



## قسم تقنيات المساحة الحقيبة تعليمية

بعنوان:

الهندسة الوصفية  
Descriptive engineering

إعداد

أ.م.د. دلير عبدالله عمر

2024 – 2023

## وصف المقرر

يوفر وصف المقرر هذا إيجازاً مقتضياً لأهم خصائص المقرر ومخرجات التعلم المتوقعة من الطالب تحقيقها مبرهنناً عما إذا كان قد حقق الاستفادة القصوى من فرص التعلم المتاحة. ولا بد من الربط بينها وبين وصف البرنامج.

1. المؤسسة التعليمية	الجامعة التقنية الشمالية
2. القسم الجامعي / المركز	الكلية التقنية الهندسية كركوك – قسم هندسة تقنيات المساحة
3. اسم / رمز المقرر	الهندسة الوصفية
4. البرامج التي يدخل فيها	المرحلة الثانية
5. أشكال الحضور المتاحة	فصلي
6. الفصل / السنة	الفصل الاول 2017
7. عدد الساعات الدراسية (الكلي)	60 ساعة
8. تاريخ إعداد هذا الوصف	2022-9-1
9. أهداف المقرر	تدريب عقل الطالب على التصور الخيالي للأجسام وتمثيلها على أرض الواقع

10. مخرجات التعلم  
11. التعلم وطرائق التعليم

أ- المعرفة والفهم

- 1- تمثيل النقاط المجسمة على المستوى
- 2- دراسة اوضاع وخصائص المستقيم في الفراغ وتمثيلها على المستوى
- 3- دراسة المستويات الرئيسية والمساعدة وتطبيقاتها العملية
- 4- دراسة المجسمات وافراد سطوحها

ب - الشرح والمناقشة وحل الامثلة

A- Knowledge and understanding

Representing solid points on a plane -1

Study the positions and properties of the rectum in space and -2  
represent them on the plane

Study the main and auxiliary levels and their practical applications -3

Study of solids and their surfaces -4

B - Explanation, discussion, and solving examples

12. طرائق التقييم

الامتحانات ن الواجبات البيتية والصفية

13. سياسة الدرجات

20% نظري: امتحانات فصلية عدد 2 ، امتحانات اسبوعية عدد 4

20% عملي : تجارب صفية عدد 10

20% عملي : تجارب بيتية عدد 10

40% الامتحان النهائي

14. بنية المقرر					
الأسبوع	الساعات	مخرجات التعلم المطلوبة	اسم الوحدة / المساق أو الموضوع	طريقة التعليم	
1	4		مقدمة عامة - تعاريف أساسية في الهندسة الوصفية والمواضيع ذات العلاقة مستويات ومحاور الإسقاط	General introduction - basic definitions in descriptive geometry and related topics: planes and axes of projection	
2	4		أنواع الإسقاط: المركزي, المائل, العمودي, الرقمي, الجسم (المتقاسم)	Types of projection: central, oblique, vertical, digital, stereoscopic (asymmetrical)	
3	4		تمثيل النقطة ذات الاحداثيات الموجبة والسالبة	Representing a point with positive and negative coordinates	
4	4		تمثيل المستقيم باتجاهاته المختلفة	Representing the straight line in its different directions	
5	4		تمثيل المستوي بمساقطه وبآثاره	Representing the plane with its projections and effects	
6	4		مسائل الوضع		
7	4		مسائل القياس		
8	4		المستويات المساعدة الاولى	Initial auxiliary levels	
9	4		المستويات المساعدة الثانوية	Secondary auxiliary levels	
10	4		الخطوط والمستويات والسطوح الهندسية- بعض الاجسام والبلورات	Lines, planes, and geometric surfaces - some objects and crystals	
11	4		دراسة عامة عن الاجسام الهندسية, ودورانها او قطعها بمستوى وايجاد شكل القطاعات الناتجة, وايجاد نقاط تقاطع مستقيم لها وحساب حجمها ومساحاتها السطحية	A general study of geometric bodies, their rotation or cutting them in a plane, finding the shape of the resulting	

	sectors, finding their straight intersection points, and calculating their volumes and .surface areas				
	Cubes, rectangles, and parallelepipeds	المكعب ومتوازي المستطيلات ومتوازي السطوح		4	12
	Prism: triangular, quadrilateral Pyramid: triangular, quadrilateral	المنشور : الثلاثي, الرباعي الهرم: الثلاثي, الرباعي		4	13
	Cylinder	الاسطوانة		4	14
	Individual flat surfaces, a cube, a prism, a pyramid... on the plane of the paper Individuals with uneven surfaces: cylinder	افراد السطوح المستوية, المكعب, المنشور, الهرم .. على مستوي الورقة افراد السطوح غير المستوية: الاسطوانة		4	15

#### 15. البنية التحتية

<ul style="list-style-type: none"> <li>مدحت فضيل, مطبعة جامعة بغداد, الهندسة الوصفية</li> <li>عمانويل فرج كريم, مطبعة التعليم العالي/بغداد, الهندسة الوصفية الجزء الاول</li> <li>عمانويل فرج كريم, مطبعة التعليم العالي/بغداد, الهندسة الوصفية الجزء الثاني</li> </ul>	<p>القراءات المطلوبة :</p> <ul style="list-style-type: none"> <li>النصوص الأساسية</li> <li>كتب المقرر</li> <li>أخرى</li> </ul>
	متطلبات خاصة ( وتشمل على سبيل المثال ورش العمل والدوريات والبرمجيات والمواقع الالكترونية )
	الخدمات الاجتماعية ( وتشمل على سبيل المثال محاضرات الضيوف والتدريب المهني والدراسات الميدانية )

#### 16. القبول

المتطلبات السابقة	اجتياز المرحلة الاولى بنجاح
-------------------	-----------------------------



أقل عدد من الطلبة	20 طالب
أكبر عدد من الطلبة	50 طالب
17. مرس المادة	أ.م.د. دلير عبدالله عمر

The Descriptive Engineering

الهندسة الوصفية



# Descriptive Engineering

## CONTENTS

CHAPTER (1)	
The Theory of Projection	5-10
CHAPTER (2)	
Projection of the Point	11-15
CHAPTER (3)	
Projection of the Straight Line	16-52
CHAPTER (4)	
The Plane Surface	53-57
CHAPTER (5)	
Projection of the Point, the Straight Line and the Plane Surface on Auxiliary Planes	58-96
CHAPTER (6)	
Projection of a Body on an Auxiliary Plane	97-129
CHAPTER (7)	
The Conic Sections	130-132
CHAPTER (8)	
Sections of Bodies and Determination of the True Shape of a Section	133-211
CHAPTER (9)	
Developments of Surfaces	212-232
CHAPTER (10)	
Loci of Points in Simple Moving Mechanisms	233-253

University of Dohuk
College of Engineering
Dept. of
Course No.:
143
Section No.:
69



## الإسقاط نظرية THE THEORY OF PROJECTION

أشعة من توجّهه  
If the eye is directed towards a body in space, rays will come from the visible parts of the body and gathered at the eye in a point.

نتيجة محال مختلف  
So, if these rays are taken from the various points of an object to meet a picture plane making an image on this plane, rdial or perspective projection is obtained, as shown in figure (1-a).

التجميع حول علما  
Also, if a source of light (as a lamp) falls on a cinema film, the rays will converge by means of a set of lenses, then diverging from point (0) as shown in figure (1-b) falling on a screen, hence a picture will then be formed.

منبع  
Also, if a plate A B C D as illustrated in figure (1-c) is situated in front of a plane of projection, and if we imagine that rays will pass from point (0) to the different points of the object, then the view a b c d will be obtained. In this type of projection, point (0) is called the *centre of projection*, and the view thus obtained is known as the *CENTRAL PROJECTION*.

محروبة  
This kind of projection is of limited use from the engineering point of view, since the size of the image depends upon many factors.

موازية  
However, if we imagine that point (0) (the centre of projection) goes far away to infinity, the rays or projectors will become parallel, all being normal to the plane of projection and the view thus obtained is *orthographic*, as illustrated in Figs. (2) and (3).

هنا هي  
In *ORTHOGRAPHIC PROJECTION*, therefore, rays are normal to the plane of projection and the projectors are all parallel to each other and in this type of projection the picture planes are called the planes of projection.

١- إذا انصاع مستقيم من مستقيم آخر نبت عنه نقطة

٢- إذا انصاع مستقيم من مستقيم نبت عنه نقطة (أي أن مستقيم

على مستوى هو نقطة) مستقيم آخر نبت عنه مستقيم

٣- إذا انصاع مستويين مستويين يكونان محدودين على

٤- المستقيم المحدودين على مستويين يكونان محدودين على

٥- إذا تجاوزا مستويين فإن المستقيم المحدودين

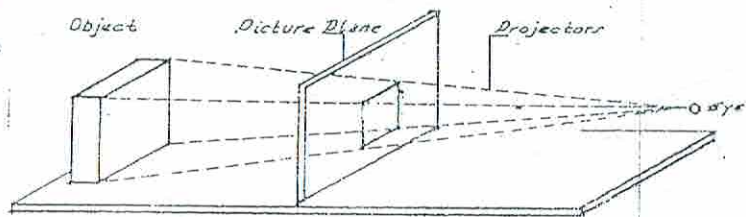
أما هما يكونان محدودين على المستويين

المنقطعة عبارة عن شكل هندسي مع استارها

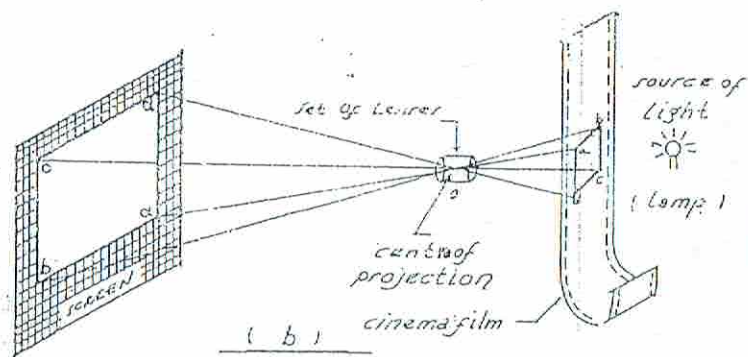
المستقيم عبارة عن مستوي هندسي مع استارها

المستقيم عبارة عن مستوي هندسي مع استارها

6



( a )



( b )

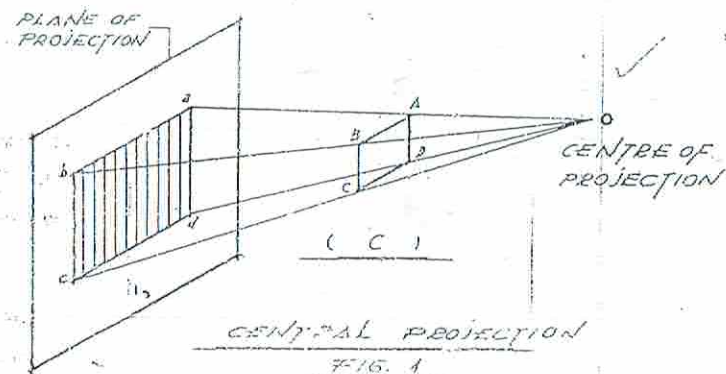


FIG. 1

7

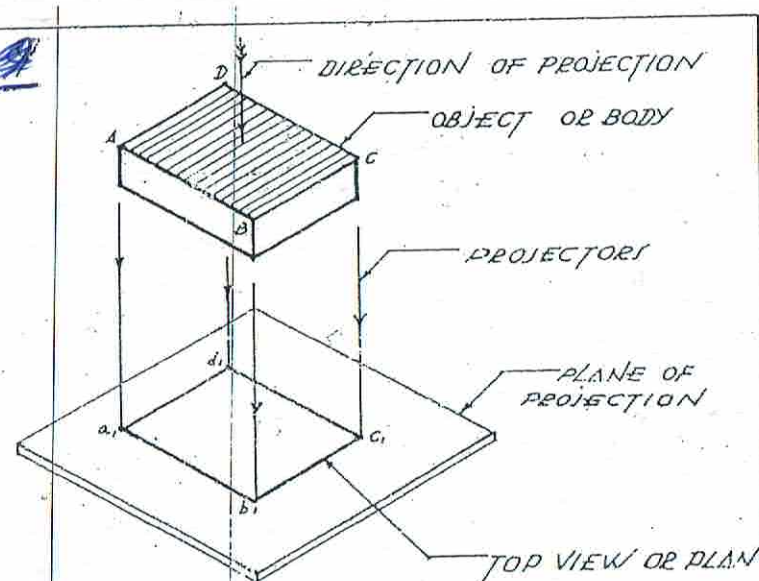


FIG. 2

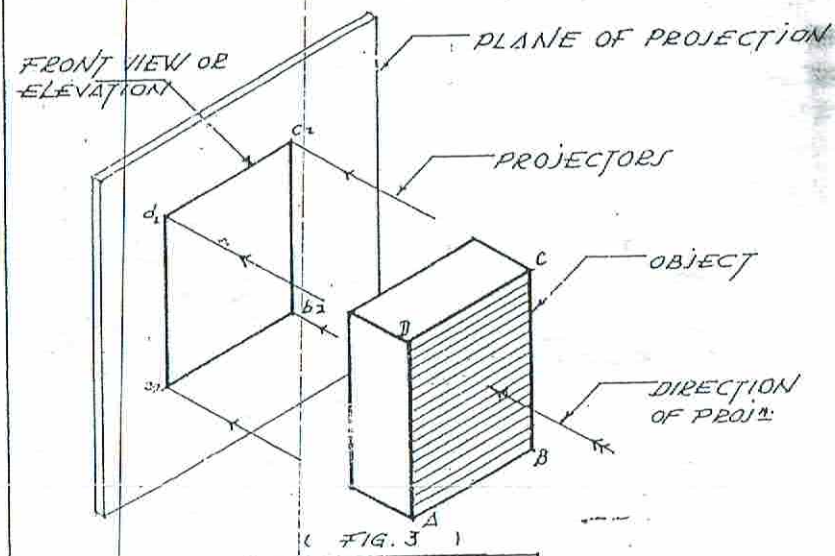


FIG. 3



المستوي  
The two **PRINCIPAL PLANES** used in **ORTHOGRAPHIC** projection

are :—

- (1) The horizontal plane known as the H.P.
- (2) The vertical plane known as the V.P.

Any plane other than these two planes is called an **auxiliary plane** and the projection on this auxiliary plane is called an **auxiliary view**.

The two principal planes namely the H.P. and the V.P. intersect at right angles and divide the space into four quadrants or dihedral angles (1), (2) (3) and (4) as shown in Figure (4). The line of intersection of these two planes is called the **ground line** or more commonly the **G.L.**

In Solid Geometry, the object may be kept in any one of the four quadrants and projected on the two principal planes of projection, but it is usual practice to use the **first angle projection**.

Sometimes the two views obtained from the projection of an object on the two principal planes are insufficient for clarifying the body, so we rely on a third plane perpendicular to both the H.P. and the V.P. The projection on this third plane is called the **SIDE VIEW** or **END VIEW**, as shown in Figures (5) and (6).

## PRINCIPAL PLANES OF PROJECTION

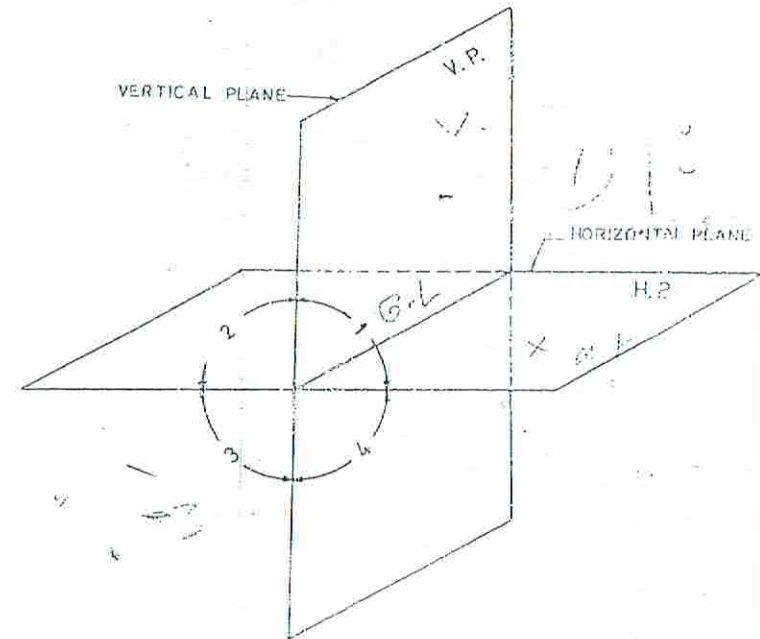


FIG (4)

ANGLE (1) SHOWS THE 1<sup>st</sup> QUADRANT  
 ANGLE (2) SHOWS THE 2<sup>nd</sup> QUADRANT  
 ANGLE (3) SHOWS THE 3<sup>rd</sup> QUADRANT  
 ANGLE (4) SHOWS THE 4<sup>th</sup> QUADRANT

10

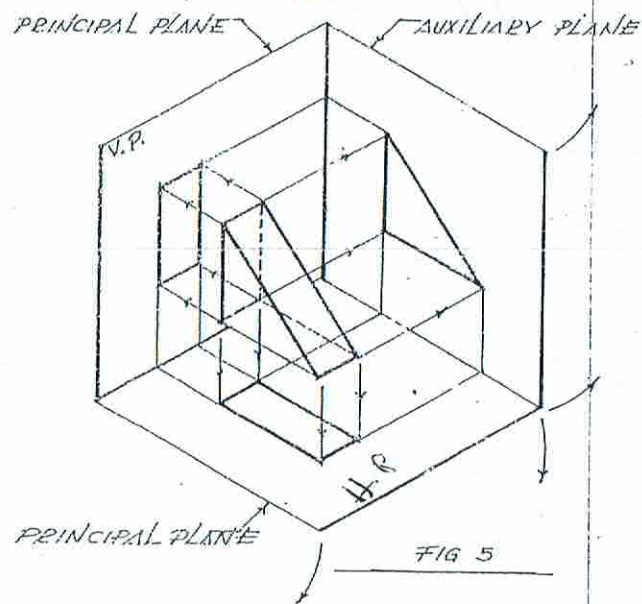
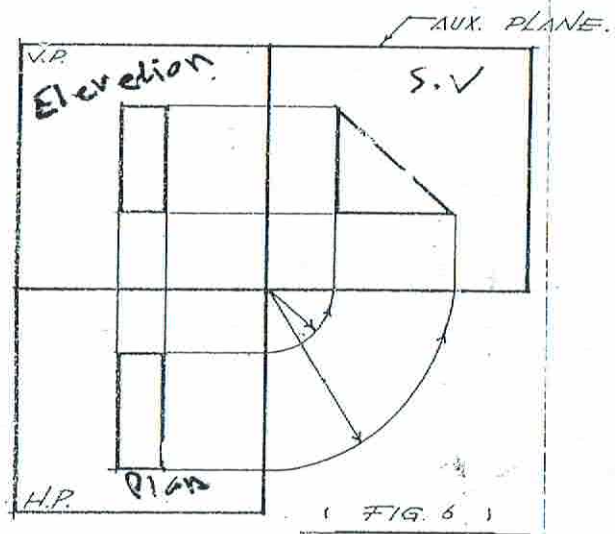


FIG 5



( FIG. 6 )

## CHAPTER (2)

11

### PROJECTION OF THE POINT

We have already mentioned that space is divided into *four quadrants* ( or dihedral angles ) resulting from the intersection of the Two Principal Planes of Projection namely the H.P. and the V.P.

A point in space is located with respect to the V.P. and the H.P. according to its coordinates . The (x) coordinate determines how far the point is away from the V.P. , while the (y) coordinate determines its latitude from the H.P.

Moreover, the sign(+) will be given to all coordinates of (x) and (y) falling *within* the first quadrant (or the first dihedral angle ).

Now, for the representation of a point, it may fall in any one of the four quadrants, depending on whether the sign of the coordinates of the point are either positive or negative . This will be illustrated numerically in the following miscellaneous examples :-

(1) Point A = (4, 3.5) Figure (1) :-

Both the (x) and (y) coordinates are positive .

Point (A) falls in the 1st Quadrant .

(2) Point B = (-4, 2) Figure (2) :-

The (x) coordinate is negative and the (y) coordinate is positive .

Point (B) falls in the 2nd Quadrant .

(3) Point C = (-3, -5) Figure (3) :-

Both the (x) and (y) coordinates are negative .

Point (C) falls in the 3rd Quadrant .

(4) Point D = (4, -3) Figure (4) :-

The (x) coordinate is positive and the (y) coordinate is negative .

Point (D) falls in the 4th Quadrant .

Answer

# PROJECTION OF THE POINT

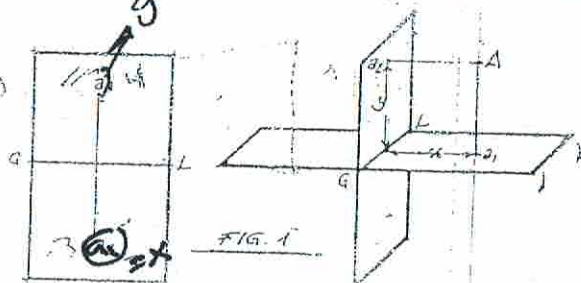


FIG. 1

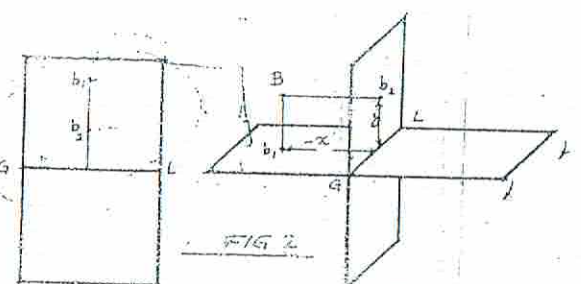


FIG. 2

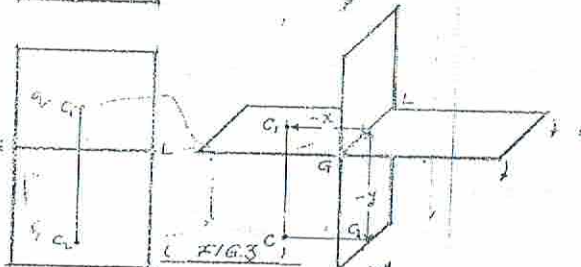


FIG. 3

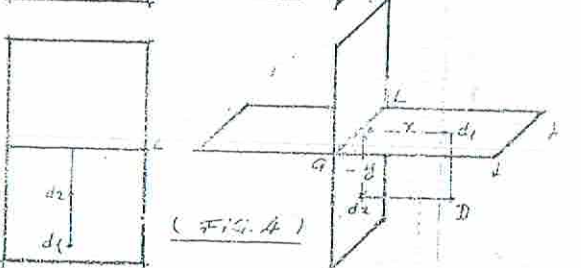


FIG. 4

Obviously, if any one of the coordinates of a point is Zero this indicates that the point *lies on* one of principal planes of projection. For illustration we may consider the following examples :-

(5) Point E = (0, 4) Figure (5) :-

∴ Point (E) lies on the V.P.

(6) Point F = (5, 0) Figure (6) :-

∴ Point (F) lies on the H.P.

(7) Point K = (0, -5) Figure (7) :-

∴ Point (K) lies on the V.P.

(8) Point P = (-3, 0) Figure (8) :-

∴ Point (P) lies on the H.P.

(9) Point N = (0, 0) Figure (9) :-

∴ Point (N) lies on G.L.

## BRIEF REVIEW CONCERNING SOME PRINCIPLES IN PROJECTION

(1) The two views namely the *ELEVATION* & *PLAN* of any point must fall on a line called the *projector* (perpendicular to the G.L.).

(2) The two views namely the *ELEVATION* & *SIDE VIEW* of any point must fall on a line *Parallel* to the G.L.

(3) If a straight line A B be *parallel* to a plane, then its projection on that plane gives its *true length*.

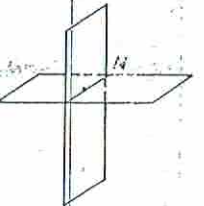
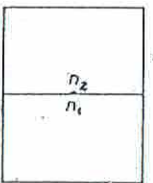
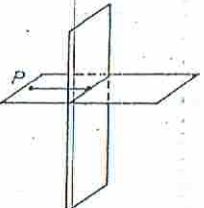
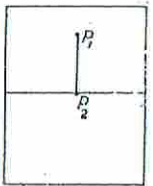
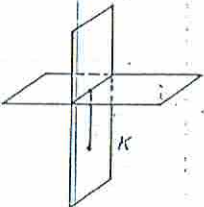
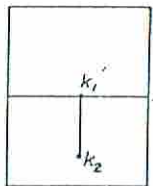
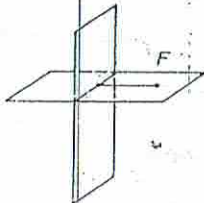
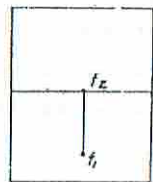
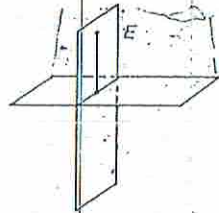
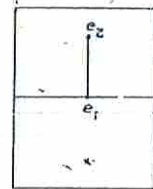
(4) If a straight line C D be *inclined* to a plane, then its projection on that plane would be shorter than its true length.

(5) If a straight line E F be *perpendicular* to a plane, then its projection on that plane will be represented by two points coinciding one on the other

Hamam



14



2014/10/2

15

- (6) If one face of a body is *parallel* to any plane, then the projection of that face on the plane will represent its true shape.
- (7) If one face of body is *inclined* to any plane, then the projection of that face on the plane will be *smaller* than its true shape.
- (8) If one face of a body is *perpendicular* to any plane, then its projection on that plane will be a *line*.

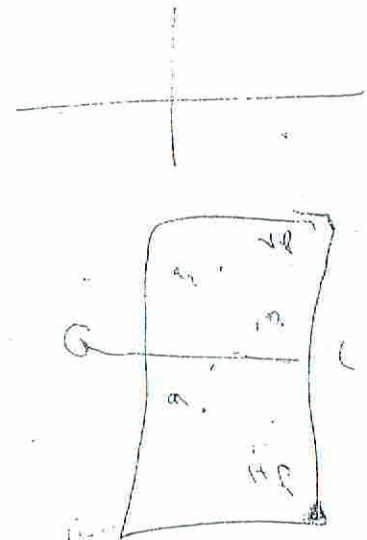
## PROBLEM

Draw the ELEVATION & PLAN of the following points (scale 1:1) and state the quadrant in which each point falls.

Give a pictorial view for each point to show its position in space

$$\begin{aligned} A &= (3, 4) & D &= (-2, -4) & K &= (-4, 0) \\ B &= (-3, 4) & E &= (4, 0) & N &= (0, -6) \\ C &= (2, -5) & F &= (0, 5) & P &= (0, 0) \end{aligned}$$

H.V.  
(H x V)



16

## PROJECTION OF THE STRAIGHT LINE

The pictorial view given in Figure (1) represents the general case of a straight line (A B) in space. This line is inclined by an angle ( $\alpha$ ) to the H.P. while its inclination to the V.P. is the angle ( $\beta$ ).

The projection ( $a_1 b_1$ ) on the H.P. is called the TOP VIEW or PLAN of the line, and the projection ( $a_2 b_2$ ) on the V.P. is called the FRONT VIEW or ELEVATION.

Figure (2) represents the two projections: ( $a_1 b_1$ ) the PLAN, and ( $a_2 b_2$ ) the ELEVATION of the straight line A B. The distance (l) between the projectors of the points (A) and (B) determines the two ends of the line.

### INCLINATION OF A STRAIGHT LINE TO ANY PLANE

Definition :

More generally, the inclination of any line to any plane is the angle between the true line in space and its projection on that plane. This definition quickly brings us to figure (1) where (A B) is the true line in space, and ( $a_1 b_1$ ) being its projection on the H.P. According to the definition just mentioned, ( $\alpha$ ) is the angle which the true line makes with the H.P. In other words ( $\alpha$ ) is the inclination of the line A B to the H.P.

Similarly, ( $a_2 b_2$ ) is the projection of the true line A B on the V.P. Hence ( $\beta$ ) is the inclination of the line A B to the V.P. This follows at once from the definition just stated above.

M:E

## TRACE OF A STRAIGHT LINE ON ANY PLANE

17

Definition :

The point at which a line (or its extension) strikes a plane is called the trace of the line on that plane.

So, if we refer to figure (1) : the point (H) at which the line A B meets the horizontal plane is called the horizontal trace of the line. Also, point (V) is the vertical trace of the line on the vertical plane. Obviously (h) and (v) are the projections of (H) and (V) respectively on the G.L. The coordinates of the horizontal trace (H) are ( $x_H, 0$ ), while that of the vertical trace (V) are ( $0, y_V$ ) as shown in figure (1).

### DETERMINATION OF BOTH THE HORIZONTAL & VERTICAL TRACES OF A STRAIGHT LINE (Given the ELEVATION & PLAN of the Line)

Figure (2) represents the general case of a straight line (A B) given by the PLAN & ELEVATION ; ( $a_1 b_1$ ) is its projection on the H.P. and ( $a_2 b_2$ ) is its projection on the V.P.

Now, it is desired to determine the coordinates of the Horizontal Trace (H) and those of the Vertical Trace (V) on both the H.P. and the V.P. respectively.

#### (i) To Determine the Horizontal Trace (H) :—

Produce the Vertical projection till it meets the G.L. at (h). From (h) draw the perpendicular to the G.L. to meet the Horizontal Projection at (H). This point (H) is the Horizontal Trace of the line. So if we refer to Figure (3) :  $a_2 b_2$  is produced till it meets the G.L. at (h) from which the perpendicular is drawn on the G.L. to meet  $a_1 b_1$  in (H).

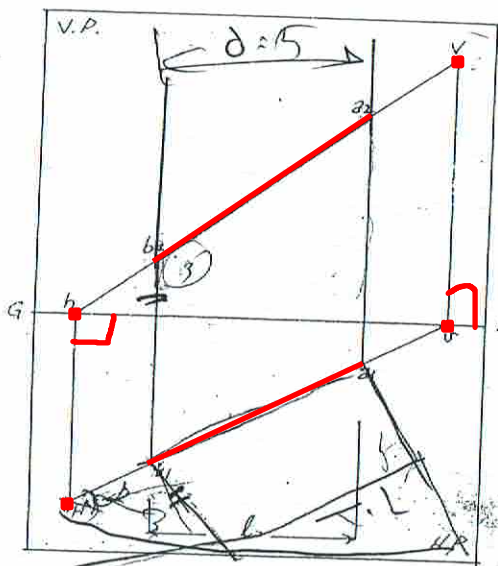
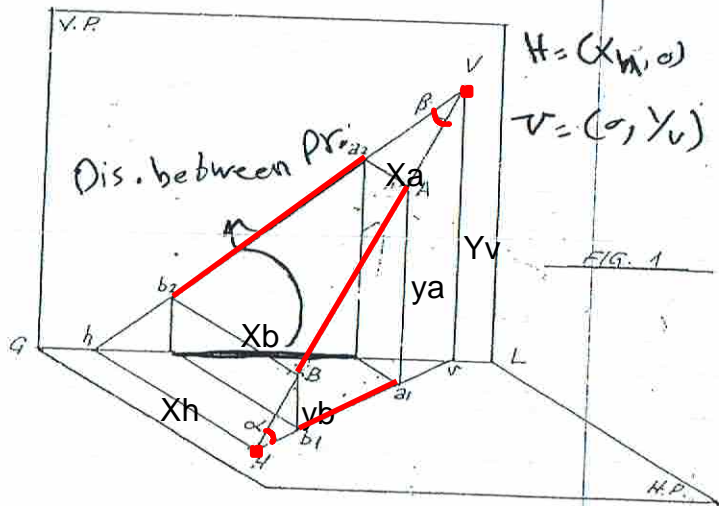
#### (ii) To Determine the Vertical Trace (V) :—

Produce the Horizontal Projection to meet the G. L. at (V).



18

PROJECTION OF THE STRAIGHT LINE

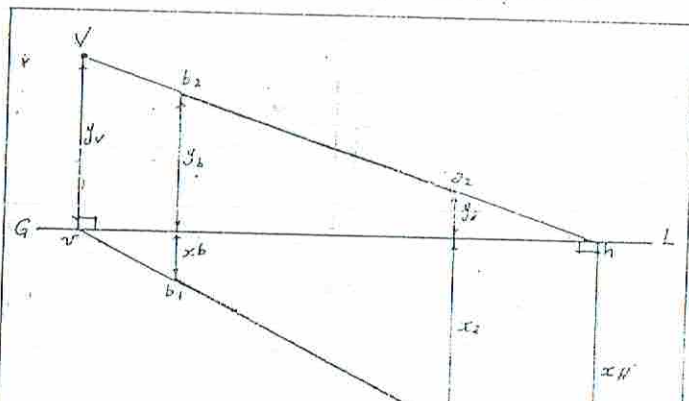


$X_a, X_b$   $\alpha$   
 $X_b, Y_b$   $B$   
 $H$  T.L  
 $V$  Dis. between  
 if 5 of them known the rest could be found.

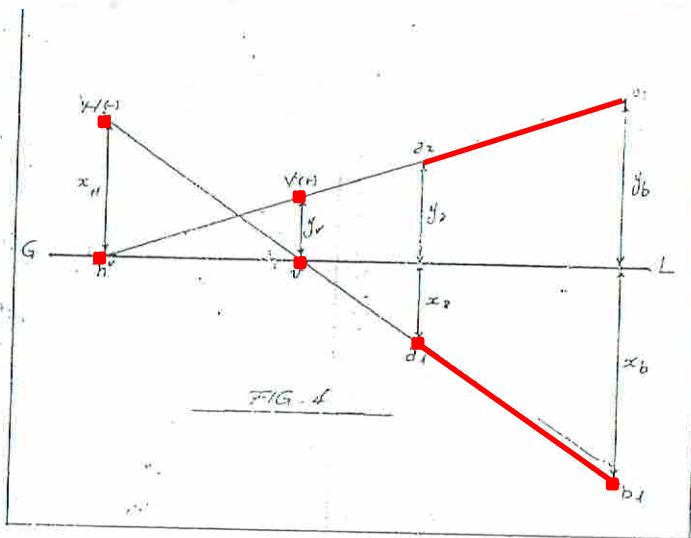
M.E

19

DETERMINATION OF THE TRACES H & V OF A STRAIGHT LINE



at :  $A = (2, 2.5)$ ,  $B = (5.5, 4.2)$  right  
 een A & B = 5 cms.



Q1

20

From (v) draw the perpendicular to the G.L. till it meets the Vertical Projection at (V). This is the Vertical Trace of the line. So, if we refer to Figure (3):  $a_1 b_1$  is produced till it meets the G.L. at (v) from which the perpendicular is drawn on the G.L. to meet  $a_2 b_2$  in (V).

Obviously the coordinates of  $H = (x_H, 0)$  and that of  $V = (0, y_V)$ , and it is clear from Figure (3) that the coordinates of both traces (H) and (V) are both positive.

But again, the case of the straight line shown in Figure (4): one of the traces may be positive while the other is negative, depending on the position of the line in space.

Example (1) : — Figure 3

The straight line A B is placed such that :  $A = (4, 8, 1.2)$ ,  $B = (1.25, 3.6)$  left of A. The distance of projectors between A & B = 6.8 cms.

Required : —

Draw, scale 1:1, the ELEVATION and PLAN of the line, hence determine the coordinates of its traces (H) & (V).

Solution : —

If we refer to Figure (3):  $H = (6.6, 0)$  and  $V = (0, 4.4)$ .

Example (2) : — Figure 4

The straight line A B is situated such that :  $A = (2, 2.5)$ ,  $B = (5.5, 4.2)$  right of A. The distance of projectors between A & B = 5 cms.

Required : —

Draw, scale 1:1, the ELEVATION and PLAN of the line, hence determine the coordinates of its traces (H) & (V).

Solution : —

If we refer to Figure (4) :  $H = (-3.5, 0)$  and  $V = (0, 1.6)$ .

## PROBLEMS

(1) The straight line A B having the following data :

$A = (5, y_A)$ ,  $B = (2, 6)$  right of A. The distance of projectors between A & B = 9 cms. The vertical trace  $V = (0, 8)$ .

Required : —

Draw, scale 1:1, the ELEVATION & PLAN of the line and determine the following : —

- The missed coordinate ( $y_A$ )  $\approx 3$
- The coordinates of the horizontal trace (H) of the line.  $H = (8, 0)$

(2) The straight line C D having the following data : —

$C = (2.5, 3)$ ,  $D = (5, 6)$  left of C. The distance of projectors between C & D = 7.5 cms.

Required : —

Draw, scale 1:1, the ELEVATION & PLAN of the line and find the coordinates of its traces.

## DIFFERENT POSITIONS OF THE STRAIGHT LINE IN SPACE

Figure (5) shows the PLAN and the ELEVATION of a straight line A B which is perpendicular to the H. P. In this case, the ELEVATION represents the true length of the line.

Figure (6) shows the PLAN and the ELEVATION of a straight line A B which is perpendicular to the V. P. In this case, the PLAN represents the true length of the line.

Figure (7) shows the PLAN and the ELEVATION of a straight line A B which is parallel to the V. P. and inclined by an angle ( $\alpha$ ) to the H. P. In this case, the ELEVATION represents the true length of the line.



22<sup>22</sup>

Figure (8) shown the PLAN & ELEVATION of a straight line A B which is Parallel to the H. P. and inclined by an angle ( ) to the V.P. In this case, the PLAN represents the true length of the line.

Figure (9) shows the PLAN & ELEVATION of a straight line A B which is parallel to both the H. P. (&) the V.P. In this case both the two projections are equal and each of them represent the true length of the line.

Figure (10) shows the PLAN & ELEVATION of a straight line A B which is falling on the V. P. and inclined by an angle ( $\alpha$ ) to the H. P. In this case, the ELEVATION represents the true length of the line.

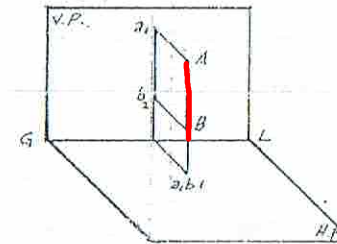
Figure (11) shows the PLAN & ELEVATION of a straight line A B which is falling on the H. P. and inclined by an angle ( $\beta$ ) to the V.P. In this case, the PLAN represents the true length of the line.

Figure (12) shows the PLAN & ELEVATION of a straight line A B which lies on the G. L. The two projections coincide one on the other. In this case both views represent the true length of the line.

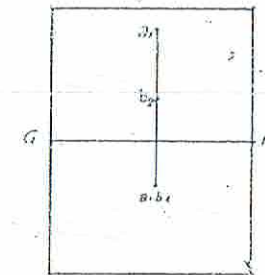
Figure (13) shows the PLAN & ELEVATION of a straight line A B which is inclined to both the planes of projection. In this case the length of each view is shorter than the true length of the line. Neither the inclinations nor the true length can be found except by different methods which will come later on

23

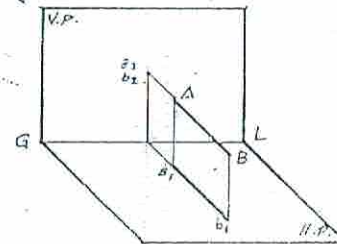
# DIFFERENT POSITIONS OF THE STRAIGHT LINE IN SPACE



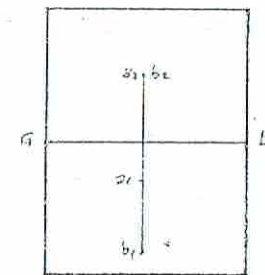
LINE IS  $\perp$  TO THE H.P.  $a_1 b_1$  IS THE TRUE LENGTH.



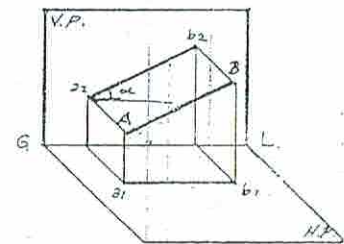
( FIG. 5 )



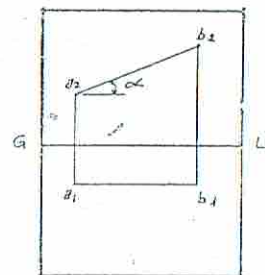
LINE IS  $\perp$  TO THE V.P.  $a_1 b_1$  IS THE TRUE LENGTH.



( FIG. 6 )



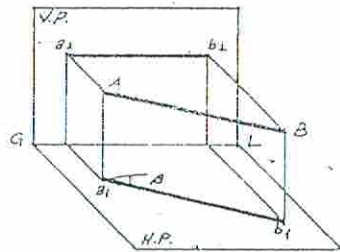
LINE IS  $\parallel$  TO THE V.P. & INCLINED  $\alpha^\circ$  TO THE H.P.  $a_1 b_1$  IS THE TRUE LENGTH.



( FIG. 7 )

سید کاتیا زانکری  
کولبرائندازیاری  
کتبخانه

24



LINE IS  $\parallel$  TO THE H.P. & INCLINED  $\beta^\circ$  TO THE V.P.  $a_1b_1$  IS THE TRUE LENGTH.

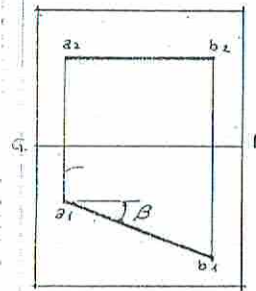
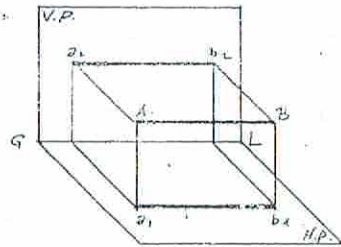
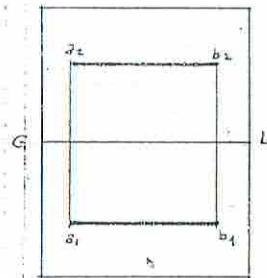


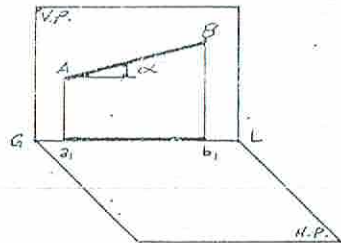
FIG. 8



LINE IS  $\parallel$  TO THE G.L. BOTH  $a_1b_1$  &  $a_2b_2$  IS THE TRUE LENGTH.



(FIG. 9)



LINE IS FALLING ON THE V.P. & INCLINED  $\alpha^\circ$  TO THE H.P.  $a_2b_2$  IS THE TRUE LENGTH

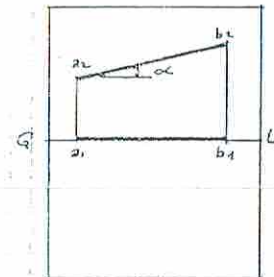
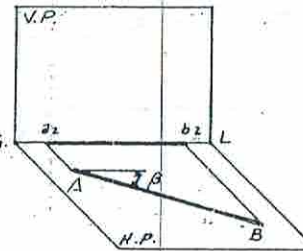


FIG. 10



LINE IS FALLING ON THE H.P. AND INCLINED  $\beta^\circ$  TO THE V.P.  $a_1b_1$  IS THE TRUE LENGTH.

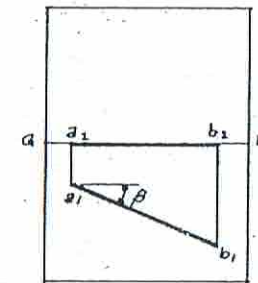
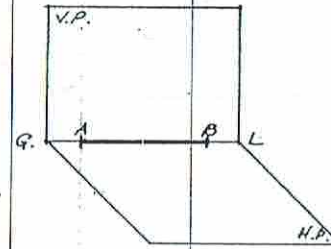


FIG. 11



LINE LIES ON THE G.L. BOTH  $a_1b_1$  &  $a_2b_2$  COINCIDE & EACH IS THE TRUE LENGTH.

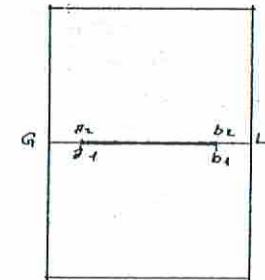
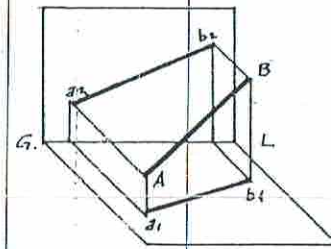


FIG. 12



LINE IS INCLINED TO BOTH THE H.P. & THE V.P. BOTH PROJECTIONS ARE SHORTER THAN THE TRUE LENGTH.

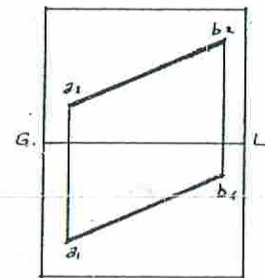


FIG. 13

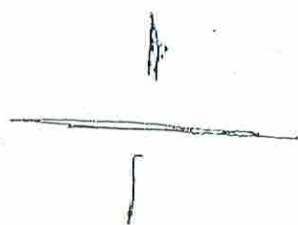
25



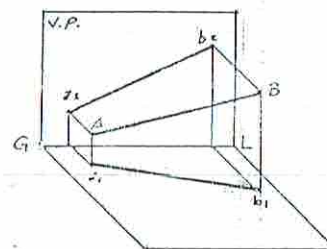
Figure (14) shows the PLAN & ELEVATION of a straight line A B which is inclined to both the H. P. & the V. P. Also in this case, the length of each view is shorter than the true length of the line. Here also, for the determination of the true length & the inclinations, we shall study later on the general case of the line.

Figure (15) shows the PLAN & ELEVATION of a straight line A B which is inclined to both of the planes of projection. Here we notice that the distance of projectors between A & B = 0. This line in that special case is called a **FRONTAL STRAIGHT LINE**.

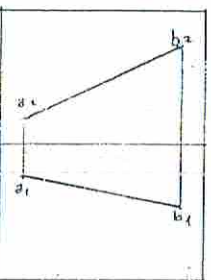
Figure (16) shows the PLAN & ELEVATION of a straight line A B which is inclined to both of the planes of projection. We notice that one of its ends (A) lies on the (H.P.) while the other end (B) lies on the V.P.



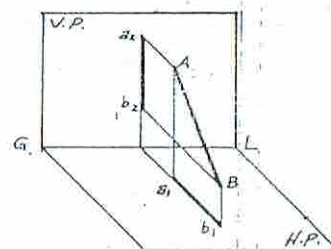
Note : The straight lines given in the figures (13), (14), (15) & (16) need certain studies for the determination of their true lengths and inclinations.



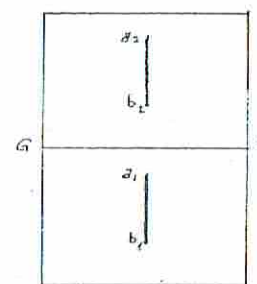
LINE IS INCLINED TO BOTH THE H.P. & THE V.P. (BOTH  $a_1b_1$  &  $a_2b_2$  ARE SHORTER THAN THE TRUE LENGTH)



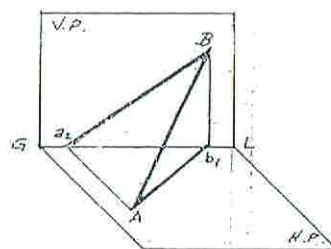
( FIG. 14 )



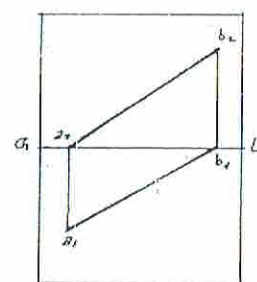
LINE IS INCLINED TO BOTH THE H.P. & V.P. HERE THE DISTANCE OF PROJECTORS BETWEEN A & B = 0.



( FIG. 15 )



LINE IS INCLINED TO BOTH THE H.P. & THE V.P. ONE END RESTS ON THE H.P. WHILE THE OTHER LIES ON THE V.P.



( FIG. 16 )

# DETERMINATION OF THE TRUE LENGTH OF A STRAIGHT LINE & ITS INCLINATIONS TO THE PLANES OF PROJECTION

Figure (17) represents the general case of a straight line  $AB$  in space,  $a_1 b_1$  being its projection on the H.P. while  $a_2 b_2$  is its projection on the V.P.

**Rabatment on the H. P. : —**

The trapezium  $AB a_1 b_1$  having  $AB$  representing the true length of the line, and the sides  $y_a$  &  $y_b$  (being the parallel sides of the trapezium) are both perpendicular to the H.P. If this trapezium be rotated (with  $a_1 b_1$  as the axis of rotation) till it is rabated on the H.P. we get the true length  $A_1 B_1$  on the H.P.

**Rabatment on the V. P. : —**

By the same way, the trapezium  $AB a_2 b_2$  having  $AB$  the true length of the line and the sides  $x_a$  &  $x_b$  are parallel and both being perpendicular to the V.P.

So by the rotation of this trapezium (about  $a_2 b_2$  as an axis) till rabatment is accomplished on the V.P. we get  $A_2 B_2$  representing the true length on the V.P.

Hence, if a straight line be given by its projections on the H.P. and the V.P. as shown in Figure (18) we can at once determine its true length either on the H.P. or on the V.P.

**Example (1) :—**

The straight line  $AB$  is placed such that:  $A = (2, 4)$ ,  $B = (5, 1)$  left of A. The distance of projectors between A & B = 6 cms.

**Required : —**

Draw, scale 1:1, the ELEVATION & PLAN of the line and hence determine:

(i) Its true length, (ii) Its inclinations, (iii) The coordinates of its traces,

## DETERMINATION OF THE TRUE LENGTH OF A STRAIGHT LINE & ITS INCLINATIONS TO THE PLANES OF PROJECTION

### (1) RABATEMENT METHOD

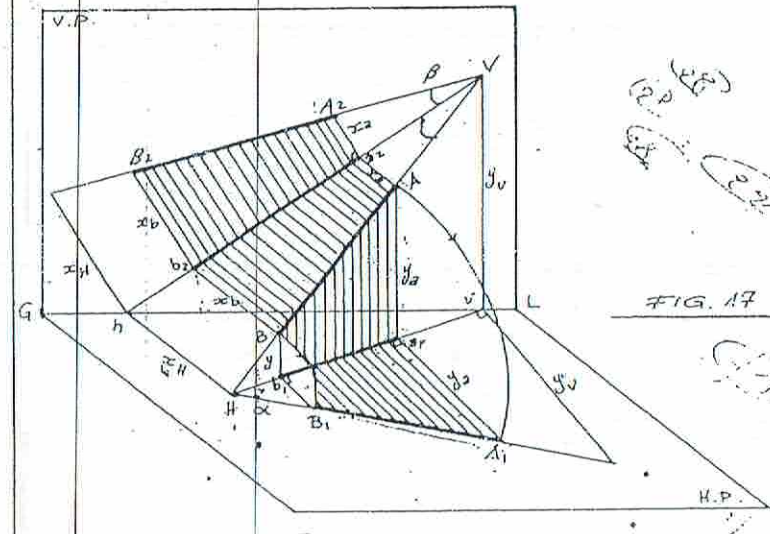


FIG. 17

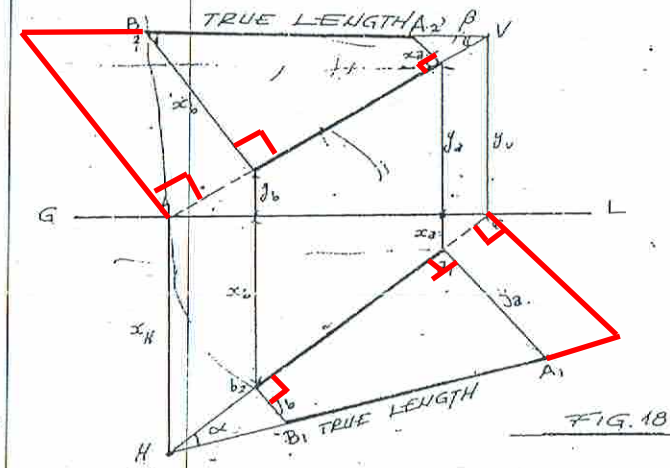


FIG. 18

2014/12/23  
HAMOD



Example (2) :—

The straight line AB is placed such that : A = (6, 4), B = (2, 2) left of A. The distance of projectors between A & B = 8 cms.

Required : —

Draw, scale 1:1, the ELEVATION & PLAN of the line & hence determine :

(i) Its true length, (ii) Its inclinations, (iii) The coordinates of its traces.

Solution of Example (1) :— See figure (19) :—

On the Horizontal Projection (  $a_1 b_1$  ) :—

We draw the perpendiculars  $a_1 A_1 = y_a$  and  $b_1 B_1 = y_b$ . Hence we get  $A_1 B_1$  ( the true length ). Also, the point at which the true line  $A_1 B_1$  meets its projection  $a_1 b_1$  gives the horizontal trace (H) of the line. Moreover, the angle between the true line  $A_1 B_1$  and its projection  $a_1 b_1$  is the inclination ( $\alpha$ ) to the H. P.

On the Vertical Projection (  $a_2 b_2$  ) :—

By the same way and using the x-coordinates on  $a_2 b_2$  we obtain the true length  $A_2 B_2$  and the vertical trace (V). The angle between  $A_2 B_2$  and  $a_2 b_2$  is the inclination ( $\beta$ ) of the line to the V. P.

Results :— as shown in figure (19) :—

- (i) The true length of the line = 7.3 cms.
- (ii) Its inclinations  $\alpha = 23^\circ$  and  $\beta = 24^\circ$ .
- (iii) Its traces: H = (5.9, 0) and V = (0, 5.8).

Solution of Example (2) — See figure ( 20 ) :—

By exactly the same procedure first by making use of the horizontal projection  $a_1 b_1$  and taking the y-coordinates on the perpendiculars through  $a_1$  &  $b_1$

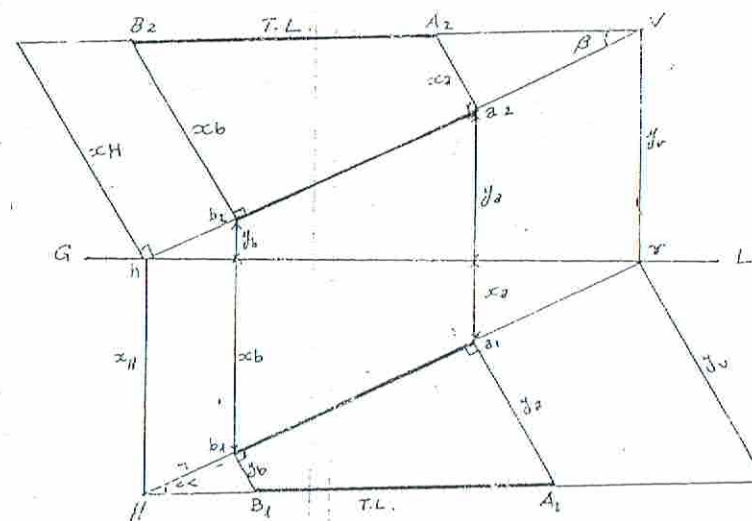


FIG. 19



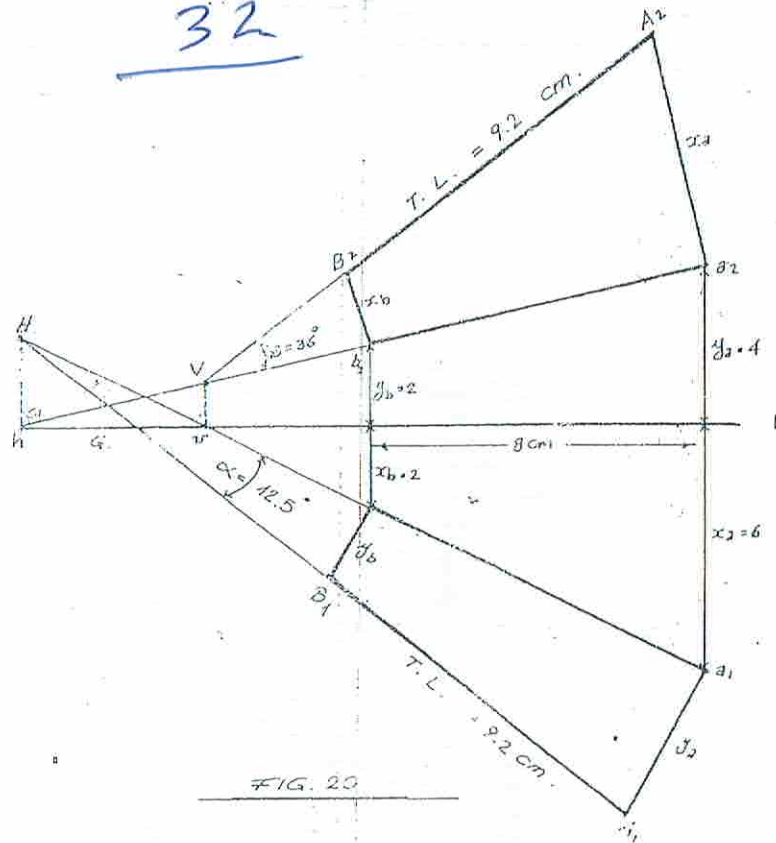
32

FIG. 20

we get the true length  $A_1 B_1$  and also the trace (H) and the inclination ( $\alpha$ ). Secondly by making use of the vertical projection  $a_2 b_2$  and taking the  $x$  coordinates on the perpendiculars through  $a_2 b_2$  we obtain all the unknown items.

Results : -

- (i) The true length of the line  $A B = 9.2$  cms.
- (ii) Its inclinations :  $\alpha = 12.5^\circ$  and  $\beta = 36^\circ$ .
- (iii) Its traces :  $H = (-2, 0)$  and  $V = (0, 1)$ .

33

33  $\downarrow$

### ROTATION METHOD

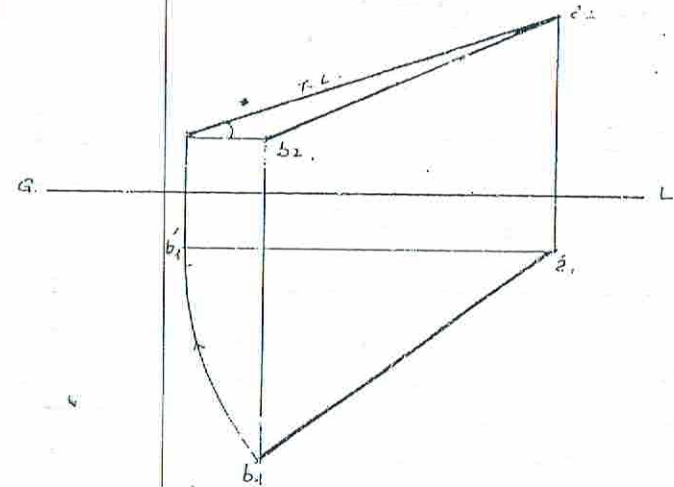


FIG. 21

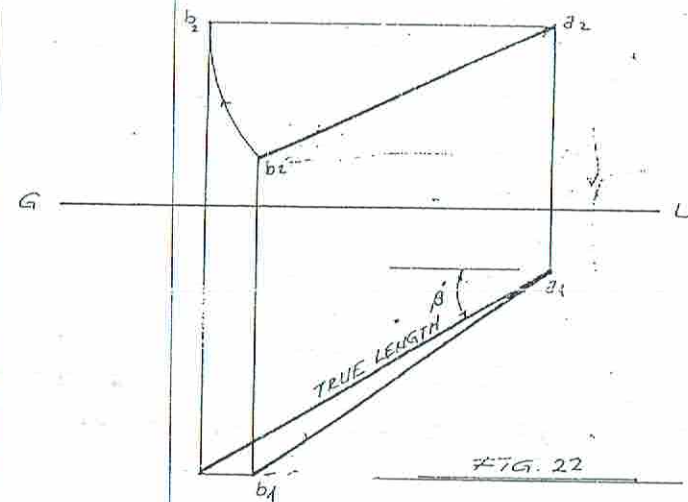
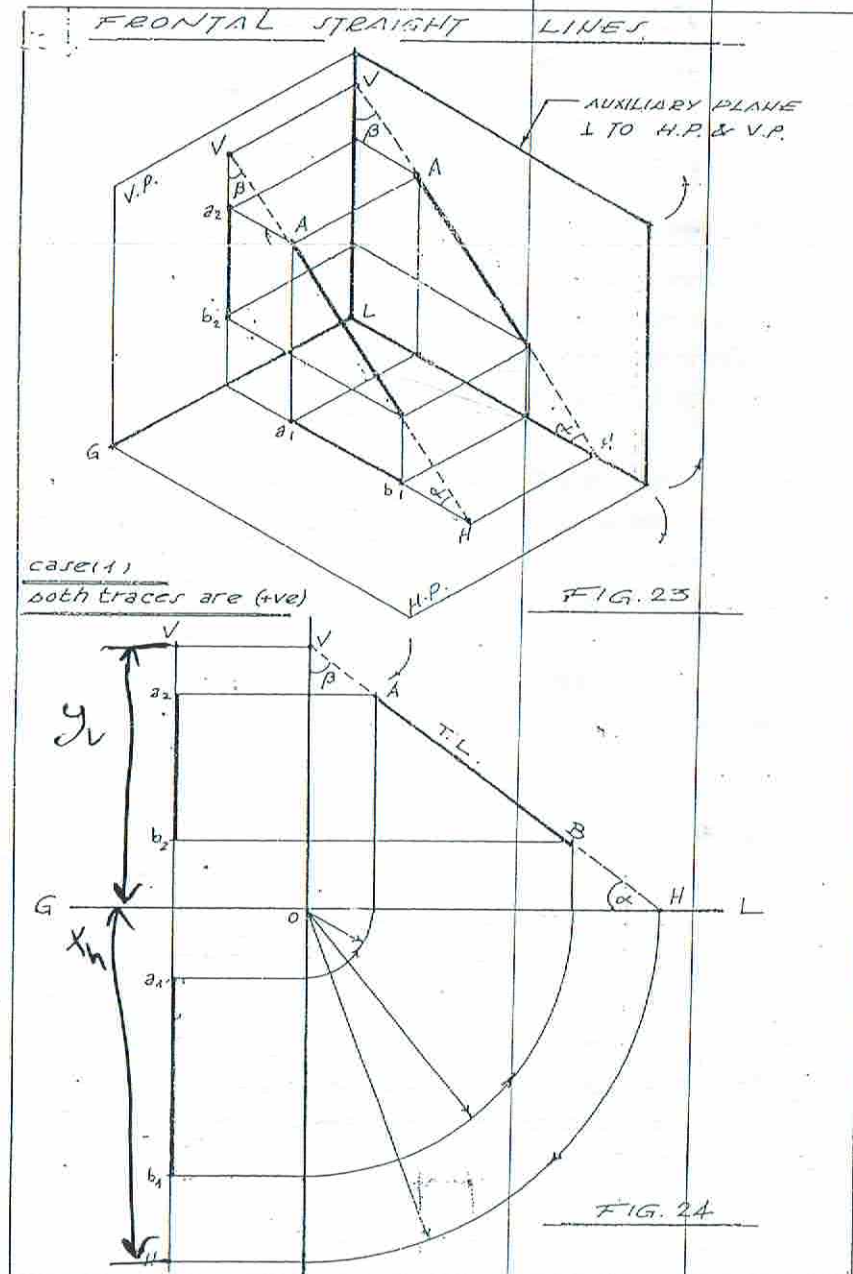


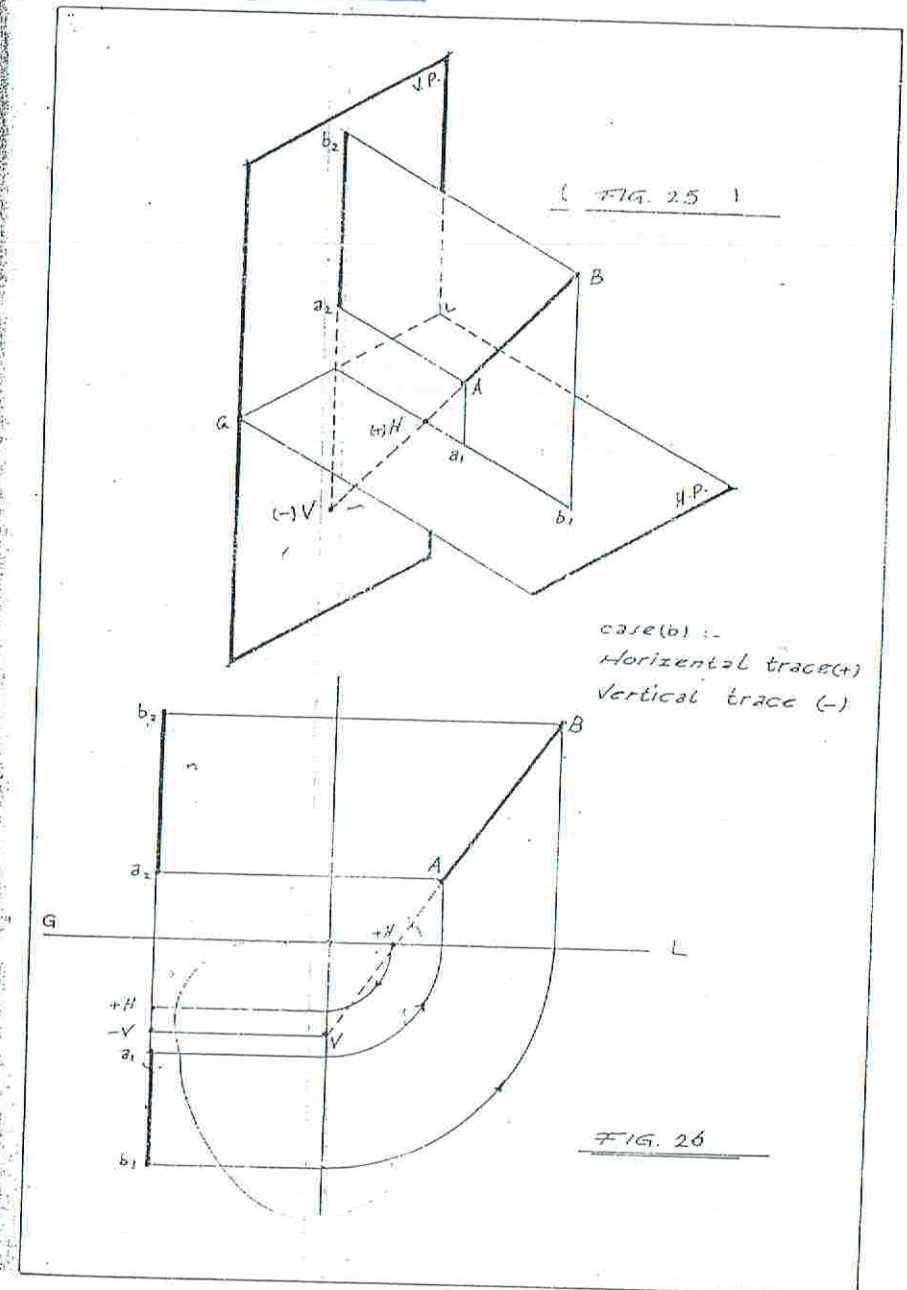
FIG. 22

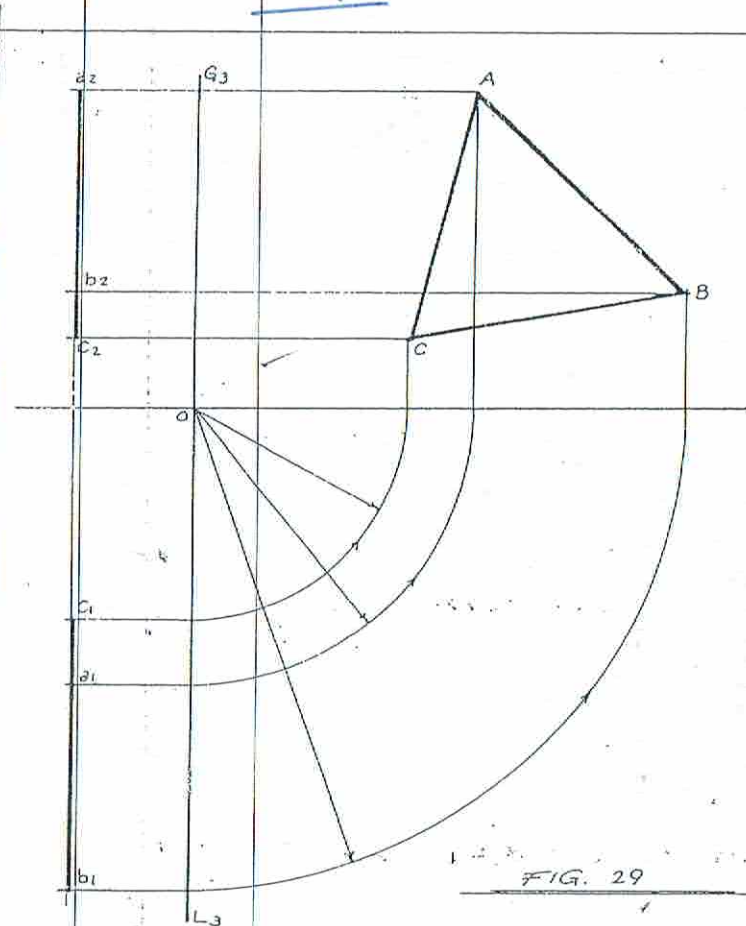
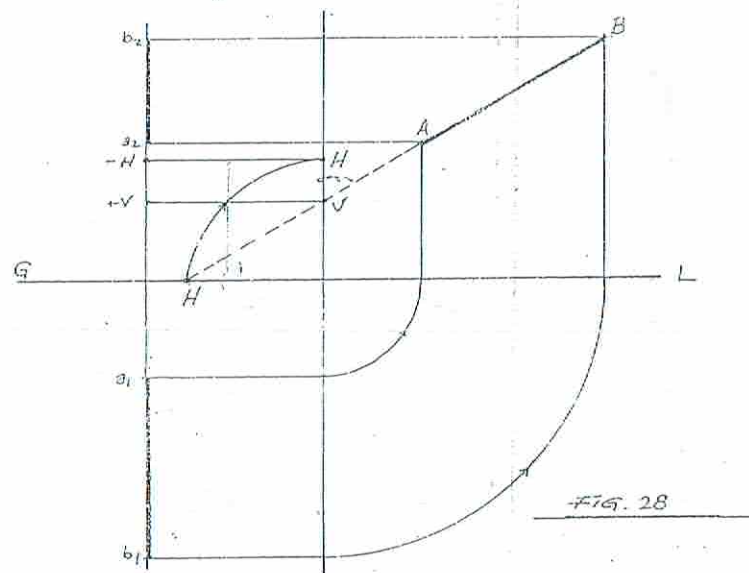
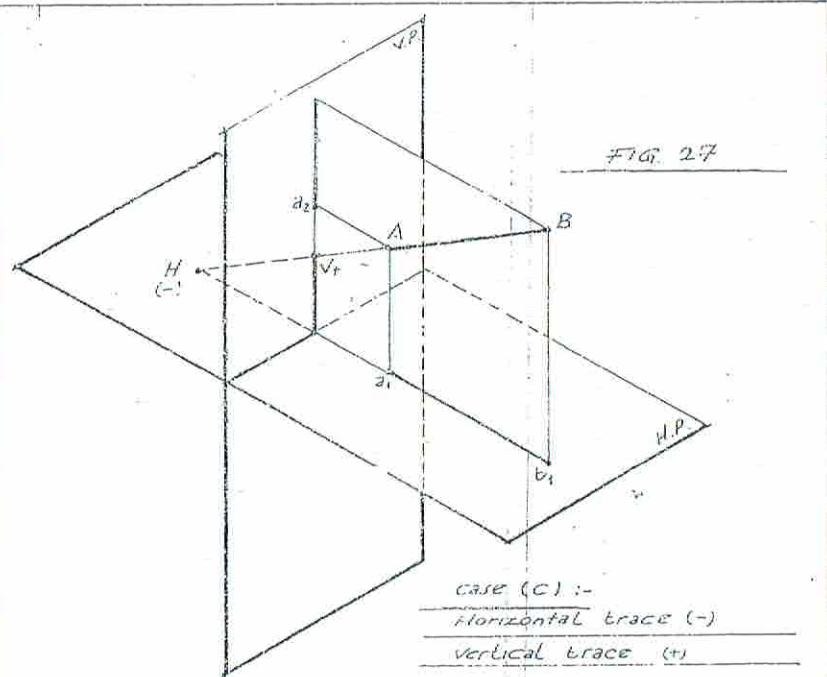
Frontal straight line  $\Rightarrow$  Dis. between two Proj. = Zero

34



35





#### EXAMPLE :-

THE TRIANGLE ABC IS PLACED SUCH THAT :-  
 $A = (7, 2.5)$   $B = (12, 3)$   $C = (5.2, 1.8)$  THE DISTANCE  
 BETWEEN ALL PROJECTORS = 0.

REQUIRED :- DRAW (SCALE 1:1) THE TRUE LENGTH  
 (SHAPE) OF THE TRIANGLE AND FIND ITS EXACT  
 AREA.

FOR DETERMINING THE TRUE SHAPE OF THE TRIANGLE  
 WE FIRST DRAW THE AUXILIARY PLANE  $G_3L_3$  WHICH  
 INTERSECTS THE G.L. AT POINT (O) THE CENTRE  
 OF ROTATION.



### PROBLEMS

Draw (scale 1 : 1) the ELEVATION & PLAN of each of the following straight lines, then answer the following :

- Determine for each line the coordinates of its traces on the principal planes of projection.
- Find the inclinations of each line to both the H.P. and the V.P.
- State the case in which the straight line appears by its true length and find that true length.

1. Straight Line A B :-

A (4, 1) B (4, 6.3) right of A. Distance between projectors = 9 cm

2. Straight Line C D :-

C (2, 3) D (7, 3) left of C. Distance between projectors = 9.5 cm

3. Straight Line E F :-

E (6, 3) F (6, 3) right of E. Distance between projectors = 10 cm

4. Straight Line K T :-

K (0, 5) T (0, 8) right of K. Distance between projectors = 9.2 cm

5. Straight Line M N :-

M (4, 0) N (9, 0) left of M. Distance between projectors = 10.7 cm

6. Straight Line A R :-

A (0, 0) R (0, 0) right of A. Distance between projectors = 10 cm

7. Straight Line P C :-

P (0, 0) C (3, 7) left of P. Distance between projectors = 9.4 cm

8. Straight Line D Q :-

D (0, 6) Q (6, 0) right of D.

Distance between projectors = 3.9 cm

9. Straight Line F M :-

F (7, 1) M (1, 6) right of F. Distance between projectors = 7 cm

10. Straight Line C R :-

C (2, 1) R (6, 7) right of C.

Distance between projectors = 8 cm

11. Straight Line N B :-

N (4, 7) B (4, 1)

Distance between projectors = 0

12. Straight Line R T :-

R (7, 3) T (2, 3)

Distance between projectors = 0

13. A B is a straight line, the coordinates of its ends are as follows:

A = (3, 7) B = (8, 2) left of A.

Distance between projectors = 10 cm.

Required :

- Draw (scale 1 : 1) a pictorial view to represent the line in space showing its traces H & V and its inclinations to the principal planes of projection.
- Draw the ELEVATION & PLAN of the line and find the following:
  - Its true length.
  - The coordinates of its traces.
  - Its inclinations to the H. P. & the V. P.

T.L = 12.4  
B = 24  
 $\alpha = 24^\circ$   
V = (0, 10)  
H = (10, 0)

مسئله 13  
الخط في الفراغ  
النقطة A (3, 7) والنقطة B (8, 2)  
المسافة بين الإسقاطين = 10 سم  
المطلوب:  
1- رسم المنظور  
2- رسم الارتفاع والخط  
3- إيجاد الطول الحقيقي  
4- إيجاد إحداثيات التتبعات  
5- إيجاد ميل الخط إلى H.P. و V.P.

40

The straight line A B having A = (1, 8) & B = (1.5, 4.5) left of A.  
The distance between the projectors = 9.5 cm.

Required :

- Draw (scale 1:1) the ELEVATION & PLAN of the line.
- Determine its true length.
- Find its inclinations to the H.P. & the V.P.
- Find the coordinates of its traces (H & V).

The straight line A B having A = (6, 4) & B = (2, 1) right of A. The distance between the projectors = 10 cm.

Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the line.
- Determine its true length.
- Find its inclinations to both the H.P. and the V.P.
- Find the coordinates of H & V (the traces of the line).

The straight line A B having A = (1, 4) & B = (5, 1) right of A. The distance between the projectors = 9 cm.

Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the line.
- On the PLAN determine its true length.
- Point N divides the line A B internally in the ratio 1:2. From N, draw the perpendicular N D on A B. Find the length N D such that  $(\overline{A D})^2 = \overline{A N} \times \overline{A B}$ .

The straight line A B having A = (4, 1.5) and B = (1, 4.5) left of A. The distance between the projectors = 10 cm.

Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the line.
- On the ELEVATION find its true length.

41

(c) Point N divides the line A B internally in the ratio 3:5. From N, the perpendicular N D is drawn on A B. Find the length N D such that  $\overline{B D}^2 = \overline{B N} \times \overline{B A}$ .

18. A B is a straight line in which A (4, 10) & B (8, 2). The distance between the projectors = 0.

Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the straight line.
- Determine its true length.
- Find the coordinates of H & V (the traces of the line).
- Find its inclinations to the H.P. & the V.P.

19. The straight line E F having : E (3, 3) & F (9, 7). The distance between the projectors = 0.

Required :

The same as in previous problem.

20. A B C is a triangle in which : A (2, 3) & B (10, 9) & C (2, 9). The distance between all projectors = 0.

Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the triangle.
- Determine its true shape.
- Prove that this triangle is right angled at C, and find its exact areas.
- Find the coordinates of H & V (the traces) of the side A B.
- Find the coordinates of the centre of the circle that passes by vertices A, B & C.

21. The lamina A B C D having A (2, 3), B (11, 3), C (8, 9) & D (5, 9). The distance between all projectors = 0.

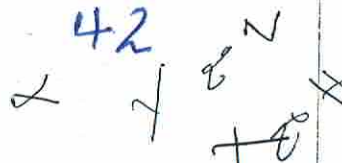
Required :

- Draw (scale 1:1) the PLAN & ELEVATION of the given figure.



(42)

42



21. Determine its true shape.

22. Prove that it is a trapezium, and find its exact area.

23. Find the coordinates H & V (the traces) of the side A D.

24. Find the coordinates of the centre of the circle which touches the sides of the triangle A B C internally.

25. A figure A B C D E having :- A = (6, 2), B = (8, 2), C = (11, 6)

26. D = (3, 6), E = (3, 6). The distance between all projectors = 0.

27. Draw (scale 1:1) the PLAN & ELEVATION of the given figure.

28. Determine its true shape.

29. Prove that it is a symmetrical figure about the line passing by the vertex D and the mid-point of the side AB.

30. Find the coordinates of H & V (the traces) of the sides A E & B F.

31. Determine the exact area of the whole figure and the coordinates of its centre of gravity.

32. A figure A B C D E F having :- A (5, 0), B (3, 0), C (11, 2), D (3, 0), E (3, 4/3), F (3, 2/3).

33. The distance between all projectors = 0.

34. Draw (scale 1:1) the PLAN & ELEVATION of the given figure.

35. Determine its true shape.

36. Prove that it is a regular hexagon, and find its exact area.

37. For the side C D :- Find its inclinations ( $\alpha$  &  $\beta$ ) and the coordinates of its traces.

38. For the sides A F & E F :- Find the coordinates of their traces.

39. The straight line A B is inclined at an angle  $27^\circ$  to the H. P. and

40. A = (4,  $y_a$ ) & B = (7,  $y_b$ ) right of A. The distance between their projectors = 8 cm. The horizontal trace (H) of the line is of coordinates (2, 0).

Dis = 8, H (2, 0)

$\alpha = 27^\circ$  A (4,  $y_a$ )

43

43

Required :

- Determine the missed coordinates  $y_a$  &  $y_b$  and draw the ELEVATION & PLAN of the line.
- Find its true length.
- Find the coordinates of its vertical trace (V).
- Determine the inclination of the line to the V. P.

25. The triangle A B C having: A = (6, 2) & B = (2, 8) & C = (10, 10). The distance between all projectors = 0. Point D divides the side A B internally in the ratio of 2:1 and point E divides the side C B internally in the ratio 1:2.

Required

- Determine the true shape of the triangle.
- Find the volume of the body generated by the rotation of the figure A D E C about A C.

26. The straight line A B is inclined at an angle  $25^\circ$  to the H. P. & having A = (2,  $y_a$ ) & B = (7,  $y_b$ ) right of A. The distance between their projectors = 9 cm. The horizontal trace (H) of the line is of coordinates (-2, 0).

Required :

- Determine the missed coordinates  $y_a$  &  $y_b$  and draw (scale 1:1) ELEVATION & PLAN of the line.
- Find its true length.
- Find the coordinates of its vertical trace (V).
- Determine the inclination of the line to the V. P.

27. The straight line C D having its true length = 10 cm. The coordinates of its ends are as follows : C = (2, 1), D = ( $x_d$ , 5), right of C.

The distance between projectors = 8 cm.

$x_d = 6.4$

H (0.9, 0)



44

Required :

(a) Find the missed coordinate ( $x_d$ ) and then draw (scale 1:1)

ELEVATION & PLAN of the line.

(b) Determine its inclinations to the H. P. & V. P.

(c) Find the coordinates of its traces H & V.

28. The straight line A B is inclined at an angle  $= 20^\circ$  to the H. P. The coordinates of its ends are as follows

A = (6, 1), B = (2, 6) right of A

Required :

(a) Determine the distance between the projectors.

(b) Draw (scale 1:1) the ELEVATION & PLAN of the line.

(c) Find its true length, its inclination to the V. P. and the coordinates of its traces H & V.

29. The straight line C D is inclined at an angle  $= 30^\circ$  to the V. P. & having C = ( $x_c$ , 3.5), D = ( $x_d$ , 6) right of C. The distance between their projectors = 9 cm. The vertical trace (V) of the line is of coordinates (0, 2).

Required :

(a) Determine the missed coordinates and draw (scale 1:1) the ELEVATION & PLAN of the line.

(b) Find its true length.

(c) Find the coordinates of its horizontal trace (H).

(d) Determine the inclination of the line to the H. P.

30. The triangle P Q R having : P = (6, 10) & Q = (9, 2) & R = (10, 8). The distance between all projectors = 0.

Required :

(a) Determine the true shape of the triangle (P, Q, R).

(b) Find the area of the triangle whose vertices are the centres of the circles which touch the sides of the triangle P Q R externally.

45

31. The straight line A B having its true length = 10 cm. The coordinates of its ends are as follows: A = (2, 1), B = (6,  $y_b$ ) left of A. The distance between projectors = 9 cm.

Required :

(a) Determine the missed coordinate ( $y_b$ ) and then draw (scale 1:1) the ELEVATION & PLAN of the line.

(b) Determine its inclinations to the H. P. & V. P.

(c) Find the coordinates of its traces H & V.

32. The straight line A B is inclined at an angle  $= 25^\circ$  to the V. P. The coordinates of its ends are as follows : A = (3, 7) & B = (1, 5) left of A.

Required :

(a) Determine the distance between the projectors.

(b) Draw ELEVATION & PLAN of the line. (Scale 1:1)

(c) Find its true length, its inclination to the V. P. and the coordinates of its traces H & V.

33. The straight line C D having its true length = 11 cm. The coordinates of its ends are as follows : C = (6, 1) & D = (2,  $y_b$ ) right of C. The distance between the projectors = 9 cm.

Required :

(a) Find the missed coordinate ( $y_d$ ) and then draw (scale 1:1) the ELEVATION & PLAN of the line.

(b) Determine its inclinations to the H. P. & V. P.

(c) Find the coordinates of its traces H & V.

34. The straight line AB is of true length = 9 cm and having : A = (6, 2), B = ( $x_b$ , 5) left of A. Its vertical trace V = (0, 7).

Required :

(a) Determine the missed coordinate ( $x_b$ ).

(b) Find the distance between the projectors.

(c) Determine the inclinations of the line to the H. P. & V. P.

(d) Find the coordinates of its horizontal trace (H).



46

The straight line CD is of true length = 10 cms having : C = (6, 2), D = (8, 0) left of C. Its horizontal trace H = (8, 0).

35

Determine the missed coordinate ( $y_d$ ).

Find the distance between the projectors.

Determine the inclinations of the line to the H. P. & V. P.

Find the coordinates of its vertical trace (V).

The straight line A B of true length 9 cms, is placed such that :-

A = (3, 2), B = ( $x_b$ ,  $y_b$ ) right of A.

Horizontal trace H = (7, 0) left of A.

The distance of projectors between A & H = 4 cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and determine the following :

(a) The missed coordinates  $x_b$  &  $y_b$ .

(b) The distance of projectors between A & B.

(c) The coordinates of the traces (H) & (V) of the line.

(d) The inclinations of the line to the H. P. and the V. P.

The straight line A B of true length 10 cms, is inclined  $20^\circ$  to the H. P.

and having the following data :

A = (3,  $y_a$ ), B = ( $x_b$ ,  $y_b$ ) left of A. The vertical trace V = (0, 8) right of A. The distance of projectors between A & V = 5 cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and determine the following :

(a) The missed coordinates  $y_a$ ,  $x_b$  &  $y_b$ .

(b) The distance of projectors between A & B.

(c) The coordinates of the horizontal trace (H).

(d) The inclination of the line A B to the V. P.

2015-2-24  
2015-2-5102

3637

11.11 (ap/9) 47 47

38. the straight line A B of true length 11 cms, is inclined  $20^\circ$  to the H. P. and having the following data :

A = ( $x_a$ ,  $y_a$ ), B = (7,  $y_b$ ) left of A.

The vertical trace V = (0, 7.5) right of A.

The distance of projectors between V & B = 12 cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and find the following :

(a) The missed coordinates  $x_a$ ,  $y_a$  &  $y_b$ .

(b) The distance of projectors between A & B.

(c) The coordinates of the horizontal trace (H).

(d) The inclination of the line A B to the V. P.

39. The straight line A B of true length 10 cms, is inclined  $20^\circ$  to the V. P. and having the following data :

A = ( $x_a$ , 3), B = ( $x_b$ ,  $y_b$ ) right of A.

The horizontal trace H = (7, 0) left of A.

The distance of projectors between A & H = 4 cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and determine the following :

(a) The missed coordinates  $x_a$ ,  $x_b$  &  $y_b$ .

(b) The distance of projectors between A & B.

(c) The coordinates of the vertical trace (V).

(d) The inclination of the line to the H. P.

40. The straight line A B of true length 12 cms, is inclined  $25^\circ$  to the H. P. Its vertical trace V = (0, 8). The line having the following data :

A = ( $x_a$ ,  $y_a$ ) left of V. B = ( $x_b$ ,  $y_b$ ) left of A.

(C) is a point on the line A B dividing it internally in the ratio C A :

C B = 1:2. The coordinates of (C) = (2,  $y_c$ ). The distance of projectors between (V) & (c) = 6.7 cms.

$$\frac{CA}{CB} = \frac{1}{2} \Rightarrow CB = 2CA$$



48

he ELEVATION & PLAN of the straight line  
following :

ordinates  $x_a, x_b, y_a, y_b$  &  $y_c$  .

projectors between A & B .

of the horizontal trace (H) .

of the line to the V . P .

3 of true length 13 cms, is inclined  $27^\circ$  to the  
of (V) = (0, 10) . The line having the following

V B =  $(x_b, y_b)$  right of A .

of A B . The coordinates of C = (3.7,  $y_c$ ) .

tors between V & C = 9 cms .

PLAN & ELEVATION of the line and determine

ordinates  $x_a, y_a, x_b, y_b$  &  $y_c$  .

projectors between A & B .

of the horizontal trace .

the line to the V . P .

of true length 10 cms, is inclined  $20^\circ$  to the V . P .

I = (6, 0) . This line is placed such that:

I B =  $(x_b, y_b)$  right of A .

nt of A B . The coordinates of C =  $(x_c, 3)$  .

actors between C & H = 7 cms .

PLAN & ELEVATION of the line and  
ing :

49

- The missed coordinates  $x_a, y_a, x_b, y_b$  &  $x_c$  .
- The distance of projectors between A & B .
- The coordinates of the vertical trace (V) .
- The inclination of the line to the H . P .

43. The straight line A B is placed such that : —

A = (1.5, 5) , B =  $(x_b, 10)$  right of A .

The horizontal trace H = (2, 0) left of A .

The distance of projectors between A & H = 9 cms .

Required :

Draw (scale 1:1) the ELEVATION & PLAN of the straight line and  
determine the following :

- The missed coordinate  $x_b$  .
- The distance of projectors between A & B .
- The true length of the line A B .
- its inclinations to the H . P & the V . P .
- The coordinates of its vertical trace .

44. The straight line A B is placed such that :

A (5.4,  $y_a$ ) , B = (1.5, 5) left of A .

The vertical trace V = (0, 3) left of B .

The distance of projectors between B & V = 4 cms .

Required :

Draw ( scale 1:1) the PLAN & ELEVATION of the line and  
determine the following :

- The missed coordinate ( $y_a$ ) .
- The distance of projectors between A & B .
- The true length of the line A B .
- its inclinations to the H . P . and the V . P .
- The coordinates of the horizontal trace (H) .

45. The straight line AB of true length 11.5 cms is placed such that :

A = (1.5, 5) B =  $(x_b, y_b)$  right of A .

DO WITH

50

The horizontal trace  $H = (-2, 0)$  left of  $A$ .

The distance of projectors between  $H$  &  $A = 9.7$  cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and determine the following :

- The missed coordinates  $x_b$  &  $y_b$ .
- The distance of projectors between  $A$  &  $B$ .
- The coordinates of the vertical trace ( $V$ ).
- The inclinations of the line to the H. P. & the V. P.

46. The straight line  $AB$  of true length 9 cms, is inclined  $30^\circ$  to the H. P. and having the following data :

$A = (5, y_a)$ ,  $B = (x_b, y_b)$  left of  $A$ .

The horizontal trace  $H = (-3, 0)$  left of  $B$ .

The distance of projectors between  $A$  &  $H = 16$  cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and determine the following :

- The missed coordinates  $y_a$ ,  $x_b$  and  $y_b$ .
- The distance of projectors between  $A$  &  $B$ .
- The coordinates of the vertical trace ( $V$ ).
- The inclination of the line to the V. P.

47. The straight line  $AB$  is placed such that :

$A = (x_a, 5.5)$ ,  $B = (5.5, y_b)$  left of  $A$ .

The horizontal trace  $H = (2.5, 0)$  left of  $B$ .

The vertical trace  $V = (0, -1.5)$  left of  $H$ .

The distance of projectors between  $H$  &  $V = 4$  cms.

Required :

Draw, scale 1:1, the ELEVATION & PLAN of the line and determine the following :

45

46

47

51

51

- The missed coordinates  $x_a$  &  $y_d$ .
- The distance of projectors between  $A$  &  $B$ .
- The true length of the line  $AB$ .
- Its inclinations to the H. P. and the V. P.

48. The straight line  $AB$  of true length 11 cms is placed such that :

$A = (2, 4)$ ,  $B = (x_b, y_b)$  right of  $A$ . ( $y_a = 4.5$ )

The vertical trace  $V = (-2.5, 0)$  left of  $A$ .

The distance of projectors between  $A$  &  $H = 8$  cms.

Required :

Draw (scale 1:1) the PLAN & ELEVATION of the line and find the following :

- The missed coordinates  $x_b$  &  $y_b$ .
- The distance of projectors between  $A$  &  $B$ .
- The coordinates of the horizontal trace ( $H$ ).
- The inclinations of the line to the H. P. and the V. P.

49. The triangle  $ABC$  is placed such that :

$A = (2, 12)$ ,  $B = (10.5, 10)$ ,  $C = (5.8, 2)$ .

The distance between all projectors = 0.

Required :

- Draw (scale 1:1) the true shape of the triangle.
- Find the area of the circle that touches the side  $AB$  and the extension of  $CB$  &  $CA$ .

50. The parallelogram  $ABCD$ , of area  $60 \text{ cm}^2$ , is placed such that :

$A = (5, 2)$ ,  $B = (9.5, 8)$ ,  $C = (x_c, y_c)$ ,  $D = (x_d, y_d)$ .

The distance between all projectors = 0. The vertical trace of the side  $AD = (0, 14)$ .

Required :

Draw (scale 1:1) the true shape of the parallelogram and determine the following :

Area =  $L \times h$

HAMOD



52

- (a) The missed coordinates of the vertices C & D.  
 (b) The horizontal and the vertical traces of the side C D.

The equilateral triangle A B C of area  $(25\sqrt{3}) \text{ cm}^2$  is placed such that:

$$A = (3, 4), B = (x_b, y_b), C = (x_c, y_c).$$

The distance between all projectors = 0.

The horizontal trace of the side A B =  $(-5, 0)$ .

Required :

Draw (scale 1:1) the true shape of the triangle and determine the following :

- (a) The missed coordinates of the vertices (B) & (C).  
 (b) Find the area of the circle that touches the side B C and the extensions of A B & A C.

52. The parallelogram A B C D is placed such that :

$$A = (5, 2), B = (9.5, 8), C = (x_c, y_c), D = (x_d, y_d).$$

The distance between all projectors = 0.

The vertical trace of the side A D =  $(0, 14)$ .

The horizontal trace of the side C D =  $(-7, 0)$ .

Required :

- (a) Draw (scale 1:1) the true shape of the parallelogram and determine its exact area.  
 (b) Find the missed coordinates  $x_c, y_c, x_d$  &  $y_d$ .

The rhombus A B C D of area  $60 \text{ cm}^2$ , is placed such that :

$$A = (4.5, 2), B = (x_b, y_b), C = (x_c, 14), D = (x_d, y_c).$$

The distance between all projectors = 0.

The diagonal A C having its vertical trace =  $(0, -4)$ .

Required :

Draw (scale 1:1) the true shape of the rhombus and determine the following:

- (a) The missed coordinates  $x_b, y_b, x_c, x_d$  &  $y_d$ .  
 (b) Find the area of the circle which touches the sides of the rhombus internally.

## CHAPTER (4)

53

### THE PLANE SURFACE

Any plane surface in space may be determined (with respect to the two principal planes of projection) by extending it to meet the H.P. and the V.P., intersecting each of them in a straight line. The line of intersection of any plane with the H.P. is called the horizontal trace of that plane, while its intersection with the V.P. is called its vertical trace.

In general, every plane has two traces: one on each of the two principal planes of projection, But, if the plane is *parallel* to one of the principal planes there would be only one trace.

It would be worth mentioned here that any plane other than the two principal planes of projection is called an AUXILIARY PLANE. These auxiliary planes will be widely used in projection. And for the purpose of projecting bodies on auxiliary planes, we must at first study their characteristics and their different positions in space in relation with the principal planes of projection, since the utility of these AUXILIARY PLANES is of vital importance in projection.

### POSITION OF THE AUXILIARY PLANE IN SPACE WITH RESPECT TO THE PRINCIPAL PLANES OF PROJECTION

These positions can be illustrated in the following :-

1. The AUXILIARY PLANE may be *perpendicular to both the H. P. and the V. P.* as shown in Figure (1).
2. The AUXILIARY PLANE may be *perpendicular to the V. P. and making an angle ( $\phi$ ) with the H. P.* as shown in Figure (2).
3. The AUXILIARY PLANE may be *perpendicular to the H. P. and making an angle ( $\psi$ ) with the V. P.* as shown in Figure (3).
4. The AUXILIARY PLANE may be *parallel to the H. P.* as shown in Figure (4).
5. The AUXILIARY PLANE may be *parallel to the V. P.* as shown in Figure (5).

These five auxiliary planes mentioned above are of the first importance in projection and by their use much more detailed views can be obtained, as will be shown later on.

Other auxiliary planes of less importance can be given in the following :

- (a) The AUXILIARY PLANE may be inclined to both the H. P. and the V. P. and making an acute angle between its traces, as shown in Figure (6).
- (b) The AUXILIARY PLANE may be inclined to both the H. P. & the V. P. and making an obtuse angle between its traces, as shown in Figure (7).
- (c) AUXILIARY PLANE may be inclined to both the H. P. & the V. P. but its traces are both parallel to the G. L. as shown in Figure (8).
- (d) AUXILIARY PLANE may be inclined to both the H. P. & the V. P. but passing by the G. L. as shown in Figure (9).

FIG. 1

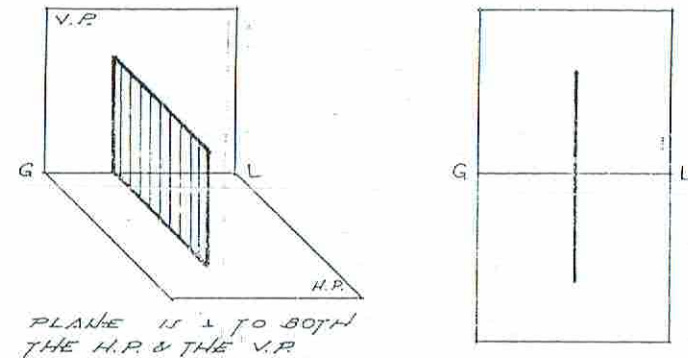


FIG. 2

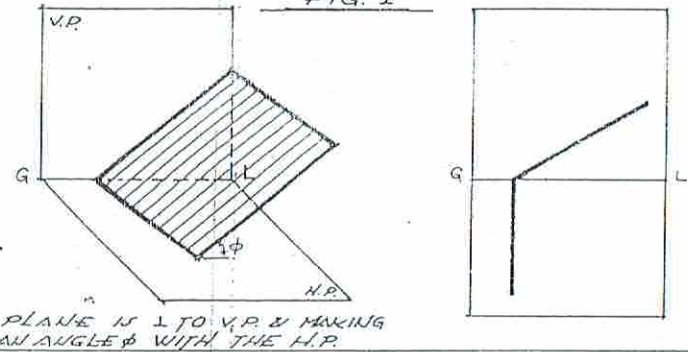
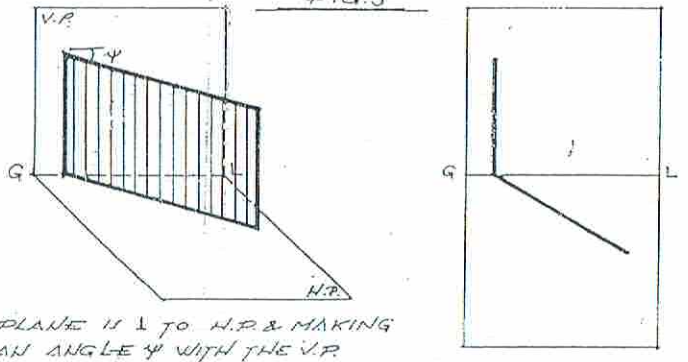


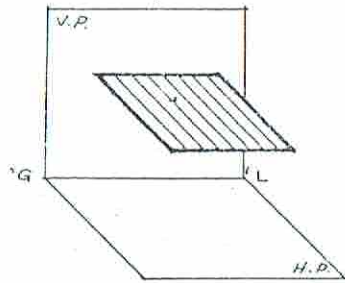
FIG. 3





56

FIG. 4



PLANE IS PARALLEL TO THE H.P.

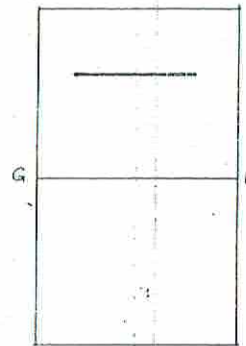
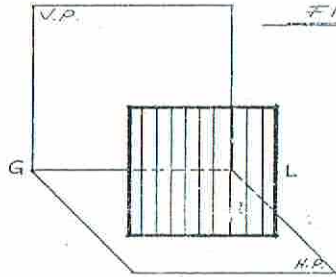


FIG. 5



PLANE IS PARALLEL TO THE V.P.

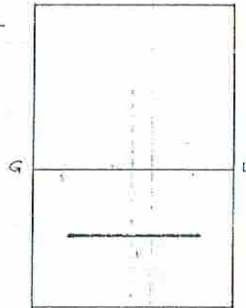
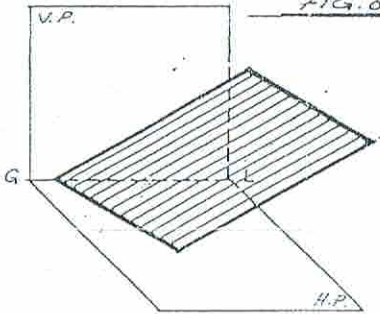
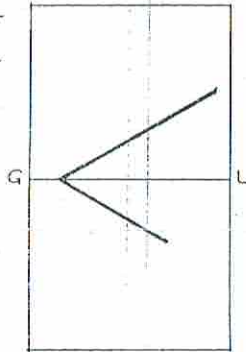


FIG. 6

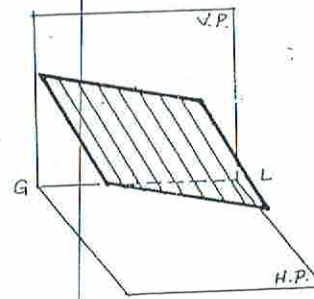


PLANE IS INCLINED TO BOTH THE H.P. & V.P.



57

FIG. 7



PLANE IS INCLINED TO BOTH THE H.P. & V.P.

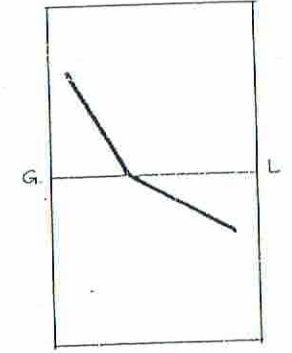
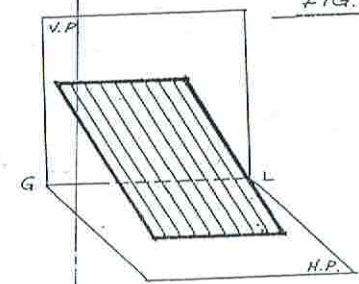


FIG. 8



PLANE IS INCLINED TO BOTH PARALLEL TO G.L.

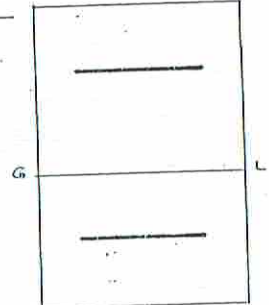
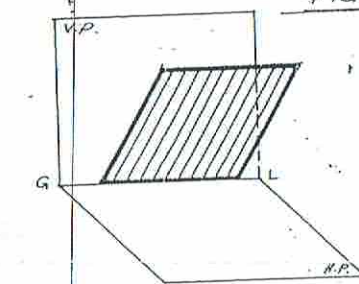
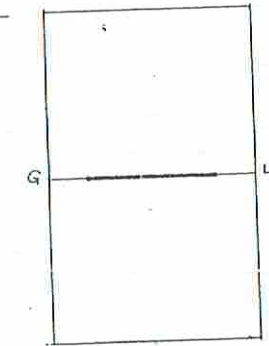


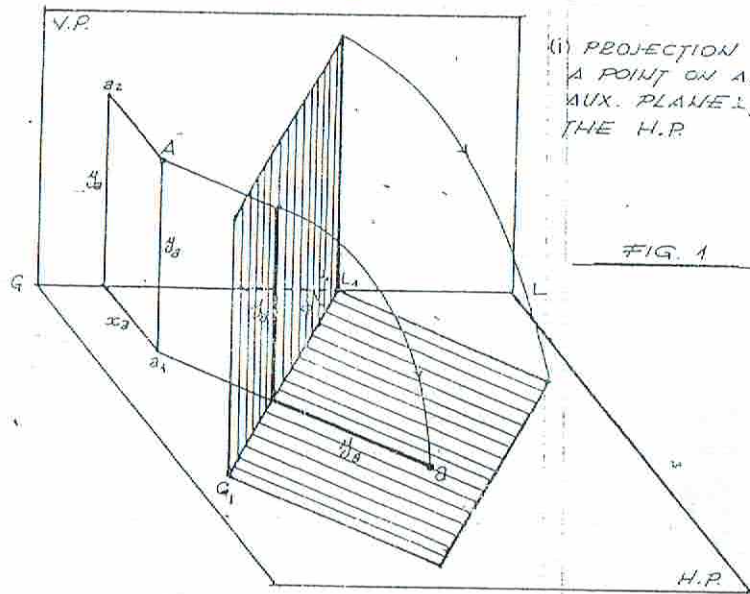
FIG. 9



PLANE IS INCLINED TO BOTH BUT PASSING BY THE G.L.

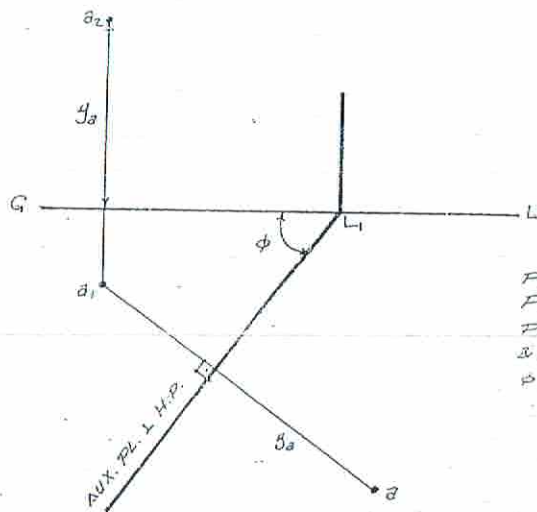


60



(i) PROJECTION OF A POINT ON AN AUX. PLANE  $\perp$  TO THE H.P.

FIG. 1



PROJECTION OF A POINT ON AN AUX. PLANE  $\perp$  TO THE H.P. & MAKING AN ANGLE  $\phi$  TO THE V.P.

FIG. 1

61

61

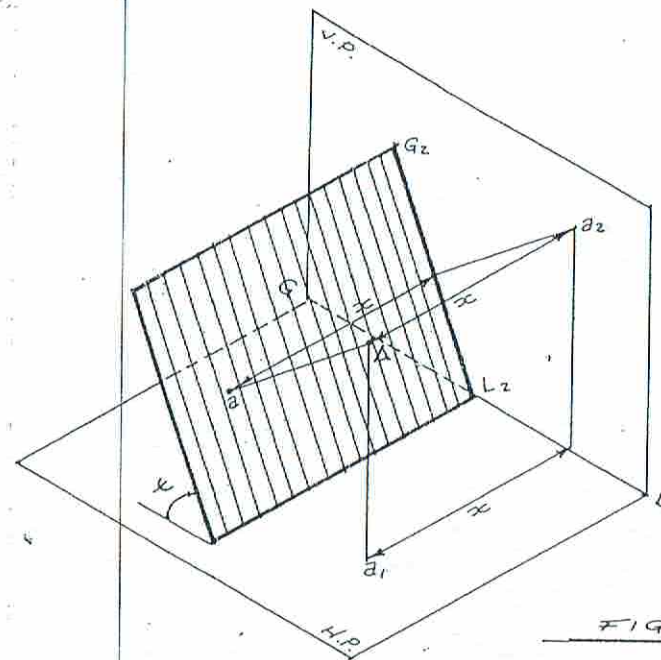
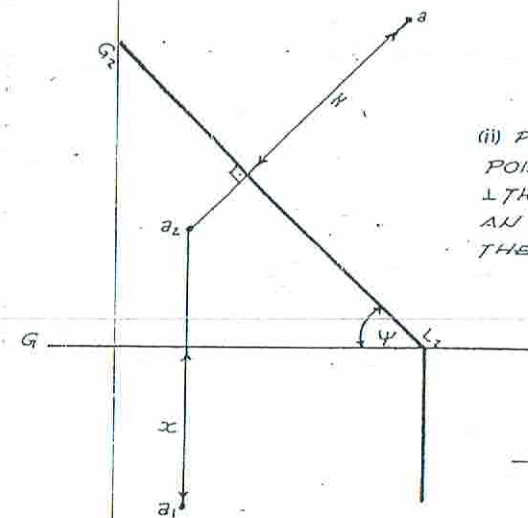


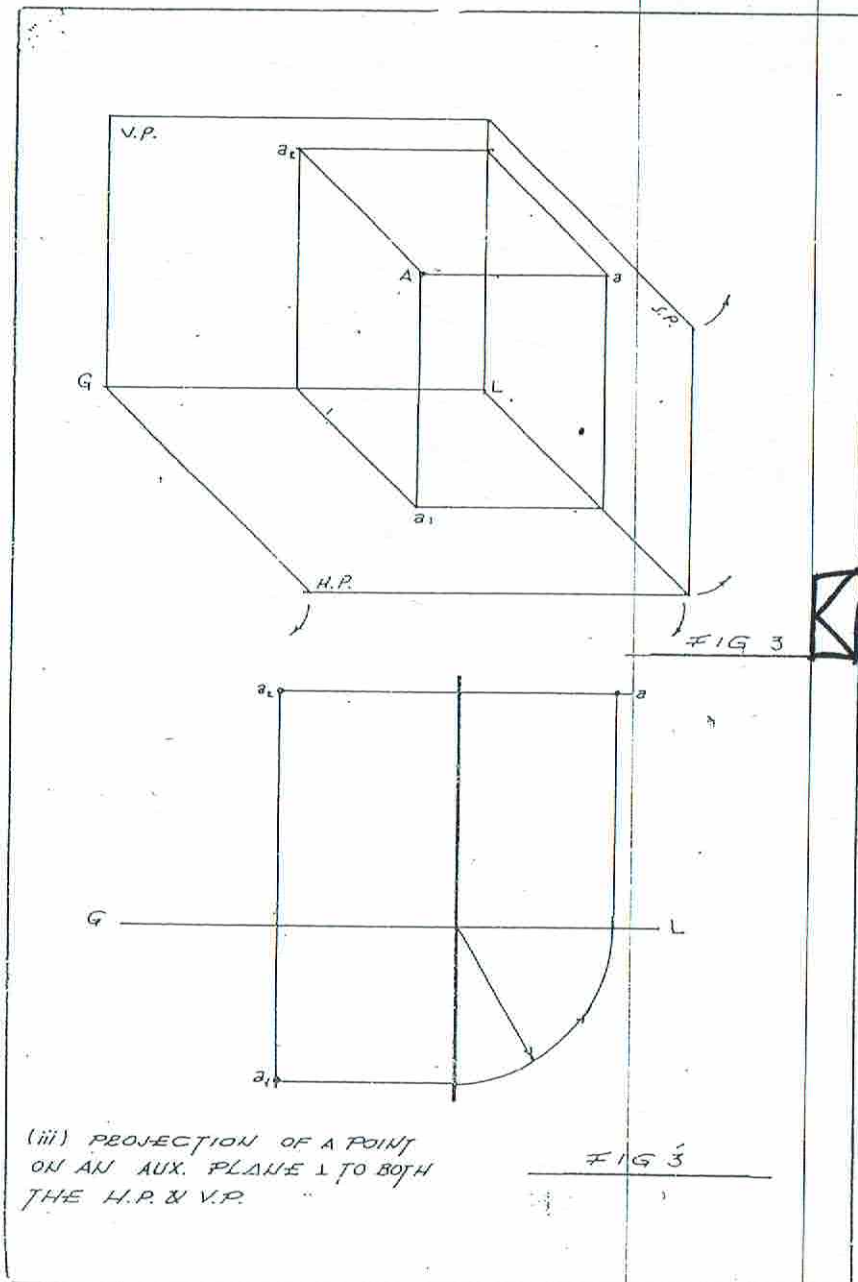
FIG. 2



(ii) PROJECTION OF A POINT ON AN AUX. PL.  $\perp$  THE V.P. & MAKING AN ANGLE ( $\psi$ ) WITH THE H.P.

FIG. 2





## (II) PROJECTION OF A STRAIGHT LINE ON AN AUXILIARY PLANE .

By exactly the same procedure in projecting the point we can follow directly the projection of any straight line *on any auxiliary plane*.

Figure (4) shows the PLAN ( $a_1 b_1$ ) and the ELEVATION ( $a_2 b_2$ ) of a straight line  $AB$ , and it is desired now to project a view : -

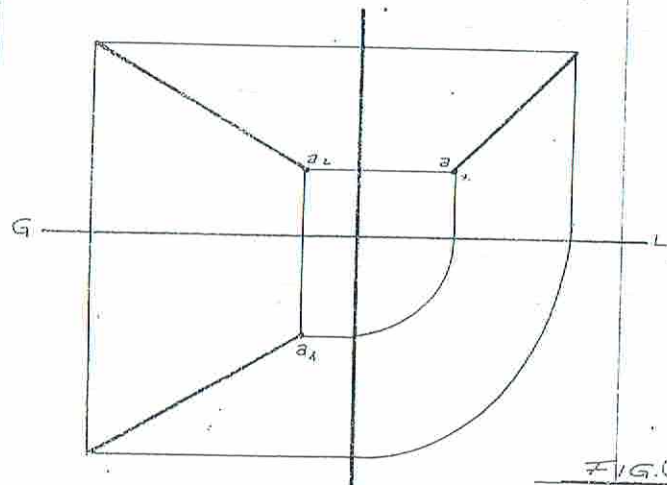
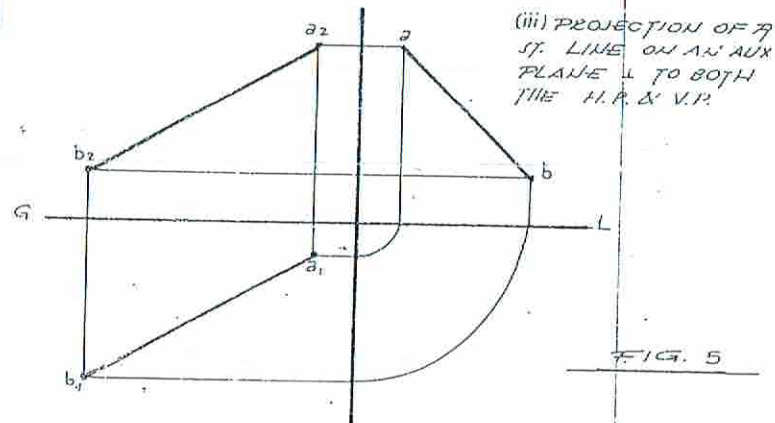
- (i) On an *AUXILIARY PLANE* perpendicular to the H. P. and making an angle  $45^\circ$  with the V. P. Here we take the 'y- coordinates on the new Ground Line ( $G_1 L_1$ ). Hence ( $a b$ ) gives the required new projection .
- (ii) On an *AUXILIARY PLANE* perpendicular to the V. P. and making an angle  $30^\circ$  with the H. P. Here we take the x- coordinates on the new Ground Line ( $G_2 L_2$ ). So, ( $a b$ ) will represent the new projection.
- (iii) On an *AUXILIARY PLANE* perpendicular to both the H. P. and the V.P. see Figure (5) and Figure (5).

### SPECIAL CASE OF THE AUXILIARY PLANE

If the *AUXILIARY PLANE* is chosen *parallel* to any of the two views of the straight line, then the projection on this auxiliary plane gives the *true length* of the straight line. In Figure (5) if we take the *AUXILIARY PLANE* ( $G_1 L_1$ ) *parallel* to ( $a_1 b_1$ ) we should obtain the new projection ( $AB$ ) representing the *true length* of the line. And, by the same way, if the *AUXILIARY PLANE* ( $G_2 L_2$ ) is chosen *parallel* to ( $a_2 b_2$ ) then the new projection ( $AB$ ) will give the *true length*.







### SOLVED EXAMPLES ON THE USE OF AUXILIARY PLANES

The following is a group of Examples illustrating the projection of some regular surface on Auxiliary planes.

Example (1) Figure (7) :-

The equilateral lamina A B C is placed *parallel to the H. P.* and having the following data : -

A = (2, 2), B = (14, 2), C = ( $x_c$ ,  $y_c$ ) right of A.

The distance of projectors between A & B = 0.

Required: -

- Find the missed coordinates of the vertex (C).
- Draw, scale 1:1 the ELEVATION & PLAN of the lamina.
- Project a view on an *AUXILIARY PLANE* perpendicular to the V. P. and making  $60^\circ$  with the H. P.

Example (2) : Figure (8) :-

The equilateral lamina A B C is placed *parallel to the H. P.* and having the following data : -

A = (2, 2), B = (2, 2). The distance of projectors between A & B = 12 cms.

Required : -

- Find the coordinates of the vertex (C).
- Draw, scale 1:1, the ELEVATION & PLAN of the lamina.
- Project a view on an *AUXILIARY PLANE* perpendicular to the V. P. and making  $30^\circ$  with the H. P.



(III) PROJECTION OF A PLANE SURFACE  
ON AN AUX. PLANE

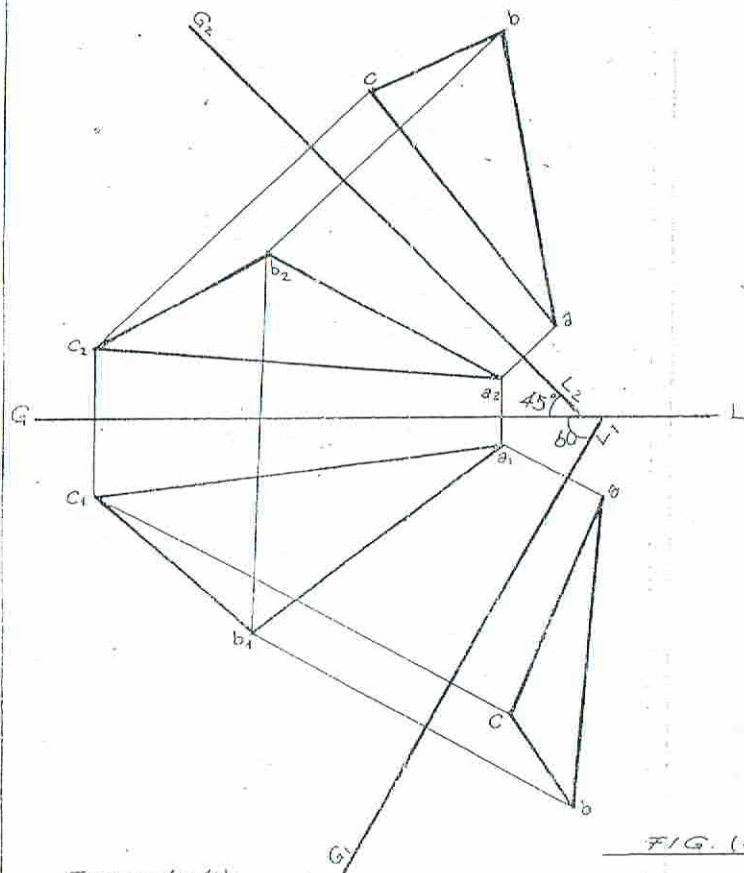


FIG. (6)

Example (1):

GIVEN: THE ELEVATION & PLAN OF A TRIANGULAR LAMINA

REQUIRED:-

- (i) PROJECT A VIEW ON AN AUX. PLANE  $\perp$  TO THE H.P. AND MAKING  $60^\circ$  WITH THE V.P.
- (ii) PROJECT A VIEW ON AN AUXILIARY PLANE  $\perp$  TO THE V.P. & MAKING  $45^\circ$  WITH THE H.P.

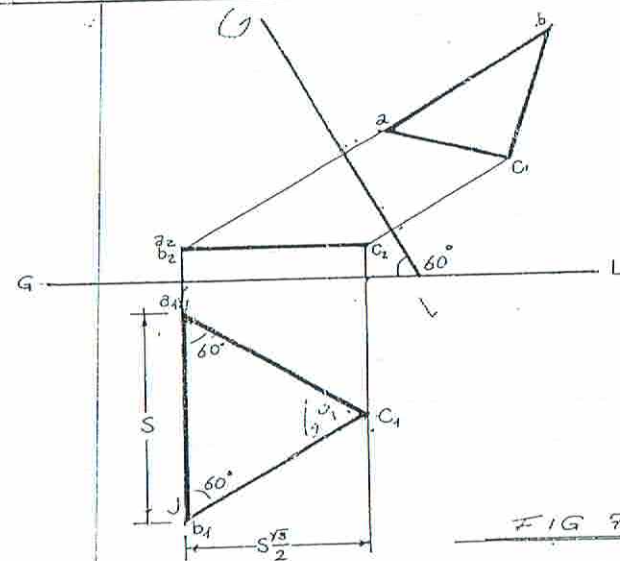


FIG 7

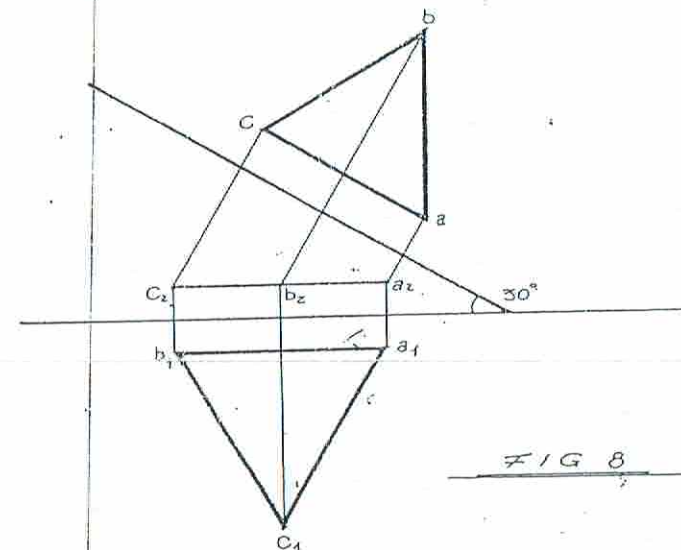


FIG 8