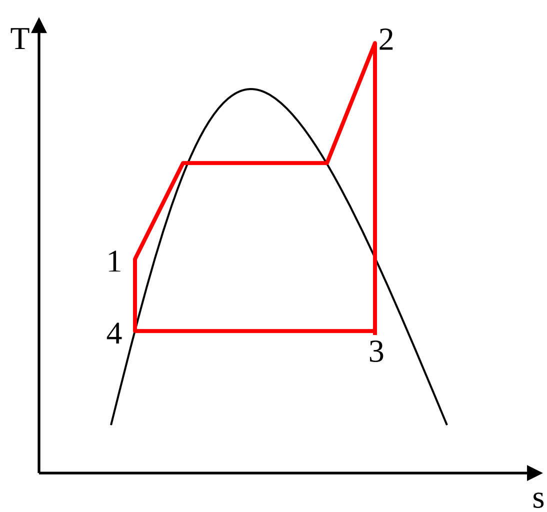
N.W.D = (3421 – 2216)

= 964 kJ/kg

EX: - Rankine cycle operates between 40bar & 0.04bar steam enter turbine with degree of superheated 49.7^{o}C neglect pump work, Calculate turbine work & Rankine efficiency.

Sol:-

At 40 bar, $t_s = 250.3^oC$, $(\Delta t)_{SH} = 49.7^oC$



$T = t_s + (\Delta t)_{SH}$

$= 250.3 + 49.7 = 300^oC$

$h_2 = 2963 kJ/kg$, $s_2 = 6.364 kJ/kg.K = s_3$

at 0.04 bar ,$s_g = 8.473 kJ/kg.K > s_2$ therefore steam is wet

$s_{f3} = 0.422 kJ/kgK$, $s_{fg3} = 8.051 kJ/kg.K$, $h_{f3} = 121 kJ/kg$, $h_{fg3} = 2433 kJ/kg$

$$x = \frac{s_3 - s_{f3}}{s_{fg3}}$$

$$x = \frac{6.364 - 0.422}{8.051} = 0.737$$

$h_3 = h_{f3} + x.h_{fg3}$

$h_3 = 121 + 0.737*2433 = 1913.64 kJ/kg$

$N.w.D = h_2 - h_3$

$= 2963 - 1913.64$

$= 1049.4 kJ/kg$

$$\eta_{Rankine} = \frac{h_2 - h_3}{h_2 - h_{f3}}$$

$$\eta_{Rankine} = \frac{2963 - 1913.64}{2963 - 121} = 0.369 = 36.9\ \%$$

EX: - Rankine cycle operates between boiler pressure 8bar & condenser pressure 1bar. Calculate Rankine efficiency if dry & saturated steam enter to the turbine with internal efficiency85%.

Sol:-

$$\eta_{Rankine} = \frac{\eta_{internal}(h_2 - h_3)}{h_2 - h_{f3}}$$

From steam table at 8 bar dry & sat.,$h_g = h_2 = 2769 kJ/kg$, $s_g = s_2 = 6.663$ kJ/kg.K

From steam table at p = 1bar, $sf_3 = 1.303$ kJ/kg.K, $s_{fg3} = 6.056 kJ/kg.K$

$$x = \frac{s_3 - sf_3}{s_{fg3}}$$

$$x = \frac{6.663 - 1.303}{6.056} = 0.885$$

From steam table at p = 1bar, hf_3 = 417 kJ/kg, h_{fg3} = 2258kJ/kg

h_3 = 417 + 0.885*2258= 2415.494 kJ/kg

$$\eta_{Rankine} = \frac{0.85\,(2769 - 2415.494)}{2769 - 417} = 0.1277 = 12.77\%$$

Problems

10 – 1 steam is supplied, dry saturated at 40bar to a turbine and the condenser pressure is 0.035 bar. If the plant operates on the Rankine cycle, calculate, per kg of steam the Rankine efficiency ; & the specific steam consumption , for the same steam condition calculate the efficiency ; & the specific steam consumption for a Carnot cycle operating with wet steam. (36.6%; 3.66kg/kW h; 43% ; 4.88kg/kWh)

10 – 2 steam is supplied, at 40bar & 350ºC to a turbine and the condenser pressure is 0.035 bar. If the plant operates on the Rankine cycle, calculate, per kg of steam the Rankine efficiency& the specific steam consumption. (37.8%; 3.2kg/kW h)

APPENDIX

STEAM TABLES

Saturated Water and Steam

t °C	p_s bar	v_g m³/kg	h_f kJ/kg	h_{fg} kJ/kg	h_g kJ/kg	s_f kJ/kg K	s_{fg} kJ/kg K	s_g kJ/kg K
0·01	0·006112	206·1	0*	2500·8	2500·8	0†	9·155	9·155
1	0·006566	192·6	4·2	2498·3	2502·5	0·015	9·113	9·128
2	0·007054	179·9	8·4	2495·9	2504·3	0·031	9·071	9·102
3	0·007575	168·2	12·6	2493·6	2506·2	0·046	9·030	9·076
4	0·008129	157·3	16·8	2491·3	2508·1	0·061	8·989	9·050
5	0·008719	147·1	21·0	2488·9	2509·9	0·076	8·948	9·024
6	0·009346	137·8	25·2	2486·6	2511·8	0·091	8·908	8·999
7	0·01001	129·1	29·4	2484·3	2513·7	0·106	8·868	8·974
8	0·01072	121·0	33·6	2481·9	2515·5	0·121	8·828	8·949
9	0·01147	113·4	37·8	2479·6	2517·4	0·136	8·788	8·924
10	0·01227	106·4	42·0	2477·2	2519·2	0·151	8·749	8·900
11	0·01312	99·90	46·2	2474·9	2521·1	0·166	8·710	8·876
12	0·01401	93·83	50·4	2472·5	2522·9	0·180	8·671	8·851
13	0·01497	88·17	54·6	2470·2	2524·8	0·195	8·633	8·828
14	0·01597	82·89	58·8	2467·8	2526·6	0·210	8·594	8·804
15	0·01704	77·97	62·9	2465·5	2528·4	0·224	8·556	8·780
16	0·01817	73·38	67·1	2463·1	2530·2	0·239	8·518	8·757
17	0·01936	69·09	71·3	2460·8	2532·1	0·253	8·481	8·734
18	0·02063	65·08	75·5	2458·4	2533·9	0·268	8·444	8·712
19	0·02196	61·34	79·7	2456·0	2535·7	0·282	8·407	8·689
20	0·02337	57·84	83·9	2453·7	2537·6	0·296	8·370	8·666
21	0·02486	54·56	88·0	2451·4	2539·4	0·310	8·334	8·644
22	0·02642	51·49	92·2	2449·0	2541·2	0·325	8·297	8·622
23	0·02808	48·62	96·4	2446·6	2543·0	0·339	8·261	8·600
24	0·02982	45·92	100·6	2444·2	2544·8	0·353	8·226	8·579
25	0·03166	43·40	104·8	2441·8	2546·6	0·367	8·190	8·557
26	0·03360	41·03	108·9	2439·5	2548·4	0·381	8·155	8·536
27	0·03564	38·81	113·1	2437·2	2550·3	0·395	8·120	8·515
28	0·03778	36·73	117·3	2434·8	2552·1	0·409	8·085	8·494
29	0·04004	34·77	121·5	2432·4	2553·9	0·423	8·050	8·473
30	0·04242	32·93	125·7	2430·0	2555·7	0·436	8·016	8·452
32	0·04754	29·57	134·0	2425·3	2559·3	0·464	7·948	8·412
34	0·05318	26·60	142·4	2420·5	2562·9	0·491	7·881	8·372
36	0·05940	23·97	150·7	2415·8	2566·5	0·518	7·814	8·332
38	0·06624	21·63	159·1	2411·0	2570·1	0·545	7·749	8·294
40	0·07375	19·55	167·5	2406·2	2573·7	0·572	7·684	8·256
42	0·08198	17·69	175·8	2401·4	2577·2	0·599	7·620	8·219
44	0·09100	16·03	184·2	2396·6	2580·8	0·625	7·557	8·182
46	0·1009	14·56	192·5	2391·8	2584·3	0·651	7·494	8·145
48	0·1116	13·23	200·9	2387·0	2587·9	0·678	7·433	8·111
50	0·1233	12·04	209·3	2382·1	2591·4	0·704	7·371	8·075
55	0·1574	9·578	230·2	2370·1	2600·3	0·768	7·223	7·991
60	0·1992	7·678	251·1	2357·9	2609·0	0·831	7·078	7·909
65	0·2501	6·201	272·0	2345·7	2617·7	0·893	6·937	7·830
70	0·3116	5·045	293·0	2333·3	2626·3	0·955	6·800	7·755
75	0·3855	4·133	313·9	2320·8	2634·7	1·015	6·666	7·681
80	0·4736	3·408	334·9	2308·3	2643·2	1·075	6·536	7·611
85	0·5780	2·828	355·9	2295·6	2651·5	1·134	6·410	7·544
90	0·7011	2·361	376·9	2282·8	2659·7	1·192	6·286	7·478
95	0·8453	1·982	398·0	2269·8	2667·8	1·250	6·166	7·416
100	1·01325	1·673	419·1	2256·7	2675·8	1·307	6·048	7·355

† and * See page 3 for footnotes

Saturated Water and Steam

p	t_s	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0·006112	0·01	206·1	0†	2375	0*	2501	2501	0†	9·155	9·155
0·010	7·0	129·2	29	2385	29	2485	2514	0·106	8·868	8·974
0·015	13·0	87·98	55	2393	55	2470	2525	0·196	8·631	8·827
0·020	17·5	67·01	73	2399	73	2460	2533	0·261	8·462	8·723
0·025	21·1	54·26	88	2403	88	2451	2539	0·312	8·330	8·642
0·030	24·1	45·67	101	2408	101	2444	2545	0·354	8·222	8·576
0·035	26·7	39·48	112	2412	112	2438	2550	0·391	8·130	8·521
0·040	29·0	34·80	121	2415	121	2433	2554	0·422	8·051	8·473
0·045	31·0	31·14	130	2418	130	2428	2558	0·451	7·980	8·431
0·050	32·9	28·20	138	2420	138	2423	2561	0·476	7·918	8·394
0·055	34·6	25·77	145	2422	145	2419	2564	0·500	7·860	8·360
0·060	36·2	23·74	152	2425	152	2415	2567	0·521	7·808	8·329
0·065	37·7	22·02	158	2427	158	2412	2570	0·541	7·760	8·301
0·070	39·0	20·53	163	2428	163	2409	2572	0·559	7·715	8·274
0·075	40·3	19·24	169	2430	169	2405	2574	0·576	7·674	8·250
0·080	41·5	18·10	174	2432	174	2402	2576	0·593	7·634	8·227
0·085	42·7	17·10	179	2434	179	2400	2579	0·608	7·598	8·206
0·090	43·8	16·20	183	2435	183	2397	2580	0·622	7·564	8·186
0·095	44·8	15·40	188	2436	188	2394	2582	0·636	7·531	8·167
0·100	45·8	14·67	192	2437	192	2392	2584	0·649	7·500	8·149
0·12	49·4	12·36	207	2442	207	2383	2590	0·696	7·389	8·085
0·14	52·6	10·69	220	2446	220	2376	2596	0·737	7·294	8·031
0·16	55·3	9·432	232	2450	232	2369	2601	0·772	7·213	7·985
0·18	57·8	8·444	242	2453	242	2363	2605	0·804	7·140	7·944
0·20	60·1	7·648	251	2456	251	2358	2609	0·832	7·075	7·907
0·22	62·2	6·994	260	2459	260	2353	2613	0·858	7·016	7·874
0·24	64·1	6·445	268	2461	268	2348	2616	0·882	6·962	7·844
0·26	65·9	5·979	276	2464	276	2343	2619	0·904	6·913	7·817
0·28	67·5	5·578	283	2466	283	2339	2622	0·925	6·866	7·791
0·30	69·1	5·228	289	2468	289	2336	2625	0·944	6·823	7·767
0·32	70·6	4·921	295	2470	295	2332	2627	0·962	6·783	7·745
0·34	72·0	4·649	302	2472	302	2328	2630	0·980	6·745	7·725
0·36	73·4	4·407	307	2473	307	2325	2632	0·996	6·709	7·705
0·38	74·7	4·189	312	2475	312	2322	2634	1·011	6·675	7·686
0·40	75·9	3·992	318	2476	318	2318	2636	1·026	6·643	7·669
0·42	77·1	3·814	323	2478	323	2315	2638	1·040	6·612	7·652
0·44	78·2	3·651	327	2479	327	2313	2640	1·054	6·582	7·636
0·46	79·3	3·502	332	2481	332	2310	2642	1·067	6·554	7·621
0·48	80·3	3·366	336	2482	336	2308	2644	1·079	6·528	7·607
0·50	81·3	3·239	340	2483	340	2305	2645	1·091	6·502	7·593
0·55	83·7	2·964	351	2486	351	2298	2649	1·119	6·442	7·561
0·60	86·0	2·731	360	2489	360	2293	2653	1·145	6·386	7·531
0·65	88·0	2·535	369	2492	369	2288	2657	1·169	6·335	7·504
0·70	90·0	2·364	377	2494	377	2283	2660	1·192	6·286	7·478
0·75	91·8	2·217	384	2496	384	2278	2662	1·213	6·243	7·456
0·80	93·5	2·087	392	2498	392	2273	2665	1·233	6·201	7·434
0·85	95·2	1·972	399	2500	399	2269	2668	1·252	6·162	7·414
0·90	96·7	1·869	405	2502	405	2266	2671	1·270	6·124	7·394
0·95	98·2	1·777	411	2504	411	2262	2673	1·287	6·089	7·376
1·00	99·6	1·694	417	2506	417	2258	2675	1·303	6·056	7·359

† u and s are chosen to be zero for saturated liquid at the triple point

* $$\frac{h_f}{\text{kJ/kg}} = \frac{pv_f}{\text{kJ/kg}} = \left(\frac{p}{\text{bar}}\right) \times \frac{10^5\text{N}}{\text{m}^2} \times \left(\frac{v_f}{\text{m}^3/\text{kg}}\right) \times \frac{\text{m}^3}{\text{kg}} \times \frac{\text{kJ}}{10^3\text{N m}} \times \frac{1}{\text{kJ/kg}}$$

Saturated Water and Steam

p	t_s	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
1·0	99·6	1·694	417	2506	417	2258	2675	1·303	6·056	7·359
1·1	102·3	1·549	429	2510	429	2251	2680	1·333	5·994	7·327
1·2	104·8	1·428	439	2512	439	2244	2683	1·361	5·937	7·298
1·3	107·1	1·325	449	2515	449	2238	2687	1·387	5·884	7·271
1·4	109·3	1·236	458	2517	458	2232	2690	1·411	5·835	7·246
1·5	111·4	1·159	467	2519	467	2226	2693	1·434	5·789	7·223
1·6	113·3	1·091	475	2521	475	2221	2696	1·455	5·747	7·202
1·7	115·2	1·031	483	2524	483	2216	2699	1·475	5·707	7·182
1·8	116·9	0·9774	491	2526	491	2211	2702	1·494	5·669	7·163
1·9	118·6	0·9292	498	2528	498	2206	2704	1·513	5·632	7·145
2·0	120·2	0·8856	505	2530	505	2202	2707	1·530	5·597	7·127
2·1	121·8	0·8461	511	2531	511	2198	2709	1·547	5·564	7·111
2·2	123·3	0·8100	518	2533	518	2193	2711	1·563	5·533	7·096
2·3	124·7	0·7770	524	2534	524	2189	2713	1·578	5·503	7·081
2·4	126·1	0·7466	530	2536	530	2185	2715	1·593	5·474	7·067
2·5	127·4	0·7186	535	2537	535	2182	2717	1·607	5·446	7·053
2·6	128·7	0·6927	541	2539	541	2178	2719	1·621	5·419	7·040
2·7	130·0	0·6686	546	2540	546	2174	2720	1·634	5·393	7·027
2·8	131·2	0·6462	551	2541	551	2171	2722	1·647	5·368	7·015
2·9	132·4	0·6253	556	2543	556	2168	2724	1·660	5·344	7·004
3·0	133·5	0·6057	561	2544	561	2164	2725	1·672	5·321	6·993
3·5	138·9	0·5241	584	2549	584	2148	2732	1·727	5·214	6·941
4·0	143·6	0·4623	605	2554	605	2134	2739	1·776	5·121	6·897
4·5	147·9	0·4139	623	2558	623	2121	2744	1·820	5·037	6·857
5·0	151·8	0·3748	639	2562	640	2109	2749	1·860	4·962	6·822
5·5	155·5	0·3427	655	2565	656	2097	2753	1·897	4·893	6·790
6	158·8	0·3156	669	2568	670	2087	2757	1·931	4·830	6·761
7	165·0	0·2728	696	2573	697	2067	2764	1·992	4·717	6·709
8	170·4	0·2403	720	2577	721	2048	2769	2·046	4·617	6·663
9	175·4	0·2149	742	2581	743	2031	2774	2·094	4·529	6·623
10	179·9	0·1944	762	2584	763	2015	2778	2·138	4·448	6·586
11	184·1	0·1774	780	2586	781	2000	2781	2·179	4·375	6·554
12	188·0	0·1632	797	2588	798	1986	2784	2·216	4·307	6·523
13	191·6	0·1512	813	2590	815	1972	2787	2·251	4·244	6·495
14	195·0	0·1408	828	2593	830	1960	2790	2·284	4·185	6·469
15	198·3	0·1317	843	2595	845	1947	2792	2·315	4·130	6·445
16	201·4	0·1237	857	2596	859	1935	2794	2·344	4·078	6·422
17	204·3	0·1167	870	2597	872	1923	2795	2·372	4·028	6·400
18	207·1	0·1104	883	2598	885	1912	2797	2·398	3·981	6·379
19	209·8	0·1047	895	2599	897	1901	2798	2·423	3·936	6·359
20	212·4	0·09957	907	2600	909	1890	2799	2·447	3·893	6·340
22	217·2	0·09069	928	2601	931	1870	2801	2·492	3·813	6·305
24	221·8	0·08323	949	2602	952	1850	2802	2·534	3·738	6·272
26	226·0	0·07689	969	2603	972	1831	2803	2·574	3·668	6·242
28	230·0	0·07142	988	2603	991	1812	2803	2·611	3·602	6·213
30	233·8	0·06665	1004	2603	1008	1795	2803	2·645	3·541	6·186
32	237·4	0·06246	1021	2603	1025	1778	2803	2·679	3·482	6·161
34	240·9	0·05875	1038	2603	1042	1761	2803	2·710	3·426	6·136
36	244·2	0·05544	1054	2602	1058	1744	2802	2·740	3·373	6·113
38	247·3	0·05246	1068	2602	1073	1729	2802	2·769	3·322	6·091
40	250·3	0·04977	1082	2602	1087	1714	2801	2·797	3·273	6·070

Saturated Water and Steam

p	t_s	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
40	250·3	0·04977	1082	2602	1087	1714	2801	2·797	3·273	6·070
42	253·2	0·04732	1097	2601	1102	1698	2800	2·823	3·226	6·049
44	256·0	0·04509	1109	2600	1115	1683	2798	2·849	3·180	6·029
46	258·8	0·04305	1123	2599	1129	1668	2797	2·874	3·136	6·010
48	261·4	0·04117	1136	2598	1142	1654	2796	2·897	3·094	5·991
50	263·9	0·03944	1149	2597	1155	1639	2794	2·921	3·052	5·973
55	269·9	0·03563	1178	2594	1185	1605	2790	2·976	2·955	5·931
60	275·6	0·03244	1206	2590	1214	1570	2784	3·027	2·863	5·890
65	280·8	0·02972	1232	2586	1241	1538	2779	3·076	2·775	5·851
70	285·8	0·02737	1258	2581	1267	1505	2772	3·122	2·692	5·814
75	290·5	0·02532	1283	2576	1293	1473	2766	3·166	2·613	5·779
80	295·0	0·02352	1306	2570	1317	1441	2758	3·207	2·537	5·744
85	299·2	0·02192	1329	2565	1341	1410	2751	3·248	2·463	5·711
90	303·3	0·02048	1351	2559	1364	1379	2743	3·286	2·393	5·679
95	307·2	0·01919	1372	2552	1386	1348	2734	3·324	2·323	5·647
100	311·0	0·01802	1393	2545	1408	1317	2725	3·360	2·255	5·615
105	314·6	0·01696	1414	2537	1429	1286	2715	3·395	2·189	5·584
110	318·0	0·01598	1434	2529	1450	1255	2705	3·430	2·123	5·553
115	321·4	0·01508	1454	2522	1471	1224	2695	3·463	2·060	5·523
120	324·6	0·01426	1473	2514	1491	1194	2685	3·496	1·997	5·493
125	327·8	0·01349	1492	2505	1511	1163	2674	3·529	1·934	5·463
130	330·8	0·01278	1511	2496	1531	1131	2662	3·561	1·872	5·433
135	333·8	0·01211	1530	2487	1551	1099	2650	3·592	1·811	5·403
140	336·6	0·01149	1548	2477	1571	1067	2638	3·623	1·750	5·373
145	339·4	0·01090	1567	2467	1591	1034	2625	3·654	1·689	5·343
150	342·1	0·01035	1585	2456	1610	1001	2611	3·685	1·627	5·312
155	344·8	0·00982	1604	2445	1630	967	2597	3·715	1·565	5·280
160	347·3	0·00932	1623	2433	1650	932	2582	3·746	1·502	5·248
165	349·8	0·00884	1641	2420	1670	895	2565	3·777	1·437	5·214
170	352·3	0·00838	1660	2406	1690	858	2548	3·808	1·373	5·181
175	354·6	0·00794	1679	2391	1711	819	2530	3·839	1·305	5·144
180	357·0	0·00751	1699	2375	1732	778	2510	3·872	1·236	5·108
185	359·2	0·00709	1719	2358	1754	735	2489	3·905	1·163	5·068
190	361·4	0·00668	1740	2339	1777	689	2466	3·941	1·086	5·027
195	363·6	0·00627	1762	2318	1801	639	2440	3·977	1·004	4·981
200	365·7	0·00585	1786	2294	1827	584	2411	4·014	0·914	4·928
202	366·5	0·00569	1796	2283	1838	560	2398	4·031	0·875	4·906
204	367·4	0·00552	1806	2271	1849	535	2384	4·049	0·835	4·884
206	368·2	0·00534	1817	2259	1861	508	2369	4·067	0·792	4·859
208	369·0	0·00517	1829	2245	1874	479	2353	4·087	0·745	4·832
210	369·8	0·00498	1842	2231	1889	447	2336	4·108	0·695	4·803
212	370·6	0·00479	1856	2214	1904	412	2316	4·131	0·640	4·771
214	371·4	0·00458	1871	2196	1921	373	2294	4·157	0·579	4·736
216	372·1	0·00436	1888	2174	1940	328	2268	4·186	0·508	4·694
218	372·9	0·00409	1911	2146	1965	270	2235	4·224	0·417	4·641
220	373·7	0·00368	1949	2097	2008	170	2178	4·289	0·263	4·552
221·2	374·15	0·00317	2014	2014	2084	0	2084	4·406	0·000	4·406

Superheated Steam

p (t_s)			t	50	100	150	200	250	300	400	500
0	$u = h - RT$		v								
			u	2446	2517	2589	2662	2737	2812	2969	3132
			h	2595	2689	2784	2880	2978	3077	3280	3489
			s								
0·006112 (0·01)	v_g	206·1	v	243·9	281·7	319·5	357·3	395·0	432·8	508·3	583·8
	u_g	2375	u	2446	2517	2589	2662	2737	2812	2969	3132
	h_g	2501	h	2595	2689	2784	2880	2978	3077	3280	3489
	s_g	9·155	s	9·468	9·739	9·978	9·193	10·390	10·571	10·897	11·187
0·01 (7·0)	v_g	129·2	v	149·1	172·2	195·3	218·4	241·4	264·5	310·7	356·8
	u_g	2385	u	2446	2517	2589	2662	2737	2812	2969	3132
	h_g	2514	h	2595	2689	2784	2880	2978	3077	3280	3489
	s_g	8·974	s	9·241	9·512	9·751	9·966	10·163	10·344	10·670	10·960
0·05 (32·9)	v_g	28·20	v	29·78	34·42	39·04	43·66	48·28	52·90	62·13	71·36
	u_g	2420	u	2445	2516	2589	2662	2737	2812	2969	3132
	h_g	2561	h	2594	2688	2784	2880	2978	3077	3280	3489
	s_g	8·394	s	8·496	8·768	9·008	9·223	9·420	9·601	9·927	10·217
0·1 (45·8)	v_g	14·67	v	14·87	17·20	19·51	21·83	24·14	26·45	31·06	35·68
	u_g	2437	u	2443	2516	2588	2662	2736	2812	2969	3132
	h_g	2584	h	2592	2688	2783	2880	2977	3077	3280	3489
	s_g	8·149	s	8·173	8·447	8·688	8·903	9·100	9·281	9·607	9·897
0·5 (81·3)	v_g	3·239	v		3·420	3·890	4·356	4·821	5·284	6·209	7·134
	u_g	2483	u		2512	2585	2660	2735	2812	2969	3132
	h_g	2645	h		2683	2780	2878	2976	3076	3279	3489
	s_g	7·593	s		7·694	7·940	8·158	8·355	8·537	8·864	9·154
0·75 (91·8)	v_g	2·217	v		2·271	2·588	2·901	3·211	3·521	4·138	4·755
	u_g	2496	u		2510	2585	2659	2734	2811	2969	3132
	h_g	2662	h		2680	2779	2877	2975	3075	3279	3489
	s_g	7·456	s		7·500	7·750	7·969	8·167	8·349	8·676	8·967
1 (99·6)	v_g	1·694	v		1·696	1·937	2·173	2·406	2·639	3·103	3·565
	u_g	2506	u		2506	2583	2659	2734	2811	2968	3131
	h_g	2675	h		2676	2777	2876	2975	3075	3278	3488
	s_g	7·359	s		7·360	7·614	7·834	8·033	8·215	8·543	8·834
1·01325 (100·0)	v_g	1·673	v			1·912	2·145	2·375	2·604	3·062	3·519
	u_g	2506	u			2583	2659	2734	2811	2968	3131
	h_g	2676	h			2777	2876	2975	3075	3278	3488
	s_g	7·355	s			7·608	7·828	8·027	8·209	8·537	8·828
1·5 (111·4)	v_g	1·159	v			1·286	1·445	1·601	1·757	2·067	2·376
	u_g	2519	u			2580	2656	2733	2809	2967	3131
	h_g	2693	h			2773	2873	2973	3073	3277	3488
	s_g	7·223	s			7·420	7·643	7·843	8·027	8·355	8·646
2 (120·2)	v_g	0·8856	v			0·9602	1·081	1·199	1·316	1·549	1·781
	u_g	2530	u			2578	2655	2731	2809	2967	3131
	h_g	2707	h			2770	2871	2971	3072	3277	3487
	s_g	7·127	s			7·280	7·507	7·708	7·892	8·221	8·513
3 (133·5)	v_g	0·6057	v			0·6342	0·7166	0·7965	0·8754	1·031	1·187
	u_g	2544	u			2572	2651	2729	2807	2966	3130
	h_g	2725	h			2762	2866	2968	3070	3275	3486
	s_g	6·993	s			7·078	7·312	7·517	7·702	8·032	8·324
4 (143·6)	v_g	0·4623	v			0·4710	0·5345	0·5953	0·6549	0·7725	0·8893
	u_g	2554	u			2565	2648	2727	2805	2965	3129
	h_g	2739	h			2753	2862	2965	3067	3274	3485
	s_g	6·897	s			6·929	7·172	7·379	7·566	7·898	8·191

Superheated Steam

p (t_s)				t 200	250	300	350	400	450	500	600
5 (151·8)	v_g	0·3748	v	0·4252	0·4745	0·5226	0·5701	0·6172	0·6641	0·7108	0·8040
	u_g	2562	u	2644	2725	2804	2883	2963	3045	3129	3300
	h_g	2749	h	2857	2962	3065	3168	3272	3377	3484	3702
	s_g	6·822	s	7·060	7·271	7·460	7·633	7·793	7·944	8·087	8·351
6 (158·8)	v_g	0·3156	v	0·3522	0·3940	0·4344	0·4743	0·5136	0·5528	0·5919	0·6697
	u_g	2568	u	2640	2722	2801	2881	2962	3044	3128	3299
	h_g	2757	h	2851	2958	3062	3166	3270	3376	3483	3701
	s_g	6·761	s	6·968	7·182	7·373	7·546	7·707	7·858	8·001	8·267
7 (165·0)	v_g	0·2728	v	0·3001	0·3364	0·3714	0·4058	0·4397	0·4734	0·5069	0·5737
	u_g	2573	u	2636	2720	2800	2880	2961	3043	3127	3298
	h_g	2764	h	2846	2955	3060	3164	3269	3374	3482	3700
	s_g	6·709	s	6·888	7·106	7·298	7·473	7·634	7·786	7·929	8·195
8 (170·4)	v_g	0·2403	v	0·2610	0·2933	0·3242	0·3544	0·3842	0·4138	0·4432	0·5018
	u_g	2577	u	2631	2716	2798	2878	2960	3042	3126	3298
	h_g	2769	h	2840	2951	3057	3162	3267	3373	3481	3699
	s_g	6·663	s	6·817	7·040	7·233	7·409	7·571	7·723	7·866	8·132
9 (175·4)	v_g	0·2149	v	0·2305	0·2597	0·2874	0·3144	0·3410	0·3674	0·3937	0·4458
	u_g	2581	u	2628	2714	2796	2877	2959	3041	3126	3298
	h_g	2774	h	2835	2948	3055	3160	3266	3372	3480	3699
	s_g	6·623	s	6·753	6·980	7·176	7·352	7·515	7·667	7·811	8·077
10 (179·9)	v_g	0·1944	v	0·2061	0·2328	0·2580	0·2825	0·3065	0·3303	0·3540	0·4010
	u_g	2584	u	2623	2711	2794	2875	2957	3040	3124	3297
	h_g	2778	h	2829	2944	3052	3158	3264	3370	3478	3698
	s_g	6·586	s	6·695	6·926	7·124	7·301	7·464	7·617	7·761	8·028
15 (198·3)	v_g	0·1317	v	0·1324	0·1520	0·1697	0·1865	0·2029	0·2191	0·2351	0·2667
	u_g	2595	u	2597	2697	2784	2868	2952	3035	3120	3294
	h_g	2792	h	2796	2925	3039	3148	3256	3364	3473	3694
	s_g	6·445	s	6·452	6·711	6·919	7·102	7·268	7·423	7·569	7·838
20 (212·4)	v_g	0·0996	v		0·1115	0·1255	0·1386	0·1511	0·1634	0·1756	0·1995
	u_g	2600	u		2681	2774	2861	2946	3030	3116	3291
	h_g	2799	h		2904	3025	3138	3248	3357	3467	3690
	s_g	6·340	s		6·547	6·768	6·957	7·126	7·283	7·431	7·701
30 (233·8)	v_g	0·0666	v		0·0706	0·0812	0·0905	0·0993	0·1078	0·1161	0·1324
	u_g	2603	u		2646	2751	2845	2933	3020	3108	3285
	h_g	2803	h		2858	2995	3117	3231	3343	3456	3682
	s_g	6·186	s		6·289	6·541	6·744	6·921	7·082	7·233	7·507
40 (250·3)	v_g	0·0498	v			0·0588	0·0664	0·0733	0·0800	0·0864	0·0988
	u_g	2602	u			2728	2828	2921	3010	3099	3279
	h_g	2801	h			2963	3094	3214	3330	3445	3674
	s_g	6·070	s			6·364	6·584	6·769	6·935	7·089	7·368
50 (263·9)	v_g	0·0394	v			0·0453	0·0519	0·0578	0·0632	0·0685	0·0786
	u_g	2597	u			2700	2810	2907	3000	3090	3273
	h_g	2794	h			2927	3070	3196	3316	3433	3666
	s_g	5·973	s			6·212	6·451	6·646	6·818	6·975	7·258
60 (275·6)	v_g	0·0324	v			0·0362	0·0422	0·0473	0·0521	0·0566	0·0652
	u_g	2590	u			2670	2792	2893	2988	3081	3266
	h_g	2784	h			2887	3045	3177	3301	3421	3657
	s_g	5·890	s			6·071	6·336	6·541	6·719	6·879	7·166
70 (285·8)	v_g	0·0274	v			0·0295	0·0352	0·0399	0·0441	0·0481	0·0556
	u_g	2581	u			2634	2772	2879	2978	3073	3260
	h_g	2772	h			2841	3018	3158	3287	3410	3649
	s_g	5·814	s			5·934	6·231	6·448	6·632	6·796	7·088

Superheated Steam

p (t_s)		t	350	375	400	425	450	500	600	700
80	v_g 0·02352	$v \times 10^2$	2·994	3·220	3·428	3·625	3·812	4·170	4·839	5·476
(295·0)	h_g 2758	h	2990	3067	3139	3207	3272	3398	3641	3881
	s_g 5·744	s	6·133	6·255	6·364	6·463	6·555	6·723	7·019	7·279
90	v_g 0·02048	$v \times 10^2$	2·578	2·794	2·991	3·173	3·346	3·673	4·279	4·852
(303·3)	h_g 2743	h	2959	3042	3118	3189	3256	3385	3633	3874
	s_g 5·679	s	6·039	6·171	6·286	6·390	6·484	6·657	6·958	7·220
100	v_g 0·01802	$v \times 10^2$	2·241	2·453	2·639	2·812	2·972	3·275	3·831	4·353
(311·0)	h_g 2725	h	2926	3017	3097	3172	3241	3373	3624	3868
	s_g 5·615	s	5·947	6·091	6·213	6·321	6·419	6·596	6·902	7·166
110	v_g 0·01598	$v \times 10^2$	1·960	2·169	2·350	2·514	2·666	2·949	3·465	3·945
(318·0)	h_g 2705	h	2889	2989	3075	3153	3225	3360	3616	3862
	s_g 5·553	s	5·856	6·014	6·143	6·257	6·358	6·539	6·850	7·117
120	v_g 0·01426	$v \times 10^2$	1·719	1·931	2·107	2·265	2·410	2·677	3·159	3·605
(324·6)	h_g 2685	h	2849	2960	3052	3134	3209	3348	3607	3856
	s_g 5·493	s	5·762	5·937	6·076	6·195	6·301	6·487	6·802	7·072
130	v_g 0·01278	$v \times 10^2$	1·509	1·726	1·901	2·053	2·193	2·447	2·901	3·318
(330·8)	h_g 2662	h	2804	2929	3028	3114	3192	3335	3599	3850
	s_g 5·433	s	5·664	5·862	6·011	6·136	6·246	6·437	6·758	7·030
140	v_g 0·01149	$v \times 10^2$	1·321	1·548	1·722	1·872	2·006	2·250	2·679	3·071
(336·6)	h_g 2638	h	2753	2896	3003	3093	3175	3322	3590	3843
	s_g 5·373	s	5·559	5·784	5·946	6·079	6·193	6·390	6·716	6·991
150	v_g 0·01035	$v \times 10^2$	1·146	1·391	1·566	1·714	1·844	2·078	2·487	2·857
(342·1)	h_g 2611	h	2693	2861	2977	3073	3157	3309	3581	3837
	s_g 5·312	s	5·443	5·707	5·883	6·023	6·142	6·345	6·677	6·954
160	v_g 0·00932	$v \times 10^2$	0·976	1·248	1·427	1·573	1·702	1·928	2·319	2·670
(347·3)	h_g 2582	h	2617	2821	2949	3051	3139	3295	3573	3831
	s_g 5·248	s	5·304	5·626	5·820	5·968	6·093	6·301	6·639	6·919
170	v_g 0·00838	$v \times 10^2$		1·117	1·303	1·449	1·576	1·796	2·171	2·506
(352·3)	h_g 2548	h		2778	2920	3028	3121	3281	3564	3825
	s_g 5·181	s		5·541	5·756	5·914	6·0[illegible]4	6·260	6·603	6·886
180	v_g 0·00751	$v \times 10^2$		0·997	1·191	1·338	1·463	1·678	2·039	2·359
(357·0)	h_g 2510	h		2729	2888	3004	3102	3268	3555	3818
	s_g 5·103	s		5·449	5·691	5·861	5·997	6·219	6·569	6·855
190	v_g 0·00668	$v \times 10^2$		0·882	1·089	1·238	1·362	1·572	1·921	2·228
(361·4)	h_g 2466	h		2674	2855	2980	3082	3254	3546	3812
	s_g 5·027	s		5·348	5·625	5·807	5·950	6·180	6·536	6·825
200	v_g 0·00585	$v \times 10^2$		0·768	0·995	1·147	1·270	1·477	1·815	2·110
(365·7)	h_g 2411	h		2605	2819	2955	3062	3239	3537	3806
	s_g 4·928	s		5·228	5·556	5·753	5·904	6·142	6·505	6·796
210	v_g 0·00498	$v \times 10^2$		0·650	0·908	1·064	1·187	1·390	1·719	2·003
(369·8)	h_g 2336	h		2500	2781	2928	3041	3225	3528	3799
	s_g 4·803	s		5·050	5·484	5·699	5·859	6·105	6·474	6·768
220	v_g 0·00368	$v \times 10^2$		0·450	0·825	0·987	1·111	1·312	1·632	1·906
(373·7)	h_g 2178	h		2300	2738	2900	3020	3210	3519	3793
	s_g 4·552	s		4·725	5·409	5·645	5·813	6·068	6·441	6·742
221·2	v_c 0·00317	$v \times 10^2$	0·163	0·351	0·816	0·978	1·103	1·303	1·622	1·895
(374·15)	h_c 2084	h	1637	2139	2733	2896	3017	3208	3518	3792
	s_c 4·406	s	3·708	4·490	5·398	5·638	5·807	6·064	6·441	6·739

Linear interpolation is not accurate near the critical point

Supercritical Steam

p	t	350	375	400	425	450	500	600	700	800
225	$v \times 10^2$	0·163	0·249	0·786	0·951	1·076	1·275	1·591	1·861	2·109
	h	1635	1980	2716	2885	3009	3203	3514	3790	4055
	s	3·704	4·470	5·369	5·616	5·790	6·050	6·430	6·729	6·988
250	$v \times 10^2$	0·160	0·198	0·601	0·789	0·917	1·113	1·412	1·662	1·890
	h	1625	1850	2580	2807	2951	3165	3491	3774	4043
	s	3·682	4·026	5·142	5·474	5·677	5·962	6·361	6·667	6·931
275	$v \times 10^2$	0·158	0·187	0·419	0·650	0·786	0·980	1·265	1·500	1·710
	h	1617	1814	2382	2718	2890	3125	3468	3758	4032
	s	3·662	3·985	4·828	5·320	5·562	5·878	6·296	6·610	6·878
300	$v \times 10^2$	0·155	0·180	0·282	0·530	0·674	0·868	1·143	1·364	1·561
	h	1610	1791	2157	2614	2823	3084	3445	3742	4020
	s	3·645	3·933	4·482	5·157	5·444	5·795	6·234	6·557	6·829
350	$v \times 10^2$	0·152	0·171	0·211	0·343	0·496	0·693	0·952	1·152	1·327
	h	1599	1762	1992	2375	2673	2998	3397	3709	3997
	s	3·614	3·875	4·219	4·776	5·197	5·633	6·120	6·459	6·741
400	$v \times 10^2$	0·149	0·164	0·191	0·255	0·369	0·562	0·809	0·993	1·152
	h	1590	1743	1935	2203	2514	2906	3348	3677	3974
	s	3·588	3·832	4·119	4·510	4·947	5·474	6·014	6·371	6·662
450	$v \times 10^2$	0·146	0·160	0·181	0·219	0·291	0·463	0·698	0·870	1·016
	h	1583	1729	1901	2115	2380	2813	3299	3644	3951
	s	3·565	3·797	4·056	4·368	4·740	5·320	5·914	6·290	6·590
500	$v \times 10^2$	0·144	0·156	0·173	0·201	0·249	0·388	0·611	0·772	0·908
	h	1577	1717	1879	2064	2288	2722	3249	3612	3928
	s	3·544	3·768	4·009	4·279	4·594	5·176	5·821	6·214	6·524
550	$v \times 10^2$	0·143	0·153	0·168	0·190	0·224	0·334	0·540	0·693	0·820
	h	1572	1709	1862	2030	2227	2641	3200	3579	3905
	s	3·525	3·742	3·971	4·218	4·494	5·047	5·731	6·144	6·462
600	$v \times 10^2$	0·141	0·151	0·164	0·182	0·209	0·295	0·483	0·627	0·747
	h	1568	1702	1848	2005	2184	2571	3152	3548	3883
	s	3·506	3·718	3·939	4·168	4·419	4·937	5·648	6·077	6·405
650	$v \times 10^2$	0·139	0·148	0·160	0·176	0·198	0·267	0·436	0·572	0·685
	h	1565	1696	1837	1986	2151	2514	3106	3517	3860
	s	3·489	3·697	3·910	4·128	4·360	4·845	5·568	6·014	6·352
700	$v \times 10^2$	0·138	0·146	0·157	0·171	0·189	0·247	0·397	0·526	0·633
	h	1561	1691	1829	1971	2127	2468	3062	3486	3839
	s	3·473	3·678	3·886	4·093	4·312	4·769	5·494	5·955	6·300
750	$v \times 10^2$	0·137	0·145	0·154	0·167	0·183	0·231	0·365	0·486	0·587
	h	1559	1687	1821	1958	2107	2431	3021	3456	3817
	s	3·459	3·659	3·863	4·064	4·272	4·705	5·425	5·899	6·252
800	$v \times 10^2$	0·136	0·143	0·152	0·163	0·178	0·219	0·338	0·452	0·548
	h	1557	1684	1815	1948	2091	2400	2983	3428	3797
	s	3·444	3·642	3·842	4·037	4·237	4·651	5·361	5·845	6·206
900	$v \times 10^2$	0·133	0·140	0·148	0·158	0·169	0·202	0·296	0·396	0·484
	h	1554	1678	1805	1932	2066	2353	2916	3373	3756
	s	3·418	3·612	3·805	3·991	4·179	4·563	5·248	5·746	6·120
1000	$v \times 10^2$	0·131	0·138	0·145	0·153	0·163	0·189	0·267	0·354	0·434
	h	1552	1674	1798	1920	2048	2319	2860	3324	3718
	s	3·394	3·584	3·773	3·951	4·131	4·493	5·153	5·656	6·042

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