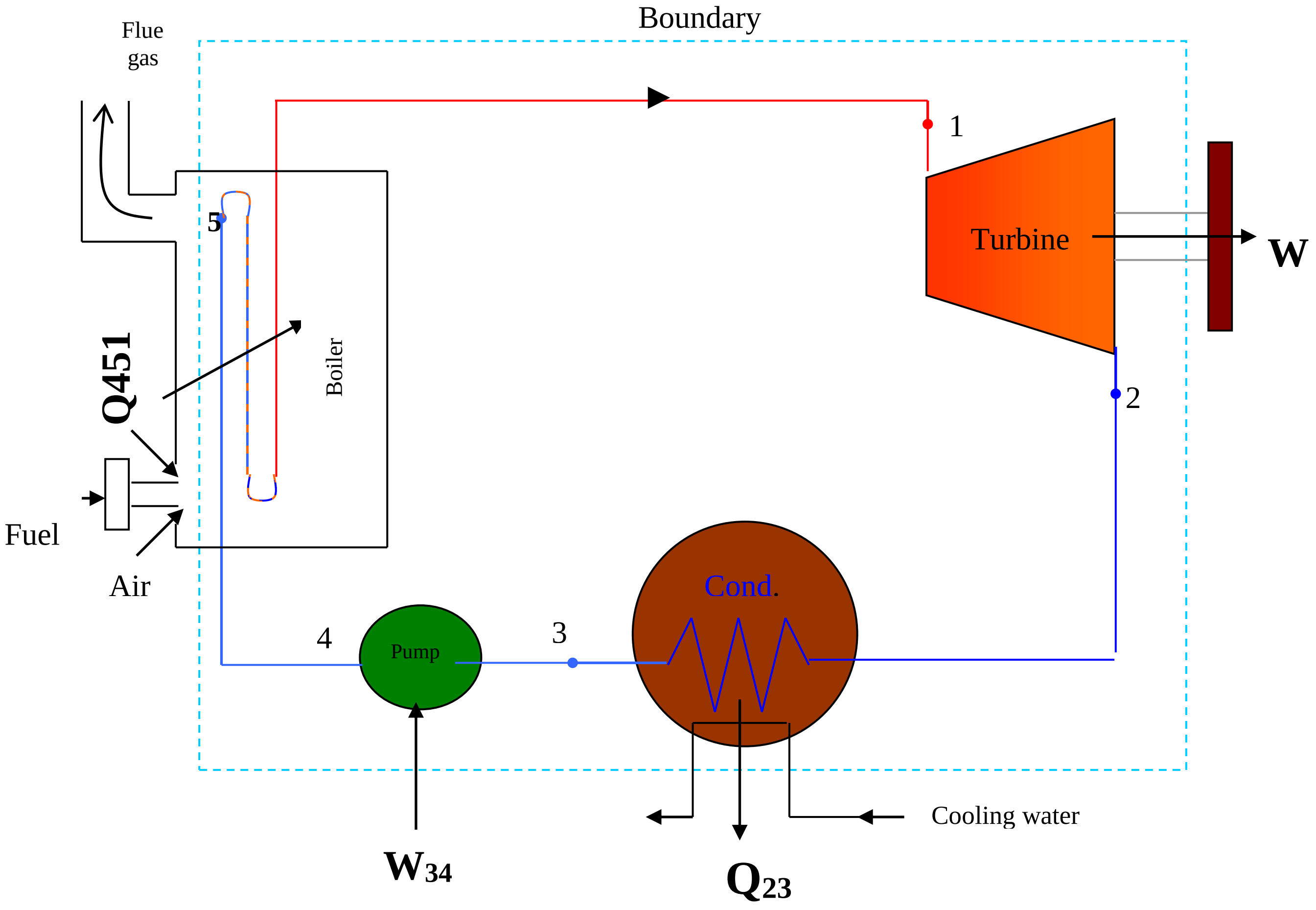
The Ministry of Higher Education
& Scientific Research
Northern Technical University
Technical College / Kirkuk
Mechanical Power Technique Eng. Dept.

Thermodynamic

Second stage



Aim of the subject: - **Define the students cycles & processes of steam with mixture & compressor & turbine.**

Week No.	Syllabus	pages
1	A brief over view of ideal gas.	2 — 4
2 — 4	Real gas & its eq.s of state (Van der Waals eq. Benedict - Webb- Rubin eq.Compressibility eq).	5 — 22
5	Pure substance ,its phases and state.	22 — 25
6	T-H ,T-V , T-P ,& P-V diagrams Of water.	25 — 26
7-8	Steam tables, Dryness friction.	26 — 29
9-10	Steam process &its diagram.	29 — 37
11	Ideal Rankine cycle.	38 — 46
12	Rankine cycle with super heat.	46 — 52
13	Reheat cycle.	52— 55
14-15	Regenerative cycle.	55 — 85
16	Binary cycle.	85 — 86
17	Mixture (Ideal gas mixture).	86 — 88
18	Dalton law.	89
19-20	Volumetric analysis & Gravimetric analysis.	89— 109
21	Air compressor, Reciprocating machine.	109—114
22	The condition for minimum work, isothermal efficiency.	114—118
23	Volumetric efficiency.	119—123
24	Multi -stage Compressor.	123—134
25	Bryton cycle. The ideal cycle of gas turbine.	134
26	Real cycle of gas turbine.	135—141
27	Application of gas turbine.	142—144
28-29	Modification of gas turbine.	145—154
30	Combined gas - steam cycle.	154—159

References:-

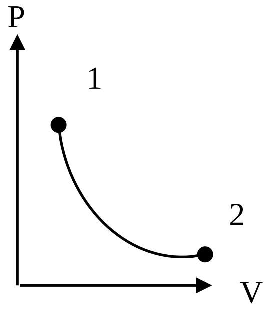
1- Fundation of thermodynamics .5th. Edition by sonntay
2- Engineering thermodynamics by Yunus A. Çengel,4th. Edition
3- Applied thermodynamic for engineering tech. by Eastop, 3th. Edition
4- Thermodynamic and transport properties of fluid SI units , arranged by Y .R . Mayhew & G.F,C. Rogers

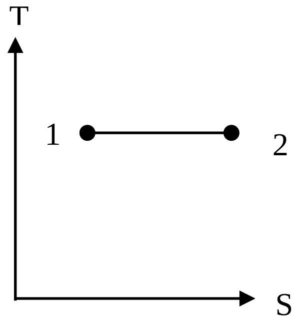
Ideal Gases:-

1- **Boyles Law** :-

$P.V = C$, at $T = C$

$P_1.V_1 = P_2.V_2$

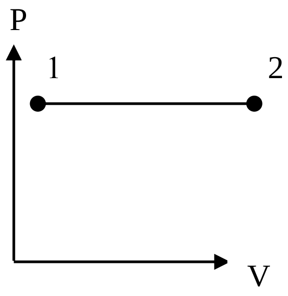




2- **Charles Law** :-

$V / T = C$, at $P = C$

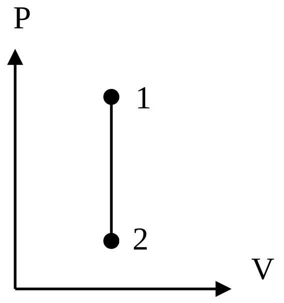
$V_1 / T_1 = V_2 / T_2$



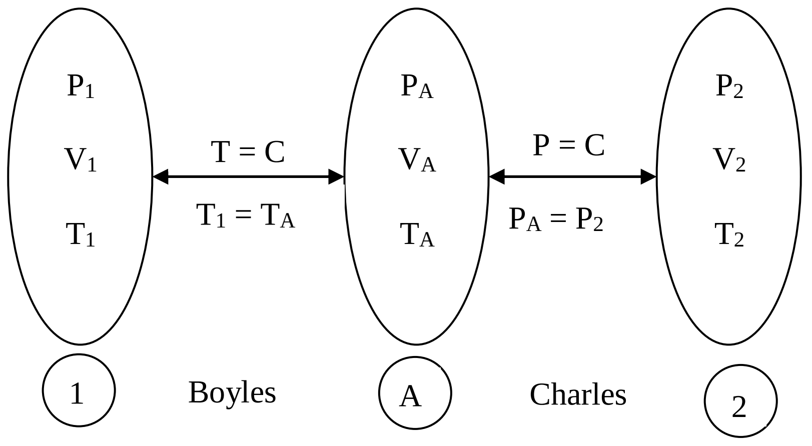
3- **Gaylosic** :-

$P / T = C$, at $V = C$

$P_1 / T_1 = P_2 / T_2$



The equation of states gas constant:-



1—A

$P_1 . V_1 = P_A . V_A$

$V_A = (P1. V_1)/ P_A$ ———— (1)

A —2

$V_A/T_A = V_2/T_2$

$V_A = (V_2 . T_A) / T_2$ ———— (2)

$(P_1 . V_1)/ P_A = (V2. T_A) / T_2$

$(P1. V_1)/ P_2 = (V_2 . T_1) / T_2$

$\boxed{(P_1 . V_1)/ T_1 = (P_2 . V_2) / T_2} \rightarrow \boxed{P. V /T = C} \div m$

$(P. V) / (m .T) = C / m = R$ = gas constant

$\boxed{P.V = m. R.T}$

$P. (V/m) = R.T$

$\boxed{P.\nu = R.T}$ ——— equation of state

EX: - The volume of an ideal gas at 2bar and 27C⁰ is 1m³ .calculate the final temperature if:-

(a)-Pressure increase to 4bar at constant volume.

(b)-Volume increase to 3bar at constant Pressure.

Solution:-

(a)-Pressure increase to 4bar at constant volume

$(P_1 . V_1)/ T_1 = (P_2 . V_2) / T_2$

$V= C \rightarrow [V_1 =V_2]$

$2 / (27 + 273) = 4 / T_2$

$T_2 = 600K^o$

(b)-Volume increase to 3bar at constant Pressure.

$P = C$

$V_1 / T_1 =V_2 / T_2$

$1/ 300 = 3/ T_2 \quad , T_2 =900K^o$

Energy Equation:-

$\boxed{Q_{12}-W_{12} = \Delta U_{12}}$ ———— Energy Equation

Specific heat:-

Cv = specific heat at constant volume.

Cp =specific heat at constant pressure.

$$\boxed{R = Cp - Cv}$$

$$\boxed{\gamma = Cp / Cv}$$

$$\boxed{Cv = R / \gamma - 1}$$

$$\boxed{Cp = \gamma R / \gamma - 1}$$

EX:- **2m^3 closed tank contain 5kg of gas at 25 C^o , 200 KJ of heat is added to the gas,**
What will be the final temperature and pressure if
R = 0.3 KJ/ Kg.K^0, Cp = 0.984 3 KJ/Kg.K^0.
Sol:-

Closed tank V = C

Cv = Cp – R

= 0.984 – 0.3

= 0.684 KJ/Kg.K^0

Q = Δ U [W = 0]

= m Cv (T_2 –T_1)

200 = 5* 0.684 *[T2 – (25+ 273)]

T_2 = 356.5 K^0

$P_2.V_2$ = m. R .T_2 or $P_1/T_1 = P_2 /T_2$

P_2 = (m. R .T_2)/ V_2

= (5*0.3*356.5) /10^2* 2 = 2.6736 bar

Real gases: -

We can be treating a gas as ideal gas at height pressure and low temperature above the critical point.

But when the pressure is low and the temperature is high we can not treat a gas as ideal gas, there fore other equation of state use to describe the relation between P.V.T.

1-Van der Waals' equation of state: - it proposed in 1873

$$\boxed{[(P+(a/v^2)](v-b) = R.T}$$

------------- Van-der Waals equation of state

The term (a/v^2) , in internal_Van der waals' equation , arises from the existence of entermolecular forces , and the term b is proportional to the volume occupied by the volume them selves .

$$\boxed{b = \frac{R\,Tcr}{8\,Pcr}}$$

$$\boxed{a = \frac{27\,R^2\,Tcr^2}{64\,Pcr}} \quad \text{----- or} \quad \boxed{a = 27\,b^2\,Pcr}$$

Where:-

Tcr:- temperature at critical point .

Pcr:- pressure at critical point .

$$R = \frac{Ru}{M}$$

Ru: - universal gas constant = 8.314 KJ/ Kmol .K°

M:- Molecular weight (Kg/ Kmol)

2- Beattie – Bridgeman equation of states :- it proposed in 1928

$$\boxed{P = \frac{Ru.\,T}{\nu`^2}\left[1-\frac{C}{\nu`\,T^3}\right](\nu` + B) - (A/\nu`^2)}$$

Where :-

$$\boxed{A = A_o\,[1-(a/\nu`)]} \qquad \boxed{B = B_o\,[1-(b/\nu`)]}$$

And A_o , a , B_o ,and C are constants having different values for different gases are given in table

$\nu`$ = molar specific volume = m^3/Kmol = M.ν

3-Benedct – Webb – Rubin equation of state :- it proposed in 1940

$$P = \frac{Ru.\,T}{v`} + \left[(Bo.\,Ru.\,T) - A_o - (C_o/\,T^2)\right](1/\,v`^2) + \left[(b.Ru\,.\,T) - a\right]/\,v`$$

$$+\,(a.\alpha)/\,v`^6 + \left[C/\,(v`^3\;T^2)\right]\left[1+(\gamma/\,v`^2)\right]\left[e^{-\gamma/v`2}\right]$$

The values of constant (a , A_o , b , B_o, C ,C_o , α , γ) are given in table.

The five constant in the beattie – Bridgeman equation are as follows:-

gas	A_o	a	B_o	b	C
Air	131.8441	0.01931	0.04611	-0.001101	$4.34*10^4$
Argon , Ar	130.7802	0.02328	0.03931	0.0	$5.99*10^4$
Carbon dioxide, Co_2	507.2836	0.07132	0.10476	0.07235	$6.60*10^5$
Helium ,He	2.1886	0.05984	0.01400	0.0	40
Hydrogen , H_2	20.0117	-0.00606	0.02096	-0.04359	504
Nitrogen , N_2	136.2315	0.02617	0.05040	-0.00691	$4.20*10^4$
Oxygen ,O_2	151.0857	0.02562	0.04624	0.004208	$4.8*10^4$

The eight constant in the benedct – Webb – Rubin equation are as follows:-

Gas	a	A_o	b	B_o
n-Butane , C_4H_{10}	190.68	1021.6	0.039998	0.12436
Carbon dioxide , CO_2	13.86	277.30	0.007210	0.04991
Carbon monoxide,CO	3.71	135.87	0.002822	0.05454
Methane ,CH_4	5.20	187.91	0.003380	0.04260
Nitrogen , N_2	2.54	106.73	0.002328	0.04074

Gas	C	C_o	α	γ
n-Butane , C_4H_{10}	$3.205*10^7$	$1.006*10^8$	$1.101*10^{-3}$	0.0340
Carbon dioxide , CO_2	$1.511*10^6$	$1.404*10^7$	$8.470*10^{-5}$	0.00539
Carbon monoxide, CO	$1.054*10^5$	$8.673*10^5$	$1.350*10^{-4}$	0.0060
Methane ,CH_4	$2.578*10^5$	$2.286*10^6$	$1.24*10^{-4}$	0.0060
Nitrogen , N_2	$7.379*10^4$	$105*8.16^4$	$1.272*10^{-4}$	0.0053

EX: - predict the pressure of (N_2) gas at T= 175 K^o and v = 0.00375m^3/Kg on basic of :-

a- The ideal equation of state.

b- The Van der waals' equation of state.

Then compare the values obtained to the actual value of 10000KPa, Take M = 28Kg/Kmol, Tcr = 126K^o, Pcr =3.39KPa.

Sol:-

a-The ideal equation

P.v = R.T

R= 8.314 / 28

= 0.297 KJ / Kg. K^o

P= R.T / v

= (0.297 * 175) / 0.00375

= 13860 KPa

$$\eta_{\text{error}} = \left| \left[(\text{Pact.} - \text{P obtained}) / \text{Pact.}\right] \right| * 100$$

$$= [(10000 - 13860) / 10000] * 100$$

$$= 38.6\ \%$$

b- The Van der waals' equation of state

$$[(P + (a / v^2)] (v - b) = R.T$$

Where:-

$$a = \frac{27 R^2 Tcr^2}{64 Pcr}$$

$$= \frac{27 * (0.297)^2 * (126.2)^2}{64 * (3.39 * 10^3)}$$

$$= 0.175$$

$$b = \frac{R\, Tcr}{8\, Pcr}$$

$$= \frac{0.297*126.2}{8(3.39*10^3)}$$

$= 0.00138$

$P = [(R.T)/ (v - b)] - (a / v^2)$

$$= \frac{0.297*175}{(0.00375-0.0138)} - \frac{0.175}{(0.00375)^2}$$

$= 9486$ Kpa

$$\eta_{\text{error}} = \frac{100000 - 9486}{10000} = 5.14\%$$

Ex:- Estimate the Pressure which would exerted by 3.7Kg of CO_2in (0.03) m^3 container at (215)K^o employing:-
a -The ideal equation of state.
b- The Van der waals' equation of state
Then compare the values obtained to the actual value of 7000 Kpa
Take M = 28Kg/Kmol, Tcr = 133K^o, Pcr =35bar.

Sol:-

a-The ideal equation

$P.v = R.T$

$R= 8.314 / 28$

$= 0.297$ KJ / Kg. K^o

$P=m. R.T / V$

$= (3.7*0.297*215) / 0.03$

$= 7848.93$ KPa

$$\eta_{\text{error}} = \left| [(P_{act.} - P_{obtained}) / P_{act.}] \right| *100$$

$$= \left| [(7000 - 7848.93) / 10000] \right| *100 = 12.12\ \%$$

b- The Van der waals' equation of state

$[(P+ (a / v^2)] (v - b) = R.T$

Where:-

$$a = \frac{27\,R^2\,Tcr^2}{64\,Pcr}$$

$$= \frac{27*(0.297)^2*(133)^2}{64*(35*10^2)}$$

$$= 0.187$$

$$b = \frac{R\,Tcr}{8\,Pcr}$$

$$= \frac{0.297*133}{8(35*10^2)} = 0.001408$$

$$P = [(R.T)/(v - b)] - (a / v^2)$$

$$= \frac{0.297*215}{((0.03/3.7) - 0.001408)} - \frac{0.187}{(0.03/3.7)^2}$$

$$= 6621.6\ Kpa$$

$$\eta_{error} = \left| [\ (Pact. - P\ obtained) / Pact.] \right| *100$$

$$\eta_{error} = \left| \frac{7000 - 6621.6}{7000} *100 \right|$$

$$= 5.4026\%$$

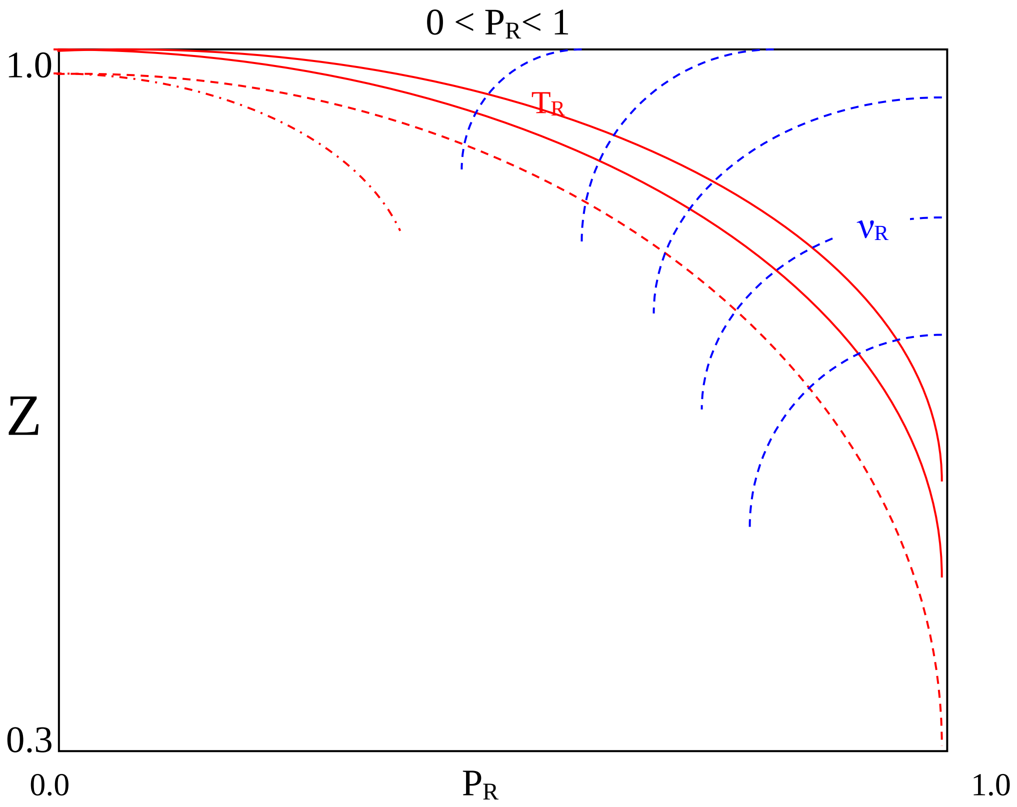
4 - Generalized compressible charts:-

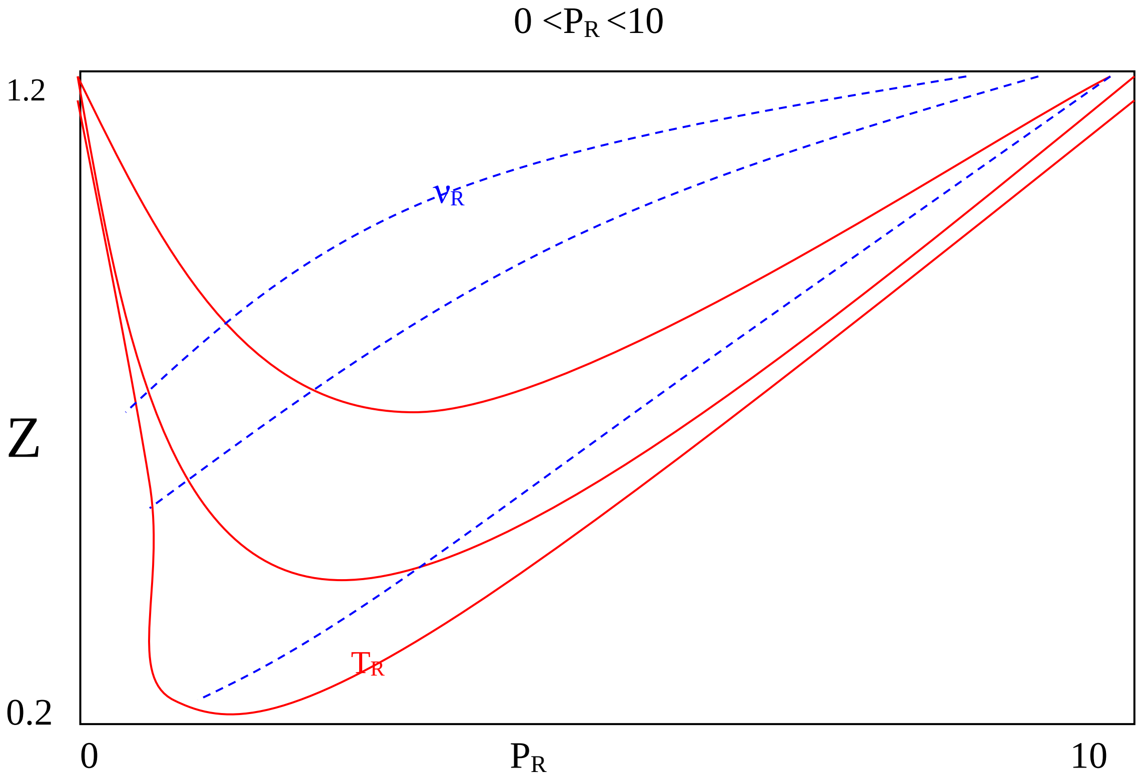
$\boxed{P.\,v = Z.R.T}$ ———————— Real gas equation

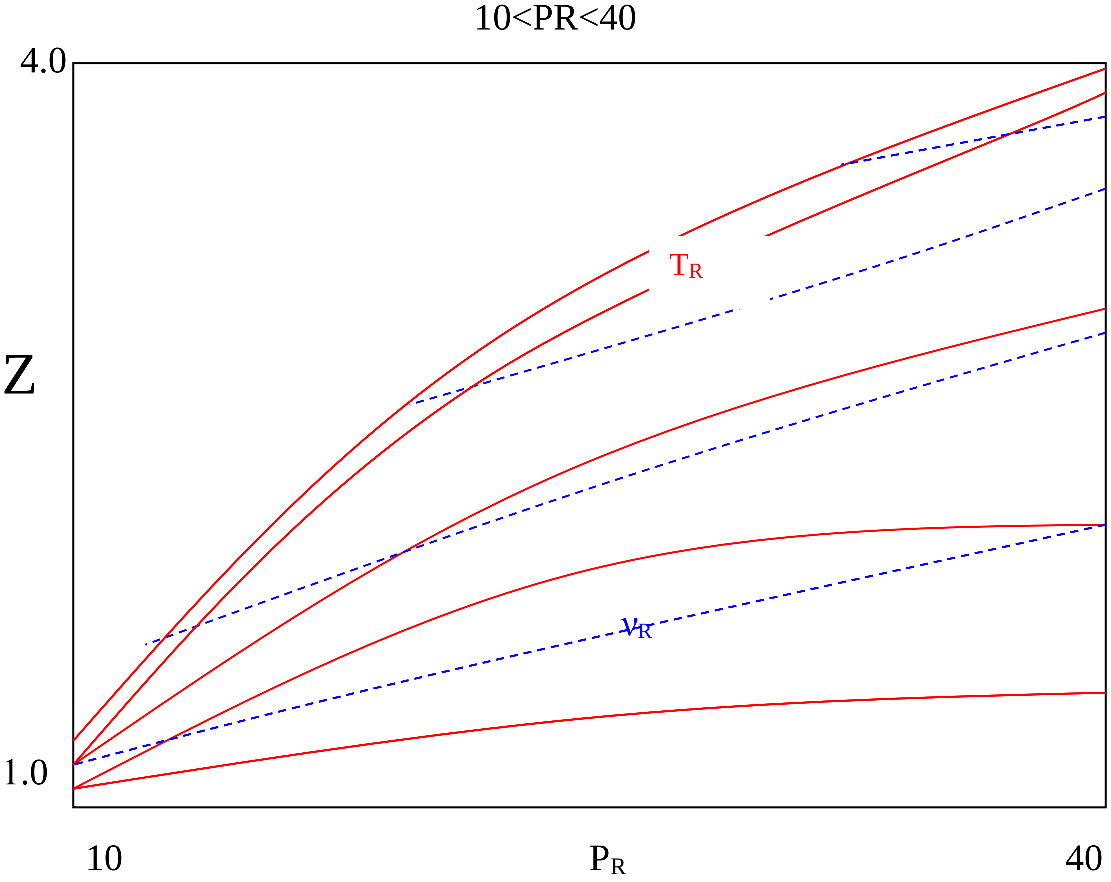
Compressibility factor (Z) is inserted in to the ideal gas equation as correction factor for real gas.

Z is function of two reduced Properties where:-

$P_R = P/ Pcr$, $T_R = T/ Tcr$, $v_R = v / vcr$


$0 < P_R < 1$


$0 < P_R < 10$


10<PR<40
4.0
1.0
Z
T$_R$
ν$_R$
10
P$_R$
40

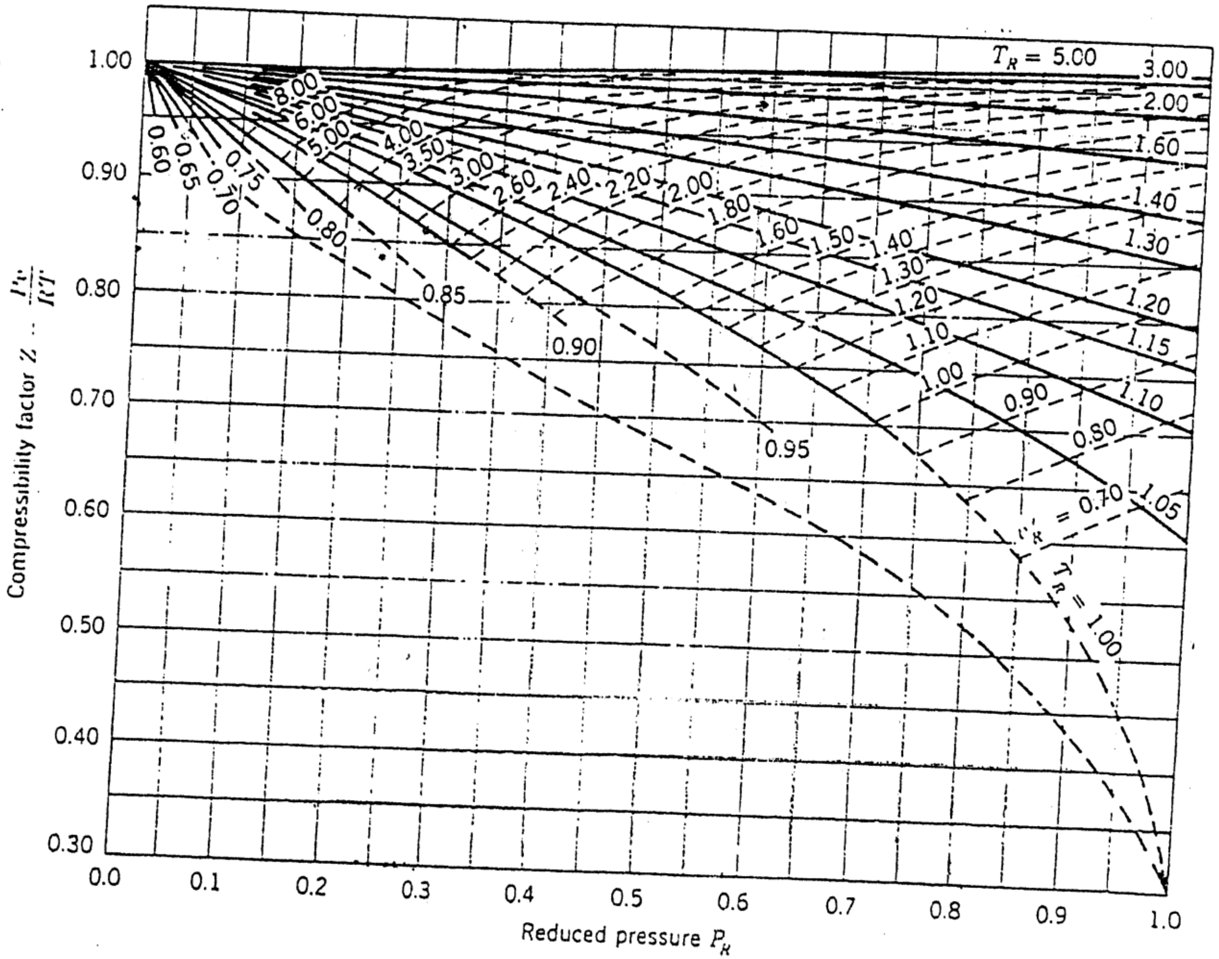


Figure 4-15 Generalized compressibility chart [L. C. Nelson and E. F. Obert: Generalized Compressibility Charts, *Chem. Eng.*, 61:203 (1954).]

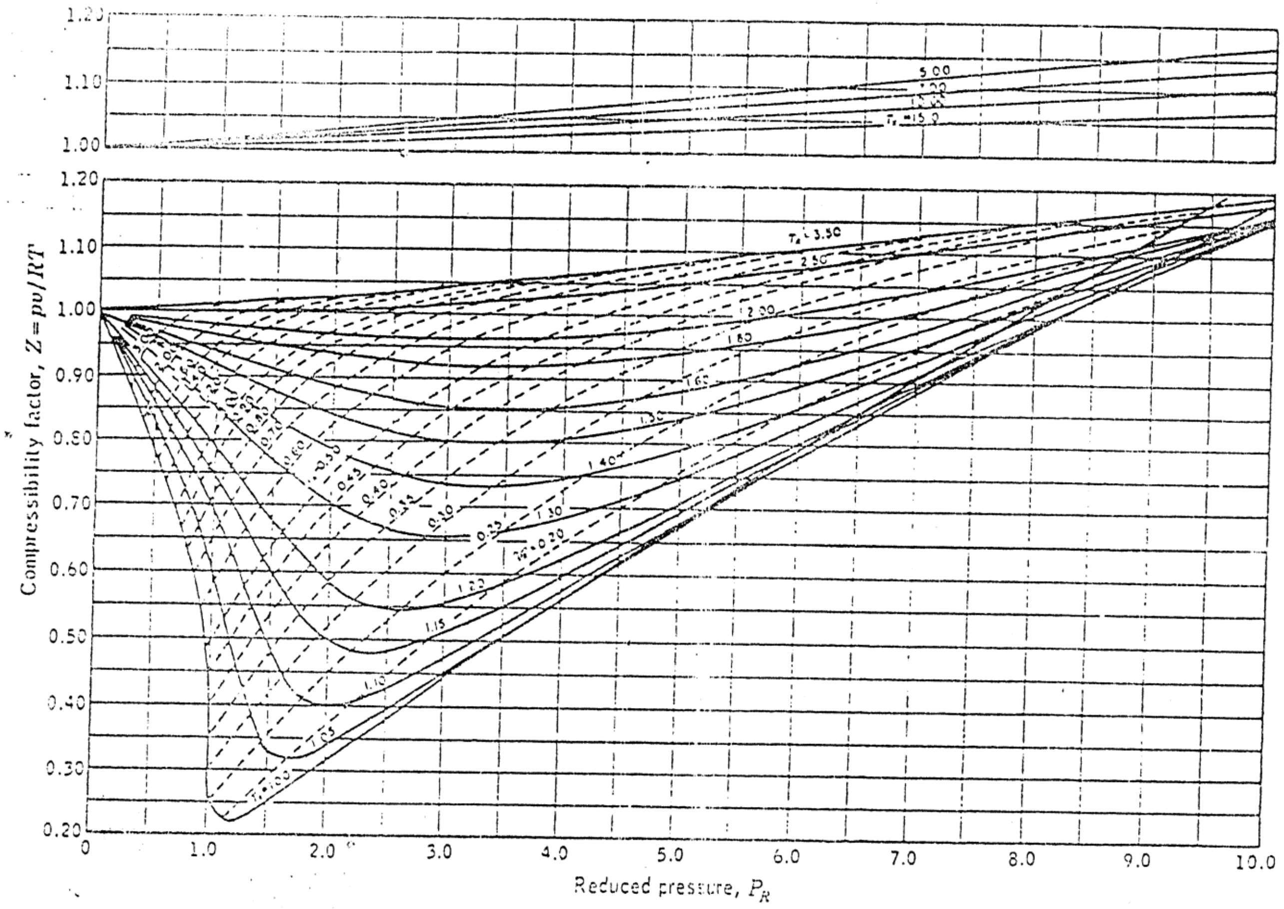


Figure A-15M Compressibility chart, low-pressure range. *Source:* V. M. Faires, "Problems on Thermodynamics," Macmillan, New York, 1962. Data from L. C. Nelson and E. F. Obert, Generalized Compressibility Charts, *Chem. Eng.*, 61 : 203 (1954).

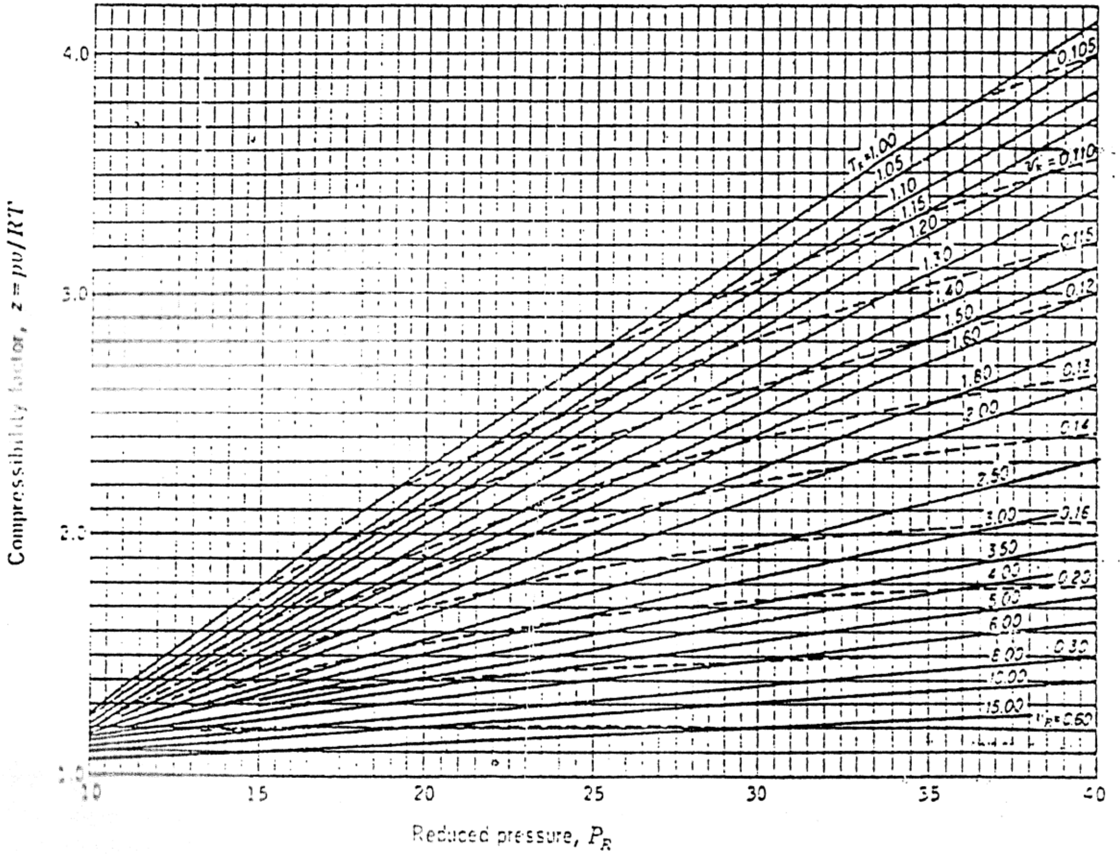


Figure A-26M Compressibility chart, high-pressure range. Adapted from E. F. Obert, "Concepts of Thermodynamics," McGraw-Hill, New York, 1960.

EX :- Determine the Specific volume of super heated water vapor at 10MPa and 400C° ,using :
a – ideal gas equation.
b- Generalize compressibility chart.
Compare the values obtained to the actual value of (0.02461m^3/Kg), take M=18.015 Kg / Kmol , Tcr = 647.3K° , Pcr = 22.09MPa .

Sol:-

a – ideal gas equation

$P.v = RT$

$R=8.314/M$

$R= 8.314/18.015$

$= 0.4615$ KJ/Kg.K°

$v_{ideal} = RT / P$

$= [0.4615 (400 + 173)]/ 10*10^3$

$= 0.03106$ m^3/Kg

$\eta_{error} = \left| [(v_{act.} – v_{obtained}) / v_{act.}] \right| *100$

$= \left| [(0.02641 – 0.03106)/ 0.02641] \right| *100 = 17.61\%$

b- Generalize compressibility chart

.

$P_R = P/Pcr$

$= 10 / 22.09$

$= 0.453$

$T_R = T/Tcr$

$= (400+273) / 647.3 = 1.04$

From the chart (1) , $Z = 0.85$

$v = Z.R.T/ P$

$=Z.\,v_{ideal}$

$= 0.85*0.03106$

$= 0.026401\,m^3/Kg$

$\eta_{error} = \left| [\,(v_{act.} - v_{obtained}) / v_{act.}] \right| *100$

$= \left| [(0.02641 - 0.026401)/\,0.02641] \right| *100 = 0.034\%$

EX :- Determine the Specific volume of R-12 at 1MPa and 50C° ,using :
a – ideal gas equation.
b- Generalize compressibility chart.
Compare the values obtained to the actual value of (0.01837m^3/Kg), take R=0.0688KJj/Kg.K°, Tcr = 384.7K° , Pcr = 4.01MPa .

Sol:-

a – ideal gas equation

$v_{ideal} = RT / P$

$= 0.0688 * 323 /1*10^3$

$= 0.02222\ m^3/Kg$

$\eta_{error} = \left| [\,(v_{act.} - v_{obtained}) / v_{act.}] \right| *100$

$= \left| [\,(0.018371 - 0.02222)/\ 0.018371\ .] \right| *100 = 20.9\ \%$

b- Generalize compressibility chart

$P_R = P/Pcr$

$= 1*10^6/4.01*10^6 = 0.249$

$T_R\ = T/Tcr$

$= 323/384.7 = 0.839$

From chart (1), Z = 0.83

$\nu = Z.\,\nu_{ideal}$

$= 0.83*0.02222$

$= 0.01844\ m^3/Kg$

$\eta_{error} = \left|\,[\,(\nu act. - \nu\ obtained) / \nu act.]\right| *100$

$= \left|\,[(0.018371 - 0.01844)/ 0.018371\right. *100 = 0.38\%$

When(p& ν) or (T & ν) are given instead of P&T we can using the same charts to determine the third properties by using another reduced property called Pseudo – reduced specific volume ν_R

$$\boxed{\nu_R = \frac{\nu}{(R.Tcr)/Pcr} = \frac{\nu\ .\ Pcr}{R.Tcr}}$$

EX:- 0.014m^3 tank contain 1Kg of R- 12 at 110C° ,determine the pressure using:
a – ideal gas equation.
b- Generalize compressibility chart.
R=0.0688KJ/Kg.K°, Tcr = 384.7K° , Pcr = 4.01MPa

Sol:-

a – ideal gas equation

$P_{ideal} = (m\ .R.T) / V$

$= (1*0.0688*383)/ 0.014$

$P = 1.8822MPa$

b- Generalize compressibility chart

$\nu_R = (\ \nu\ .\ Pcr) / R.Tcr$

$= (0.014*4.01*10^3) / (0.0688*384.7) = 2.121$

$T_R = T/Tcr$

$= 383/384.7 = 0.995$

From chart (1)

$Z = 0.87,\ P_R = 0.42$

$P = P_{ideal*}Z$

$= 1.8822 * 0.87$

$= 1.6441 MPa$

$P_R = P/Pcr$

$P = P_R . Pcr$

$= 0.42 * 4.01 = 1.6441$ MPa

EX:- Estimate the temperature of Ethan gas (C_2H_6) when it has pressure of 68.4bar and v = 0.0104 m^3/Kg by using :
a – ideal gas equation.
b- Generalize compressibility chart.
Take M = 30Kg / Kmol , Pcr = 4.88MPa , Tcr = 305.5
Sol:-

a – ideal gas equation

$R = 8.314/30$

$= 0.277$ KJ/Kg.K°

$T = (P.v) / R$

$= (68.4*10^2 * 0.0104) / 0.277 = 257$ K°

b- Generalize compressibility chart

$P_R = P/Pcr$

$= 68.4*10^5/4.88*10^6 = 1.4$

$v_R = (v. Pcr) / R.Tcr$

$= (0.0104*4.88*10^3) / 0.277*305.5 = 0.6$

From chart (2)

$Z = 0.7, T_R = 1.2$

$T_R = T/Tcr$

$T = T_R.Tcr$

$= 1.2 * 305.5 = 367$K°

Or ..

$P.v = Z.R.T$

$T = P.v / Z.R$

$= (10^2 * *68.4*0.0104) / (0.7*0.277) \quad = 366.869\ K^o$

EX :- A 3.2 m^3 tank contains 100Kg of N_2 at 225K° Determine the pressure in the tank using :-

a – ideal gas equation.

b- The Van der waals' equation of state

c- Generalize compressibility chart

Compare the values obtained to the actual value of (2000KPa), take M= 28Kg/Kmol, Tcr = 126.2K°, Pcr = 3.39MPa.

Sol:-

$R = 8.314/28$

$= 0.297\ KJ/Kg.K^o$

$P_{ideal} = (m.R.T) / V$

$= (100*0.297*225) / 3.2 = 2043.6KPa$

$\eta_{error} = |[(P_{act.} - P_{obtained}) / P_{act.}]| *100$

$= |[(2000-2043.6) / 2000]| *100 = 2.18\%$

b- The Van der waals' equation of state.

$[P + (a / v^2)](v - b) = R.T$

$$a = \frac{27 R^2 Tcr^2}{64 Pcr}$$

$$= \frac{27*(0.297)^2 *(126.2)^2}{64*(3.39*10^3)}$$

$= 0.175$

$$b = \frac{R\ Tcr}{8\ Pcr}$$

$$= \frac{0.297*126.2}{8(3.39*10^3)} = 0.00138$$

$\nu = V/m$

$= 3.27/100 = 0.0327 m^3/Kg$

$P = [(R.T)/(\nu - b)] - (a/\nu^2)$

$$= \frac{0.297 * 225}{(0.0327 - 0.00138)} - \frac{0.175}{(0.0327)^2} = 1970 KPa$$

$ŋ_{error} = |[(P_{act.} - P_{obtained}) / P_{act.}]| * 100$

$= |[(2000-1970)/2000]| * 100 = 1.5\%$

c- Generalize compressibility chart

$P = Z.R.T/\nu$

$\nu_R = (\nu . P_{cr}) / R.T_{cr}$

$= (0.0327 * 3.39 * 10^3) / 0.297 * 305.5 = 2.957$

$T_R = T/T_{cr}$

$= 225/126.2 = 1.78$

From chart

$P_R = 0.59, Z = 0.98$

$P = Z * P_{ideal}$

$P = 0.98 * 2043.6 = 2002.1 KPa$

$ŋ_{error} = |[(P_{act.} - P_{obtained}) / P_{act.}]| * 100$

$= |[(2000-2002.7)/2000]| * 100 = 0.135\%$

EX: - Predict the Pressure of CO_2 at 313 K° &0.00455m^3/Kg on basic of a – ideal gas equation.
b- The Van der waals' equation of state
c- Generalize compressibility chart
Take M= 44Kg / Kmol Tcr = 304.2, Pcr = 7.39 MPa

Sol:-
a-The ideal equation

$P.\nu = R.T$

R= 8.314 / 44 = 0.189 KJ / Kg. K°

P= R.T / ν

= (0.189 * 313) / 0.00455= 12.9984 MPa

The Van der waals' equation of state

$$\left[\left(P+ (a / \nu^2)\right)\right] (\nu - b) = R.T$$

Where:-

$$a = \frac{27\,R^2\,Tcr^2}{64\,Pcr}$$

$$= \frac{27*(0.189)^2*(304.2)^2}{64*(7.39*10^3)}$$

= 0.1887

$$b = \frac{R\,Tcr}{8\,Pcr}$$

$$= \frac{0.189*304.2}{8(7.39*10^3)} = 0.0009725$$

$$P = [(R.T)/ (\nu - b)] - (a / \nu^2)$$

$$= \frac{0.189*313}{(0.0045 - 0.000973)} - \frac{0.1887}{(0.00455)^2} = 16.497\text{Mpa}$$

Generalize compressibility chart

T_R = T/Tcr

= 313/304.2= 1.029

ν_R = (ν . Pcr) / R.Tcr

= $(0.00455*7.39*10^3) / (0.189*304.2) = 0.585$

From table Z = 0.56 &P_R = 1.0
P. ν =Z.R.T

P = Z.R.T / ν

= (0.56*0.189*313)/0.00455=7.28 MPa

P = P_R *Pcr

= 1.0*7.39

= 7..39MPa

Substance can be classified to:-

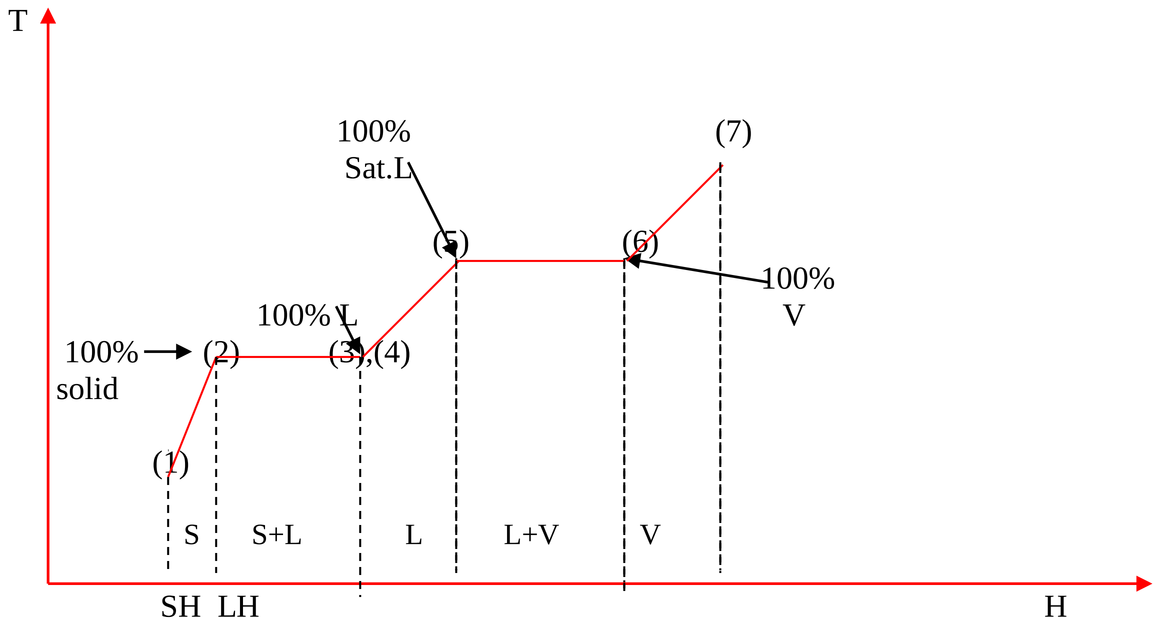
1- **Pure Substance**
2- **Mixture Substance can be classified to:-**

a- homogenous
b- Acrogenous

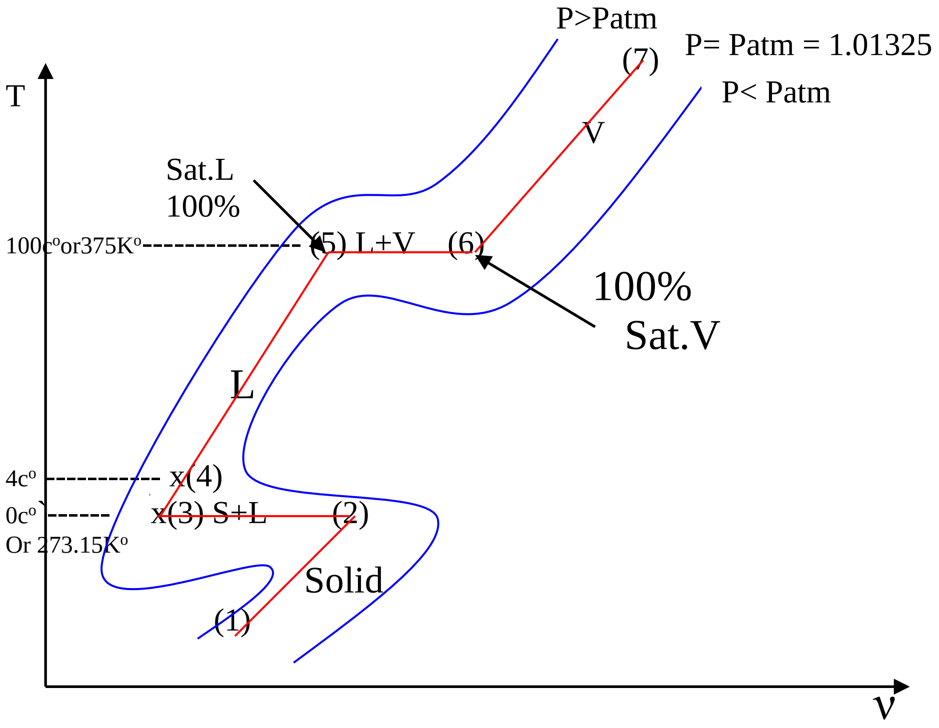
Phase of the pure substance:-

Solid ⇄ Liquid ⇄ Gas

Freezing or solidification: - Liquid → solid
Melting or fusion: - solid → Liquid
Vaporization: - Liquid → Gas
Condensation: - Gas → Liquid
Sublimation: - solid → Gas



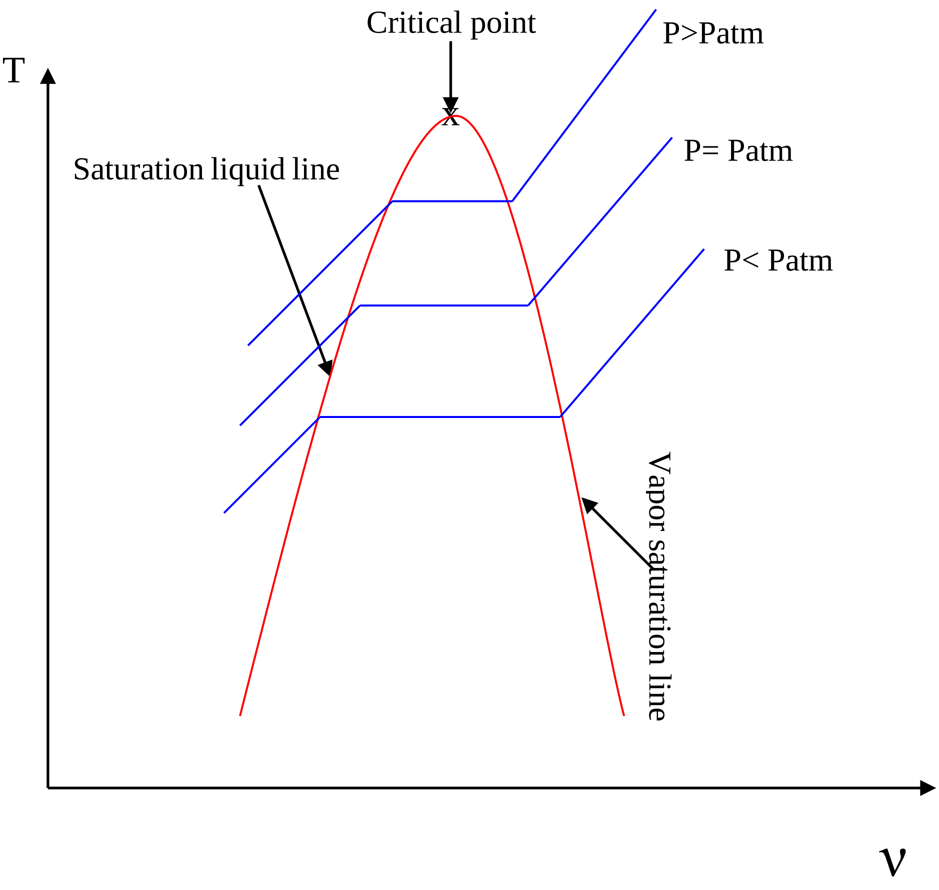
SH = sensible heat , LH: - latent heat



$T<T_S$: - un saturated Liquid

$T = T_S$: - saturated Liquid, dry &saturated steam = wet steam

T> Ts :- super heat



Tcr =temperature at Critical point = 647.15 ^oK

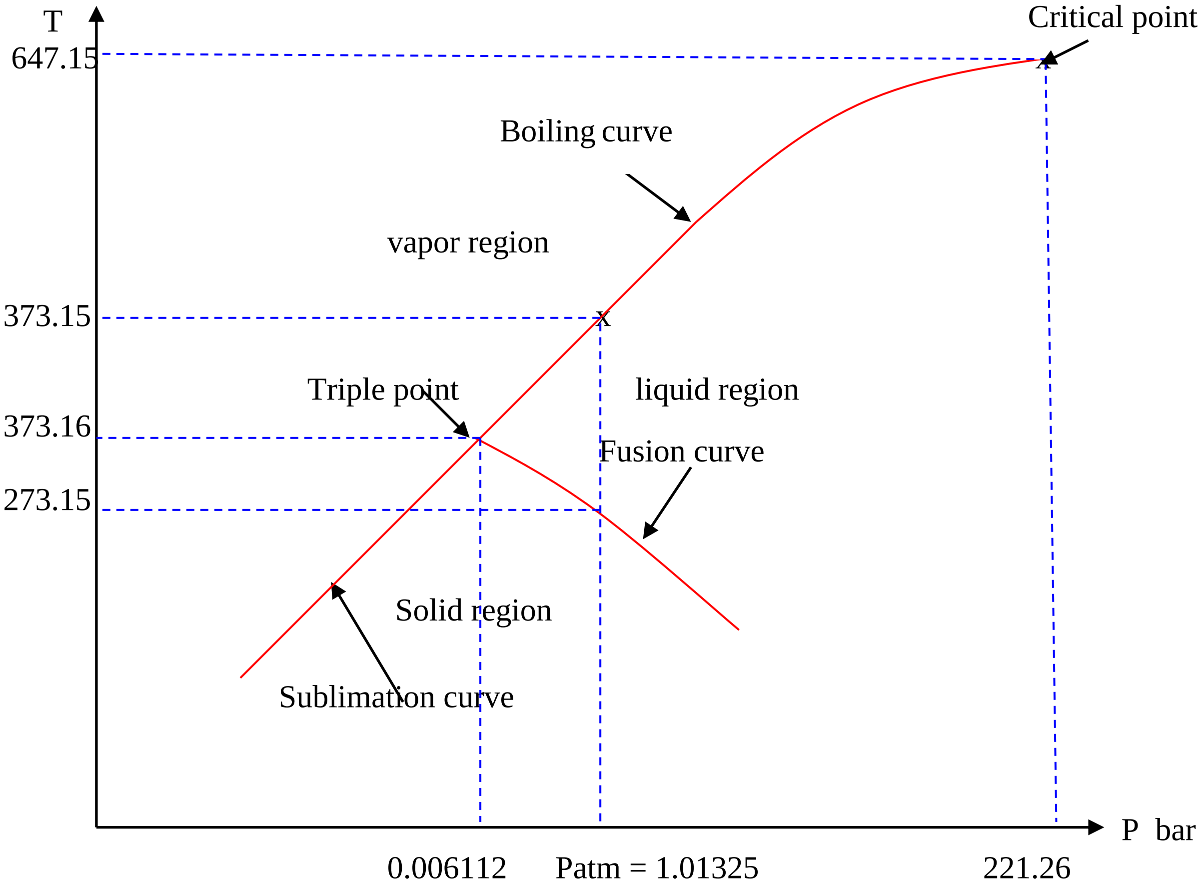
Pcr =pressure at Critical point = 221.2 bar

P>Patm

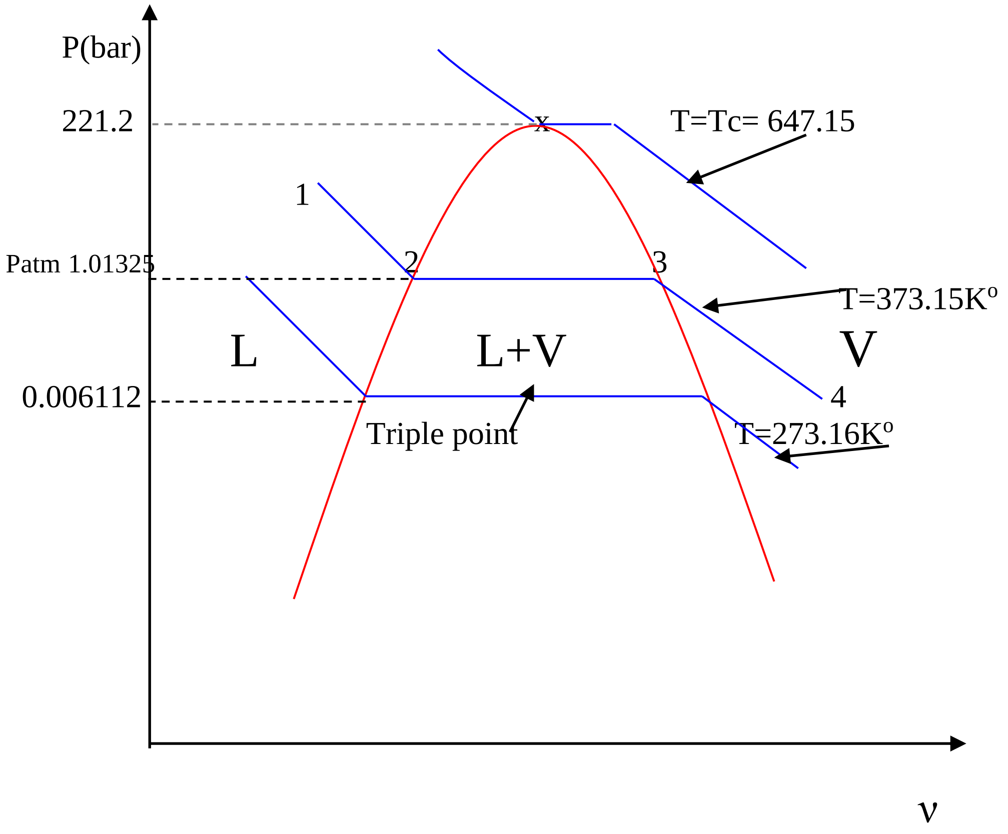
1- Tmelt <T
2- Tboil = T_s >>T
3- ν << ν

P<Patm

1- Tmelt >T
2-Tboil = T_s <<T
3-ν << ν

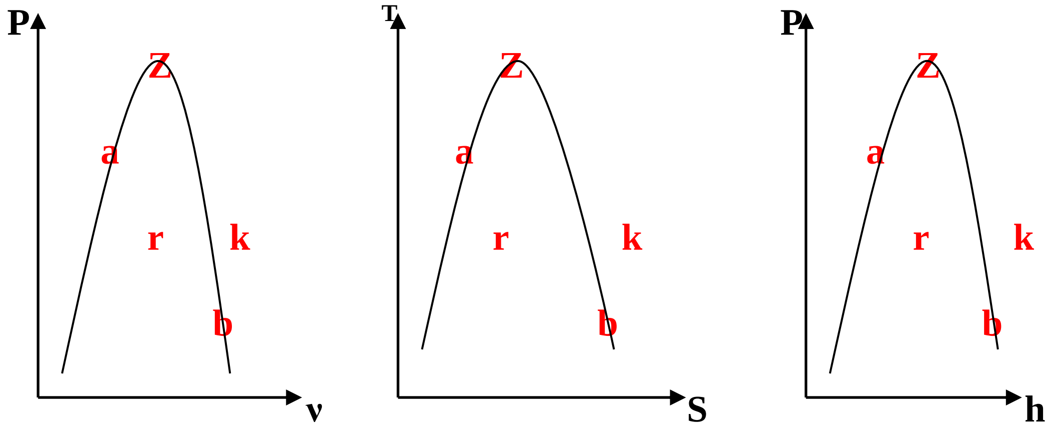


`



1—— 2: un saturated water.
2: saturated water.
2——3: wet steam.
3: saturated steam.
3:—— 4 super heat steam.

Phase pure substance at P.v &T.S & P.h diagram .



point	Condition
Z	Critical point
r	Wet mixture
a	Sat. liquid
b	Sat. vapor
k	Super vapor

Dryness fraction ,X :-The mass of dry vapor in 1 Kg of the mixture .
Wetness fraction ,(1 - X):- is defined as the mass of the liquid in 1 Kg of the mixture.
Note that for dry and saturated vapor ,X= 1 and that for saturated liquid ,X= 0.

Steam tables:-

Saturated state properties:-

The saturation pressure and corresponding saturation temperatures of steam are tabulated in parallel columns in the first table, for pressure ranging from 0.006112bar to the critical pressure of 221.2bar.
The specific volume, internal energy, enthalpy, and entropy are also tabulated for the dry saturated vapor at each pressure and corresponding saturation temperature.

The suffix (g) is used to denote the dry saturated state (u_g , h_g , v_g , s_g)
The suffix (f) is used to denote the saturated liquid state (u_f , h_f , v_f , s_f)

Properties of wet vapor:-

The volume of the liquid is usually negligibly small compared to the volume of dry saturated vapor.

$$\boxed{v = x.v_g}$$

$$\boxed{u = u_f + x\,(u_g - u_f)}$$

$$\boxed{h = h_f + x\, h_{fg}}$$

$$\boxed{s = s_f + x\; s_{fg}}$$

Properties of superheated vapor:-

For steam in the superheat region temperature and pressure are in dependent properties.
The tables of properties of super heated steam range in pressure from 0.006112bar to the critical pressure of 221.2 bar, and there is an additional table of supercritical pressure up to 1000bar.
For pressure above 70bar the internal pressure is not tabulated ,it can be found when required using equation as following:-

$$\boxed{u = h - p.\,v}$$

For example steam at 80bar, 400C°has an enthalpy, h= 3139KJ/Kg, and specific volume of $3.428*10^{-2}$ m^3/Kg therefore,
$u = h - p.\,v$

$= 3139 – (80*10^2 * 0.03428)$

u $=2864.8$KJ/Kg

Interpolation

for properties which are not tabulated exactly in the tables it is necessary to interpolate between the values tabulated for example ,dry sat. steam at 9.8bar , it is necessary to interpolate between the values given in the tables .

h_g at 9.8bar = h_g at 9bar + (9.8 -9)/(10 -9) *(h_g at 10bar - h_g at 9bar)

EX:- find the specific volume , enthalpy , and internal energy of wet steam at 18bar , dryness fraction 0.9.

Sol:-

$\mathbf{v = x.v_g}$

$= 0.9*0.1104 = 0.0994 m^3/Kg$

$\mathbf{h = h_f + x\, h_{fg}}$

$= 885+ 0.9*1912 =2605.8$KJ/Kg

$\mathbf{u = u_f +x\,(u_g – u_f)}$

$= 883 +0.9(2598 – 883) = 2426.5$KJ/Kg

EX: - find the specific volume, and internal energy oft steam at 18bar, &h=1841KJ/Kg .

Sol:-

$\mathbf{h = h_f + x\, h_{fg}}$

$x= \mathbf{(h - h_f)/ h_{fg}}$

$= (1841 – 885) / 1912 = 0.5$

$\mathbf{v = x.v_g}$

$= 0.5*0.1104 = 0.0552 m^3/Kg$

$\mathbf{u = u_f +x\,(u_g – u_f)}$

$= 883 +0.5(2598 – 883) = 1740.5$KJ/Kg

EX:- find properties of steam at 110bar , &v = 0.0235 m^3 /Kg .

Sol:-

From steam table at 18bar, $v_g = 0.01598$

0.0235 >0.01598, Steam is super heat

$v = 2.035*10^{-2} m^3/Kg$, h = 3075 KJ/Kg , s = 6.143 KJ/Kg .K°, T= 400C°

u = h -p. v

$= 3075 - 110*10^{2} * 0.0235 = 2816.5 KJ/Kg$

$(\Delta T)_s = T_s - T_{SH}$

$= 400 - 318 = 82C^o$

EX: - find properties of steam at 110bar, & $(\Delta T)_s = 92K^o$.

Sol:-

$T_{SH} = T_s + (\Delta T)_s$

$= 92 + (318+273)$

$= 683K^o$

$= 683 - 273$

$= 410\ C^o$

$$h_{410} = h_{400} + \frac{410 - 400}{425 - 400}(h_{425} - h_{400})$$

$$h_{410} = 3075 + \frac{410 - 400}{425 - 400}(3153 - 3075) \qquad = 3106.2\ KJ/Kg$$

$$v_{410} = 2.350*10^{-2} + \frac{410 - 400}{425 - 400}(2.514*10^{-2} - 2.350*10^{-2}) == 0.024156\ m^3/Kg$$

$$s_{410} = 6.143 + \frac{410 - 400}{425 - 400}(6.257 - 6.143) \qquad = 6.1886\ KJ/Kg.K^o$$

$u = h - P.v$

$= 3106.2 - (110*10^{2}*0.024156)$

$= 2840.484\ KJ/Kg$

EX: - complete the tables:-

P(bar)	T (C^o)	v (m^3/Kg	X	ΔT	h(KJ/Kg)	u(KJ/Kg.)
20	212.4	0.09957	1	0	**2799**	2600
12	**188**	0.1461	0.895	-	2575.41	**2400**
3	**200**	0.7166	-	66.5	2866	2651
40	450	0.08	**-**	**199.7**	3330	3010
7	**165**	0.2728	**1**	0	2764	2573
60	**570**	0.060034	-	294.4	3586.2	3210.5
11	184.1	0.1774	1	0	**2781**	2586
26	**226**	0.0485	0.6304	-	2126.2	**2000**
10	200	0.2061	-	**20.1**	2829	2623
50	**540**	0.0725	-	276.1	3526.2	3103.2
40	250.3	0.0498	**1**	0	2801	2602
30	400	0.0993	-	**166.2**	3231	2933
7	**165**	0.2728	1	**0**	2764	2573
170	**485**	0.01602	-	136.7	3233	2960.66

Steam process &it is drawing on P - v &T-S diagram:-

1- isochoric or constant volume process :-

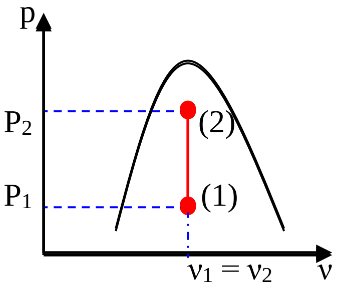
$Q12 = (U_2-U_1) +W$

Since no work is done, we therefore have

$$\boxed{q12 = u_2 - u_1}$$

or for mass ,m, of the working substance
$Q12 = U_2-U_1$

$$\boxed{\mathbf{Q12 = m\ Cv(T_2-T_1)}}$$



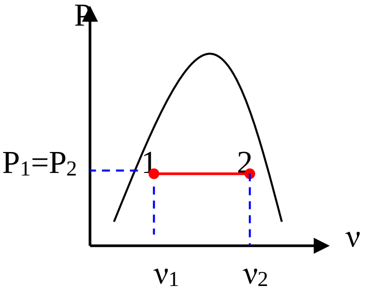
2-isobaric or constant pressure Process:-

$$\boxed{\mathbf{Q12 = \Delta U_{12}+W_{12}}}$$

$$\boxed{q_{12} = h_2 - h_1}$$

$$\boxed{W_{12} = P\,(v_2 - v_1)}$$

$$\boxed{Q12 = m\,Cp\,(T_2 - T_1)}$$

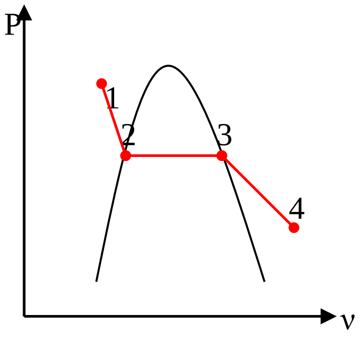


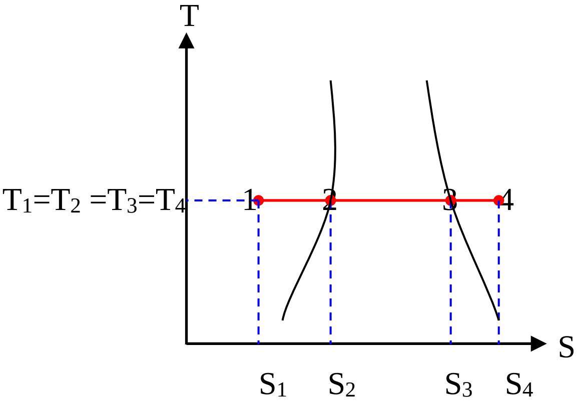
3-isothermal or constant temperature process:-

$$\boxed{q_{12} = T.\,\Delta S_{12} = T.\,(S_2 - S_1)}$$

$$\boxed{Q12 = \Delta U_{12} + W_{12}}$$

$$\boxed{P.\,v = C}$$





4-reversible adiabatic or isentropic (constant entropic) process:-

$Q12 = (U_2 - U_1) + W$

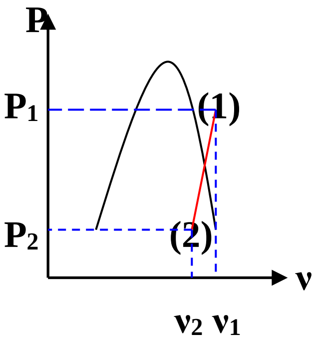
And for an adiabatic process:-

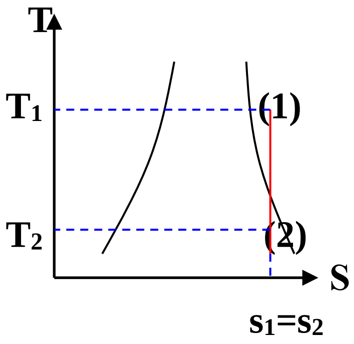
$Q = 0$

$$\boxed{W_{12} = u_1 - u_2}$$

$$\boxed{P.\,v^{\gamma} = C}$$

$$\boxed{W_{12} = \frac{P_1 .v_1 - P_2 . v_2}{\gamma - 1} = \frac{R\,(T_1 - T_2)}{\gamma - 1}}$$



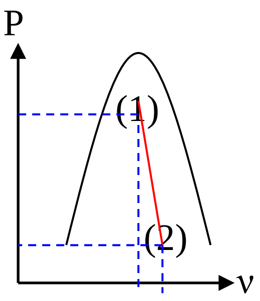


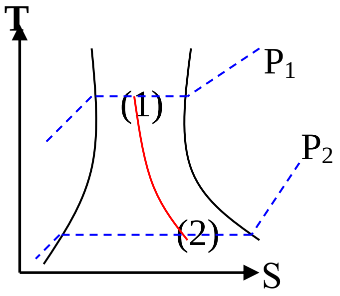
5- Polytropic processes:-

$$\boxed{P.v^n = C}$$

$$\boxed{w_{12} = \frac{P_1.v_1 - P_2.v_2}{n - 1}}$$

$$\boxed{q_{12} = \frac{P_1.v_1 - P_1.v_1}{n - 1} + (u_2 - u_1)}$$





EX:- 0.05 Kg of dry sat. Steam is heated at constant pressure of 2bar until volume occupied is 0.0658 m^3 ,calculate the heat supplied and the work done .

Sol:-

$h_1 = h_g$ at 2bar = 2707KJ/Kg dry &sat.

$v_1 = v_g$ at 2bar = 0.8856 m^3/Kg

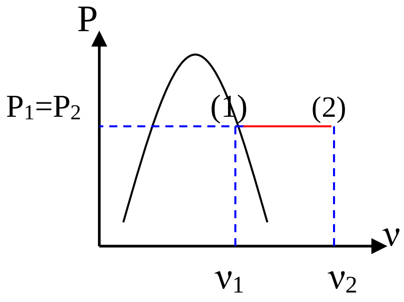
$P_1 = P_2$= 2bar at constant pressure

$v_2 = V_2 / m$

= 0.065/0.05

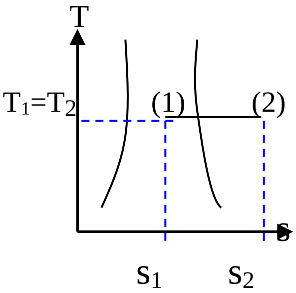
v_2 = 1.316m^3/Kg &P= 2bar from superheated table the temperature of steam is 300°C &h_2 = 3072KJ/Kg

$Q_{12} = m (h_2 - h_1)$
= 0.05(3072-2707)
= 18.25KJ
$W = P_2.v_2 - P_1 v_1$
$P_1 = P_2$= 2bar at constant pressure
$W = P (v_2 - v_1)$
= $2*10^5$ (1.316 -0.8856)
= 4.304KJ



EX: - steam at 7bar and dryness friction 0.9 in cylinder behind a piston isothermally and reversibly to a pressure of 1.5bar. Calculate the change of internal energy & change of enthalpy per Kg of steam work done. The heat supplied during the process is found to be 400KJ/Kg.

Sol:-



7bar & x=0.9 from table

$t_1 = t_2$ =165°C at constant temperature

u_{f1}= 696 KJ/Kg, u_g = 2573, h_{f1}= 697, h_{fg} = 2067 KJ/Kg

$u_1 = u_{f1} + x (u_g - u_{f1})$

$= 696 + 0.9\,(2573 - 696) = 2385.3$ KJ/Kg

$h_1 = h_{f1} + x\,h_{fg}$

$= 697 + 0.9 * 2067 = 2557.3$ KJ/Kg

$t_1 = t_2 = 165^oC$ at constant temperature

$P_2 = 1.5$bar, $t_2 = 165^oC$ there for point 2 is super heat

$$u_2 = 2580 + \frac{165-150}{200-150}(2656-2580) = 2602.8 \text{ KJ/Kg}$$

$$h_2 = 2773 + \frac{165-150}{200-150}(2873-2773) = 2803 \text{ KJ/Kg}$$

$\Delta u_{12} = u_2 - u_1$

$= 2602.8 - 2385.3 = 217.5$ KJ/Kg

$\Delta h_{12} = h_2 - h_1$

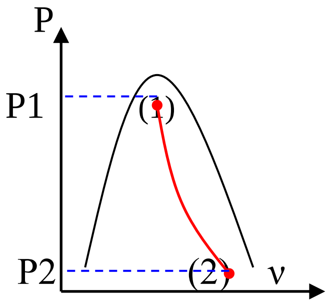
$= 2803 - 2557.3 = 245.7$ KJ/Kg

$w_{12} = q_{12} - \Delta u_{12}$

$= 400 - 217.5 = 182.5$ KJ/Kg

EX: - in steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.95 , and the expansion follows the low P. $v^{1.1}$ = constant ,down to the pressure of 0.34 bar. Calculate the work done per kg of steam during the expansion, and the heat flow per kg of steam to or from the cylinder walls during the expansion.

Sol:-



$P_1 = 7$bar, $x = 0.95$, from table

$v_{g1} = 0.2728$ m^3/kg, $u_{f1} = 696$ KJ/Kg, $u_{g1} = 2573$ KJ/Kg,

$v_1 = x.\,v_{g1}$

$= 0.95 * 0.2728 = 0.2592 m^3/kg$

$u_1 = u_{f1} + x\,(u_g - u_{f1})$

$= 696 + 0.95(2573-696) = 2476.8$ KJ/Kg

$P1/P2 = (\mathbf{v}_2 / \mathbf{v_1})^n$

$\mathbf{v}_2 = \mathbf{v_1}\,(P_1/P_2)^{1/n}$

$= 0.2592*(7/0.34)^{1/1.1} = 4.054\ m^3/kg$

From steam table at$P_2 = 0.34$ bar, $v_{g2} = 4.649\ m^3/kg > \mathbf{v}_2$ there for steam is wet at state2

$v_2 = x.\ v_{g2}$

$x = v_2 / v_{g2}$

$= 4.054 / 4.649 = 0.873$

$u_{f2} = 302$ KJ/Kg, $u_{g2} = 2472$ KJ/Kg,

$u_2 = u_{f2} + x\,(u_g - u_{f2})$

$= 302 + 0.873(2472-302) = 2196.4$KJ/Kg

$$\mathbf{w_{12}} = \frac{\mathbf{P_1.v_1 - P_2.v_2}}{\mathbf{n - 1}}$$

$$\mathbf{w_{12}} = \frac{10^2(7*0.2592 - 0.34*10^5*4.05)}{1.1 - 1} = 436 \text{ KJ/Kg}$$

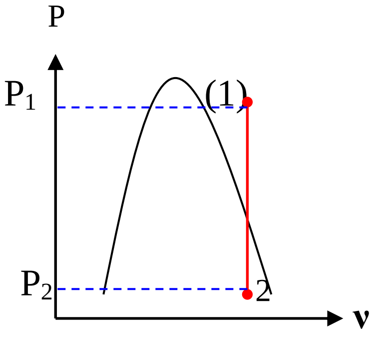
$q_{12} = (u_2 - u_1) + w$

$= (2196.4 - 2476.8) + 436$

$= 155.6$ KJ/Kg

EX: - closed vessel has a volume 0.025m^3 contained a steam at 80bar &350°C is cooled until pressure is 50bar calculate mass of steam &heat removing during the process.

Sol:-



Closed vessel ν = constant, $\boldsymbol{\nu_1 = \nu_2}$

P_1 =80bar &350°C from superheated table , $\nu_1 = 0.02994\ m^3/kg$,

$h_1 = 2990$ KJ/Kg , $s_1 = 6.133$ KJ/Kg.°k,

$u_1 = h_1 - P.\ \nu_1$

$= 2990 - 80*10^2\ 0.02994 = 2750$ KJ/Kg

$m = V/\ \nu$

$= 0.025/0.02994 = 0.835$Kg, mass of steam

Point 2 , $\boldsymbol{\nu_2} = 0.02994\ m^3/kg$,& $P_2 = 50$bar , $\boldsymbol{\nu}g_2 = 0.03944 > \boldsymbol{\nu_2}$ there for stem is wet

$x = \nu_2 / \nu_{g2}$

$= 0.02994 / 0.03944 = 0.75912$

$U_2 = u_{f2} + x\ (u_g - u_{f2})$

$= 1149 + 0.75912(2597-1149) = 2248.20$ KJ/Kg

$q_{12} = u_2 - u_1$

$= 2248.20 - 2750 = -502.27$ KJ/Kg

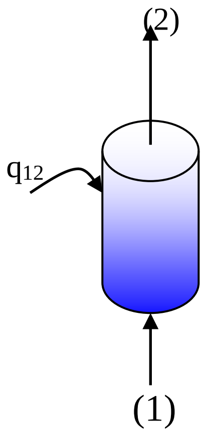
Steam power plant

$\boxed{\mathbf{q_{12} - w_{12}} = \Delta h_{12} + \Delta P.E_{12} + \Delta K.E_{12}}$ ..steady flow energy equation (S.F.E.E)

Application of steady flow energy equation:-

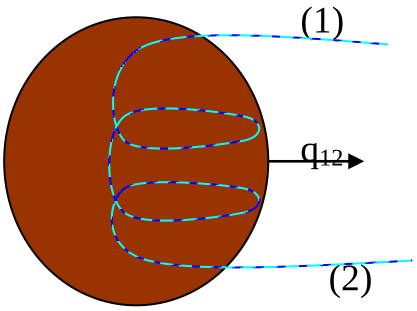
1- Steam boiler:-

$\boxed{\mathbf{q_{12} = h_2 - h_1}}$ …..at constant pressure $h_2 > h_1$, q_{12} = +ve

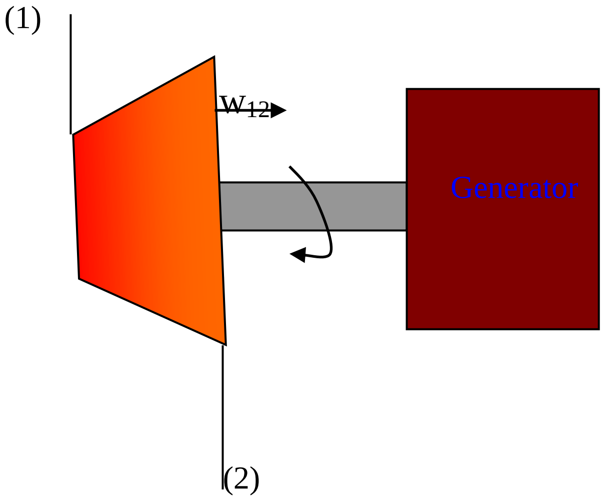


2- condenser:-

$\boxed{q_{12} = h_2 - h_1}$ ………at constant pressure $h_2 < h_1$, $q_{12} = -$ ve



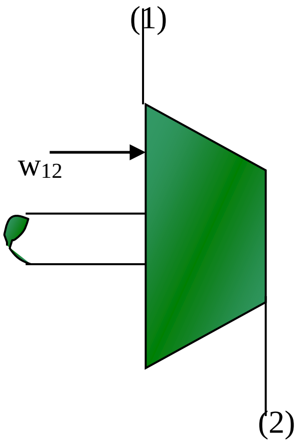
3-Steam turbine:-



$\boxed{w_{12} = h_1 - h_2}$ ………………$h_1 > h_2$, $w_{12} = +$ ve , $P_1 > P_2$

4- Compressor:-

$\boxed{w_{12} = h_1 - h_2}$ ……….. $h_1<h_2$, $w_{12} = -$ ve, $P_1<P_2$



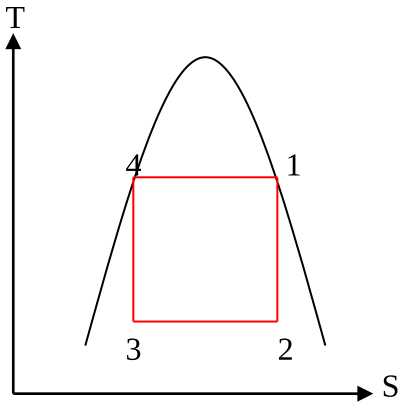
5-Throttle:-

$\boxed{h_1 = h_2}$ ……… $P_1>P_2$

Stem cycles

1-The Carnot cycle:-

.



Carnot french engineer ,showed in his paper 1824 the most efficient possible cycle is one in which all the heat supplied is suplied at one fixed temperature, and all the heat rejected is rejected at lower fixed temperature , from T-S diagram:-

Process 1 to 2 is isentropic expansion from T_1 toT_2.

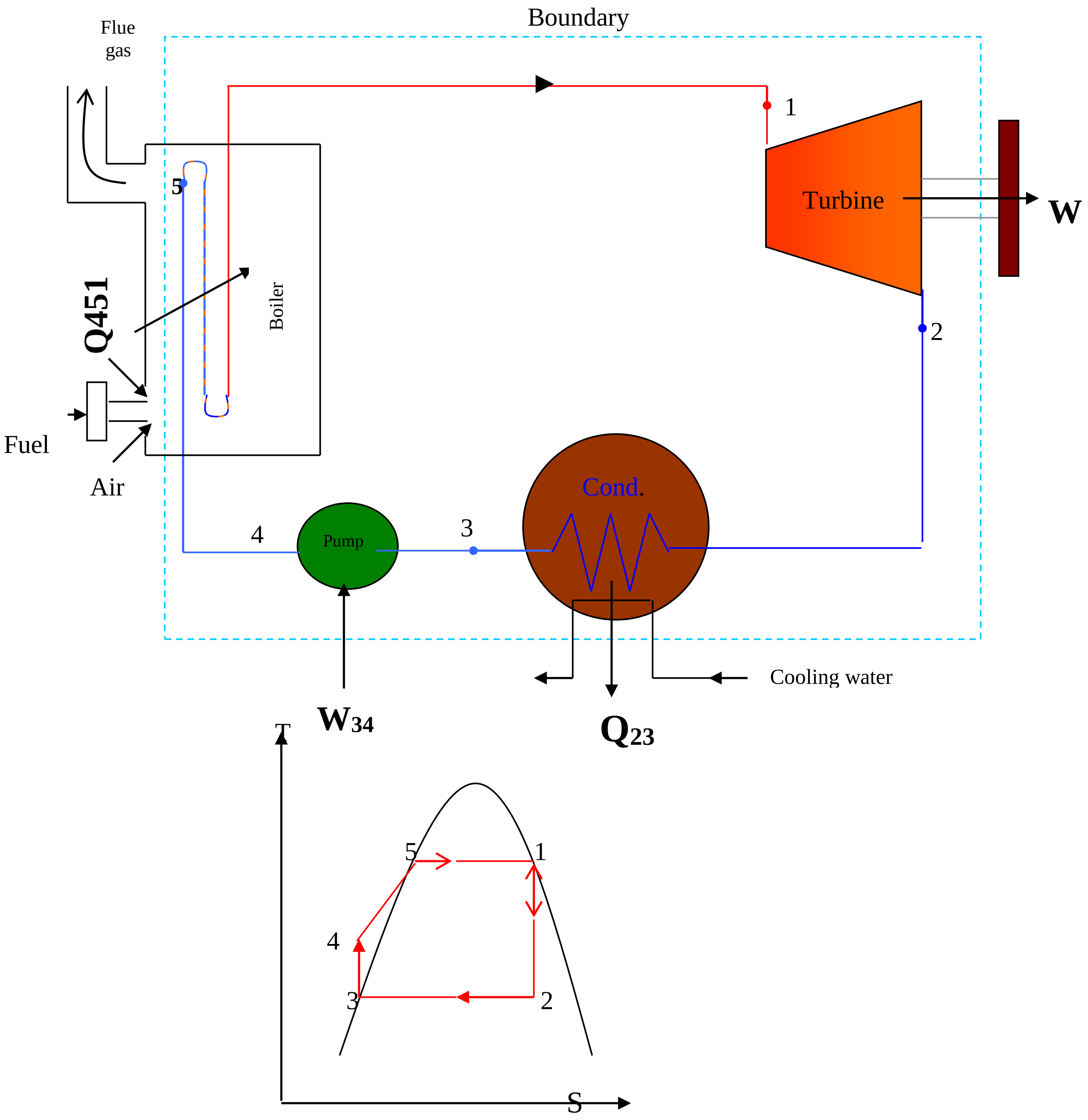
Process 2 to 3is isothermal heat rejection .

Process 3 to 4 is isentropic expansion from T_2 toT_1

Process 4 to 1is isothermal heat supply .

$$\boxed{\eta_{Car} = \frac{T_{max} - T_{min}}{T_{max}}}$$

2- The Rankine cycle:-



Process (1— 2) isentropic expansion in turbine , $S_1=S_2$,$P_1>P_2$,& adiabatic (Q = 0) therefore ,

$w_{12} = h_1 - h_2 = w$ turbine

Process (2— 3)isothermal constant temperature in condenser ,$P_3=P_2$,

$q_{rej} = q_{23} = h_3 - h_2 = -ve$, $h_2>h_3$, W =0

Process (3— 4) isentropic compresion in pump ,& adiabatic (Q = 0) $S_3=S_4$,$P_4>P_3$, therefore,

$W_{34} = -(h_4 - h_3) = h_3 - h_4 = w\ pump$
Process (4— 1)heat supply at constant pressure in the boiler ,$P_4=P_1$,
$Q_{add} = q_{41} = h_1 - h_4 = +ve,\ h_1 > h_4$,

Note, It is usually work pump neglected , it is asmall quantity in comparision with turbine work , & ,It neglected especielly when boiler pressure are low .

Net work done in the cycle , $\mathbf{W = W_{12} + W_{34}}$

$$\mathbf{= W_{turbine} - W_{pump}}$$

$$\mathbf{= (h_1-h_2) - (h_4-h_3)}$$

Rankine efficiency, $\eta_R = \dfrac{\textbf{Net work out put}}{\textbf{Heat supplied in the boiler}}$

$$= \frac{(h_1-h_2) - (h_4-h_3)}{h_1-h_4}$$

$$= \frac{(h_1-h_2) - (h_4-h_3)}{(h_1-h_3) - (h_4-h_3)} \quad \text{or} = \frac{(h_1-h_2)}{(h_1-h_3)} \text{ if } W_{Pump} = 0$$

Pump work :-

For reversible process ,

$dQ = du + p\ dv$ …….………. (a)

enthalpy is defind by equation ,

$h = u + p\ v$

therefore,

$dh = du + d(pv)$

$dh = du + pdv + vdp$

therefore,

$du = dh - pd\nu - \nu dp$ ……………………(b)

$dQ = dh - pd\nu - \nu dp + p\,d\nu$

$= dh - \nu dp$

Therefore for isentropic process (Q=0),

$0 = dh - \nu dp$ Therefore,

$dh = \nu dp$

$\int_3^4 dh = \int_3^4 \nu dp$

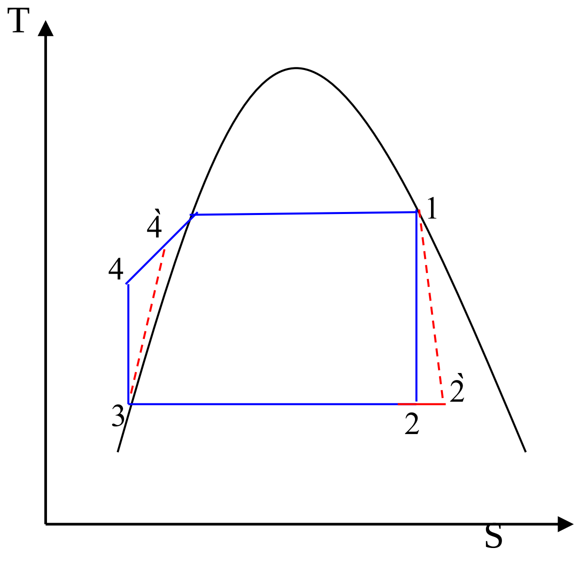
For aliquid , since ν is approximatly constant ,we have

$h_4 - h_3 = \nu (p_4 - p_3) = w$ therfor work pump ,

$w = \nu (p_4 - p_3)$

(where ν can be taken from tables for water at the pressure p_3)

Actual rankine cycle :-



$$\text{Isentropic Efficiency} = \frac{\text{Actual work}}{\text{Isentropic work}} \text{, for an expansion process} = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{w_{12'}}{w_{12}}$$

$$\text{Isentropic Efficiency} = \frac{\text{Isentropic work input}}{\text{Actual work input}}\ , \quad \text{for an compression process} = \frac{h_4 - h_3}{h_{4'} - h_3} = \frac{W_{34}}{W_{34'}}$$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{gross work}} \quad \text{or} \quad \frac{\text{Net work}}{\text{Turbine work}}$$

Specific steam consumption, (S.S.C):- is the steam flow in Kg /h required to develop 1KW,

Wnet KJ/Kg * S.S.C = 1KW*3600 KJ/h

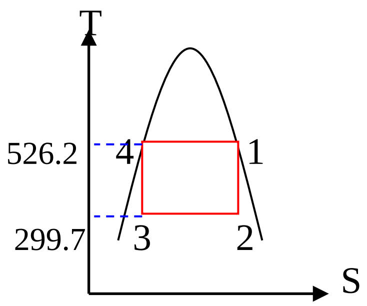
$$\text{S.S.C} = \frac{3600}{\text{Wnet}} \ (\text{Kg / KW h})$$

EX: - a steam power plant operates between a boiler pressure of 42bar and condenser pressure of 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption:

- **(a) for Carnot cycle using wet steam ;**
- **(b) for Rankine cycle with dry saturated steam at entry to the turbine ; and**
- **(c) for Rankine cycle of(b) when the expansion process has an isentropic efficiency of 80%**

Sol:-

(a) For Carnot cycle using wet steam



T_1= saturation temperature at42 bar

= 253.2+273 =526.2°K

T_2= saturation temperature at0.035 bar

= 26.7+273 =299.7^{o}K

$$ŋ_{Car} = \frac{T_{max} - T_{min}}{T_{max}}$$

$$ŋ_{Car} = \frac{526.2 - 299.7}{526.2}$$

= 0.432 or 43.2%

Heat supplied = $h_1 – h_4 = h_{fg}$ at 42 bar = 1698 KJ/Kg

$ŋ_{Car}$ = W / Q = 0.432 therefore W = 0.432 * 1689

W = 734 KJ / Kg

h_1= 2800 KJ / Kg & S_1=S_2 = 6.049 KJ/Kg.^{o}K

$S_2 = S_{f2} + X_2S_{fg2}$

6.049 = 0.391+ X_2 8.13

X_2 = 0.696

$h_2 = h_{f2} + X_2h_{fg2}$ at 0.035bar

=112 + 0.696 *2438

= 1808 KJ/Kg

$W_{12} = (h_1-h_2)$

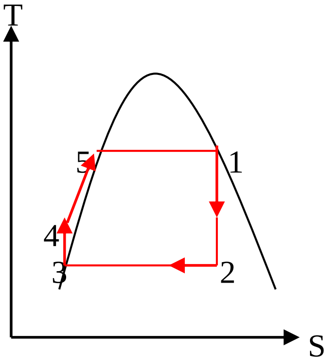
= 2800 – 1808

= 992 KJ/Kg

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = 734 / 992 = 0.739$$

$$\text{S.S.C} = \frac{3600}{\text{Wnet}} = \frac{3600}{734} = 4.9 \text{ (Kg / KW h)}$$

(b) For Rankine cycle with dry saturated steam at entry to the turbine



$h_1 = 2800$ KJ/kg & $h2 = 1808$ KJ/kg

$h_3 = h_f = 112$ KJ/kg

$w = \nu\,(p_4 - p_3)$

$= 0.001 * (42 - 0.035) * 10^2$

$= 4.2$ KJ/Kg

$w = h_4 - h_3$

$4.2 = h_4 - 112$

$h_4 = 116.2$ KJ/Kg

$$\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3) - (h_4 - h_3)}$$

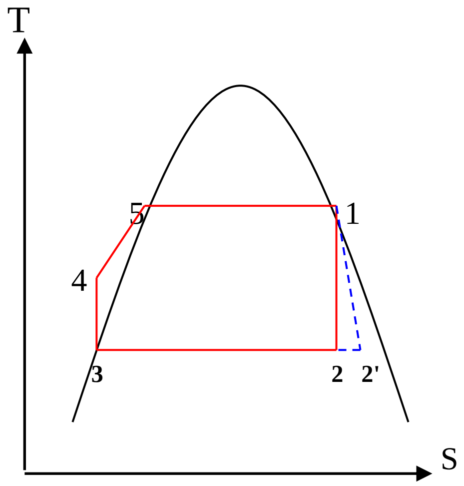
$$\eta_R = \frac{992 - 4.2}{(2800 - 112) - 4.2}$$

$\eta_R = 36.8\%$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{992 - 4.2}{992} = 0.996$$

$$\text{S.S.C} = \frac{3600}{\text{Wnet}} = \frac{3600}{992 - 4.2} = 3.64\ (\text{Kg / KW h})$$

(c) For Rankine cycle of (b) when the expansion process has an isentropic efficiency of 80%



$$\text{Isentropic Efficiency} = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{w_{12'}}{w_{12}}$$

$$0.8 = \frac{w_{12'}}{992}$$

$$w_{12'} = 0.8 * 992$$

$$= 793.6 \text{ KJ/Kg}$$

$$\eta_R = \frac{(h_1 - h_{2'}) - (h_4 - h_3)}{\text{Heat supplied}}$$

$$= \frac{793.6 - 4.2}{(2800 - 112) - 4.2}$$

$$\eta_R = 0.294 = 29.4\%$$

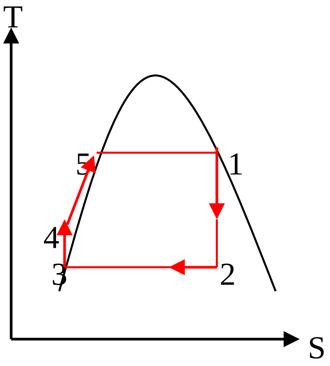
$$\text{Work ratio} = \frac{w_{12'} - \text{pump work}}{w_{12'}} = \frac{793.6 - 4.2}{793.6} = 0.995$$

$$\text{S.S.C} = \frac{3600}{\text{Wnet}} = \frac{3600}{793.6 - 4.2} = 4.56 \text{ (Kg / KW h)}$$

EX: - steam power plant leaves the boiler dry saturated steam at 30bar and its condensing state at 1 bar. If plant operates on Rankine cycle, Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption, neglete work pump.

Sol:

For Rankine cycle with dry saturated steam leaves the boiler



P_1=30bar dry saturated steam

$h_1 = h_g = 2803$KJ/kg

$h_1 = 2803$ KJ / Kg & $S_1 = S_g = S_2 = 6.186$ KJ/Kg.°K

$S_2 = 6.186$ KJ/Kg.°K&$P_2 = 1$bar

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.186 = 1.303 + X_2\ 6.056$

$X_2 = 0.8063$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 15bar

$=417 + 0.8063 * 2258$

$= 2237.642$ KJ/Kg

$h_3 = h_f = 417$KJ/kg ….at $P_2 = 1$bar

$h_4 = h_3$ ……neglete work pump

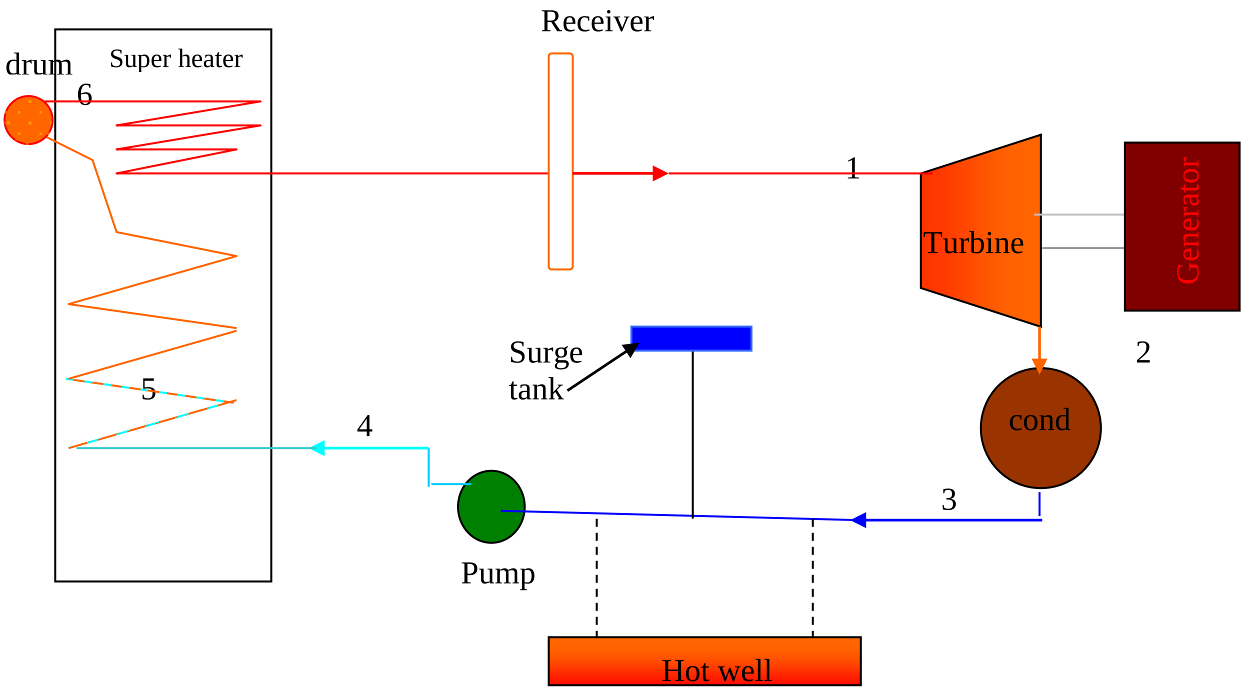
$$\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_3) - (h_4 - h_3)}$$

$$\eta_R = \frac{(2803 - 2237.642) - 0}{(2803 - 417) - 0} = 23.7\%$$

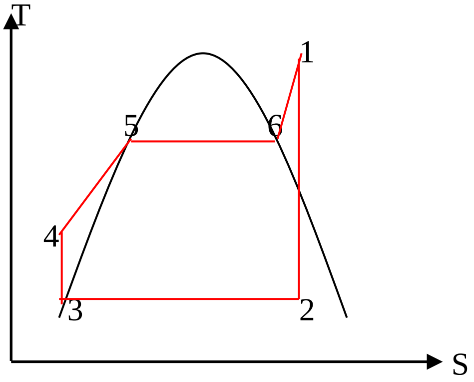
$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{(h_1-h_2) - (h_4-h_3)}{(h_1-h_2)} = \frac{(2803 - 2237.642) - 0}{2803 - 2237.642} = 1$$

$$\text{S.S.C} = \frac{3600}{W_{net}} = \frac{3600}{(h_1-h_2) - (h_4-h_3)} = \frac{3600}{(2803 - 2237.642) - 0} = 6.3676 \text{ (Kg / KW h)}$$

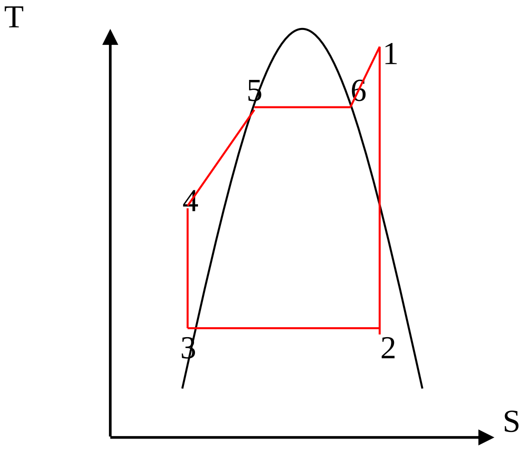
Rankine cycle with super heat :-



- The average temperature at which heat is supplied in the boiler can be increased by super heating the steam, the dry sat. steam from the boiler drum is passed through a second bank of smaller bore tubes with in the boiler , the bank is situated such that is heated by the hot gases from the furnace until the steam reaches the required temperature
- In modern plant receiver is used with one boiler and is placed between the boiler & turbine
- Since the quantity of feed water varies with the different demands on the boiler ,it is necessary to provide feed pump , this storage may be either a surge tank or hot well



EX: - a steam power plant operates on super heat Rankine cycle, if the inlet to turbine at 42bar&500°C and condenser pressure 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption, neglete work pump.



Sol:-

h_1= h at 42bar & T_1 =500°C

$$h_1 = h_{40} + \frac{42-40}{50-42}(h_{50} - h_{40})$$

$= 3445 + 0.2 (3433 - 3445)$

$= 3442.6$ KJ/Kg

$$S_1 = S_{40} + \frac{42-40}{50-42}(S_{50} - S_{40})$$

$= 7.089 + 0.2(6.975 - 7.089)$

$S_1 = 7.066$KJ/Kg $= S_2$

$S_2 = S_{f2} + X_2 S_{fg2}$

$7.066 = 0.391 + X_2\ 8.13$

$X_2 = 0.821$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 0.035bar

$= 112 + 0.821 * 2438$

$= 2113$ KJ/Kg

$h_3 = h_f = 112$KJ/kg ….at P_2 =0.035bar

$h_4=h_3$ ……neglete work pump

$$\eta_R = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_3)-(h_4-h_3)}$$

$$\eta_R = \frac{(3442.6-2113)-0}{(3442.6-112)-0} = 0.399 = 39.9\%$$

$$S.S.C = \frac{3600}{W_{net}} = \frac{3600}{(h_1-h_2)-(h_4-h_3)} = \frac{3600}{(3442.6-2113)-0} = 2.71\ (Kg / KW\ h)$$

EX: - a steam power plant operates on Ideal Rankine cycle, steam inters to the turbine at 3MPa&350°C and condenser pressure at 10KPa. Calculate

(a) the cycle efficiency, the work ratio, and the specific steam consumption,

(b) the cycle efficiency, the work ratio, and the specific steam consumption if steam enter at 600°C instead of 350°C,

(c) The cycle efficiency, the work ratio, and the specific steam consumption if boiler pressure is raised to (15MPa) while turbine inlet temperature remains 600°C.

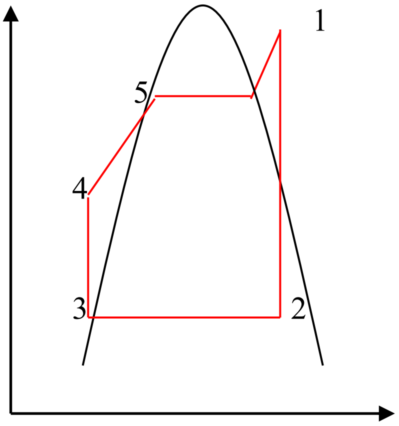
Take $v_{f3} = 0.001\ m^3/kg$

Sol:-

(a)the cycle efficiency, the work ratio, and the specific steam consumption,

at 30bar & $T_1 = 350°C$

T



$h_1 = 3117$ KJ/Kg

$S_1 = 6.744 KJ/Kg = S_2$

$S_2 = 6.744$ KJ/Kg °K & $P_2 = 0.1$bar

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.744 = 0.649 + X_2\ 7.509$

$X_2 = 0.81$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 0.1bar

$= 192 + 0.81 * 2392$

$= 2130.82$ KJ/Kg

$h_3 = h_f = 192$KJ/kg ….at $P_2 = 0.1$bar

$w = v\,(p_4 - p_3)$

$= 0.001 * (30 - 0.1) * 10^2$

$= 3.0199 KJ/Kg$

$W = h_4 - h_3$

$3.0199 = h_4 - 192$

$h_4 = 195.0199 KJ/Kg$

$$\eta_R = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_4)}$$

$$\eta_R = \frac{(3117-2130.83)-3.0199}{3117-195.0199} = 0.3364$$

$\eta_R = 33.64\%$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_2)}$$

$$= \frac{(3117-2130.83)-3.0199}{(3117-2130.83)}$$

$= 0.9969$

$W_{net} = (3117 - 2130.83) - 3.0199 = 983.15 KJ/Kg$

$$S.S.C = \frac{3600}{W_{net}} = \frac{3600}{983.15} = 3.661 \text{ (Kg / KW h)}$$

(b)the cycle efficiency, the work ratio, and the specific steam consumption if steam enter at 600°C instead of 350°C,

at 30bar & $T_1 = 600^oC$

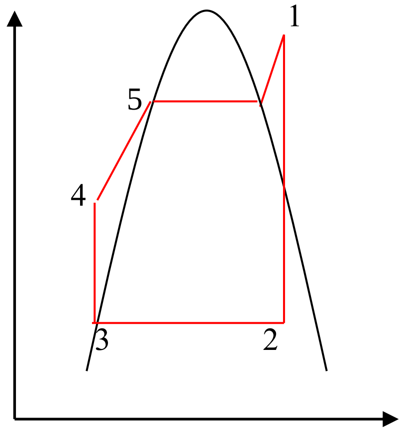
$h_1 = 3682$ KJ/Kg

$S_1 = 7.507 KJ/Kg = S_2$

$S_2 = 6.744$ KJ/Kg °K & $P_2 = 0.1bar$

$S_2 = S_{f2} + X_2 S_{fg2}$

$7.507 = 0.649 + X_2\, 7.509$



$X_2 = 0.9144$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 0.1bar

$=192 + 0.9144 * 2392$

$= 2379.2$ KJ/Kg

$h_3 = h_f = 192$KJ/kg ….at $P_2 = 0.1$bar

$w = \nu\ (p_4 - p_3)$

$= 0.001 * (30 – 0.1) * 10^2$

$= 3.0199$KJ/Kg

$w = h_4 - h_3$

$3.0199 = h_4 - 192$

$h_4 = 195.0199$KJ/Kg

$$\eta_R = \frac{(h_1 - h_2) – (h_4 - h_3)}{(h_1 – h_4)}$$

$$\eta_R = \frac{(3682 – 2379.2) - 3.0199}{3682 – 195.0199} = 0.3742$$

$\eta_R = 37.42\%$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{(h_1 - h_2) – (h_4 - h_3)}{(h_1 - h_2)}$$

$$= \frac{(3682 – 2379.2) - 3.0199}{(3682 – 2379.2)}$$

$= 0.9977$

$W_{net} = (3682 – 2379.2) - 3.0199 = 1307.78$ KJ/Kg

$$\text{S.S.C} = \frac{3600}{W_{net}} = \frac{3600}{1307.78} = 2.752\ (\text{Kg / KW h})$$

(c)The cycle efficiency, the work ratio, and the specific steam consumption if boiler pressure is raised to (15MPa) while turbine inlet temperature remains 600°C

at 150bar & T_1 =600°C

h_1=3581 KJ/Kg

S_1 =6.667KJ/Kg = S_2

S_2 = 6.744 KJ/Kg °K & P_2 = 0.1bar

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.667 = 0.649 + X_2\ 7.509$

$X_2 = 0.804$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 0.1bar

$=192 + 0.804 * 2392$

= 2114.53 KJ/Kg

$h_3 = h_f$ = 192KJ/kg ….at P_2 =0.1bar

$w = \nu\ (p_4 - p_3)$

$= 0.001 * (150 - 0.1) * 10^2$

= 15.00499KJ/Kg

$w = h_4 - h_3$

$15.00499 = h_4 - 192$

h_4 = 207.00499KJ/Kg

$$\eta_R = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$\eta_R = \frac{(3581 - 2114.53) - (207.00499 - 192)}{3682 - 207.00499} = 0.0.43$$

η_R = 43%

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_2)}$$

$$= \frac{(3581 - 2114.53) - (207.00499 - 192)}{(3581 - 2114.53)}$$

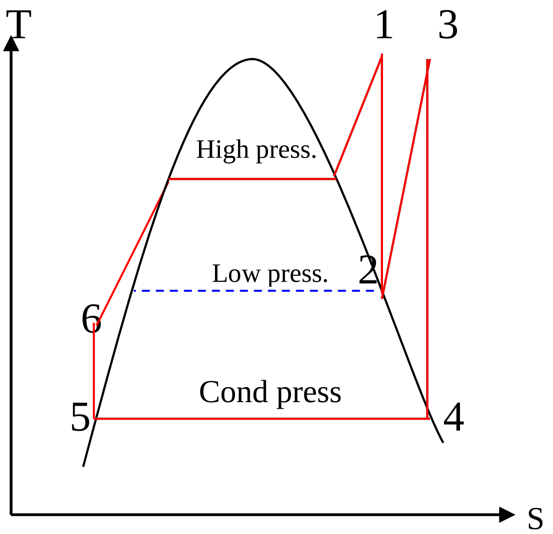
$$= 0.9898$$

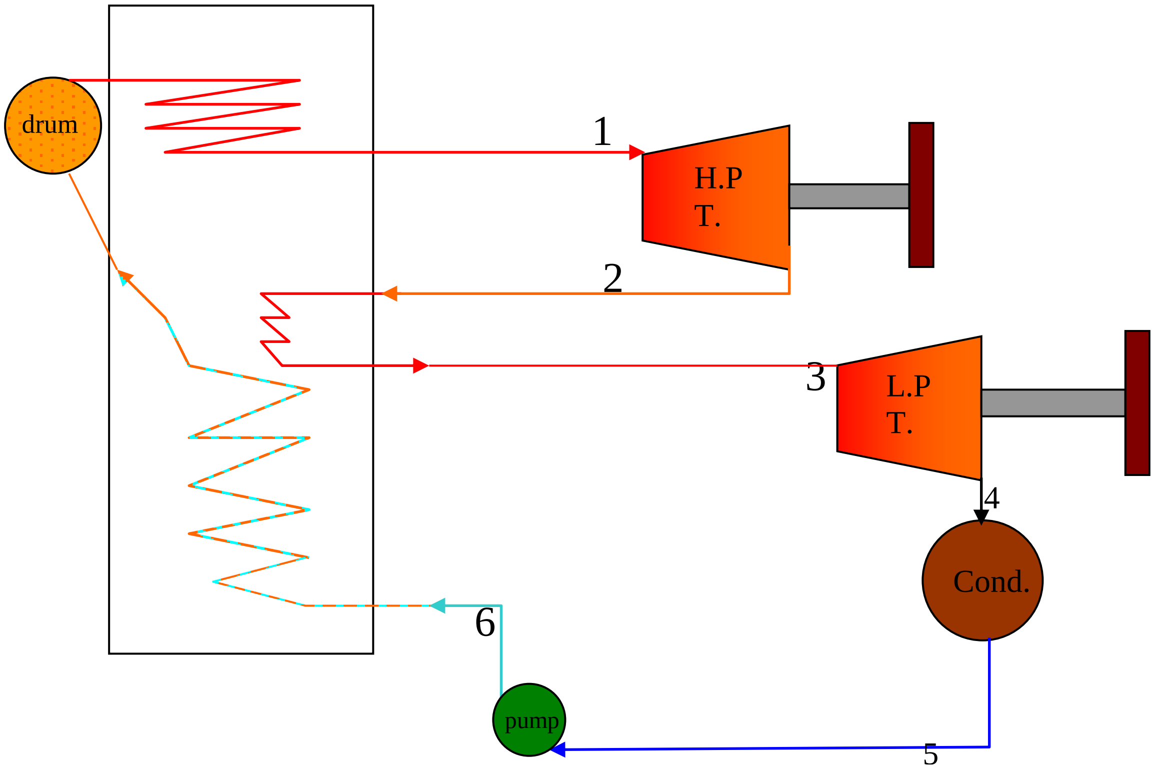
$$\text{Wnet} = (3581 - 2114.53) - (207.00499 - 192) = 1451.4650 \text{ KJ/Kg}$$

$$\text{S.S.C} = \frac{3600}{\text{Wnet}} = \frac{3600}{1451.4650} = 2.48 \text{ (Kg / KW h)}$$

The reheat cycle: -

it is described to increase the average temperature at which heat is supplied to the steam, and also to keep steam as dry as possible in the lower pressure stages of the turbine.





H.P.T. = high pressure turbine

L.P.T. = low pressure turbine

$P_6 = P_1$, $P_3 = P_2$, $P_5 = P_4$

$S_1 = P_2$, $S_3 = S_4$, $S_5 = S_6$

Work turbine = work high pressure turbine + work low pressure turbine

$$= W_{12} + W_{34}$$

$$= (h_1 - h_2) + (h_3 - h_4)$$

Work pump = $h_6 - h_5$

$$= v_{f5} (P_6 - P_5)$$

Work net = Work turbine - Work pump

Heat supply = $q_{61} + q_{23}$

$$= (h_1 - h_6) + (h_3 - h_2)$$

$$\eta R = \frac{W_{12} + W_{34}}{q_{61} + q_{23}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_6) + (h_3 - h_2)}$$

$$S.S.C = \frac{3600}{Wnet}$$

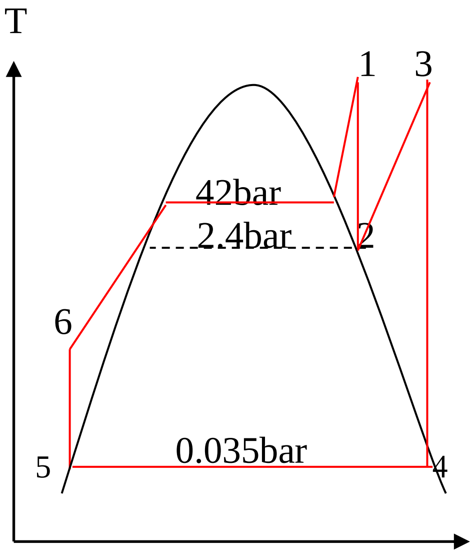
EX :- Calculate the cycle efficiency, and the specific steam consumption, if reheated is included in the plant . the steam condition at inlet to the turbine are 42 bar & 500 °C , & condenser pressure is 0.035 bar assume that the steam is just dry saturated on leaving first turbine & is reheated to its initial temperature . Neglect the feed pump term.

Sol :-

$P_1 = 42bar$ & $t_1 = 500^oC$

$$h_1 = h_{40} + \frac{42 - 40}{50 - 40} (h_{50} - h_{40})$$

$$= 3445 + 0.2 (3433 - 3445)$$
$$= 3442.6 KJ/Kg$$



$$S_1 = S_{40} + \frac{42 - 40}{50 - 40} (S_{50} - S_{40})$$

$= 7.089 + 0.2 (6.975 - 7.089)$

$S_1 = 7.066 KJ/Kg .^oK = S_2$

$S_2 = 7.066 KJ/Kg .^oK$ dry saturated $= S_{g2}$,therefore …..

$$P_2 = 2.4 + \frac{7.066 - 7.067}{7.053 - 7.067} (2.5 - 2.4)$$

$P_2 = 2.407 bar \approx 2.4$ bar

$h_2 = h_{g2}$ at $P_2 = 2.4 bar = 2715$ KJ/Kg

$P_2 = P_3 = 2.4$ bar &$T_3 = 500^oC$

$$h_3 = h_{2bar} + \frac{2.4 - 2}{3 - 2} (h_{3bar} - h_{2bar})$$

$h_3 = 3487 + 0.4 (3486 - 3487)$

$= 3486.6$ KJ/Kg

$$S_3 = S_{3bar} + \frac{2.4 - 2}{3 - 2} (S_{3bar} - S_{2bar})$$

$= 8.513 + 0.4 (8.324 - 8.513)$

$S_3 = 8.437 KJ/Kg.^oK = S_4$

$S_4 = 8.437 KJ/Kg.^oK$ & $P_4 = 0.035$ bar

$S_4 = S_{f4} + X_4 S_{fg4}$

$8.437 = 0.391 + X_4\ 8.13$
$X_4 = 0.99$

$h_4 = h_{f4} + X_4 h_{fg4}$ at 0.035bar

$= 112 + 0.99 * 2438$

$= 2525.6$ KJ/Kg

$h_5 = h_f = 112$ KJ/kg ….at $P_2 = 0.035$bar

$h_5 = h_6 = 112$ KJ/kg

$w_{net} = (h_1 - h_2) + (h_3 - h_4)$

$= (3442.6 - 2715) + (3486.6 - 2525.6)$

$= 1688.6$ KJ/kg

Heat supply = $q_{61} + q_{23}$

$= (h_1 - h_6) + (h_3 - h_2)$

$= (3442.6 - 112) + (3486.6 - 2715)$

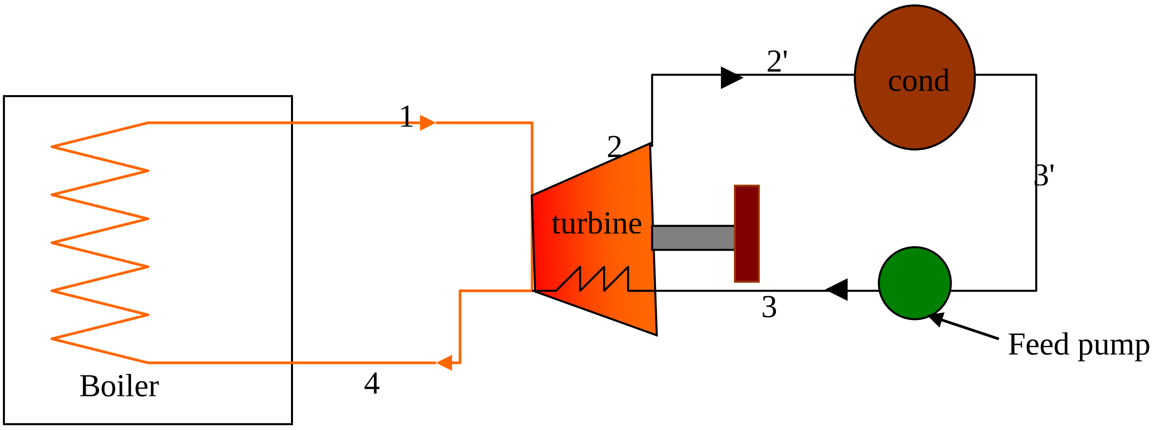
$= 4102.2$ KJ/kg

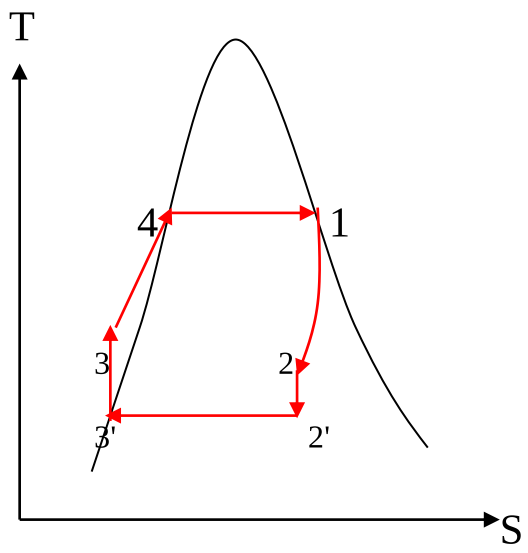
$$\eta R = \frac{w_{net}}{\text{Heat supply}} = \frac{1688.6}{4102.2} = 0.412 = 41.2\%$$

$$\text{S.S.C} = \frac{3600}{\text{Wnet}} = \frac{3600}{1688.6} = 2.13 \text{ (Kg / KW h)}$$

The regenerative cycle (bleed cycle):-

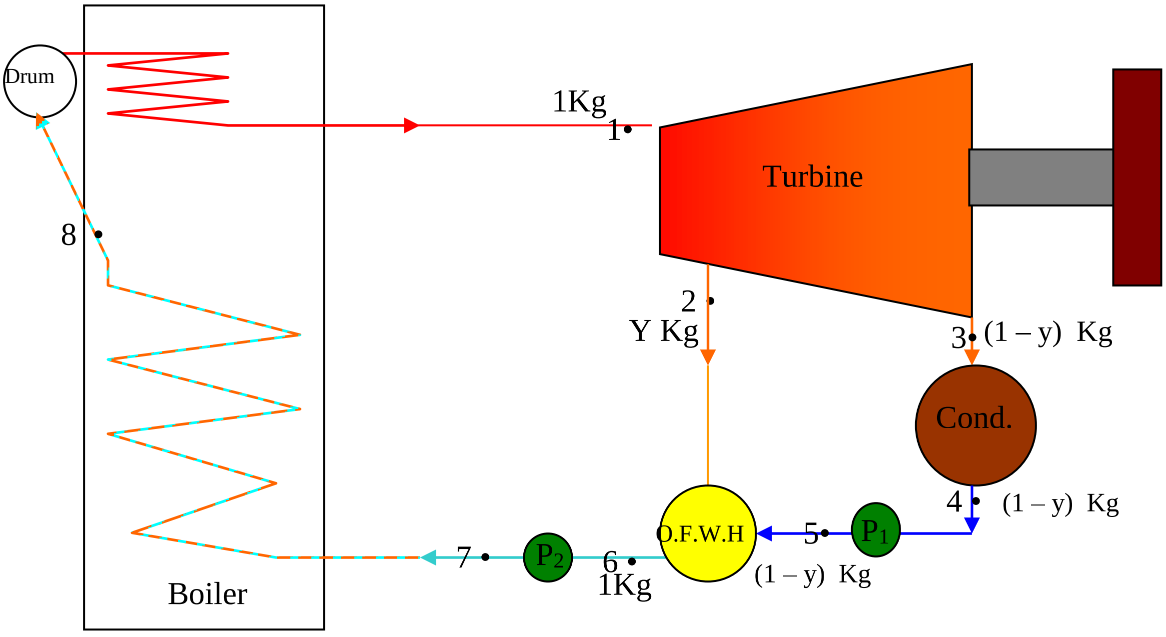
In order to achieve the Carnot efficiency it is necessary to supply and reject heat at single fixed temperature, one method of doing this is by raising the feed water to the saturation temperature corresponding to the boiler pressure before it enter to the boiler .this regenerative cycle has an efficiency equal to the Carnot , since the heat supplied and rejected externally is done at constant temperature .



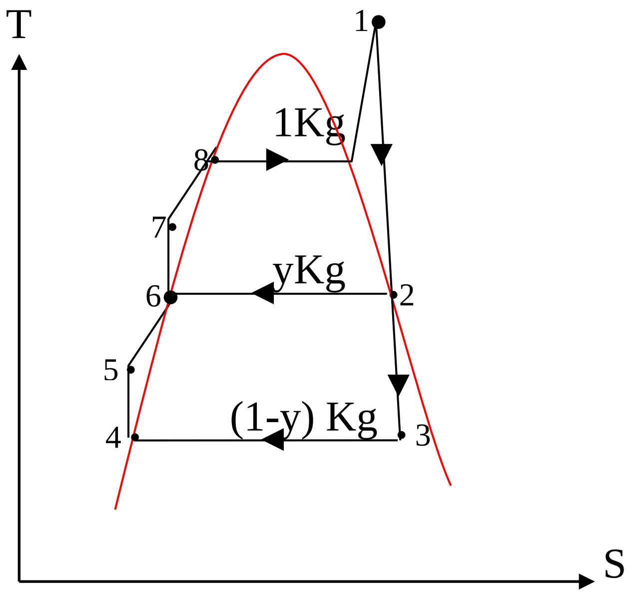


The expansion process is represented by line 1 −2−2'
This cycle is clearly not a practical proposition, and in addition it can be seen that the turbine operates with wet steam which is to be avoided if possible.

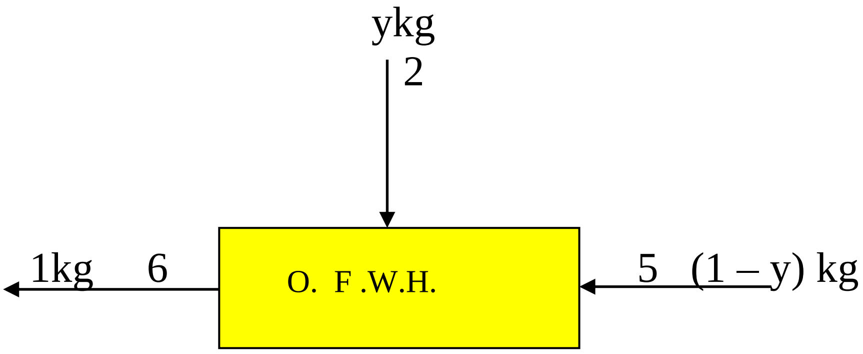
Open feed water heater (O.F.W.H.) :-



The rankine efficiency can be improved by bleeding of some the steam at intermediate pressure during the expansion and mixing this steam with feed water which has been pumped to the same pressure. The mixing process is carried out in a feed heater; several feed heaters can be used.



The steam expands from condition 1 through the turbine, at the pressure corresponding to point 2 a quantity of steam say y Kg per Kg of steam supplied to the boiler, is bled off for feed heater purposes, the rest of the steam (1- y) kg, completes the expansion and is exhausted at state 3 this amount of steam is then condensed and pumped to the same pressure as the bled steam. The bled steam & the feed water are mixed in the feed heater &the quantity of bled steam, y kg, is such that after mixing &being pumped in a second feed pump.



Q in = Q out

$$y * h_2 + (1-y)\, h_5 = 1 * h_6$$

$$y = \frac{h_6 - h_5}{h_2 - h_5}$$

The heat to be supplied in the boiler is given by $(h_1 – h_7)$

Work turbine $= 1*(h_1 – h_2) + (1 – y)\,(h_2 – h_3)$

Work pump = $(1 - y)(h_5 - h_4) + 1*(h_7 - h_6)$

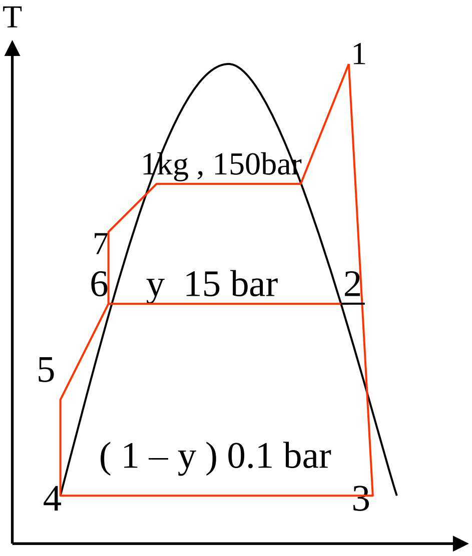
$= (1 - y)v_{f4}(P_5 - P_4) + 1*v_{f6}(P_7 - P_6)$

Note :- if the bleeding pressure is not given it can be obtained from the law below

$$t_s \text{ bleed} = \frac{t_s \text{ boiler} + t_s \text{ condenser}}{2}$$

EX :- consider a steam power plant operating on the regenerative Rankine cycle with O.F.W.H the steam enters the turbine at 15 MPa & 600°C &is condensed at a pressure 10KPa , some steam bleed at 1.5MPa & enters to open feed water heater ,determine the amount of bleed steam & cycle efficiency .

Sol:-



P_1 = 15 MPa = 150 bar , T_1= 600°C

h_1 = 3581 KJ /Kg & S_1= 6.677 KJ/Kg.°K = $S_2 = S_3$

P_2 = 1.5 MPa = 15 bar & S_2= 6.677 KJ/Kg.°K

$S_2 > S_{g2}$ there fore steam is super heat

$$h_2 = h_{200} + \frac{6.677 - 6.452}{6.711 - 6.452}(h_{250} - h_{200})$$

$$h_2 = 2796 + \frac{6.677 - 6.452}{6.711 - 6.452}(2925 - 2796)$$

=2908.1kJ/kg

P_3 = 0.1bar & S_3= 6.677 KJ/Kg.°K

$S_3 < S_{g3}$ = 8.149 KJ/Kg.°K there fore steam is wet

$S_3 = S_{f3} + X_3 S_{fg3}$

$6.677 = 0.649 + X_3\ 7.5$

$X_3 = 0.804$

$h_3 = h_{f3} + X_3 h_{fg3}$ at 0.1bar

=192 + 0.804*2392

= 2115.2 KJ/Kg

$h_4 = h_f = 192 KJ/kg$ ….at $P_2 = 0.1 bar$

$$\nu_{f4} = 0.0010099 + \frac{0.1 - 0.09582}{0.01233 - 0.09582} (0.00102 - 0.0010099)$$

$\nu_{f4} = 0.0010102 m^3/Kg$

$w_1 = (1 - y) \nu_{f4} (p_5 - p_4) = (1 - y) h_5 - h_4$

$\nu_{f4} (p_5 - p_4) = h_5 - h_4$

$0.0010102 * (15 - 0.1) * 10^2 = h_5 - 192$

$1.5 = h_5 - 192$

$h_5 = 193.5 KJ/Kg$

$P_6 = 15$ bar

$h_6 = h_{f6} = 845$ KJ/Kg at P = 15 bar

$$\nu_{f6} = 0.001142 + \frac{15 - 12.55}{15.55 - 12.55} (0.0011572 - 0.001142)$$

$\nu_{f6} = 0.0011543 m^3/Kg$

$w_2 = \nu_{f6} (p_7 - p_6)$

$= 0.0010102 * (150 - 15) * 10^2$

$= 15.6 KJ/Kg$

$W_2 = h_7 - h_6$

$15.6 = h_5 - 845$

$h_7 = 860.6 KJ/Kg$

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{845 - 193.5}{2908.1 - 193.5} = 0.24 \text{ Kg /Kg of steam}$$

Work turbine = $1 * (h_1 - h_2) + (1 - y)(h_2 - h_3)$

$= 1*(3581 – 2908.1) + (1 –0.24) (2908.1 – 2115.2)$

$= 1275.5 KJ/Kg$

Work pump = $wp_1 - wp_2$

$= (1 –y) (h_5 – h_4) - 1*(h_7 – h_6)$

$= (1 –0.24) (193.5 – 192) - 1*(860.6 – 845)$

$= 16.7 KJ /Kg$

Work net = Work turbine + Work pump

$=1275.5 +16.7$

$= 1258.8 KJ /Kg$

Heat supply = $h_1 - h_7$

$=3581 - 860.6$

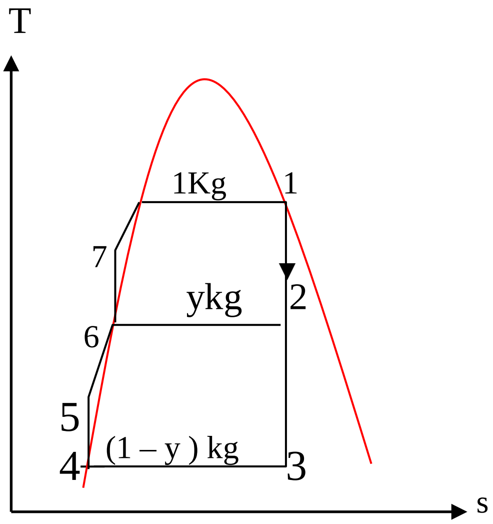
$= 2720.4 KJ /Kg$

$$ŋ_{th} = \frac{w\ net}{heat\ supply}$$

$$ŋ_{th} = \frac{1258.8}{2720.4} = 0.463 = 46.3\%$$

EX: - Calculate the cycle efficiency, and the specific steam consumption for regenerative Rankine cycle with one open feed water heater, the steam enters the turbine at 42bar dry saturated, and condenser pressure of 0.035 bar. (Neglect pumps work).

Sol:-



$$t_2 = \frac{t_s \text{ boiler} + t_s \text{ condenser}}{2}$$

$$t_2 = \frac{253.2 + 26.7}{2} = 139.95^oC$$

$$P_2 = 3.5 + \frac{139.95 - 138.9}{143.6 - 138.9}(4 - 3.5)$$

$P_2 = 3.56 \approx 3.5$ bar ….. Bleed pressure

At P = 42bar dry sat.

$h_1 = h_g = 2800$KJ/Kg

$S_1 = S_g = 6.049$KJ/Kg.^{o}K $= S_2 = S_3$

At P = 3.5bar & $S_2 = 6.049$KJ/Kg.^{o}K

$S_{g2} = 6.941$KJ/Kg.^{o}K > S_2 therefore steam is wet

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.049 = 1.727 + X_2\ 5.214$

$X_2 = 0.8289$

$h_2 = h_{f2} + X_2 h_{fg2}$ at 3.5bar

$= 584 + 0.8289 * 2148$

$= 2364$ KJ/Kg

P condenser = 0.035bar & $S_3 = 6.049$KJ/Kg.^{o}K

$S_{g3} = 8.521$KJ/Kg.^{o}K > S_3 therefore steam is wet

$S_3 = S_{f3} + X_3 S_{fg3}$

$6.049 = 0.391 + X_3\ 8.130$

$X_3 = 0.6959$

$h_3 = h_{f3} + X_3 h_{fg3}$ at 0.035bar

=112 +0.6959 *2438

= 1808.6 KJ/Kg

$h_4 = h_f = 112$KJ/kg ….at P_2 =0.035bar

$h_4 = h_5$ =112KJ/kg …….neglect work pump

$h_6 = h_f = 584$KJ/kg ….at P_2 =3.5bar

$h_6 = h_7$ =584KJ/kg …….neglect work pump

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{584 - 112}{2364.47 - 112} = 0.209 \text{ Kg /Kg of steam}$$

Work turbine = $1*(h_1 - h_2) + (1 - y)(h_2 - h_3)$

= 1*(2800 – 2364.47) + (1 –0.209) (2364.47 – 1808.6)

= 874.9KJ/Kg

Heat supply = $h_1 - h_7$

=2800 – 584

= 2216 KJ /Kg

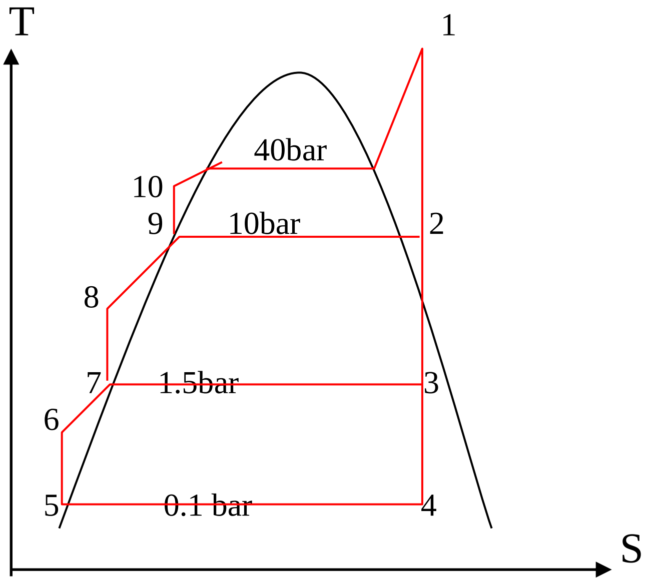
$$\eta_{th} = \frac{w\ net}{heat\ supply}$$

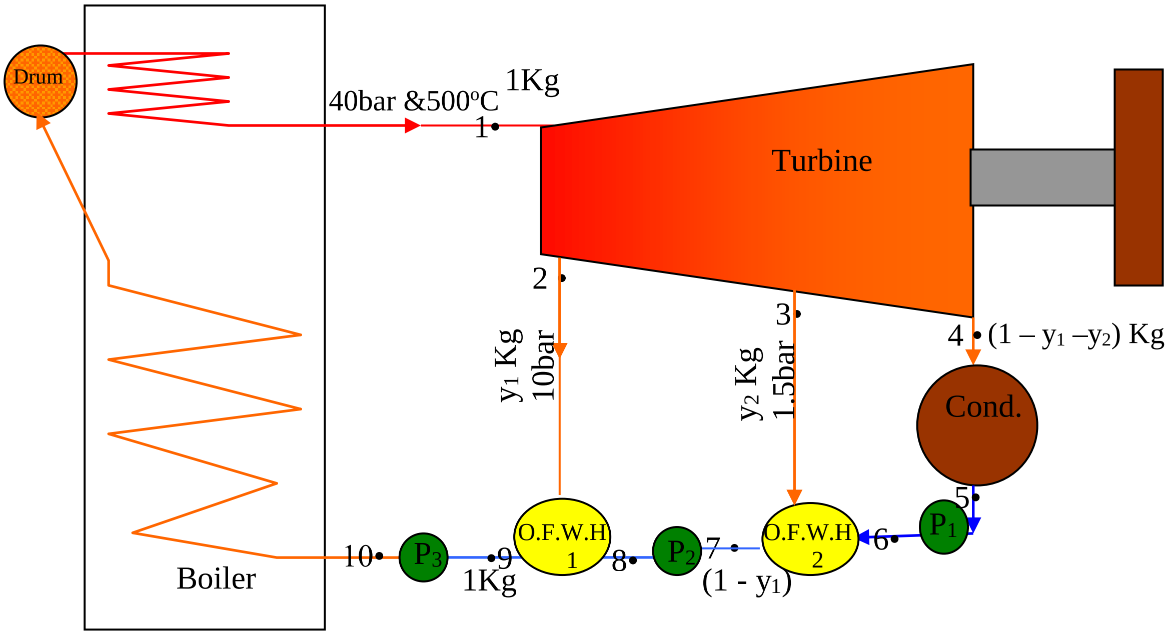
$$\eta_{th} = \frac{874.9}{2216} = 0.3948 = 39.48\%$$

$$S.S.C = \frac{3600}{Wnet} = \frac{3600}{874.9} = 4.1147 \text{ (Kg / KW h)}$$

EX :- consider a steam power plant operating on the regenerative Rankine cycle with two(O.F.W.H)the steam enters the turbine at 40bar & 500°C &is condensed at a pressure 0.1 , intermediate pressure are 10bar & 1.5bar,determine the amount of bleed steam at each stage & cycle efficiency .

Sol: -





P_1 = 40 bar & 500°C

$h_1 = 3445$ KJ/Kg , $S_1 = 7.089$ KJ/Kg .°K $= S_2 = S_3 = S_4$

$P_2 = 10$ bar , $S_2 = 7.089$ KJ/Kg .°K

$S_g = 6.586$ KJ/Kg .°K $< S_2 = 7.089$ KJ/Kg .°K therefore steam is super heated

$$h_2 = h_{250} + \frac{7.089 - 6.926}{7.124 - 6.926} \, (h_{300} - h_{250})$$

$h_2 = 2944+0.823\ (3052 - 2944)$

$h_2 = 3032.9091 KJ/Kg$

$P_3 = 1.5 bar$ & $S_3 = 7.089 KJ/Kg .^oK$

$S_g = 7.223 KJ/Kg .^oK > S_3 = 7.089 KJ/Kg .^oK$ therefore steam is wet

$S_3 = S_{f3} + X_3 S_{fg3}$

$7.089 = 1.434 + X_3\ 5.789$

$X_3 = 0.97$

$h_3 = h_{f3} + X_3 h_{fg3}$

$= 467 + 0.97 * 2226$

$= 2640 KJ /Kg$

$P_4 = 0.1 bar$ & $S_4 = 7.089 KJ/Kg .^oK$

$S_{g4} = 8.149 KJ/Kg .^oK > S_4 = 7.089 KJ/Kg .^oK$ therefore steam is wet

$S_4 = S_{f4} + X_4 S_{fg4}$

$7.089 = 0.649 + X_4\ 7.500$

$X_4 = 0.86$

$H_4 = h_{f4} + X_4 h_{fg4}$

$= 192 + 0.86 * 2392 = 2245.93 KJ /Kg$

$h_5 = h_{f5} = 192$ KJ/Kg …… at 0.1 bar

$wp_1 = h_6 – h_5 = v_{f5} (P_6 – P_5)$

$P_6 = P_3 , P_5 = P_4$

$$v_{f5} = 0.0010099 + \frac{0.1 – 0.09582}{0.12333 – 0.09582} (0.001012 – 0.0010099)$$

$v_{f5} = 0.00101022 m^3 / kg$

$wp_1 = h_6 – h_5 = v_{f5} (P_6 – P_5)$

$= h_6 – 192 = 0.00101022 * 10^2 (1.5 – 0.1) = 0.1414 KJ/Kg$

$0.1414 = h_6 – 192$

h_6 = 192.1414 KJ/Kg

$h_7 = h_{7f}$ = 467 KJ/Kg ……at P = 1.5bar $P_7 = P_3$

$wp_2 = h_8 - h_7 = \nu_{f7} (P_8 - P_7)$

$P_8 = P_2$ = 10bar

$P_7 = P_3$= 1.5bar

$$\nu_{f7} = 0.001052 + \frac{1.5 - 1.433}{1.691 - 1.433} (0.001056 - 0.001052)$$

ν_{f5} = 0.0010533 m^3 / kg

$wp_2 = h_8 - h_7 = \nu_{f7} (P_8 - P_7)$

$= h_8 - 467 = 0.0010533 * 10^2 (10 - 1.5) = 0.8950$ KJ/Kg

$0.8950 = h_8 - 467$

h_8= 467.8950 KJ/Kg

$h_9 = h_{f9}$ = 763 KJ/Kg …… at 10 bar

$wp_3 = h_{10} - h_9 = \nu_{f9} (P_{10} - P_9)$

$P_{10} = P_1$ = 40bar

$P_9 = P_2$= 10bar

$$\nu_{f9} = 0.001251 + \frac{40 - 39.78}{46.94 - 39.78} (0.001276 - 0.001251)$$

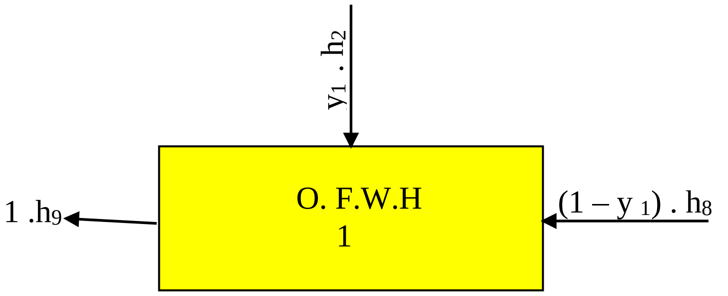
ν_{f9} = 0.001126 m^3 / kg

$wp_3 = h_{10} - h_9 = \nu_{f9} (P_{10} - P_8)$

$= h_{10} - 763 = 0.001126 * 10^2 (40 - 10) = 3.378$ KJ/Kg

$3.378 = h_{10} - 763$

h_{10}= 766.378 KJ/Kg

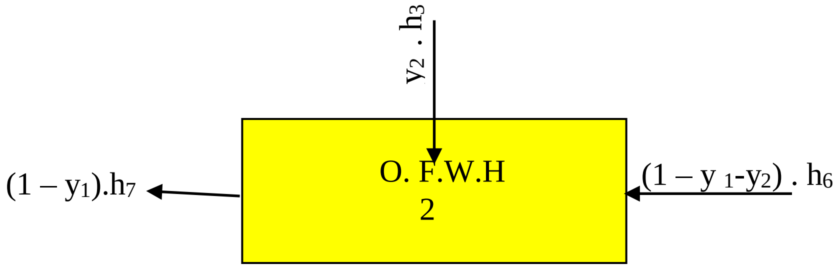


Q in = Q out

$$y_1 * h_2 + (1-y_1) h_8 = 1*h_9$$

$$y = \frac{h_9 – h_8}{h_2 – h_8}$$

$$y = \frac{763 – 467.896}{3033– 467.896} = 0.11504 \text{ Kg /Kg of steam}$$



Q in = Q out

$$y_2 * h_3 + (1-y_1 – y_2) h_6 = (1 – y_1)h_7$$

$$y_2 * h_3 + h_6 - y_1 h_6 - y_2 h_6 = h_7 - y_1 h_7$$

$$y_2 = \frac{h_7 - h_6 + y_1 (h_6 – h_7)}{h_3 - h_6}$$

$$= \frac{467 – 192.1414 + 0.115 (192.1414 – 467)}{2640 – 192.1414} = 0.09895 \text{ Kg /Kg of steam}$$

$$wT = 1*(h_1 – h_2) + (1-y_1)*(h_2 – h_3) + (1-y_1 –y_2)*(h_3 – h_4)$$

= (3445 – 3032) + (1- 0.115)(3032 – 2640) + (1- 0.115 -0.09895) (2640-2245.93)

= 1069.64 KJ/Kg

wpump = 1*wp_3+ (1-y_1)* wp_2+ (1-y_1 –y_2)* wp_1

= 1*3.378 +(1- 0.115) 0.895 +(1- 0.115 -0.09895) 0.1414

= 3.378 + 0.792075+0.11114747

= 4.28122247 KJ/Kg

Wnet = Wturbine - wpump

= 1069.64 - 4.28122247

= 1065.35877753 KJ/Kg

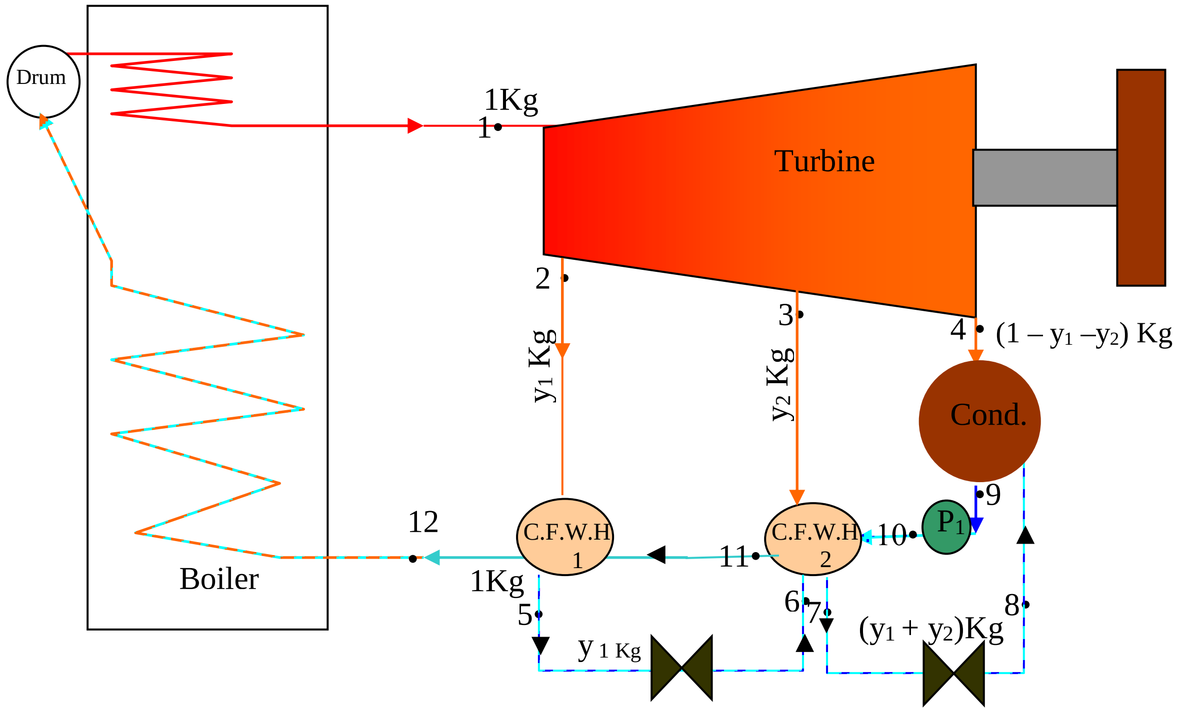
Heat supply = $h_1 – h_{10}$

= 3445 - 766.378

= 2678.622 KJ/Kg

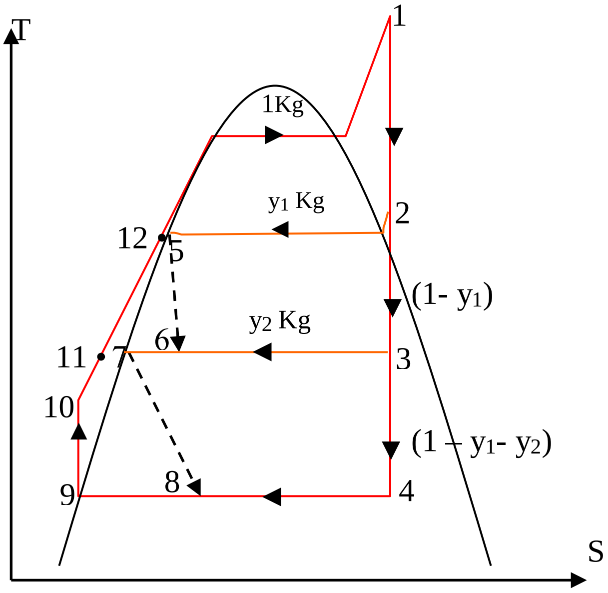
$$\eta R = \frac{w_{net}}{\text{Heat supply}} = \frac{1065.35877753}{2678.622} = 0.3977 \quad = 39.77\%$$

Closed feed water heater(C.F.W.H) :-



The methode is indicated for two feed heaters , but the number used could be as high as eight , an amount of bleed steam ,y_1, is passed to feed heater 1 , and the feed water receives heat from it by the transfer of heat through the separating tubes , the condensed steam is then throttled to the next feed heater which is also supplied with a second quantity of bleed steam , y_2 , and lower temperature heating of the feed water is carried out . When the final feed heating has been accomplished the condensed steam is then feed to the condenser.

The feed water leaves the feed heater at the temperature of the bleed steam supplied to it.



$P_{11}=P_{12} = P_{10}=P_1$

$P_2=P_5$

$P_3=P_6 = P_7$

$P_9=P_8 = P_4$

$S_1=S_2 = S_3=S_4$

$T_{12}=T_5$

$T_{11}=T_7=T_6=T_3$

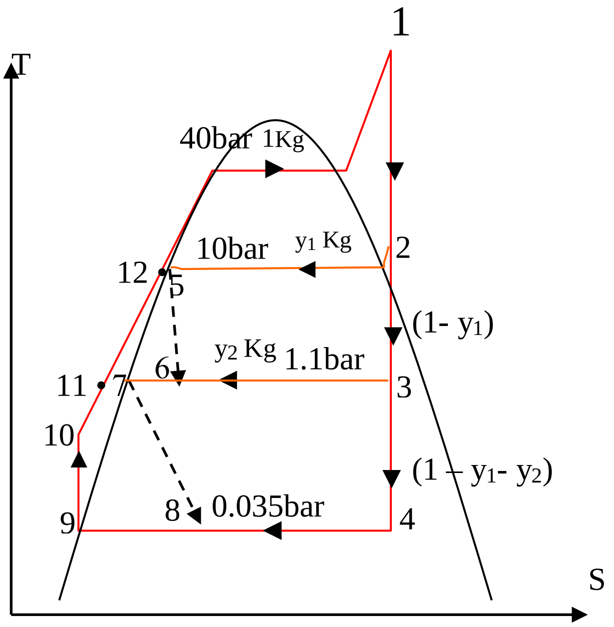
$h_5 = h_6$

$h_7 = h_8$

EX :- consider a steam power plant operating on the regenerative Rankine cycle with two(C.F.W.H)the steam enters the turbine at 40bar & 500°C &is exhaust to condenser at 0.035bar , intermediate bleed pressure are obtained such that the saturation temperature intervals are approximately equal ,giving pressure

10bar & 1.5bar,determine the amount of bleed steam at each stage & cycle efficiency .

Sol:-



P_1 = 40 bar & 500°C

h_1 = 3445 KJ/Kg , S_1 = 7.089 KJ/Kg .°K = S_2 = S_3 = S_4

P_2 = 10 bar , S_2 = 7.089 KJ/Kg .°K

S_g = 6.586 KJ/Kg .°K < S_2 = 7.089 KJ/Kg .°K therefore steam is super heated

$$h_2 = h_{250} + \frac{7.089 - 6.926}{7.124 - 6.926} (h_{300} - h_{250})$$

h_2 = 2944+0.823 (3052 – 2944)

h_2 = 3032.9091KJ/Kg

P_3 = 1.1bar & S_3 = 7.089 KJ/Kg .°K

$S_{g3} > S_3$ therefore steam is wet

$S_3 = S_{f3} + X_3 S_{fg3}$

7.089 = 1.333+ X_3 5.994

X_3 = 0.961

$h_3 = h_{f3} + X_3h_{fg3}$

$= 429 + 0.961 * 2251$

$= 2590.6$KJ /Kg

$P_4 = 0.035$bar & $S_4 = 7.089$ KJ/Kg .^{o}K

$S_{g4} > S_4$ therefore steam is wet

$S_4 = S_{f4} + X_4S_{fg4}$

$7.089 = 0.391 + X_4\ 8.130$

$X_4 = 0.824$

$h_4 = h_{f4} + X_4h_{fg4}$

$= 112 + 0.824 * 2438 = 2120.9$KJ /Kg

$P_5 = 10$bar

$h_{f5} = h_5 = 763$KJ/Kg

$h_6 = h_5 = 763$KJ/Kg ……throttle process

$P_7 = 1.1$bar

$h_{f7} = h_7 = 429$KJ/Kg

$h_8 = h_7 = 429$KJ/Kg ……throttle process

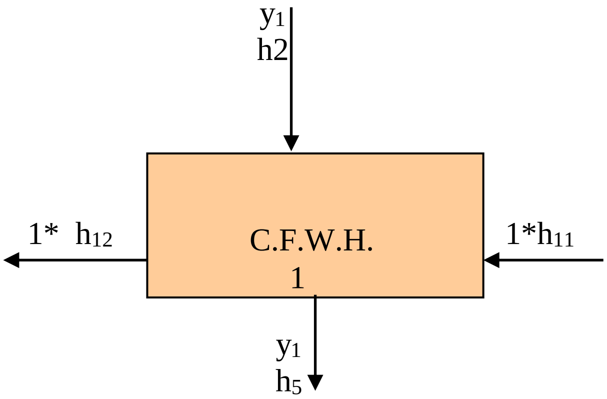
$P_9 = 0.035$bar

$h_{f9} = h_9 = 112$KJ/Kg

$h_9 = h_{10} = 112$KJ/Kg ……neglected work pump

$h_{11} = h_7 = 429$KJ/Kg ……throttles process…. C.F.W.H

$h_{12} = h_5 = 763$KJ/Kg ……throttles process…. C.F.W.H

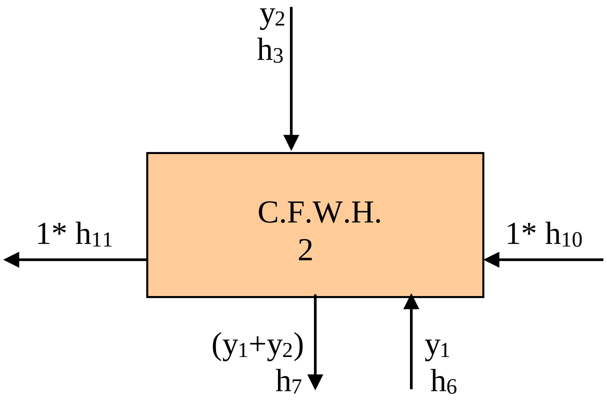


Q in = Q out

$y_1 h_2 + h_{11} = y_1 h_5 + h_{12}$

$y_1 (h_2 - h_5) = h_{12} - h_{11}$

$$y_1 = \frac{(h_{12} - h_{11})}{h_2 - h_5} = \frac{763 - 429}{3029.09 - 763} = 0.147 \text{ Kg /Kg of steam}$$



Q in = Q out

$y_2 h_3 + h_{10} + y_1 h_6 = (y_1 + y_2) h_7 + h_{11}$

$y_2 (h_3 - h_7) + y_1 (h_6 - h_7) = h_{11} - h_{10}$

$$Y_2 = \frac{h_{11} - h_{10} - y_1 (h_6 - h_7)}{h_3 - h_7} = \frac{(429 - 112) - 0.147 (763 - 429)}{2591 - 429} = 0.124 \text{ Kg /Kg of steam}$$

$$wT = 1*(h_1 - h_2) + (1-y_1)*(h_2 - h_3) + (1-y_1 - y_2)*(h_3 - h_4)$$

$$= (3445 - 3032) + (1- 0.147)(3032.9 - 2591) + (1- 0.147 -0.124) (2591-2120.9)$$

$$= 1131.7 \text{ KJ/Kg}$$

Wnet = Wturbine - wpump

= 1131.7 - 0

=1131.7 KJ/Kg

Heat supply = $h_1 - h_{12}$

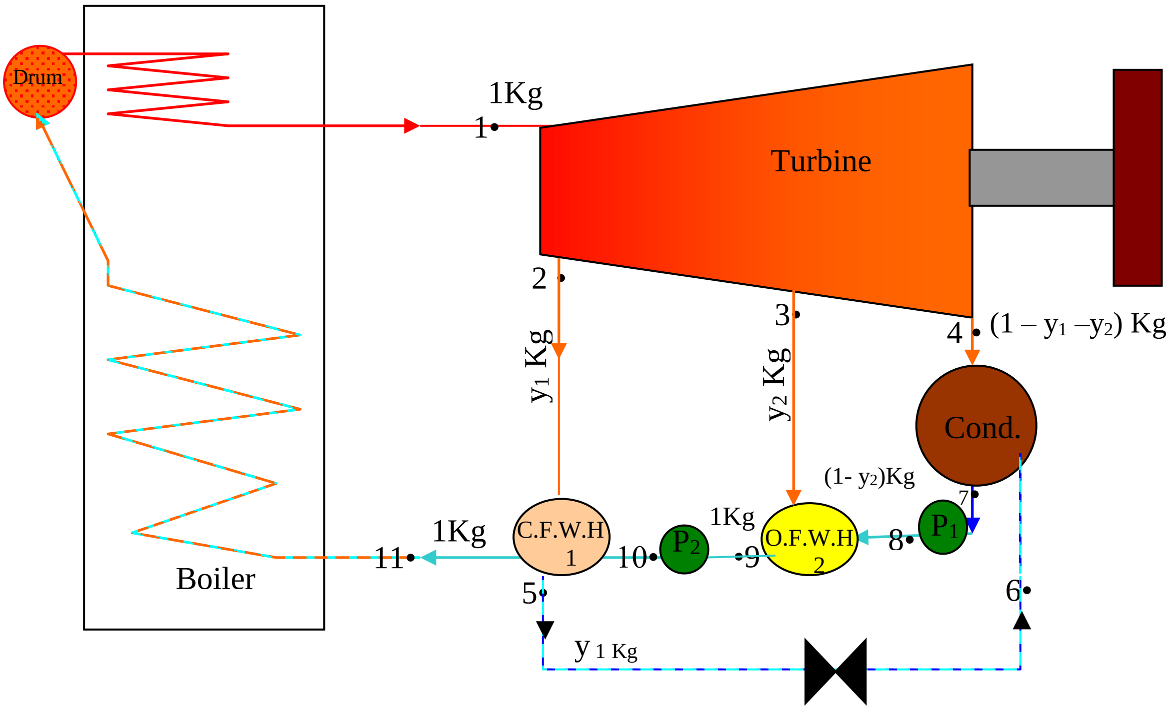
= 3445 – 763

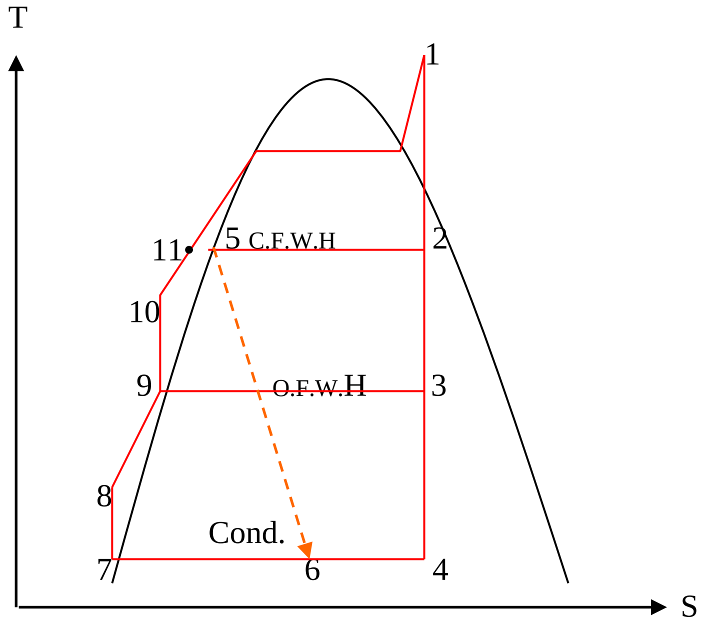
= 2682 KJ/Kg

$$\eta R = \frac{w_{net}}{\text{Heat supply}} = \frac{1131.7}{2682} = 0.422 = 42.2\%$$

Combination open &closed system :-

Ex :- consider regenerative Rankine cycle with two feed heaters one closed & one open ,steam enter the turbine at 12MPa & 550°C & exhausted to condenser at 10KPa steam extracted from turbine at 0.8 MPa for closed & at 0.3 MPa for open the condensate steam leaves closed feed water heater & throttle to condenser , determine mass flow rate if power out put is 600KW , thermal efficiency .





Sol:-

$P_1 = 120bar$, $T_1 = 550^oC$

$$h_1 = \frac{h_{500} + h_{600}}{2} = \frac{3348 + 3607}{2}$$

$h_1 = 3477.5$ KJ/Kg , $S_1 = 6.644$ KJ/Kg .^{o}K = $S_2 = S_3 = S_4$

$P_2 = 8$ bar , $S_2 = 6.644$ KJ/Kg .^{o}K

$S_g > S_2 = 6.644$ KJ/Kg .^{o}K therefore steam is wet

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.644 = 2.049 + X_2\ 4.617$

$X_2 = 0.996$

$h_2 = h_{f2} + X_2 h_{fg2}$

$= 721 + 0.9961 * 2048$

$= 2760.28$ KJ /Kg

$P_3 = 3bar$ & $S_3 = 6.644$ KJ/Kg .^{o}K

$S_{g3} > S_3$ therefore steam is wet

$S_3 = S_{f3} + X_3S_{fg3}$

$6.644 = 1.672 + X_3\ 5.321$

$X_3 = 0.935$

$h_3 = h_{f3} + X_3h_{fg3}$

$= 561 + 0.935 * 2164$

$= 2584.34$KJ /Kg

$P_4 = 0.1$bar & $S_4 = 6.644$ KJ/Kg .^{o}K

$S_{g4} > S_4$ therefore steam is wet

$S_4 = S_{f4} + X_4S_{fg4}$

$6.644 = 0.649 + X_4\ 7.5$

$X_4 = 0.799$

$h_4 = h_{f4} + X_4h_{fg4}$

$= 192 + 0.799 * 2392 = 2103.2$KJ /Kg

$P_5 = 8$bar

$h_{f5} = h_5 = 721$KJ/Kg

$h_6 = h_5 = 721$KJ/Kg ……throttle process

$P_7 = 0.1$bar

$h_{f7} = h_7 = 192$KJ/Kg

$P_8 = P_3 = 3$bar

$Wp_1 = h_8 - h_7 = \nu_{f7}\ (P_8 - P_7)$

$$\nu_{f7} = 0.0010099 + \frac{0.1 - 0.09882}{0.1233 - 0.09582}\ (0.001012 - 0.001099)$$

$\nu_{f7} = 0.0010102\ m^3 / kg$

$wp_1 = h_8 - h_7 = \nu_{f7}\ (P_8 - P_7)$

$= h_8 - 192 = 0.0010102 * 10^2(3 - 0.1) = 0.292958$KJ/Kg

$0.292958 = h_8 - 199$

$h_8 = 192.292958 KJ/Kg$

$h_9 = h_{f9} = 561$ KJ/Kg …….at $P_9 = P_3 = 3bar$

$P_{10} = P_1 = 120bar$

$Wp_2 = h_{10} - h_9 = v_{f9}\,(P_{10} - P_9)$

$$v_{f9} = 0.00107 + \frac{3 - 2.701}{3.131 - 2.701}\,(0.001075 - 0.00107)$$

$v_{f7} = 0.001072\ m^3 / kg$

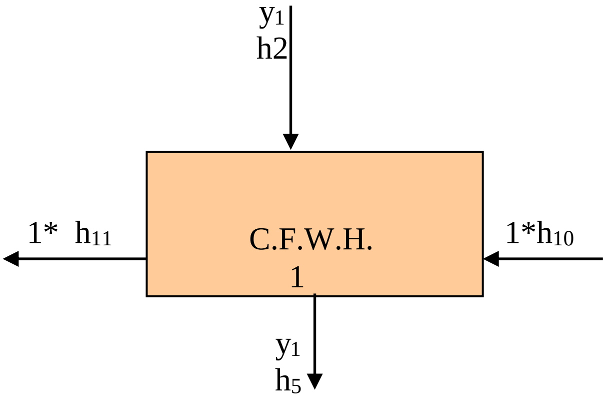
$wp_2 = h_{10} - h_9 = v_{f9}\,(P_{10} - P_9)$

$= h_{10} - 561 = 0.001 * 10^2 (120 - 3) = 11.7 KJ/Kg$

$11.7 = h_{10} - 561$

$h_{10} = 572.7 KJ/Kg$

$h_{11} = h_5 = 721 KJ/Kg$

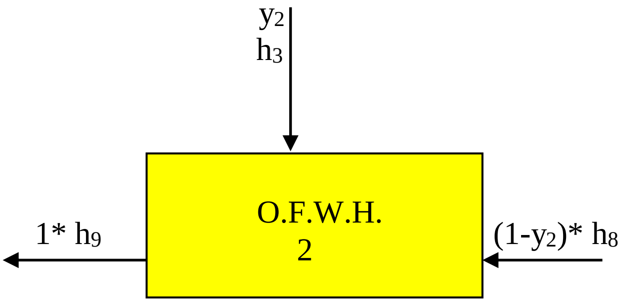


Q in = Q out

$y_1 h_2 + h_{10} = y_1 h_5 + h_{11}$

$y_1 (h_2 - h_5) = h_{11} - h_{10}$

$$y_1 = \frac{(h_{11} - h_{10})}{h_2 - h_5} = \frac{721 - 572.7}{2760.28 - 721} = 0.07272\,Kg/Kg \text{ of steam}$$



Q in = Q out

$y_2 h_3 + (1- y_2) h_8 = h_9$

$$y_2 = \frac{h_9 - h_8}{h_3 - h_8} = \frac{561 - 192.292958}{2584.34 \quad - 192.292958} = 0.1541 \text{Kg /Kg of steam}$$

$wT = 1^*(h_1 - h_2) + (1-y_1)^*(h_2 - h_3) + (1-y_1 - y_2)^*(h_3 - h_4)$

$=(3477.5-2760.28)+(1-0.07272)(2760.28-2584.34)+(1-0.07272-0.1541)(2584.34 - 2103.2)$

$=717.22+0.92728^*175.94+ 0.77318^*481.14$

$= 1252.373468$KJ/Kg

$wpump = 1^*wp_2 + (1-y_2)^* wp_1$

$= 1^*11.7+(1- 0.1541)\ 0.292958$

$= 11.992958$KJ/Kg

Wnet = Wturbine - wpump

$= 1252.37- 11.992958$

$=1240.377042$KJ/Kg

Power out = $m^{\bullet}_s$ * Wnet

$600^*10^*10^*10= m^{\bullet}_s * 1240.377042$

$m^{\bullet}_s =483.72$Kg/sec

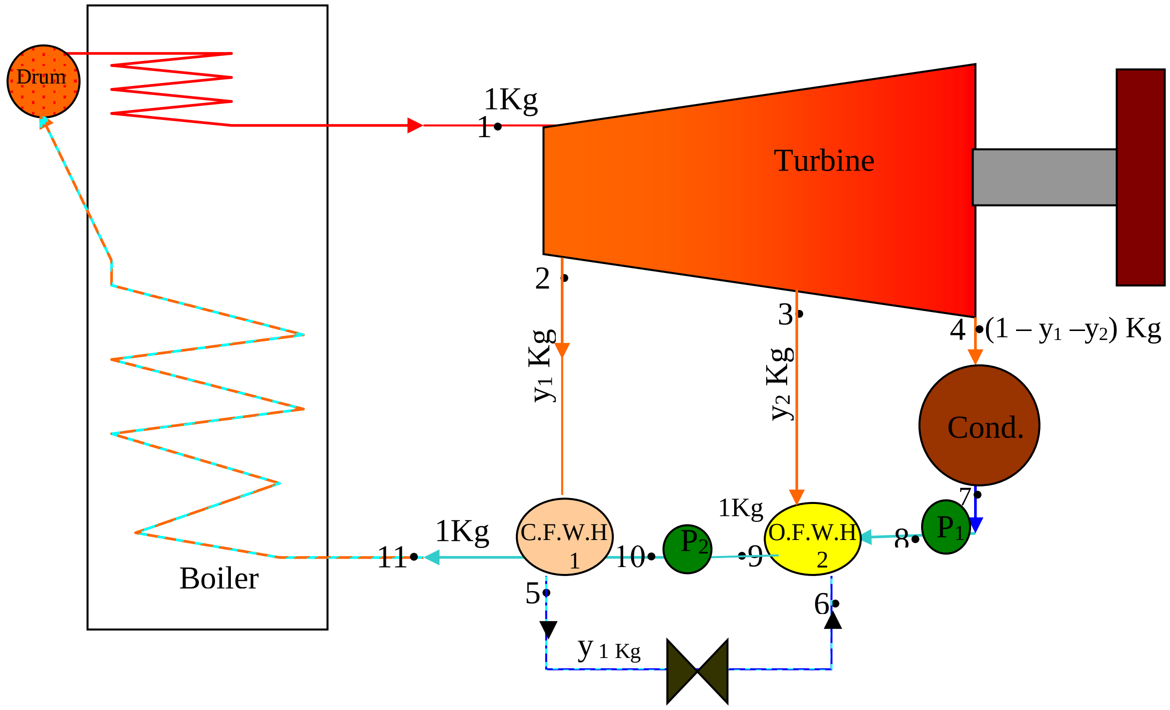
Heat supply = $h_1 - h_{11}$

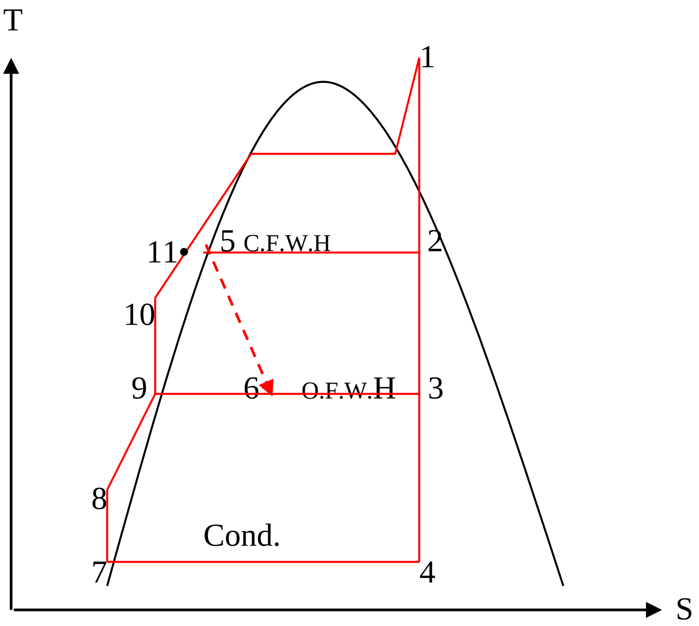
$= 3477.5 - 721$

$= 2756.5$ KJ/Kg

$$\eta R = \frac{w_{net}}{\text{Heat supply}} = \frac{1240.377042}{2756.5} = 0.4499 = 44.99\%$$

Ex: - consider regenerative Rankine cycle with two feed heaters one closed & one open ,steam enter the turbine at 12MPa & 550°C & exhausted to condenser at 10KPa steam extracted from turbine at 0.8 MPa for closed & at 0.3 MPa for open the condensate steam leaves closed feed water heater & throttle to open feed water heater , determine mass flow rate if power out put is 600KW , thermal efficiency . Neglected pump work





Sol:-

P_1= 120bar , T_1= 550°C

$$h_1 = \frac{h_{500} + h_{600}}{2} = \frac{3348 + 3607}{2}$$

h_1 = 3477.5 KJ/Kg , S_1 = 6.644 KJ/Kg .°K = $S_2 = S_3 = S_4$

P_2 = 8 bar , S_2 = 6.644 KJ/Kg .°K

$S_g > S_2$ =6.644 KJ/Kg .°K therefore steam is wet

$S_2 = S_{f2} + X_2 S_{fg2}$

$6.644 = 2.049 + X_2\, 4.617$

$X_2 = 0.996$

$h_2 = h_{f2} + X_2 h_{fg2}$

= 721 + 0.9961* 2048

= 2760. 28KJ /Kg

P_3 = 3bar & S_3 = 6.644 KJ/Kg .°K

$S_{g3} > S_3$ therefore steam is wet

$S_3 = S_{f3} + X_3 S_{fg3}$

$6.644 = 1.672 + X_3\ 5.321$

$X_3 = 0.935$

$h_3 = h_{f3} + X_3 h_{fg3}$

$= 561 + 0.935 * 2164$

$= 2584.34$KJ /Kg

$P_4 = 0.1$bar & $S_4 = 6.644$ KJ/Kg .°K

$S_{g4} > S_4$ therefore steam is wet

$S_4 = S_{f4} + X_4 S_{fg4}$

$6.644 = 0.649 + X_4\ 7.5$

$X_4 = 0.799$

$h_4 = h_{f4} + X_4 h_{fg4}$

$= 192 + 0.799 * 2392 = 2103.2$KJ /Kg

$P_5 = 8$bar

$h_{f5} = h_5 = 721$KJ/Kg

$h_6 = h_5 = 721$KJ/Kg ……throttle process

$P_7 = 0.1$bar

$h_{f7} = h_7 = 192$KJ/Kg

$P_8 = P_3 = 3$bar

$h_8 = h_7$ ……..neglected work pump

$h_8 = 192$KJ/Kg

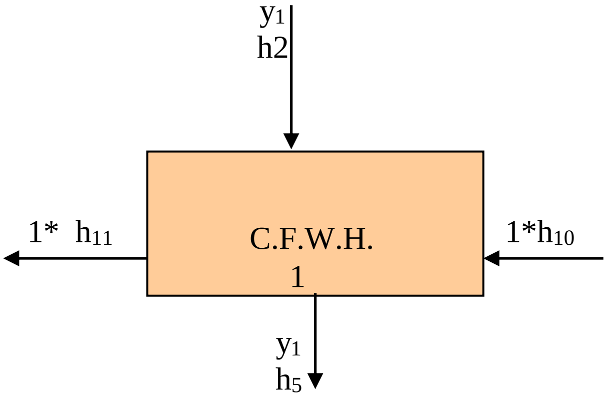
$h_9 = h_{f9} = 561$ KJ/Kg …….at $P_9 = P_3 = 3$bar

$P_{10} = P_1 = 120$bar

$H_{10} = h_9$ ……..neglected work pump

$h_{10} = 561$KJ/Kg

$h_{11} = h_5 = 721$KJ/Kg

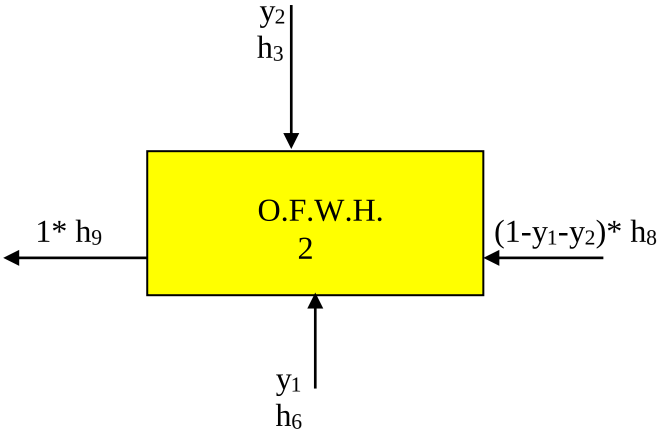


Q in = Q out

$y_1 h_2 + h_{10} = y_1 h_5 + h_{11}$

$y_1 (h_2 - h_5) = h_{11} - h_{10}$

$$y_1 = \frac{(h_{11} - h_{10})}{h_2 - h_5} = \frac{721 - 561}{2760.28 - 721} = 0.0784 \text{ Kg /Kg of steam}$$



Q in = Q out

$y_2 h_3 + (1 - y_1 - y_2) h_8 + y_1 h_6 = h_9$

$$y_2 = \frac{(h_9 - h_8) - y_1(h_6 - h_8)}{h_3 - h_8} = \frac{561 - 192 - 0.0784\,(721 - 192)}{(2584.34 - 192)} = 0.157 \text{ Kg /Kg of steam}$$

$wT = 1^*(h_1 - h_2) + (1 - y_1)^*(h_2 - h_3) + (1 - y_1 - y_2)^*(h_3 - h_4)$

$= 3477.5 - 2760.28 + (1 - 0.0784)(2760.28 - 2584.34) + (1 - 0.0784 - 0.157)$

(2584.34 -2103.2)

= 717.5+ 0.9216* 175.94 +0.7646 * 481.14 =717.5+162.146304 + 367.879

= 1247.525304KJ/Kg

Wnet = Wturbine – wpump , = 1247.525304 – 0=1247.525304KJ/Kg

Power out = $m^{\bullet}_{s}$ * Wnet

600*10*10*10= $m^{\bullet}_{s}$ * 1247.52

$m^{\bullet}_{s}$ =440.42Kg/sec

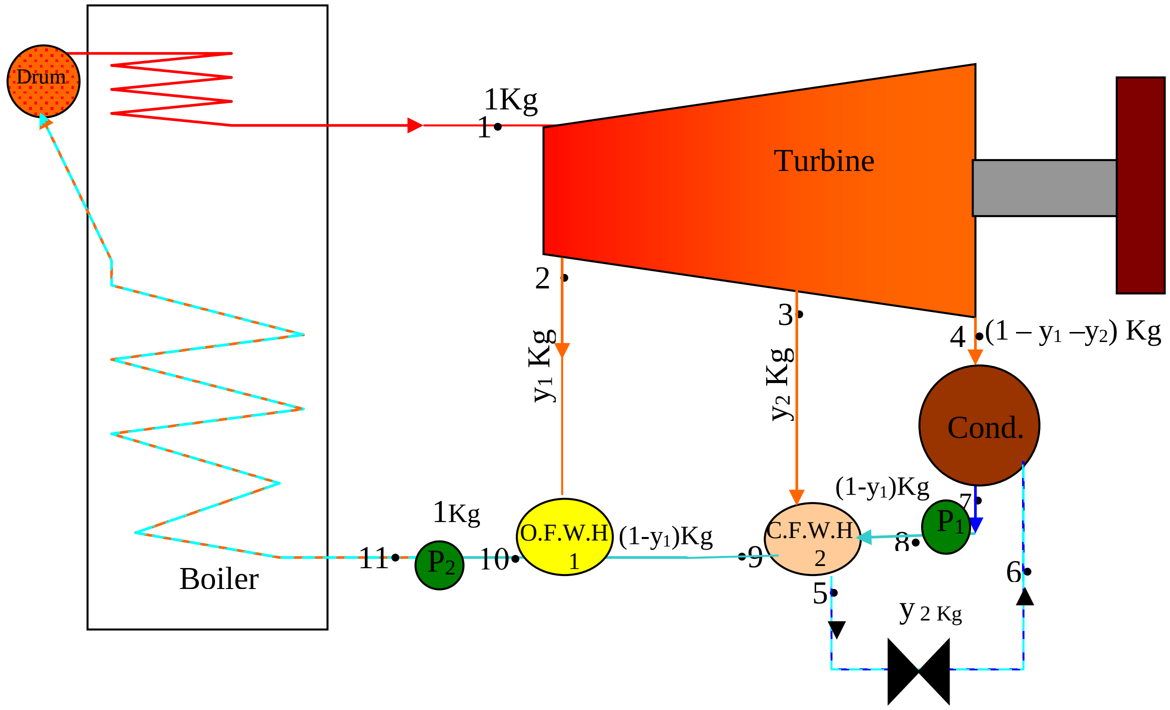
Heat supply = $h_1 - h_{11}$

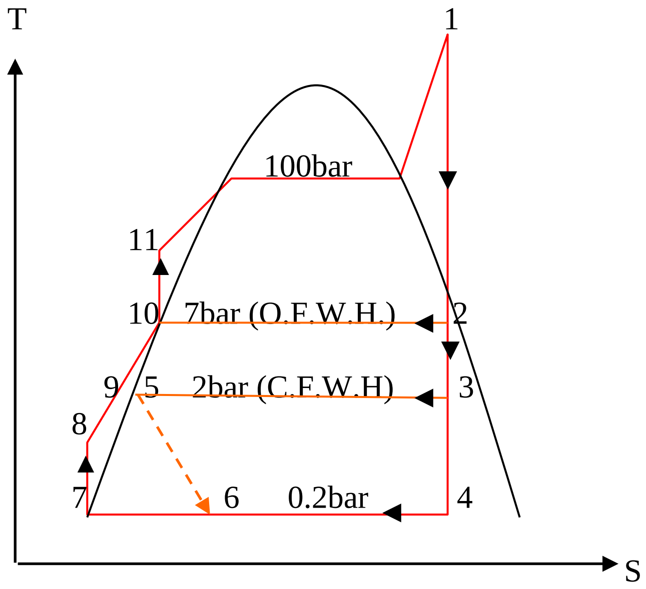
= 3477.5 – 721

= 2756.5 KJ/Kg

$$\eta R = \frac{W_{net}}{\text{Heat supply}} = \frac{1247.52530}{2756.5} = 0.452 = 45.2\%$$

Ex :- consider regenerative Rankine cycle with two feed heaters one closed & one open ,steam enter the turbine at 10MPa & 500°C & exhausted to condenser at 20KPa steam extracted from turbine at 0.7 MPa for open & at 0.2 MPa for close the condensate steam leaves closed feed water heater & throttle to condenser , determine mass flow rate if power out put is 250MW , thermal efficiency . Neglected pump work.





Sol:- P_1= 100bar , T_1= 500°C

h_1 = 3373 KJ/Kg , S_1 = 6.596 KJ/Kg .°K = S_2 = S_3 = S_4

P_2 = 7 bar , S_2 = 6.596 KJ/Kg .°K

$S_g > S_2$ ………..therefore steam is wet

$S_2 = S_{f2} + X_2S_{fg2}$

$6.596 = 1.992 + X_2\ 4.717$

$X_2 = 0.976$

$h_2 = h_{f2} + X_2h_{fg2}$

$= 697 + 0.976 * 2067$

$= 2714.4$ KJ /Kg

P_3 = 2bar & S_3 = 6.596 KJ/Kg .°K

$S_{g3} > S_3$ therefore steam is wet

$S_3 = S_{f3} + X_3S_{fg3}$

$6.596 = 1.53 + X_3\ 5.597$

$X_3 = 0.905$

$h_3 = h_{f3} + X_3h_{fg3}$

$= 505 + 0.905 * 2202$

$= 2498$KJ /Kg

$P_4 = 0.7$bar & $S_4 = 6.596$ KJ/Kg .°K

$S_{g4} > S_4$ therefore steam is wet

$S_4 = S_{f4} + X_4 S_{fg4}$

$6.596 = 0.832 + X_4\ 7.075$

$X_4 = 0.814$

$h_4 = h_{f4} + X_4 h_{fg4}$

$= 251 + 0.814 * 2358 = 2172$KJ /Kg

$P_5 = 2$bar

$h_{f5} = h_5 = 505$KJ/Kg

$P_6 = P_4 = 2$bar

$h_6 = h_5 = 505$KJ/Kg ……throttle process

$P_7 = 0.2$bar

$h_{f7} = h_7 = 251$KJ/Kg

$P_8 = P_3 = 2$bar

$h_8 = h_7$ ……..neglected work pump

$h_8 = 251$KJ/Kg

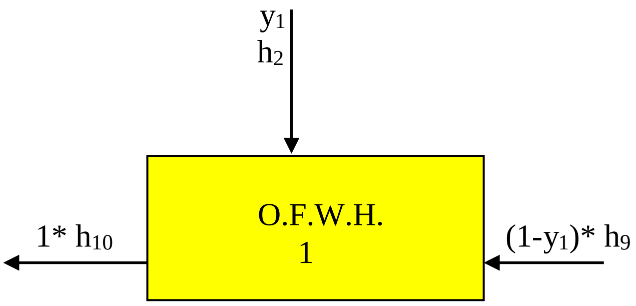
$h_9 = h_{f9} = 505$ KJ/Kg …….at $P_9 = P_3 = 2$bar

$P_{10} = P_2 = 7$bar

$h_{f10} = h_{10} = 697$KJ/Kg

$h_{10} = h_1$ ……..neglected work pump

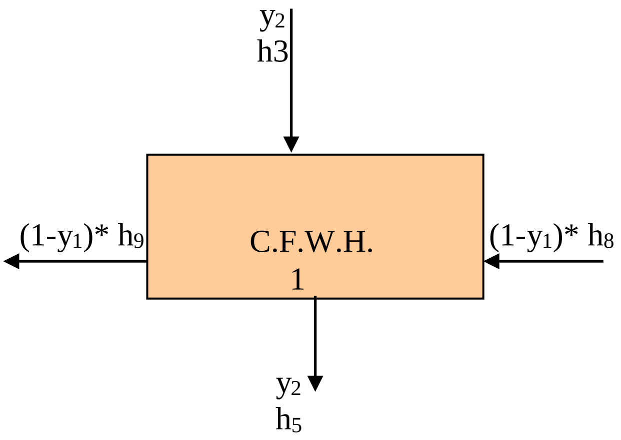
$h_{11} = 697$KJ/Kg



Q in = Q out

$$y_1h_2 + (1-y_1)h_9 = h_{10}$$

$$y_2 = \frac{h_{10} - h_9}{h_2 - h_9} = \frac{697 - 505}{2714.4 - 505} = 0.0869 \text{ Kg/Kg of steam}$$



Q in = Q out

$$y_2h_3 + (1-y_1)h_8 = h_9(1-y_1) + y_2h_5$$

$$y_2 = \frac{(1-y_1)\ (h_9 - h_8)}{h_3 - h_5} = \frac{(1-0.0869)\ (505 - 251)}{(2498 - 505)} = 0.1163 \text{ Kg/Kg of steam}$$

$$wT = 1^*(h_1 - h_2) + (1-y_1)^*(h_2 - h_3) + (1-y_1 - y_2)^*(h_3 - h_4)$$

$$= (3373 - 2714.4) + (1 - 0.0869)(2714.4 - 2498) + (1 - 0.0869 - 0.1163)(2498 - 2172)$$

= 658.6+ 0.9131* 216.4+0.7968* 326

=658.6+197.59484+ 259.7568

= 1115.95164KJ/Kg

Wnet = Wturbine – wpump

= 1115.95164 – 0

=1115.95164KJ/Kg

Power out = $m^{\bullet}_{s}$ * Wnet

250*10*10*10= $m^{\bullet}_{s}$ * 1115.95164

$m^{\bullet}_{s}$ =224.024 Kg/sec

Heat supply = $h_1 - h_{11}$

= 3373- 697

= 2676KJ/Kg

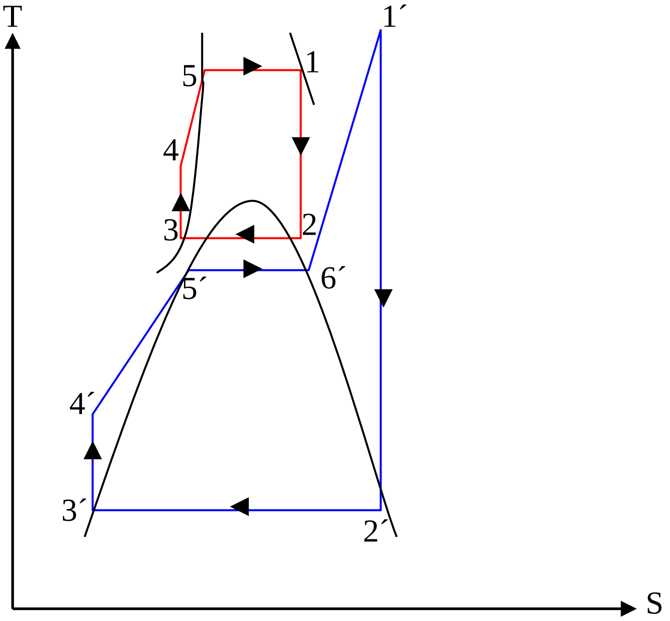
$$\eta R = \frac{w_{net}}{\text{Heat supply}} = \frac{1115.95164}{2676} = 0.4170 = 41.7\%$$

The binary vapour cycle

It has been stated that the efficiency of the Rankine cycles increases as the maximum temperature increases. The maximum value at present is about 600°C . The critical temperature of steam is374.15°C , which is well within this limits ; the corresponding saturation pressure is high at 221.2 bar . These considerations show some of the limitation of steam as the working substance. It would be better to have a working substance which has a critical temperature well above 600°C , and a low corresponding saturation pressure . Mercury is such a substance; from tables at p = 24 bar, t = 604.6 °C, and v_g = 0.01518 m^3 /kg. At the normal temperature of condensation (i.e. about 30 °C) the vapour pressure is very low and v_g is very high; at 109.2 °C, p = 0.0006 bar and v_g 259.6 m^3 /kg. It is evident then that mercury is satisfactory at the upper temperature limits but the lower temperatures the properties of steam are preferable. This indicates the possibility of combining the desirable properties of mercury and steam in one plant. This is done in the binary vapour plant, and the T-s diagram for such a system is shown in fig.

The upper cycle is Rankine cycle for mercury, and the lower is Rankine cycle for steam. The two cycle have different scales, and the connecting link between the two is that the heat required to evaporate the steam is obtained by condensing the mercury the temperature difference, $(T_3 - T_{5'}) = (T_2 - T_{6'})$, being that required for the heat transfer process . The superheat to the steam is provided by the hot gases in the mercury boiler, as is the preheating of the feed water.

If the steam pressure is 42 bar, say, the temperature of steam generation is 253.2 ^{o}C . Allowing 22K for the heat transfer process gives a temperature of 275.2 ^{o}C for the mercury condensation, with a corresponding saturation pressure of 0.19 bar approximately.



Several binary vapour plants using mercury and steam have been built, but there are several disadvantages in such plants. Mercury is very expensive, heavy, and poisonous, so that the capital cost of plant is high. Mercury in its normal state does not wet steel, and this impairs the transfer of heat to the mercury; this can be improved by adding magnesium and titanium to the mercury.

Mixtures :-

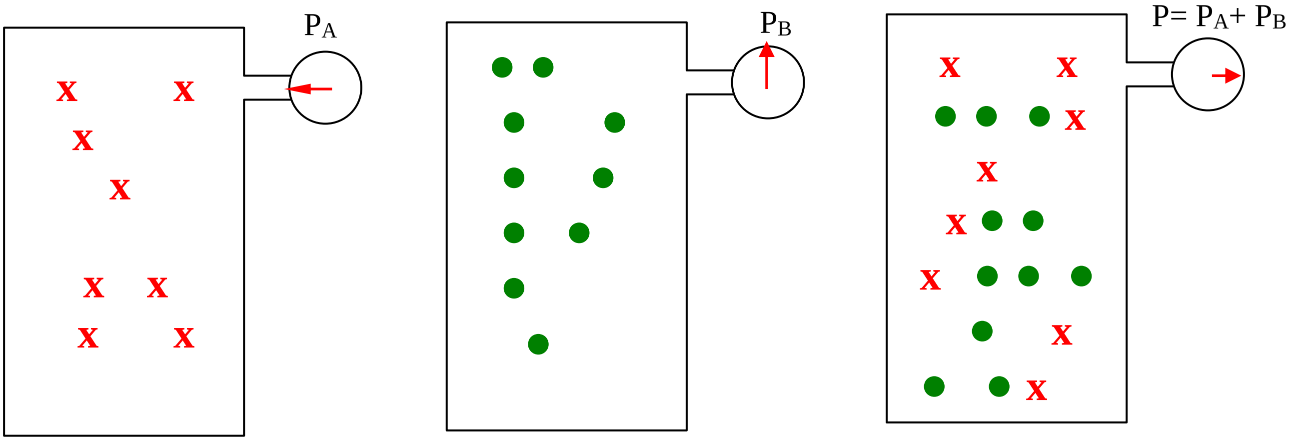
A pure substance is defind as a substance having a constant and and uniform chemical composition , and this definition can be extended to include a homogeneuos Mixture of gases when there is no chemical reaction taking place .
The thermodynamic properties of amixture of gases can be determined in the same wayas for single gases .
The most common example of this is dry air , which is amixture of oxygen , nitrogen , asmall percentage of argon , and traces of other gases .
The properties of air have been detemined and it is considered as asingle substance .

Dalton̉ s law and the gibbs – Dalton law

Consider a closed vessel of volume V at temperature T , which contains a mixture of perfect gases at a known pressure .

The pressure of a mixture of gases is equal to the sum of the partial pressures of the consitituents .
The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixture at the same temperature .

This is expressed in fig. the gases A&B, originally occupying volume V at temperature T are mixed in the third vessel which is of the same volume and is at the Same temperature .



Gas A
mass = m_A

Gas B
mass = m_B

mixture of A + B
mass = $m_A + m_B$

for a number of gases …

$m = m_A + m_B + m_C + \ldots\ldots\ldots$

$m = \sum m_i$ (where m_i is the mass of constituent)

$P = P_A + P_B + P_C + \ldots\ldots\ldots$

$P = \sum P_i$

(where P_i is the partial pressure of constituent).

EX:- A vessel of volume (0.4 m^3) contain 0.45kg of carbon monixide CO (28) &1kg of air , at 15°C . calculate the partial pressure of each constituent &the the total pressure in the vessel . the gravimetric analysis of air is to be taken as 23.3% oxygen (32) , and 76.7% nitrogen (28) .

Sol :-

Mass of oxygen present $= \frac{23.3}{100} * 1 = 0.233$ Kg

Mass of nitrogen present $= \frac{76.7}{100} * 1 = 0.767$ Kg

PV = mRT

$$R = \frac{R_u}{M}$$

$$P_i = \frac{m_i R_u T}{M_i V}$$

The volume V is $0.4m^3$ and the temperarure T is (15+273) = 288K. therefore we have,

For O_2 ,

$$P_{O_2} = \frac{0.233 * 8.314 * 288}{32 * 0.4} = 43.59 KN / m^2$$

$$= \frac{43.59 * 10^3}{10^5} = 0.4359 \text{ bar}$$

For N_2 ,

$$P_{N2} = \frac{0.767 * 8.314 * 288}{28 * 0.4} = 164 KN / m^2$$

$$= \frac{164 * 10^3}{10^5} = 1.64 \text{ bar}$$

For CO,

$$P_{CO} = \frac{0.45 * 8.314 * 288}{28 * 0.4} = 96.2 KN / m^2$$

$$= \frac{96.2 * 10^3}{10^5} = 0.962 \text{ bar}$$

The total pressure in the vessel is given by

$$P = \sum P_i = P_{O_2} + P_{N2} + P_{CO}$$

$$= 0.436 + 1.64 + 0.962$$

$$= 3.038 \text{ bar}$$

Gibbs – Dalton law :-

Dalton law was reformulated by gibbs to include a second statement on the properties of mixture . and is as follows :

The internal energy , enthalpy , and entropy , of agaseous mixture are respectively equal to the sums of the internal energies , enthalpies , and entropies , of the constituents .

Each constituent has that internal energy ,enthalpy, and entropy , which it would have if it occupied alone that volume occupied by the mixture at the temperature of the mixture .

This statement lead to the equations .

$$mu = m_A u_A + m_B u_B + etc. \quad or \quad mu = \sum m_i u_i$$

$$mh = m_A h_A + m_B h_B + etc. \quad or \quad mh = \sum m_i h_i$$

$$ms = m_A s_A + m_B s_B + etc. \quad or \quad ms = \sum m_i s_i$$

volumetric analysis of gas mixture

Consider a volume V of a gaseous mixture at a temperature T, consisting of three constituents A,B, and C let each of the constituents be compressed to apressure P equal to the total pressure of the mixture , and let the temperature remain constant .the partial volumes then occupied by the constituents will be V_A , V_B , V_C.

$m = m_A + m_B + m_C = \sum m_i$
$p = p_A + p_B + p_C = \sum p_i$
$n = n_A + n_B + n_C = \sum n_i$

V_A	V_B	V_C
P	p	P
m_A	m_B	m_C
n_A	n_B	n_C

from equation pV =mRT therefore

$$m_A = p_A V / R_A T = p V_A / R_A T$$

i.e.

$$p_A V = p V_A \quad or \quad V_A = (p_A / p) V$$

ingeneral therefore ,

$$V_i = (p_i / p) V$$

$$\sum V_i = \sum (p_i / p) V$$

$$\sum V_i = (V / p) \sum p_i$$

$$P = \sum p_i$$

$$V = \sum V_i$$

$pV = mRT = n.M.R.T$ (n = number of mole) ,$n = m/M$

$MR = p.v / n.T$ ($R = Ru/M$)

$p.V = Ru \, . \, n \, .T$

$p_i.V = Ru \, . \, n_i \, .T$

$\sum p_i.V = Ru \, .\sum n_i \, .T$

$p.V = Ru \, . \, n \, .T$

$n=\sum n_i$

$n_i = m_i / M_i$

molecular weight & gas constant .

to find the gas constant for the mixture in terms of the gas constants of the constituents ,

$pV = mRT$

$p_i.V = R_i \, . \, m_i \, .T$

$\sum p_i.V = R_i \, .\sum m_i \, .T$

$V \sum p_i = T.\sum m_i \, .R_i$

$P=\sum p_i$

$V \, p = T.\sum m_i \, .R_i = m.R.T$

$m.R = \sum m_i \, .R_i$ **or…**

$R = \sum \frac{m_i}{m} R_i$ **(where m_i/m is the mass fraction of a constituents) .**

$$\frac{p_i \, .V}{p \, .V} = \frac{n_i \, .Ru \, .T}{n.Ru \, .T}$$

$$\frac{p_i}{p} = \frac{n_i}{n}$$

$$\boxed{\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V}}$$

…. This is an important result which means that the

molar analysis is identical with the volumetric analysis ,and both are equal to the ratio of the partial pressure to the total pressure .

Another method of determining the apparent molecular weight is as follows :

$m_i = p_iV/ R_iT$ & $m = pV/ RT$ $m = \sum m_i$ therefore…….

$$\frac{p\,.V}{R\,.T} = \sum \frac{p_i\,.V}{R_i\,.T}$$

$$\frac{p}{R} = \sum \frac{p_i}{R_i}$$

using equation $R = Ru /M$ thterefore ….

$$\frac{P\,M}{Ru} = \sum \frac{p_i\,M_i}{Ru}$$

$$P\,M = \sum p_i\,M_i$$

$$\boxed{M = \sum \frac{p_i}{P} M_i}$$

$$\boxed{M = \sum \frac{V_i}{V} M_i}$$

$$\boxed{M = \sum \frac{n_i}{n} M_i}$$

Gravimetric analysis :-

$m = m_A + m_B + m_C + \ldots\ldots Mn$

divide by m

$$1 = \frac{m_A}{m} + \frac{m_B}{m} + \frac{m_C}{m} + \ldots\ldots \frac{m_n}{m}$$

$$g_A = \frac{m_A}{m}$$

$$g_B = \frac{m_B}{m}$$

$$g_C = \frac{m_C}{m}$$

$$\boxed{1 = g_A + g_B + g_C + \ldots\ldots\ g_n}$$

$$\mathbf{V = m \cdot \nu}$$

$$\mathbf{V = V_A = m_A \nu_A} \longrightarrow \mathbf{m_A = \frac{V}{\nu_A}}$$

$$\mathbf{V = V_B = m_B \nu_B} \longrightarrow \mathbf{m_B = \frac{V}{\nu_B}}$$

$$\mathbf{V = V_C = m_C \nu_C} \longrightarrow \mathbf{m_C = \frac{V}{\nu_C}}$$

$$\mathbf{V = V_n = m_n \nu_n} \longrightarrow \mathbf{m_n = \frac{V}{\nu_n}}$$

$$\mathbf{m = m_A + m_B + m_C + \ldots\ldots\ mn}$$

$$\mathbf{\frac{V}{\nu} = \frac{V}{\nu_A} + \frac{V}{\nu_B} + \frac{V}{\nu_C}\ \ldots\ \frac{V}{\nu_n}} \qquad \mathbf{\div V}$$

$$\boxed{\mathbf{\frac{1}{\nu} = \frac{1}{\nu_A} + \frac{1}{\nu_B} + \frac{1}{\nu_C}\ \ldots\ \frac{1}{\nu_n}}}$$

$$\frac{1}{v} = \sum_{i=1}^{i=n} \frac{1}{v_i}$$

$$\rho = \frac{1}{v}$$

therefore…..

$$\boxed{\rho = \rho_A + \rho_B + \rho_C + \ldots\ldots\ \rho n}$$

$$\rho = \sum_{i=1}^{i=n} \rho_i$$

E.X :- the gravimetric analysis of air is 23.14% oxygen (31.999) , 75.53 % nitrogen (28.013) , 1.28% argon (34.948) , 0.05% carbon dioxide (44.01) , calculate the gas constant for air and the molecular weight .

Sol:-

R = Ru/M

$$R_{O2} = \frac{8.314}{31.999} = 0.2598 \text{ KJ/Kg.K}$$

$$R_{N2} = \frac{8.314}{28.013} = 0.2968 \text{ KJ/Kg.K}$$

$$R_{Ar} = \frac{8.314}{39.948} = 0.208 \text{ KJ/Kg.K}$$

$$R_{CO2} = \frac{8.314}{44.01} = 0.1889 \text{ KJ/Kg.K}$$

$$R = \sum \frac{m_i}{m} R_i$$

R= 0.2314 * 0.2598 + 0.7553 * 0.2968 + 0.0128 * 0.2081 + 0.0005 * 0.1889 = 0.2871 KJ / Kg.K

Apperent gas constant for air **= 0.2871 KJ / Kg.K**

R = Ru / M

= 8.314 / 0.287 = 28.96

E.X :- the gravimetric analysis of air is 23.14% oxygen (31.999) , 75.53 % nitrogen (28.013) , 1.28% argon (34.948) , 0.05% carbon dioxide (44.01) , calculate the analysis by volume and the partial pressure of each constituent when the total pressure is 1 bar .

sol:-

constituent	m_i	M_i	$n_i= m_i/ M_i$	$n_i/n *100 = V_i/V *100$
O_2	0.2314	31.999	0.00723	(0.00723/0.03452) * 100 = 20.95%
N_2	0.7553	28.013	0.02696	(0.02696/0.03452) * 100 = 78.09%
Ar	0.0128	39.948	0.00032	(0.00032/0.03452) * 100 = 0.93%
CO_2	0.0005	44.01	0.00001	(0.00001/0.03452) * 100 = 0.03%
$n = \sum n_i = 0.03452$				

$$\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V}$$ **therefore , $p_i =(n_i / n)*p$ & p = 1 bar**

for O_2 $p_{O2} = 0.2095 *1 = 0.2095$ **bar**

for N_2 $p_{N2} = 0.7809 *1 = 0.7809$ **bar**

for Ar $p_{Ar} = 0.0093 *1 = 0.0093$ **bar**

for CO_2 $p_{CO2} = 0.0003 *1 = 0.0003$ **bar**

EX:- A mixture of 1 mole CO_2 (44) & 3.5 moles of air is contained in avessel at 1bar & 15°C . the volumetric analysis of air can be taken as 21% O_2 & 79% N_2 . calculate for the mixture,

- **(a) the masses of CO_2 ,O_2 &N_2 , & total mass.**
- **(b) The percentage carbon content by mass .**
- **(c) The apparent molecular weight & the gas constant for the mixture.**
- **(d) The specific volume of the mixture .**

Sol:-

(a)

$n_i = (V_i/V)\ n$

$n_{O2} = 0.21 * 3.5 = 0.735$

$n_{N2} = 0.79 * 3.5 = 2.765$

$m_i = M_i\ n_i$

$m_{CO2} = 1* 44 = 44$ Kg

$m_{O2} = 0.735* 32 = 23.55$ Kg

m_{N2} = 2.765* 28 = 77.5 Kg

the total mass,m = $m_{O2} + m_{N2} + m_{CO2}$

= 23.55 + 77.5 + 44 = 145.05 Kg

(b)

the molecular weight of carbon is 12 , therefore there are 12Kg of carbon present for every mole of carbon dioxide ,

$$\text{perecentage carbon in mixture} = \frac{12 * 100}{145.05} = 8.27\ \%\ \text{by mass}$$

(C)

$n = \sum n_i$

$n = n_{CO2} + n_{O2} + n_{N2}$

= 1+ 0.735 + 2.765 = 4.5

$$M = \sum \frac{n_i}{\mathbf{n}} M_i$$

$$\mathbf{M} = \frac{\mathbf{1}}{4.5} \mathbf{*}44 + \frac{\mathbf{0.735}}{4.5} \mathbf{*}32 + \frac{\mathbf{2.765}}{4.5} \mathbf{*}28 = \mathbf{32.2}$$

apperent molecular weight = 32.2

R = Ru /M , we have

R = 8.314 / 32.2 = 0.2581 KJ/Kg .K

Gas constant for the mixture = 0.2581 KJ/Kg .K

(d)

Pv = RT

T = 15 + 273 = 288 K

v = RT /P

=(0.2581 * 288)/ $1*10^2$

= 0.7435 m^3 / Kg

specific volume of the mixture at 1bar & 15 ^oC is 0.7435 m^3/kg

EX:- A mixture of H_2 (2), and O_2 (32) , is to be made so that the ratio of H_2 to O_2 is 2 to 1 by volume .calculate the mass of O_2 required and the volume of the container , per Kg of H_2 ,if the pressure and temperature are 1 bar & 15 °C respectively .

Sol:-
Let the mass of O_2 per Kg of H_2 be x kg .

$n_i = m_i / M_i$

$n_{H2} = ½ = 0.5$ kmol & $n_{O2} = x/32$

$V_i / V = n_i / n$,

$$\frac{V_{H2}}{V_{O2}} = \frac{n_{H2}}{n_{O2}} \quad \& \quad \frac{V_{H2}}{V_{O2}} = 2 \text{ (given)}$$

$$\frac{0.5}{x/32} = 2$$

$x = (32*0.5)/ 2 = 8$

mass of oxygen per kg of hydrogen = 8kg

the total number of moles in the vessel per kg of H_2 is

$n = n_{H2} + n_{O2} = 0.5 + x/32 = 0.5 + 8/32 = 0.75$ Kmol

$pV = nRu\,T$

$$V = \frac{0.75 * 8.314 * 288 *10^2}{1*10^5} = 17.96 \text{ m}^3$$

EX:- A vessel contains a gaseous mixture of composition by volume , 80% H_2 (2) , and 20% CO (28). It is desired that the mixture should be made in the proportion 50% H_2 and 50% CO by removing some of the mixture and adding some CO .calcculate per mole of mixture the mass of mixture to be removed , and the mass of CO to be added the pressure and temperature in the vessel remain constant during the procedure .

Sol:-
Since the pressure and temperature remain constant , then the number of moles in the vessel remain the same throughtout . therefore ,
moles of mixture removed = moles of CO added
Let x kg of mixture be removed &y kg of CO be added .

$$M = \sum \frac{V_i}{V} M_i$$

M = 0.8 * 2 + 0.2 * 28 =7.2

n = m / M

Moles of mixture removed = x / 7.2 = moles of CO added = y/28

$V_i / V = n_i / n$ therefore ,

moles of H_2 in the mixture removed = 0.8 * (x /7.2)= x / 9

moles of H_2 initially = 0.8 * 1 = 0.8

moles of H_2 remaining in the vessel = 0.8 – (x / 9)

1 mole of the new mixture is 50% H_2 & 50% CO ,therefore

0.8 – (x – 9) = 0.5

x = 2.7

mass of mixture removed = 2.7 kg

also since

x / 7.2 = y / 28

y = (28 / 7.2)* x = (28 * 2.7)/ 7.2 = 10.5

mass of CO added = 10.5 kg

EX:- Given that standerd pressure and temperature may be taken as 1.01 bar and 0 °C respectively calculate the volume of one kg mole of aperfect gas at S.T.P . and if we assume that air containes 21% of oxygen by volume the remainder being Nitrogen . calculate

a – the average molecular mass of air .

b- the value of the characterestic gas constant .

c- the mass of one cubicmeter of air at S.T.P.

Sol:-

PV =n Ru T

$$V = \frac{n\ Ru\ T}{10^2 * P} = \frac{1 * 8.314 * (0 + 273\ )}{10^2 * 1.01} = 22.47\ m^3$$

$$M = \sum \frac{V_i}{V} M_i$$

$M = (0.2 * 32) + (0.79 * 28) = 28.84 kg/mol$

the average molecular mass of air = 28.84 kg / mol

$R = Ru / M$

$= 8.314 / 28.84 = 0.2882$ kJ / kg.K

the value of the characterestic gas constant = 0.2882 kJ / kg.K

$PV = m R T$

$$m = \frac{PV}{T R} = \frac{1 * 10^2 * 1}{273 * 0.288} = 1.2895 \text{ kg} / m^3$$

the mass of one cubicmeter of air at S.T.P. = 1.2895 kg / m^3

EX :- A vessel contains 8kg of oxygen , 7 kg of Nitrogen and 22 kg of carbon dioxide. The total pressure in the vessel is 4.15 bar and the temperature is 60 °C . calculate,

- **a- the partial pressure of each gas in the vessel .**
- **b- the volume of the vessel**
- **c- the total pressure in the vessel when the temperature is raised to 280 °C .**

Sol :-

$PV = m R T$

$P_{O2} V = m_{O2} R_{O2} T$

$$R = \sum \frac{m_i}{m} R_i$$

$m = m_{O2} + m_{N2} + m_{CO2}$

$= 8 + 7 + 22 = 37$ kg

$$R = \left(\frac{8}{37} * \frac{8.314}{32}\right) + \left(\frac{7}{37} * \frac{8.314}{28}\right) + \left(\frac{22}{37} * \frac{8.314}{44}\right)$$

$R = 0.0561 + 0.0561 + 0.112$

$R = 0.2242$ kJ / kg .K

$$V = \frac{m R T}{10^2 * P} = \frac{37 * 0.2242 * (60 + 273)}{10^2 * 4.15} = 6.656 \ m^3$$

$R_{O2} = 8.314 / 32 = 0.2598$ kJ / kg .K

$R_{N2} = 8.314 / 28 = 0.2969$ kJ / kg .K

$R_{CO2} = 8.314 / 44 = 0.188$ kJ / kg .K

$$P = \frac{m\,R\,T}{V}$$

$$P_{O2} = \frac{8 * 0.2598 * (60 + 273)}{6.656 * 10^2} = 1.\,039 \text{ KN / m}^2$$

$$P_{N2} = \frac{7 * 0.2969 * (60 + 273)}{6.656 * 10^2} = 1.\,039 \text{ bar}$$

$$P_{CO2} = \frac{22 * 0.188 * (60 + 273)}{6.656 * 10^2} = 2.\,069 \text{ bar}$$

$$P_{total} = \frac{37 * 0.2242 * (280 + 273)}{6.656 * 10^2} = 6.\,89 \text{ bar}$$

EX :- A sample of a mixture of gases has the following gravemetric analaysis CO_2 20.2% ,O_2 5.3% , N_2 74.5% if the mixture pressure is 10 bar determine the volumetric analaysis and the partial pressure of each constituent .

Sol:-

For 1 Kmole

Constituent	**m_i**	**M_i**	**$n_i = m_i / M_i$**	**$n_i / n = V_i / V$**
CO_2	0.202	44	0.00459	0.00459/ 0.0328= 0.1493= 13.93 %
O_2	0.053	32	0.00165	0.00165/ 0.0328= 0.0502 = 5.02 %
N_2	0.745	28	0.0266	0.0266/ 0.0328= 0.9101 = 91.01 %
$n=\sum n_i = 0.0328$				

$$\frac{P_i}{P} = \frac{V_i}{V}$$

$$P_i = \frac{V_i}{V} P$$

$P_{O2} = 0.1393 * 10 = 1.393$ bar

$P_{N2} = 0.05 * 10 = 0.5$ bar

$P_{CO2} = 0.81 * 10 = 8.1$ bar

EX :- A gas consists of 20% CO_2 , 70% N_2 and 10% O_2 by volume . Determine the average analysis of the gas by mass and the average relative molecular mass of the gas . mixture .

Sol:-

$$\frac{n_i}{n} = \frac{V_i}{V}$$

$$n_i = \frac{V_i}{V} n$$

Constituent	**n_i**	**M_i**	**$m_i = n_i M_i$**	**m_i / m = fraction by mass**
CO_2	0.2	44	8.8	8.8 / 31.6 = 0.278 = 27.8%
N_2	0.7	28	19.6	19.6 / 31.6 = 0.62 = 62 %
O_2	0.1	32	3.2	3.2/ 31.6 = 0.1012 = 10.12 %
$m = \sum m_i = 31.6$				

$$M = \sum \frac{V_i}{V} M_i = (0.2 * 44) + (0.7 * 28) + (0.1 * 32) = 31.6 \text{ kg / Kmol}$$

Specific heats of a gas mixture

From gibbs dalton law , the internal energy of amixture of gases is given by equation $mu = \sum m_i u_i$
Also for aperfect gas is given by equation , $u = CvT$.
Hence substituting we have

$$mCv\ T = \sum m_i Cv_i\ T$$

$$m\ Cv = \sum m_i Cv_i$$

$$Cv = \sum \frac{m_i}{m} Cv_i$$

Similary from equation

$mh = \sum m_i h_i$, & $h = CpT$, therefore ,

$$mCp\,T = \sum m_i Cp_i\,T$$

$$m\,Cp = \sum m_i Cp_i$$

$$Cp = \sum \frac{m_i}{m} Cp_i$$

$$Cp - Cv = \sum \frac{m_i}{m} Cp_i - \sum \frac{m_i}{m} Cv_i = \sum \frac{m_i}{m} (Cp_i - Cv_i)$$

$Cp_i - Cv_i = R_i$, therefore

$$Cp - Cv = \sum \frac{m_i}{m} R_i$$

i.e.

$$\gamma = \frac{Cp}{Cv} \quad ; Cv = \frac{R}{\gamma - 1} \quad ; Cp = \frac{\gamma R}{\gamma - 1}$$

EX :- the gas in an engine cylinder has a volumetric analysis of 12% CO_2 (44) , 11.5% O_2 (32), and 76.5% N_2 (28)the temperature at the beginning of expannsion 1000 °C & the gas mixture expands reversibly through a volume ratio of 7 to 1 , according to alow $pv^{1.25}$ = constant . calculate the work done and the heat flow per kg of gas& the change of entropy per kg of mixture . the values of Cp for the constituents are as follows :-

Cp for CO_2 = 1.235 kJ/kg K ; Cp for O_2 = 1.088 kJ/kg K ; Cp for N_2 = 1.172kJ/kg K.

Sol:-

$m_i = n_i M_i$ therefore a conversion from volume fraction to mass fraction is as follows: consider 1 mole of the mixture .

Constituent	**n_i**	**M_i**	**$m_i = n_i M_i$**	**m_i / m = fraction by mass**
CO_2	0.12	44	5.28	5.28 / 30.36 = 0.174 = 17.4 %
O_2	0.115	32	3.68	3.68 / 30.36 = 0.121 = 12.1 %
N_2	0.765	28	21.40	21.40/ 30.36 = 0.705 = 70.5 %
			$\sum m_i = 30.36$	

$$Cp = \sum \frac{m_i}{m} Cp_i$$

$Cp = 0.174 * 1.235 + 0.121 * 1.088 + 0.705 * 1.172$

$= 1.173$ kJ /kg .K

$$R = \sum \frac{m_i}{m} R_i$$

$R_i = Ru / M_i$

$R = 0.174 * (8.314 / 44) + 0.121 * (8.314 / 32) + 0.705 (8.314 / 28)$

$= 0.2737$ kJ/kg .K

$Cp - Cv = R$

$Cv = 1.173 - 0.2737$

$= 0.899$ kJ/kg.K

the work done per kg of gas can be obtained from equation

$$W = \frac{R (T_1 - T_2)}{n - 1}$$

T_2 can be found using equation ,

$$\frac{T_2}{T_1} = (\nu_{1/} \nu_2)^{n-1} = (1/7)^{0.25}$$

Where $T_1 = 1000 + 273 = 1273$ K

$T_2 = T_1 / 7^{0.25} = 1273 / 1.625 = 783.2$ K

$$W = \frac{0.2739 (1273 - 783.2)}{1.25 - 1} = 536 .3 \text{ kJ/kg}$$

also from equation , for 1 kg ,

$u_2 - u_1 = Cv\ (T_2 - T_1)$, therefore ,

$u_2 - u_1 = 0.899 - (783.2 - 1273) = - 440.3$ kJ / kg

finally from the non – flow energy equation ,

$Q = (u_2 - u_1) + W = - 440.3 + 536 .3 = 96$ kJ/kg

To find change in entropy

$$\Delta s = \frac{R\,(\,n-\gamma\,)}{(\,n-1\,)\,(\,\gamma-1\,)}\ \ln \frac{T_2}{T_1}$$

R = 0.2737 kJ/kg .K

n = 1.25

Cv = 0.899 kJ/kg.K

Cp = 1.173 kJ /kg .K

γ = Cp / Cv = 1.173 / 0.899 =1.3047 kJ/kg .K

$$\Delta s = \frac{0.2737\ (\,1.25-1.3047\,)}{(\,1.25-1\,)\,(\,1.3047-1\,)}\ \ln \frac{783.2}{1237}$$

Δs = 0.095 KJ/Kg .K

EX :- A mixture of gases has the following composition by mass :- N_2 60% , O_2 30% , CO_2 10% , 0.9kg of mixture initially at 1 bar and 27 °C are compressed to 4 bar polytropically with n = 1.2 , find the work done , heat transferred and change of entropy . the values of cp for the constituents are N_2 =1.04 kJ/kg .K , O_2 = 0.918 KJ/kg.K , CO_2 = 0.846 KJ/kg.K :-

Sol:

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$P_1V_1 = mRT$

$$R = \sum \frac{m_i}{m} R_i$$

$R_i = Ru / M_i$

R_{N2} = 8.314 / 28 = 0.269

R_{O2} = 8.314 / 32 = 0.259

R_{CO2} = 8.314 /44 = 0.188

m_i / m for N_2 = 60/100 = 0.6

m_i / m for O_2 = 30/100 = 0.3

m_i / m for $CO_2 = 10/100 = 0.1$

$R = 0.6 * 0.296 + 0.3 * 0.259 + 0.1 * 0.188$

$R= 0.2741 kJ/kg.K$

$P_1V_1 = mRT$

$$V_1 = \frac{m\,RT}{p_1}$$

$$V_1 = \frac{0.9 * 0.274\ (\ 27 + 273\ )}{1*10^5 *10^{-3}}$$

$V_1= 0.731\ m^3$

$p_1V_1^n = p_2V_1^n$

$(V_2 / V_1)^n = p_1 / p_2$

$V_2 /V_1 = (p_1 / p_2)^{1/n}$

$V_2 = (p_1 / p_2)^{1/n} * V_1$

$= (1/4)^{1/1.2} * 0.731$

$V_2 = 0.23\ m^3$

$$W = \frac{P_1 V_1 - P_2 V_2}{n -1} = \frac{(\ 1* 10^2 * 0.731\ ) - (\ 4 * 10^2 * 0.23\ )}{1.2 - 1} = -95\ kJ$$

$$Q = W_{12} * \frac{\gamma - n}{\gamma -1} \quad , \gamma = Cv / Cp\ , Cv = Cp - R$$

$$Cp= \sum \frac{m_i}{m} Cp_i$$

$= 0.6 * 1.04 + 0.3 * 0.918 + 0.1 * 0.846$

$Cp = 0.9912\ kJ / kg .K$

$Cv = Cp - R$

$$= 0.9912 - 0.271 = 0.72 \text{ KJ /kg .K}$$

$$\gamma = Cp / Cv = 0.72 / 0.9912 = 0.726$$

$$Q = -95 * \frac{0.726 - 1.2}{0.726 - 1} = -194\text{kJ}$$

$$\Delta s = \frac{R (n - \gamma)}{(n - 1) (\gamma - 1)} \ln \frac{T_2}{T_1}$$

T_2 can be found using equation ,

$$\frac{T_2}{T_1} = (V_1/ V_2)^{n-1} = (0.731/0.23)^{1.2 -1}$$

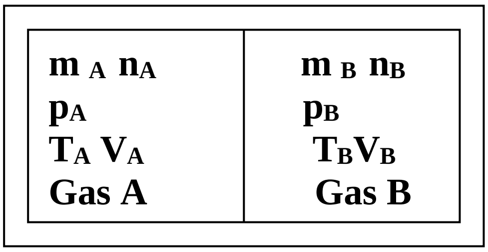
Where $T_1 = 27 + 273 = 300$ K

$T_2 = T_1 * 1.26 = 300 * 1.26 = 378$K

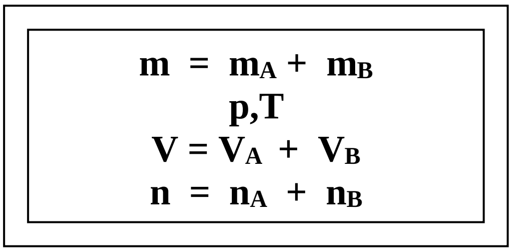
$$\Delta s = \frac{0.271 \ (1.2 - 0.726)}{(1.2 - 1) (0.726 - 1)} \ln \frac{378}{300} = - 0.5417 \text{ kJ/kg.K}$$

Adiabatic mixing of perfect gases

Consider two gases A and B seperated from each other in aclosed vessel by athin diaphragm , as shown in fig.



If the diaphragm is punctured or removed then the gases mix and each then occupies the total volume , behaving as if the other gas were not present . this process is equivalent to a free expansion of each gas ,and is irreversible . the process can be simplified by the assumption that it is adiabatic ;



This mean that the vessel is perfectly thermally insulated and there will therefore be to increase in intropy of the system .
Iternal inergy initially is equal to the internal energy finally . in this case From equation

$U_1 = n_A C_{vA} T_A + n_B C_{vB} T_B$
&
$U_2 = (n_A C_{vA} + n_B C_{vB})T$

Where C_v is molar heat ,which specific heat can be expressed in term of the mole .is defind as follow $C_v = M\, Cv$

$U_1 = \sum n_i C_{vi} T_i$ & $U_2 = T \sum n_i C_{vi}$

Then $U_1 = U_2$

$\sum n_i C_{vi} T_i = T \sum n_i C_{vi}$

$$T = \sum \frac{n_i \; C_{vi} \; T_i}{n_i \; C_{vi}}$$

EX:-**Avessel of 1.5 m^3 capacity contains oxygen at 7 bar and 40 °C. the vessel is connected to another vessel of 3 m^3 capacity containing carbon monoxide at 1 bar and 15 °C . Aconnecting valve is opened and the gases are allowed to mix . Assuming the process is adiabatic determine the final temperature & pressure of mixture and change of entropy of the system , for oxygen C_v = 21 kJ / mole K ; for carbon monixide C_v = 20.86 kJ / mole K .**
Sol :-

$$n = \frac{pV}{Ru\,T}$$

$$n_{O2} = \frac{7 * 10^5 * 1.5}{8.314 * 313 * 10^3} = 0.4035 \quad (\text{ where } T_{O2} = 40 + 273 = 313 \text{ K })$$

$$n_{CO} = \frac{1 * 10^5 * 3}{8.314 * 288 * 10^3} = 0.1253 \quad (\text{ where } T_{Co} = 15 + 273 = 288\text{K })$$

befor mixing

$U_1 = \sum n_i C_{vi} T_i = 0.4034 * 21.07 * 313 + 0.1253 * 20.86 * 288$

$U_1 = 3413.8$ kJ

After mixing ,

$U_2 = T \sum n_i C_{vi} = T (0.4035 * 21.07 + 0.1253 * 20.86)$

$U_2 = 11.118 * T$

For adiabatic mixing , $U_1 = U_2$, therefore ,

$$3413.8 = 11.118 * T$$

$$T = 3413.8 / 11.118 = 307 \text{ K}$$

Temperature of mixture = 307 -273 = 34 ^oC

$PV = n\,RuT$

$$P = \frac{(0.4035 + 0.1253) * 8.314 * 307 * 10^3}{10^5 * (1.5 + 3.0)} = 3 \text{ bar}$$

pressure after mixing = 3 bar

the change of entropy of the system is equal to the change of entropy of the oxygen plus the change of entropy of the carbon monoxide; this follows from the Gibbs-Dalton law .

change of entropy of oxygen can be calculated ,

$\Delta s = R \ln(v_2/v_1) - Cv \ln(T_1/T_2)$

$\Delta S = mR \ln(v_2/v_1) - m\,Cv \ln(T_1/T_2)$,where m= n M , R = Ru / M , C_v = M Cv therefore,

$\Delta S_{O2} = n\,Ru \ln(v_2/v_1) - n\,C_v \ln(T_1/T_2)$

$= 0.4035 * 8.314 * \ln (4.5 / 1.5) - 0.4035 * 21.07 * \ln (313/307)$

$= 3.518 \text{kJ/K}$

change of entropy of the carbon monixide can be calculated ,

$\Delta S = n\,Ru \ln(v_2/v_1) - n\,C_v \ln(T_1/T_2)$,or

$\Delta S_{CO} = n\,Ru \ln(v_2/v_1) + n\,C_v \ln(T_2/T_1)$

$= 0.1253 * 8.314 * \ln (4.5 / 3) + 0.1253 * 8.314 * \ln (307 / 288)$

$= 0.5897 \text{ kJ/K}$

Hence the change of entropy of the whole system is given by

$$\Delta S = \Delta S_{O2} + \Delta S_{CO}$$

change of entropy of system = 3.518 +0.5897 = 4.108 kJ/K

EX:- **Two cotainers each 1.4 m^3 in volume . are isolated from one another by a valve . one contains oxygen at 7 bar and 150 °C and the other carbon dioxide at 2 bar and 15 °C . the valve is opened and the gases are allowed to mix . Assuming the process is adiabatic determine the equilibrium temperature & pressure of mixture and change of entropy associeted with system as awhole the values of heat at aconstant volume for constituent are O_2 0.656 , CO_2 0.643 KJ/Kg °k .**

Sol:-

$$R_{O2} = \frac{Ru}{M_i} = \frac{8.314}{32} = 0.2598 \text{ KJ/kg.K}$$

PV =m RT

$$m_{O2} = \frac{7 * 10^2 * 1.4}{0.2598 * (150 + 273)} = 8.917 \text{ kg}$$

$$R_{CO2} = \frac{Ru}{M_i} = \frac{8.314}{44} = 0.1889 \text{ KJ/kg.K}$$

$$m_{CO2} = \frac{2 * 10^2 * 1.4}{0.1889 * (15 + 273)} = 5.146 \text{ kg}$$

$$T = \sum \frac{m_i \ cv_i \ T_i}{m_i \ cv_i}$$

$$T = \frac{(8.917 * 0.656 * 423) + (5.146 * 0.643 * 288)}{(8.917 * 0.656) + (5.146 * 0.643)}$$

T = 374 K

$m = m_{O2} + m_{CO2} = 8.917 + 5.146 = 14.063$ kg

$$R = \sum \frac{m_i}{m} R_i$$

R =(8.917 / 14.06) * (8.314 / 32) + (5.146 / 14 . 06) * (8.314 / 44)

= 0.2338 kJ/kg .K

$V = V_{O2} + V_{CO2} = 1.4 + 1.4 = 2.8$ m^3

PV =m RT

$$P = \frac{14.063 * 0.2338 * 374.2}{10^2 * 2.8} = 4.3948 \text{ bar}$$

the change of entropy of the system is equal to the change of entropy of the oxygen plus the change of entropy of the carbon monoxide; this follows from the Gibbs-Dalton law .
change of entropy of oxygen can be calculated ,

$$\mathbf{\Delta s = R \ln(v_2/v_1) - Cv \ln(T_1/T_2)}$$

$\Delta S_{O2} = mR \ln(v_2/v_1) - m Cv \ln(T_1/T_2)$,

$= 8.917 * 0.2598 * \ln (2.8 / 1.4) – 8.917 * 0.656 *\ln (423 /374)$

$= 1.605 - 6.6159$

$= -5.0109$ kJ/K

change of entropy of the carbon monixide can be calculated ,

$\Delta S_{CO2} = mR \ln(v_2/v_1) - m Cv \ln(T_1/T_2)$

$= 5.146 * 0.1889 \ln(2.8 /1.4) – 5.146 * 0.643 \ln (288 / 374)$

$= 0.6737 - (-0.864)= 0.6737 + 0.864 = 1.5377$ kJ/K

Hence the change of entropy of the whole system is given by

$$\Delta S = \Delta S_{O2} + \Delta S_{CO2}$$

change of entropy of system = -5.0109 + 1.5377= -3.4732kJ/K

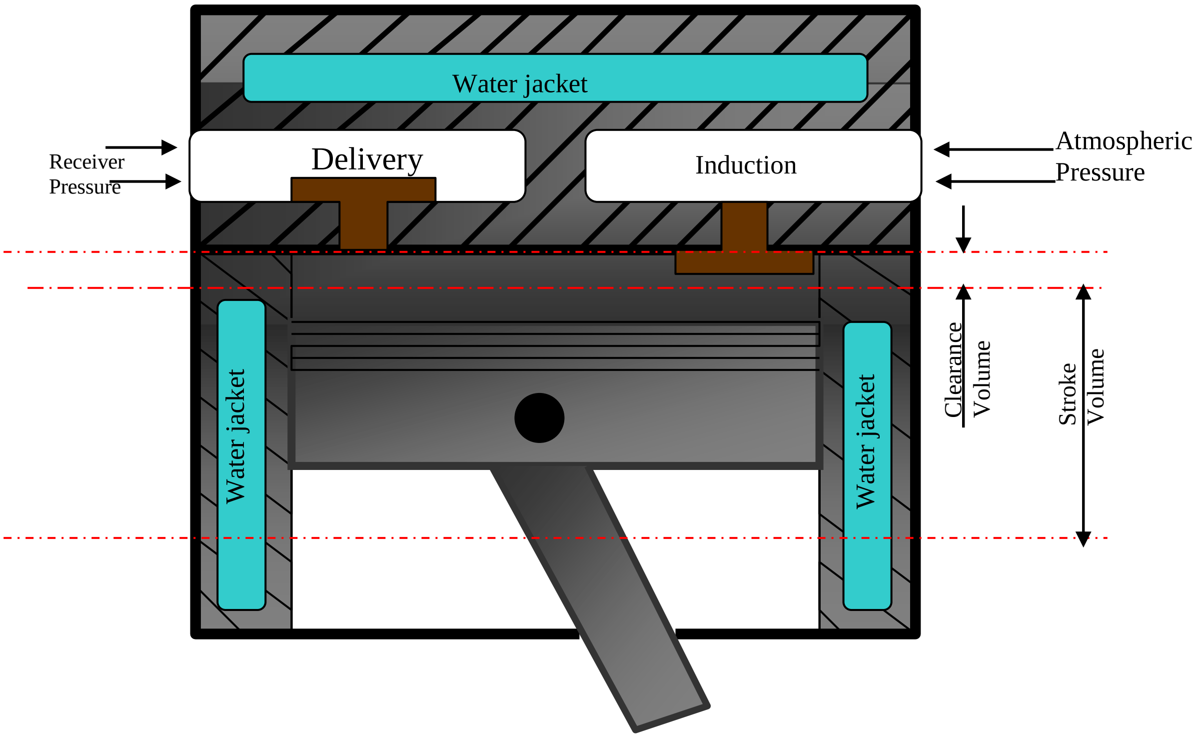
Positive Displacement Compressors

The function of acompressor is to take a definite quantity of fluid (usually a gas , and most often air) and deliver it at arequired pressure .
Both reciprocating and rotary positiive displacement machines are used for avariety of purposes .
They may be single or multi stage and have air or water cooling .
On the basis of performance ageneral distinction can be made between the two types:-

Reciprocating characteristics :-

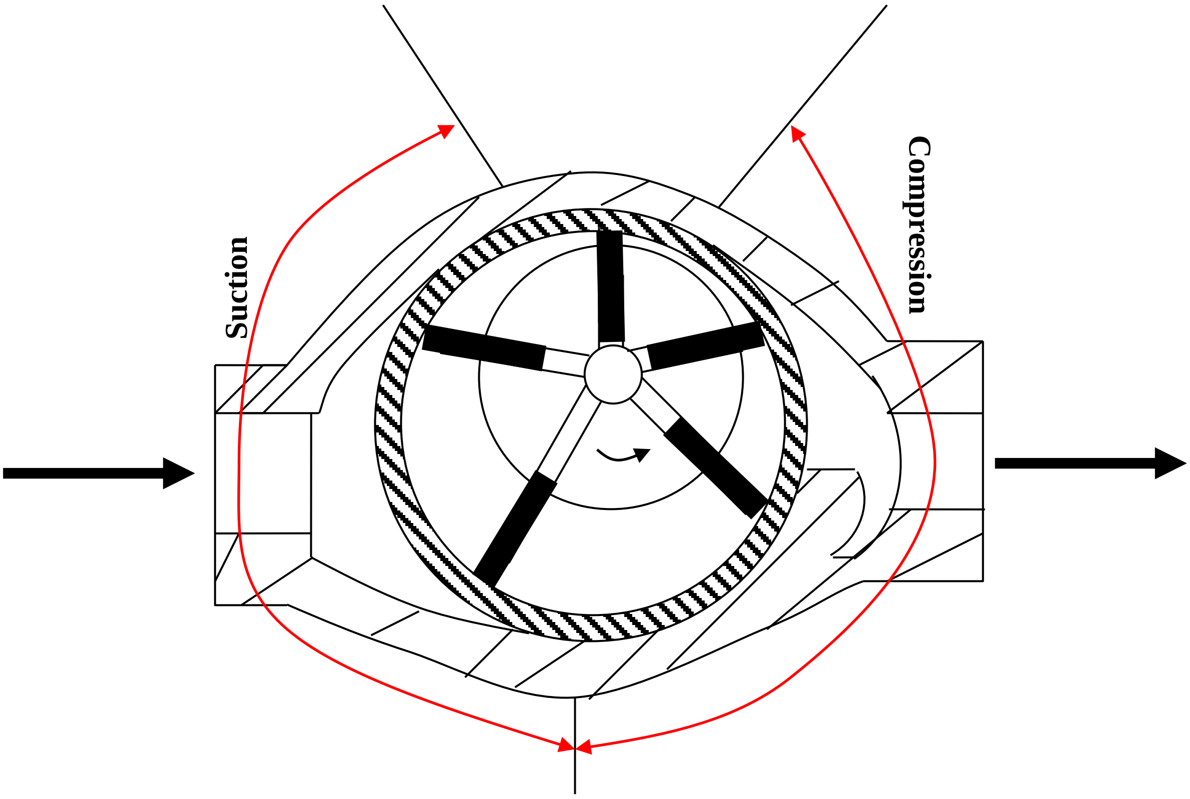
1- Low mass flow rate .
2- High pressure ratios , high pressure work up to 485 bar and above.
3- Heavy in weight.
4- big size .

5- complex in mechanically .



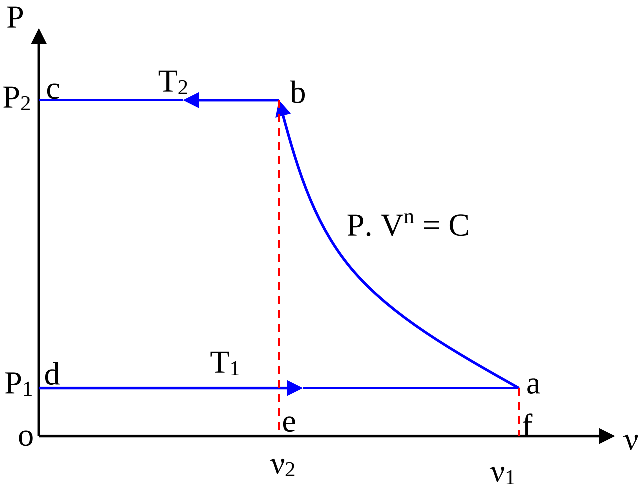
Rotary characteristics :-

1- high mass flow rate .
2- Low pressure ratios , high pressure work up to 485 bar and above.
3- Light in weight.
4- Small size .
5- Simple in mechanically .



Reciprocating machines :-

The mechanism involved is the basic piston , connecting – rod , crank , and cylinder arrangement .
Initially the clearance volume in the cylinder will be considered negligible .
Also the working fluid will be assumed to be perfect gas .
The cycle takes one revolution of the crankshaft complection .
The valves employed in most air compressors are designed to give automatic action . they are of the spring – loaded type operated by asmall differnce in pressure across them , the light spring pressure giving arapid closing action . the lift of the valve to give the required air flow should be as small as possible and should operate without shock



Line d - a represents the induction stroke . the temperature is constant atT_1 for this process and there is no heat exchange with the surroundings in the ideal case .
Line a- b- c represents compression and delivery stroke . as the piston begins its return stroke the pressure in the cylinder rises and closes the inlet valve .
the pressure rise continues with the returing piston by line a- b until the pressre is reached at which the delivery valve open .
the delivery takes place by the line b – c , which is aprocess at constant temperature T_2 , constant pressure P_2 , zero heat exchange and decreasing mass .
the general form of compression is the reversible polytropic (PV^n = constant)

Indicated work done on the air per cycle = area (abcd)

= area (abef) + area (bcoe) – area (adof)

$$\text{Work in put} = \frac{p_2V_b - p_1V_a}{n-1} + p_2V_b - p_1V_a$$

$$= (p_2V_b - p_1V_a)\left[\frac{1}{n-1} + 1\right]$$

$$= (p_2V_b - p_1V_a) \left[\frac{1 + n - 1}{n - 1} \right]$$

$$= n / n\text{-}1 (p_2 V_2 - p_1V_1)$$

$p_1V_1 = m\cdot RT_1$, $p_2V_2 = m\cdot RT_2$

where m· is the mass induced and delivered per cycle .

$$\text{work input per cycle} = \frac{n}{n - 1} \; m\cdot R\,(T_2 - T_1)$$

work done on the air per unit time is equal to the work done per cycles times the number of cycles per unit time .

the delivery temperature is given by the equation , $T_2 = T_1 (p_2 / p_1)^{(n-1) / n}$

EX :- A single – stage reciprocating compressor takes 1m^3 of air per minute at 1.013 bar and 15°C and delivers it at 7bar . Assuming that the law of compression is pV$^{1.35}$ = constant , and that clearance is negligible , calculate the indicated power . Rair = 0.287 kJ/kg .K

Sol:-

$$\text{indecated power} = \frac{n}{n - 1} \; m\cdot R\,(T_2 - T_1)$$

$$\text{Mass delivered per sec, } m\cdot = \frac{p_1V_1}{RT_1} = \frac{1.031 * 10^2 * 1/60}{287 * 288} = 0.02 \text{ kg/sec}$$

(Where T_1= 15 + 273 = 288 K) .

$$T_2 = T_1 (p_2 / p_1)^{(n-1) / n} = 288(7/ 1.013)^{(1..35 -1) / 1..35} = 475.2 \text{ K}$$

$$\text{indecated power} = \frac{1.35 * 0.02 * 0.\ 287 \ (475.2 - 288)}{(1.35 - 1) *10^2} = 4.149 \text{ KW}$$

The actual work input to the compressor is larger than the indicated work , due to the work necessary to overcome the losses due to friction ,
Shaft work = indicated work + friction work
Shaft work = i .p. + f.p.
(where f.p. is the friction power)

The mechanical effciency of the machine is given by

$$\text{Compressor mechanical efficiency} = \frac{\text{Indicated work}}{\text{Shaft work}}$$

$$\text{Efficiency of motor and drive} = \frac{\text{Shaft work}}{\text{Input power}}$$

EX :- If the compressor of last example is to be driven at 300 rev /min and is a single- acting , single cylinder machine , calculate the cylinder bore required , assuming a stroke to bore ratio of 1.5/1 , also calculate the power of the motor required to drive the compressor if the mechanical efficiency of the compressor is 85 % and that of the motor transmission is 90% .

Sol :-
Volume dealt with per minute at inlet = $1\text{m}^3/\text{min} = V^{\cdot}$ from last exam

Vol drawn in per cycle = $V^{\cdot} / N = 1 / 300 = 0.00333\ \text{m}^3/$ cycle

Cylinder volume = $0.00333\ \text{m}^3$

V = Area * L

$V = (\Pi /4)d^2 * L= 0.00333\ \text{m}^3$ = Cylinder volume

(where d = bore ; L = stroke) ,

L / d = 1.5 / 1 , L = 1.5 d

$(\Pi /4)d^2 * (1.5\ d) = 0.00333$

d = 0.1415 m

cylinder bore = 141.5 mm

$$\text{Compressor mechanical efficiency} = \frac{\text{Indicated work}}{\text{Shaft work}}$$

$$S.P = \frac{I.P.}{\eta m\ .\ comp} = \frac{4.149}{0.85} = 4.88\ \text{KW}$$

power input to the compressor = 4.881 KW

$$\text{Efficiency of motor and drive} = \frac{\text{Shaft work}}{\text{Input power (motor power)}}$$

motor power = 4.881 / 0.9 = 5.423 KW

other expressions for the indicated work can be derived ,

$$\text{work input per cycle} = \frac{n}{n-1}\, m{\cdot}R\,(T_2 - T_1)$$

$$\text{work input per cycle} = \frac{n}{n-1}\, m{\cdot}R\, T_1\,[(T_2 / T_1) - 1]$$

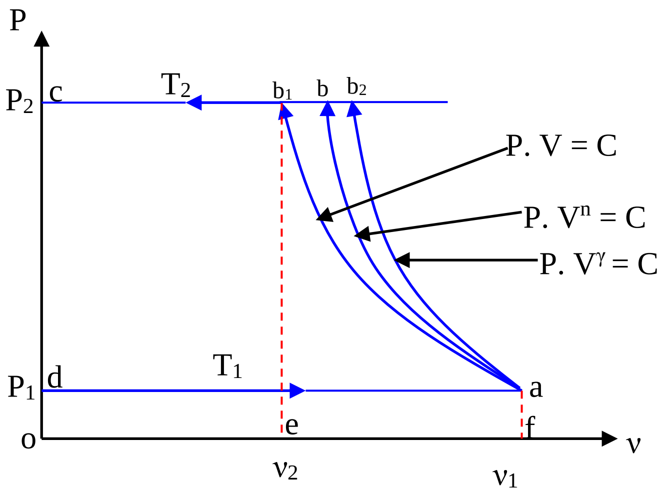
$$T_2 / T_1 = (p_2 / p_1)^{(n-1)/n}$$

$$\text{work input per cycle} = \frac{n}{n-1}\, m{\cdot}R\, T_1\,[(p_2 / p_1)^{(n-1)/n} - 1]$$

$$\text{work input per cycle} = \frac{n}{n-1}\, p_1 V^{\cdot}_1\,[(p_2 / p_1)^{(n-1)/n} - 1]$$

The condition for minimum work

The work done will be a minimum when the area of the diagram is a minimum .



Line ab_1 is according to the law PV = const .(i.e. isothermal) ,
Line ab_2 is according to the law PV = const .(i.e . isentropic) , both processes are reversible .
Isothermal compression is the most desirable process between a and b giving the minimum work to be done on the air . this means that in an actual compressor the gas temperature must be kept as close as possible to its initial value , and a means of cooling the gas is always provided ,either by air or by water .

Indicated work done when the gas is compressed isothermally = area (ab_1cd)
= area (ab_1ef) + area (b_1coe) – area (adof)

$$\text{area} (ab_1ef) = p_2 V_{b1} \ln(p_2 / p_1)$$

$$\text{Indicated work per cycle} = p_2 V_{b1} \ln(p_2 / p_1)+ p_2V_{b1} - p_1V_a$$

Also , $p_1V_a = p_2V_{b1}$, since the process ab_1 is iso thermal .

$$\text{Indicated work per cycle} = p_2 V_{b1} \ln(p_2 / p_1)$$

$$= p_2 V_a \ln(p_2 / p_1)$$

$$= m{\cdot}R\, T_1 \ln(p_2 / p_1)$$

then these equations give the isothermal power

Isothermal efficiency

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Indicated work}}$$

EX :-Using data of two last example calculate the isothrmal efficiency of the compressor .

Sol:-

Isothermal power = $m{\cdot}R\, T_1 \ln(p_2 / p_1)$

$$= 0.02 * 0.287 * 288 \ln(7 / 1.013)$$

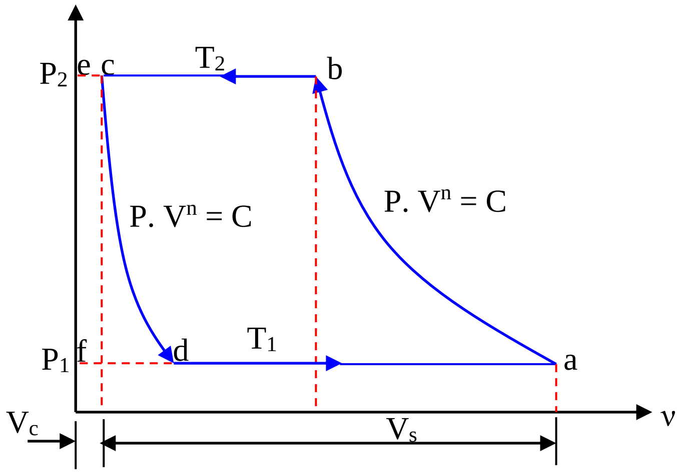
$$= 3.195 \text{ kw}$$

Indicated work = 4.149 from last example

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Indicated work}} = \frac{3.195}{4.149} = 0.77 * 100 = 77\%$$

Reciprocating compressor including clearance

Clearance is necessary in acompressor to give mechanical freedom to the working parts and allow the necessary space for valve operations.



When the delivery stroke bc is completed the clearance volume V_c is full of gas at pressure p_2 and temperatureT_2 , Ideally as soon as the pressure reaches p_1 , the induction of fresh gas will begin and continue to the end of this stroke at a .
The effect of clearance is to reduce the induced volume at p_1 from V_s to (V_a - V_d).

Indicated work = area (abcd)
= area (abef) – area (cefd)

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}_a R\,(T_2 - T_1) - \frac{n}{n-1}\dot{m}_d R\,(T_2 - T_1)$$

$$\text{Indicated power} = \frac{n}{n-1} R\,(\dot{m}_a - \dot{m}_d)\,(T_2 - T_1)$$

$$\text{Indicated power} = \frac{n}{n-1} R\ \dot{m}\,(T_2 - T_1)$$

Where $\dot{m}$ = the mass induced per unit time = ($\dot{m}_a$ - $\dot{m}_d$) .

$$\text{Indicated power} = \frac{n}{n-1} p_1 V^{\cdot}\left[\left(p_2 / p_1\right)^{(n-1)/n} - 1\right]$$

Where, $V^{\cdot}$ = ($V_a - V_d$)
The mass delivered per unit time can be increased by designing the machine to be double acting, i.e. gas is dealt with on both sides of the piston the induction stroke for one side being the compression stroke for the other.

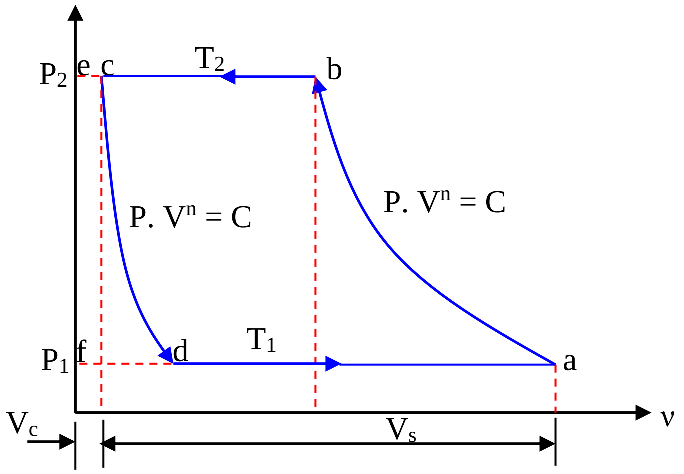
EX :- A single – stage double – acting air compressor is required to deliver 14m^3 of air per min measured at 1.013 bar and 15°C . the delivery pressure is 7 bar and the speed 300 rev/min .take the clearance volume as 5% of the swept volume with a compression index of n = 1.3 . Calculate the swept volume of the cylinder, the delivery temperature and the indicated power.

Sol:-

Swept volume = $(V_a – V_c) = V_s$

Clearance volume, $V_c = 0.05\ V_s$

$V_a = 1.05\ V_s$



Volume induced per cycle = $(V_a – V_d)$

$$(V_a – V_d) = \frac{14}{300 * 2} = 0.0233\ m^3$$

Cycle double

Cycle per min = Rev. per min * cycle per rev .

Now,

$V_a = 1.05\ V_s$, and

$V_d = V_c (p_2 / p_1)^{1/n}$

$= 0.05\ V_s (7 / 1.013)^{1/1.3}$

$V_d = 0.221\ V_s$

$(V_a – V_d) = (1.05\ V_s - 0.221\ V_s) = 0.0233\ m^3$

Therefore , $V_s = \dfrac{0.0233}{0.\ 829} = 0.0281 m^3$

Delivery temp,

$T_2 = T_1 (p_2 / p_1)^{(n-1)/n}$

And , $T_1 = 15 + 273 = 288$ K

$T_2 = 288 (7 / 1.013)^{(1.3-1)/1.3}$

$= 288 * 1.563 = 450$ K

delivery temp. = 450 – 273 = 177 °C

Indicated power $= \dfrac{n}{n-1}\ p_1.\ V[(p_2 / p_1)^{(n-1)/n} - 1]$

Indicated power $= \dfrac{1.3}{1.3-1} * 1.013 * 10^2 * (14 / 60)[(7 / 1.o13)^{(1.3-1)\ 1.3} - 1]$

$= 57.622 kW$

0r …

Indicated power $= \dfrac{n}{n-1} R\ m\cdot (T_2 - T_1)$

Indicated power $= \dfrac{n}{n-1} R \dfrac{p_1.\ V_1}{R\ T_1} (T_2 - T_1)$

Indicated power $= \dfrac{1.3}{1.3-1} * 0.287 * \dfrac{1.013 * 10^2 * 14 / 60}{0.287 * 288} (450 - 288)$

$= 57.622 kW$ (as before)

Note: - when it given bore to stroke ratio we obtain the value V_s from law ,

$V_s = (Л /4)d^2 * L$
(where d = bore ; L = stroke) ,

Volumetric efficiency

It has been shown that one of the effects of clearance is to reduce the induced volume to a value less than that of the swept volume. This means that for a required induction the cylinder size must be increased over that calculated on the assumption of zero clearance.

The volumetric efficiency is defined as:

The volumetric efficiency: - the mass of air delivered, divided by the mass of air which would fill the swept volume at the free air conditions of the pressure and temperature .

The volumetric efficiency: - the volume of air delivered measured at the free air pressure and temperature, divided by the swept volume of the cylinder.

The volume of air dealt with by an air compressor is quoted as the free air delivery (F. A .D), and is the volume delivered, measured at the pressure and temperature of the atmosphere in which the machine is situated.
If the free air delivery (F. A .D) is V, at p and T, then the mass delivered is

$$m^{\cdot} = \frac{pV}{RT}$$

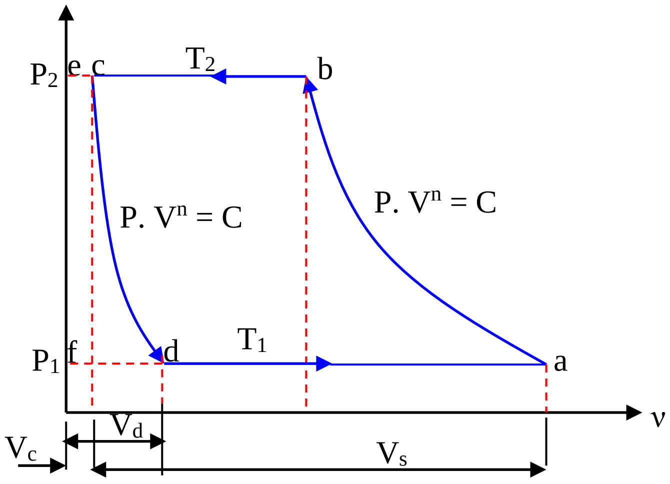
The mass required to fill the swept volume, V_s ,at p and T is given by,

$$m^{\cdot}_s = \frac{pV_s}{RT}$$

Therefore,

$$\eta_v = \frac{m^{\cdot}}{m^{\cdot}_s} = \frac{pV}{RT} * \frac{RT}{pV_s} = \frac{V}{V_s}$$

The volumetric efficiency can be obtained from the indicator diagram.



Volume induced = $V_a - V_d = V_s + V_c - V_d$

$V_d / V_c = (p_2 / p_1)^{1/n}$

$V_d = V_c (p_2 / p_1)^{1/n}$

Volume induced $= V_s + V_c - V_c (p_2 / p_1)^{1/n}$

$= V_s - V_c [(p_2 / p_1)^{1/n} - 1]$

$$\eta_v = \frac{V_a - V_d}{V_s} = \frac{V_s - V_c [(p_2 / p_1)^{1/n} - 1]}{V_s}$$

$$\eta_v = 1 - \frac{V_c}{V_s} [(p_2 / p_1)^{1/n} - 1]$$

In fact the gas will be heated by the cylinder walls, and there will be a reduction in pressure due to the pressure drop required to induce the gas in to the cylinder against the inevitable resistance to flow.

When as before the F.A.D. per cycle is denoted by V at p and T, then,

$$m^{\cdot} = \frac{pV}{RT} = \frac{p_1(V_a - V_d)}{RT_1}$$

F.A.D./ cycle , $V = (V_a - V_d) \frac{T}{T_1} * \frac{p_1}{p}$

(Where p_1 and T_1 are the suction conditions).

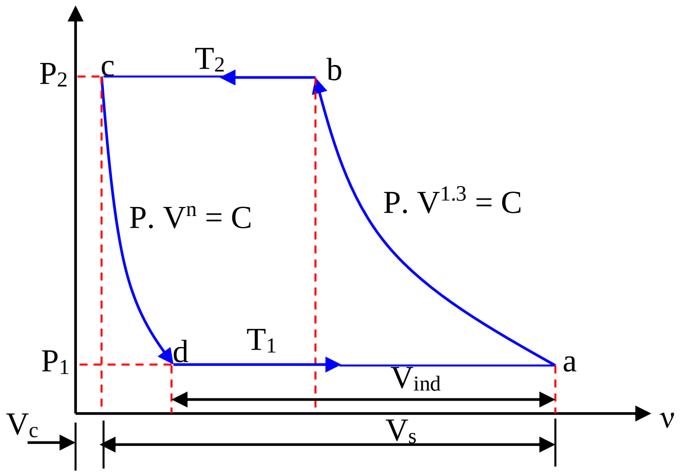
EX:- A single – atage , double acting air compressor has a F. A.D. of 14m^3/ min measured at 1.013 bar and 15°C . the pressure and temperature in the cylinder during induction are 0.95 bar and 32°C . the delivery pressure is 7 bar and the index of compression and expansion , n = 1.3 . calculate the indicated power required and the volumetric efficiency . the clearance volume is 5 % of the swept volume, also determine the bore and stroke of cylinder if N = 300rev/min and stroke to bore = 1.5 /1 .

Sol:-

Mass delivered per minute,

$$m^{\cdot} = \frac{pV}{RT} = \frac{1.013 * 10^2 * 14}{0.287 * 288} = 17.16 \text{kg / min}$$

Where T = 15 + 273 = 288 K



$T_2 = T_1 \left(p_2 / p_1 \right)^{(n-1)/n}$

And , $T_1 = 32 + 273 = 305$ K

$T_2 = 305 \left(7 / 0.95 \right)^{(1.3 - 1)/1.3}$

$= 305 * 1.586 = 483.7$ K

$$\text{Indicated power} = \frac{n}{n-1} R \; m\cdot \left(T_2 - T_1 \right)$$

$$\text{Indicated power} = \frac{1.3}{1.3 - 1} * 17.16 * 0.287 * (483.6 - 305)$$

$= 3813$ kJ / min

Indicated power = 63.5 kW

$V_d = V_c \left(p_2 / p_1 \right)^{1/n}$

$= 0.05 \, V_s \left(7 / 0.95 \right)^{1/1.3}$

$= 0.233 \, V_s$

$V_a = V_s + V_c = V_s + 0.05 \, V_s$

$= 1.05 V_s$

Volume induced = $V_a - V_d$

$= 1.05 V_s - 0.233 \, V_s$
$= 0.817 \, V_s$

$$\text{F.A.D./ cycle , } V = (V_a - V_d) \frac{T}{T_1} * \frac{p_1}{p}$$

$$V = 0.817\, V_s \frac{288}{305} * \frac{0.95}{1.013}$$

$$= 0.724\, V_s$$

$$\eta_v = \frac{V}{V_s} = \frac{0.724\, V_s}{V_s} = 0.724 \text{ or } 72.4\%$$

While the η_v measured from indicator diagram is

$$\eta_v = 1 - \frac{V_c}{V_s} [(p_2 / p_1)^{1/n} - 1]$$

$$\eta_v = 1 - \frac{0.05\, V_s}{V_s} [(7 / 0.95)^{1/1.3} - 1]$$

$$= 1- 0.05 (4.65 - 1)$$

$$= 0.817 \text{ or } 81.7\%$$

to find stroke & bore

$$\text{F.A.D./ cycle , } V = (V_a - V_d) \frac{T}{T_1} * \frac{p_1}{p}$$

$$\text{F.A.D./ cycle , } 14/60 = V_{ind} \frac{288}{305} * \frac{0.95}{7}$$

$V_{ind} = 0.2635\ m^3/sec$

$V_d = 0.233\, V_s$

$V_s = V_a - V_c = V_{ind} + V_d - V_c$

$= 0.2635 + 0.233\, V_s - 0.05\, V_s$

$= 0.322\ m^3 / sec$

$V_s = (\text{Л}\, d^2/ 4) * L$

$$V_s = \frac{0.322 * 60 \ m^3/ \min}{300 \ rev/\min * 2 \ cycle \ /rev} = 0.0322 \ m^3/cycle$$

$1.5/1 = L/d$

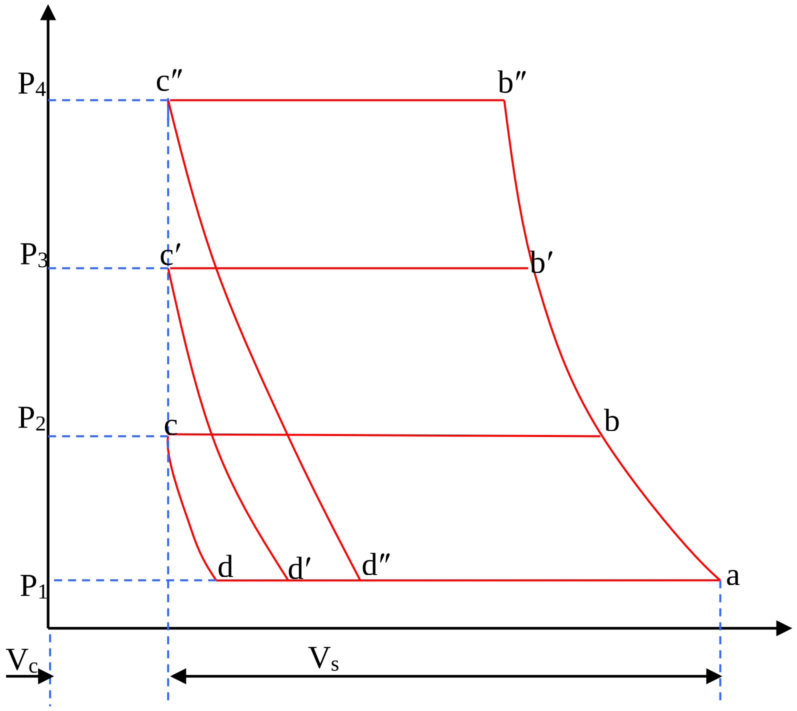
$0.0322 = (\text{Л}\, d^2/ 4) * 1.5 \ d$

$d = 0.3013 \ m = 301.3 \ mm$

$L = 1.5d = 1.5 * 0.3013 = 0.452 \ m$

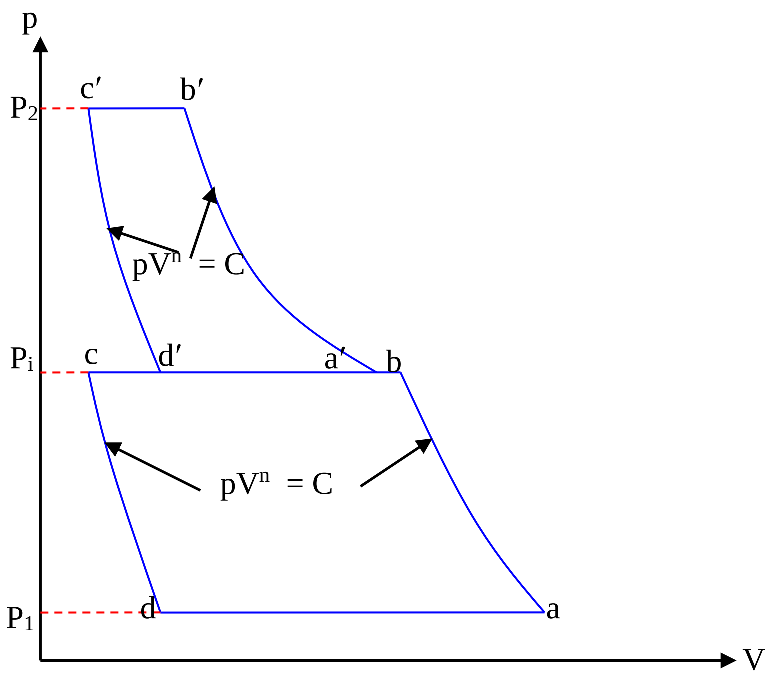
Multi-stage compressor :-

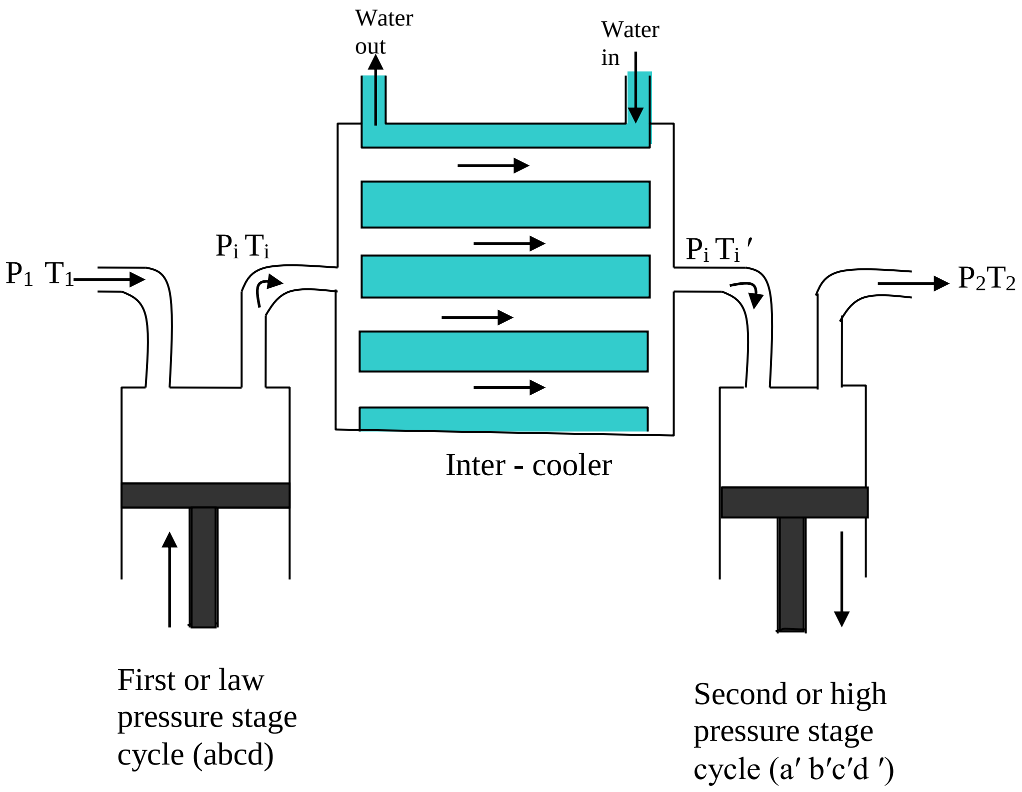
When pressure ratio increases the F.A.D. per cycle decrease therfore for required F.A.D , the cylinder size must increase as the pressure ratio increase the volumetric efficiency decrease ,



For compression from p_1 to p_2 the cycle is abcd and F.A.D. per cycle is $V_a - V_d$; for compression from p_1 to p_3 the cycle is ab′c′d′ and the F.A.D. per cycle is $V_a - V_{d'}$; for compression from $p_1$to p_4 the cycle is ab″c″d″ and the F.A.D. per cycle is $V_a - V_{d''}$.

The volumetric efficiency can be improved by carrying out the compression in two stages . After the first stage of compression the fluid is passed into a smallrer cylinder in which the gas is compressed to the required final pressure . if the machine has two stages , the gas will be delivered at the end of this stage , but it could be delivered to a third cylinder for higher pressure ratios . The cylinder of the successive stages are proportioned to take the volume of gas delivered from the previous stage .



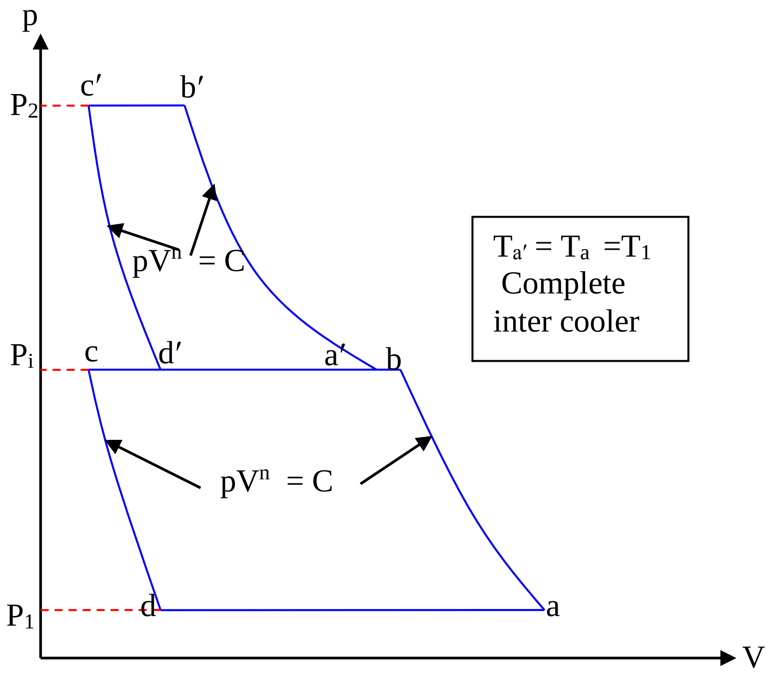


The delivery process of first or low pressure stage and the induction process of the second or high – pressure stage, are at the same pressure (p_i or intermediate pressure).

The gas is cooled as it transfers from one cylinder to the next, by passing through an intercooler. If inter cooling is complete, the gas temp enter the second stage at the same temp. At which it entered first stage.

EX:- in a single – acting two stage reciprocating air compressor (4.5) kg of air per min are compressed from 1.013 bar and 15°C through a pressure ratio of 9 to 1 . Both stages have the same pressure ratio, and the law of compression and expansion in both stages is $pV^{1.3}$ = constant. If the inter cooling is complete, calculate the indicated power and the cylinder swept volumes required , assume that the clearance volumes of both – stages are 5% of their respective swept volumes and that the compressor runs at 300 rev / min .

Sol:-



$p_2/p_1 = 9,$

$p_2 = 9\, p_1$

$p_i / p_1 = p_2 / p_i ,$

$p_i^2 = P_2\, p_1 = p_1\, 9\, p_1,$

$p_i^2 = 9\ p_1^2$,

$p_i / p_1 = \sqrt{9} = 3$

$T_i / T_1 = (p_i / p_1)^{(n-1)/n}$

$T_i / T_1 = (p_i / p_1)^{(n-1)/n}$

Where $T_1 = (15 + 273) = 288$ K ,

$T_i / 288 = 3^{(1.3 - 1)/1.3}$

$T_i = 371$ K

Now as n,m, and the temperature difference are the same for both stages , then the work done in each stage is the same .

$$\text{Total work required per min} = 2 * \frac{n}{n-1}\ m^{\cdot}\ R\ (T_i - T_1)$$

$$\text{Total work required per min} = 2 * \frac{1.3}{1.3-1} * 4.5 * 0.287 * (371 - 288)$$

$$= 930.1 \text{ kJ/min} = 930.1/60 = 15.502 \text{ KW}$$

The mass induced per cycle is ,

$$m^{\cdot} = 4.5 \text{ (kg/min)} / 300 \text{ (rev / min)} = 0.015 \text{ kg / cycle}$$

This mass is passed through each stage,

For law pressure cylinder,

$$V_a - V_d = \frac{m^{\cdot}\ R\ T}{p_1} = \frac{0.015 * 287 * 288}{1.013 * 10^5} = 0.0122 \text{ m}^3 \text{ / cycle}$$

$$\eta_v = 1 - \frac{V_c}{V_s}\ [(p_i / p_1)^{1/n} - 1] = \frac{V_a - V_d}{V_s}$$

$$= 1 - 0.05\ (3^{0.769} - 1)$$

$$= 0.934$$

$$V_s = \frac{V_a - V_d}{\eta_v} = \frac{0.0122}{0.934} = 0.0131 \text{ m}^3 \text{ / cycle}$$

Swept volume of law pressure cylinder = 0.0131 m^3 / cycle

For high - pressure stage a mass of 0.015 kg /cycle is drawn in at 15°C and a pressure of $p_i = 3\ p_1 = 3 * 1.013 = 3.039$ bar

$$V_{a'} - V_{d'} = \frac{m^{\cdot}\ R\ T}{P_i} = \frac{0.015 * 287 * 288}{3.039 * 10^5} = 0.00407\ m^3 / \text{cycle}$$

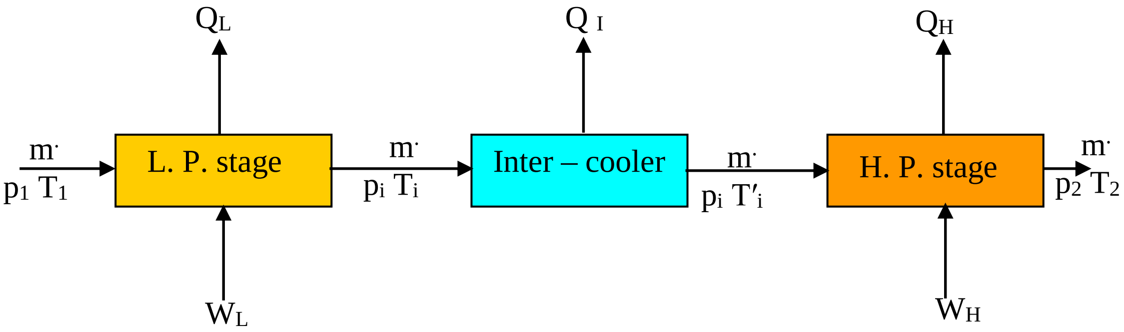
$$\eta_v = 1 - \frac{V_c}{V_s}\ [(p_i / p_1)^{1/n} - 1] = \frac{V_{a'} - V_{d'}}{V_s}$$

Since V_c/V_s is the same as for the law – pressure stage and also $p_i / p_1 = p_2 / p_i$ then η_v is 0.934

Therefore swept volume of high – pressure stage = 0.00407 / 0.934= 0.00436m^3/cycle

Energy balance for a two stage machine with inter cooler

The steady – flow energy equatioin can be applied to the law – pressure stage , the inter cooler and the high pressure stage , in turn .



Changes in kinetic energy can be neglected,

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

For the law – pressure stage, for unit mass,

$$h_1 - Q_L = h_i - W_L$$

Or for mass flow $m^{\cdot}$

$$m^{\cdot}\ Cp\ T_1 - Q_L = m^{\cdot}\ Cp\ T_i - W_L$$

$$Q_L = W_L - m^{\cdot}\ Cp\ (T_i - T_1)$$

For the inter cooler, for unit mass,

$$h_i - Q_I = h_1$$

Or for mass flow $m^{\cdot}$

$$m^{\cdot}\, Cp\, T_i - Q_I = m^{\cdot}\, Cp\, T_1$$

$$Q_I = m^{\cdot}\, Cp\, (T_i - T_1)$$

For the high – pressure stage, for unit mass,

$$h_1 - Q_H = h_i - W_H$$

Or for mass flow $m^{\cdot}$

$$m^{\cdot}\, Cp\, T_1 - Q_H = m^{\cdot}\, Cp\, T_2 - W_H$$

$$Q_H = W_H - m^{\cdot}\, Cp\, (T_2 - T_1)$$

$$W_L = W_H = \frac{n}{n-1}\, m^{\cdot}\, R\, (T_2 - T_1)$$

EX:- using the data of last example determine the heat loss to the cylinder jacket cooling water and the heat loss to the intercooler circulating water , per minute.

Sol:-

$W_L = W_H = (930 / 2)$ kJ /min

$T_2 = T_i = 371K$

$Q_L = W_L - m^{\cdot}\, Cp\, (T_i - T_1)$

$= (930/ 2) - 4.5 * 1.005\, (371 - 288)$

$= 90$ kJ/min

$Q_H = W_H - m^{\cdot}\, Cp\, (T_2 - T_1)$

$W_L = W_H$, & $T_2 = T_i$, therefore

$Q_L = Q_H = 90$ kJ/ min

Heat loss from the cylinder in each stage = 90 kJ/ min

$Q_I = m^{\cdot}\, Cp\, (T_i - T_1)$

$= 4.5 * 1.005\, (371 - 288)$

$= 375$ kJ/ min

The ideal intermediate pressure:-

The value chosen for the intermediate pressure p_i, influences the work to be done on the gas and its distribution between the stages. The condition for the work done to be a minimum will prove for two – stage compression but can be extended to any number of stages.

Total work = low pressure stage work + high pressure stage work ,

$$\text{Total work} = \frac{n}{n-1}\, m{\cdot}R\, T_1\left[\left(p_i / p_1\right)^{(n-1)/n} - 1\right] +$$

$$\frac{n}{n-1}\, m{\cdot}R\, T_i'\left[\left(p_2 / p_i\right)^{(n-1)/n} - 1\right]$$

It is assume that inter cooling is complete therefore the temperature at the start of each stage = T_1 or $T_i' = T_1$,

$$\text{Total work} = \frac{n}{n-1}\, m{\cdot}R\, T_1\left\{\left[\left(p_i / p_1\right)^{(n-1)/n} - 1\right] + \left[\left(p_2 / p_i\right)^{(n-1)/n} - 1\right]\right\}$$

If p_1, T_1, and p_2 are fixed, then the optimum value of p_i which makes the work a minimum.

Optimum value of p_i when

$$\frac{d}{dp_i}\,(\text{work}) = \text{zero}$$

$$\frac{d}{dp_i}\left\{\left(p_i / p_1\right)^{(n-1)/n} + \left(p_2 / p_i\right)^{(n-1)/n} - 2\right\} = 0$$

Therefore …

$$p_1^{-(n-1)/n}\left(\frac{n-1}{n}\right) p_i^{[(n-1)/n]-1} + p_2^{(n-1)/n}\left(\frac{1-n}{n}\right) p_i^{[(1-n)/n]-1} = 0$$

$$p_1^{-(n-1)/n}\left(\frac{n-1}{n}\right) p_i^{[(n-1)/n]-1} = p_2^{(n-1)/n}\left(\frac{n-1}{n}\right) p_i^{[(1-n)/n]-1}$$

$$p_1^{\,-(n-1)/n} \left(\frac{n-1}{n} \right) p_i^{\,-1/n} = p_2^{\,(n-1)/n} \left(\frac{n-1}{n} \right) p_i^{\,(1-2n)/n}$$

therefore .

$$p_i^{\,-1/n}\; p_i^{\,-(1-2n)/n} = p_1^{\,(n-1)/n}\; p_2^{\,(n-1)/n}$$

$$p_i^{\,2(n-1)/n} = (p_1p_2)^{(n-1)/n}$$

$$p_i^{\,2} = p_1p_2$$

$$p_i / p_1 = p_2 / p_i$$

the pressure ratio is the same for each stage .

Total minimum work = 2 * (work required for one stage)

$$= 2 * \frac{n\, m^{\cdot}\, R\, T}{n-1} \left([(p_i / p_1)^{(n-1)/n} - 1] \right)$$

$$\frac{p_i}{p_1} = \frac{\sqrt{p_1p_2}}{p_1} = \sqrt{\frac{p_1p_2}{p_1}}$$

therefore,

$$\text{Total minimum work} = 2 * \frac{n\, m^{\cdot}\, R\, T}{n-1} \left((p_2/ p_1)^{(n-1)/2n} - 1 \right)$$

This can be shown to extended to Z stages giving in general, therefore,

$$\text{Total minimum work} = Z * \frac{n}{n-1}\, m^{\cdot}\, R\, T \left((p_2/ p_1)^{(n-1)/z\,n} - 1 \right)$$

Pressure ratio for each stage = $(p_2/ p_1)^{1/z}$

Hence the condition for minimum work is that the pressure ratio in each stage is the same and that inter cooling is complete.
(Note that in last example the information given implies minimum work.)

EX:- A three – stage single – acting air compressor running in an atmosphere at 1.013 bar and 15°C has a free air delivery of 2.83 m^3/ min .the suction pressure and temperature are 0.98 bar and 32°C respectively . Calculate the indicated power required, assuming complete inter cooling, n = 1.3, and that the machine is designed for minimum work. The delivery pressure is to be 70 bar.

Sol:-

$$\text{Mass of air delivered} = \frac{pV}{R\,T} = \frac{1.013 * 10^5 * 2.83}{287 * 288} = 3.47 \text{ kg / min}$$

(Where T = 15 + 273 = 288 K).

$$\text{Total minimum work} = Z * \frac{n}{n-1} m^{\cdot} R\,T_1 \left[(p_2/ p_1)^{(n-1)/z\,n} - 1 \right]$$

(Where T_1 = 32 + 273 = 305 K).

$$\text{Total minimum work} = 3* \frac{1.3}{1.3-1} * 3.47 * 0.287 * 305 \left[(70/ 0.98)^{(1.3-1)/3* 1.3} - 1 \right]$$

$$= 1534.8 \text{ kJ /min}$$

Indicated power = 1534.8 / 60 = 25.654 kW

Problems 9.3 pg 285 T.D. Eastop, third edition :- A single stage , single acting air compressor running at 1000 rev / min delivers air at 25 bar . For this purpose the induction and free air conditions can be taken as 1.013 bar and 15°C , and the free air delivery as 0.25 m^3 / min . The clearance volume is 3% of the swept volume and the stoke / bore ratio is (1.2 / 1). Calculate the bore & stroke and volumetric efficiency of this machine. Take the index of compression and expansion as 1.3. Calculate also the indicated power and the isothermal efficiency.

Sol:-

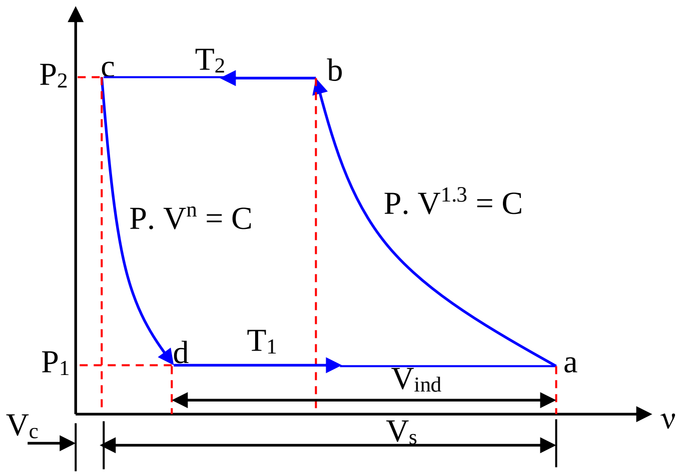
Volume induced per cycle = (0.25 m^3/min)/(1000 cycle / min)

$= 0.25 * 10^{-3}$ m^3

$V_a - V_d = 0.25 * 10^{-3}$ m^3

$V_s = V_a - V_c = V_a - 0.03\, V_s$

$V_a = 1.03\, V_s$



$$\frac{V_d}{V_c} = \left(\frac{p_2}{p_1}\right)^{1/n}$$

$$\frac{V_d}{0.03V_s} = \left(\frac{25}{1.013}\right)^{1/1.3}$$

$V_d = 0.3533V_s$

$V_a – V_d = 1.03\ V_s - 0.3533V_s = 0.25 * 10^{-3} = 0.677\ V_s$

Swept volume = $V_s = 0.00037m^3 = \frac{\text{Л}}{4} d^2 * L$, $L / d = 1.2$

$$0.00037 = \frac{\text{Л}}{4} d^2 * (1.2d)$$

d = 0.0732m = 73.2 mm, therefore L = 87.82 mm

$$\eta_v = \frac{F.A.D}{V_s} = \frac{\text{volume induced}}{V_s} = \frac{0.25 * 10^{-3}}{0.00037} * 100\% = 67.67\ \%$$

Mass of air delivered = $\frac{p_1V_1}{R\ T_1} = \frac{1.013 * 10^5 * 0.25}{287 * (15 + 273)} = 0.3064$ kg / min

$T_2 / T_1 = (p_2 / p_1)^{(n-1)/n}$

Where $T_1 = (15 + 273) = 288$ K ,

$T_2 / 288 = (25 / 1.013)^{(1.3-1)/1.3}$

$T_2 = 603.5$ K

$$I.P. = \frac{n}{n-1}\ m \cdot R\ T_1 [(p_2 / p_1)^{(n-1)/n} - 1]$$

$$I.P. = \frac{1.3}{1.3-1} * 0.3064 * 0.287 * 288 [(25 / 1.013)^{(1.3-1)/1.3} - 1]$$

= 120.2 kJ/ min =2kW

Isothermal power = $p_1V_1 \ln(p_2 / p_1)$

$$= 1.013 * 10^2 * 0.25 \ln(25 / 1.013)$$

$$= 81.19 \text{kJ/min} = 1.353 \text{ kW}$$

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Indicated work}} = \frac{1.353}{2} = 0.6766 * 100 = 67.66\%$$

Problem 9.8 , pg 287 T.D. Eastop, third edition: - A single cylinder, single acting air compressor running at (300) rev / min is driven by a (23)kW electric motor . The mechanical eff . Of the drive between motor and compressor is (87) %. The air inlet condition are (1.013) bar and 15°C and delivery pressure = 8 bar. Calculate F.A.D. in (m^3 / min), η_v & bore of the compressor. Assume n = 1.3, V_c = 7% V_s 7& bore is equal to the stroke.

Sol:-

$$\eta_v = \frac{\text{Out put}}{\text{In put}} = \frac{I . P}{23} = 0.87$$

I . P = 20.01 kW

$$T_2 / T_1 = (p_2 / p_1)^{(n-1)/n}$$

Where $T_1 = (15 + 273) = 288$ K ,

$$T_2 / 288 = (8 / 1.013)^{(1.3 - 1)/1.3}$$

T_2 = 464 K

$$I . P = \frac{n}{n - 1} m^{\cdot} R(T_2 - T_1)$$

$$20.01 = \frac{1.3}{1.3 - 1} * m^{\cdot} * 0.287(464 - 288)$$

$m^{\cdot}$ = 0.09142 kg / sec * 60 = 5.458 = kg / min

$$\text{F.A.D.} = \frac{m^{\cdot} R T}{P_1} = \frac{5.485 * 287 * 288}{1.013 * 10^5} = 4.476 \text{ m}^3/\text{min}$$

$$\eta_v = 1 - \frac{V_c}{V_S}\left\{\left(\frac{P_2}{p_1}\right)^{1/n} - 1\right\}$$

$$\eta_v = 1 - \frac{0.07\,V_S}{V_S}\left\{\left(\frac{8}{1.013}\right)^{1/n} - 1\right\} = 0.727 = 72.7\%$$

$$\eta_v = \frac{F.A.D}{V_s}\ ,\ V_s = \frac{F.A.D}{\eta_v} = \frac{4.476\ m^3/min}{0.727} = 6.1574\ m^3/min$$

$$\text{Swept volume per cycle} = \frac{6.1574\ m^3/min}{300\ cycle/min} = 0.02052\ m^3$$

$$\text{Swept volume} = V_s = 0.0205m^3 = \frac{Л}{4} d^2 * L\ ,\ L = d$$

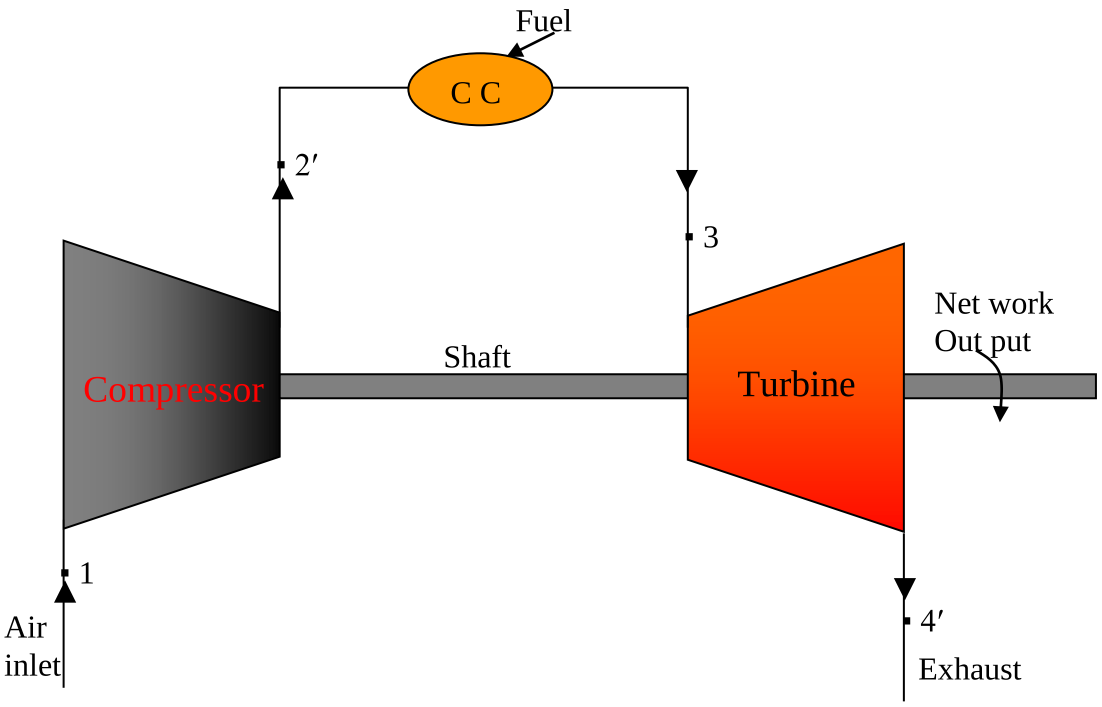
$$0.0205 = \frac{Л}{4} d^2 * d$$

d = 0.2968m = 296.8 mm

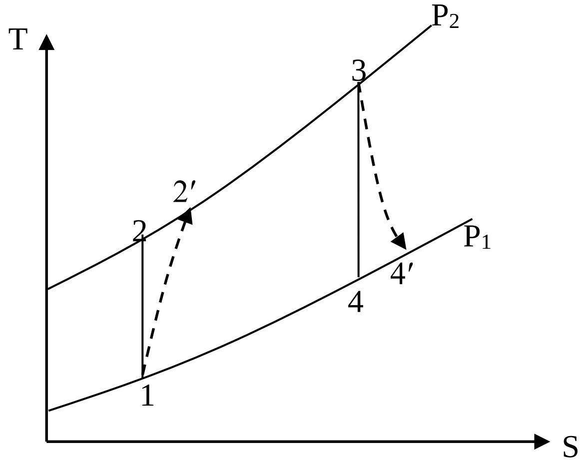
Gas turbine: - (Bray ton cycle)

The most basic gas turbine is open cycle in which a compressor and turbine are mounted on a common shaft,

- 0pen cycle of gas turbine



Air is drawn in to compressor and after compression passes to a combustion chamber, C C. Energy is supplied in the combustion chamber by spraying fuel in to the air stream, and the resulting hot gases expand through the turbine, to the atmosphere , in the composition chamber the chemical energy released on combustion is equivalent to a transfer of heat at constant pressure to a working fluid of constant mean specific heat , and it is usually the mass of fuel is neglect in comparison with the mass of air .



Line 1- 2′ represents irreversible adiabatic compression.
Line 2′ - 3 represents constant pressure heat supply in the combustion chamber.

Line 3- 4′ represents irreversible adiabatic expansion.

The process 1 – 2 represents the ideal isentropic compression process between the same pressures p_1 and p_2 ,

The process 3 – 4 represents the ideal isentropic expansion process between the same pressures p_2 and p_1 ,

The change in kinetic energy between the various points in the cycle is negligibly small compared with the enthalpy changes. The applying the flow equation to each part of the cycle,

$q - w = \Delta h + \Delta k.E + \Delta p.E.$

For the compressor: work input = $Cp_a (T_{2'} - T_1)$

For the combustion chamber: Heat supplied = $Cp_g(T_3 - T_{2'})$

For the turbine: work output = $Cp_g (T_3 - T_{4'})$
Net work out put = work turbine – work compressor

$$= Cp_g (T_3 - T_{4'}) - Cp_a (T_{2'} - T_1)$$

$$\text{Thermal efficiency } (\eta_{th}) = \frac{\text{Net work out put}}{\text{Heat supply}}$$

$$= \frac{Cp_g\,(T_3 - T_{4'}) - Cp_a\,(T_{2'} - T_1)}{Cp_g(T_3 - T_{2'})}$$

$$\text{Compressor isentropic efficiency } (\eta_{isent.C}) = \frac{\text{Isentropic work input}}{\text{Actual work required}}$$

$$= \frac{Cp_a\,(T_2 - T_1)}{Cp_a\,(T_{2'} - T_1)} = \frac{(T_2 - T_1)}{(T_{2'} - T_1)}$$

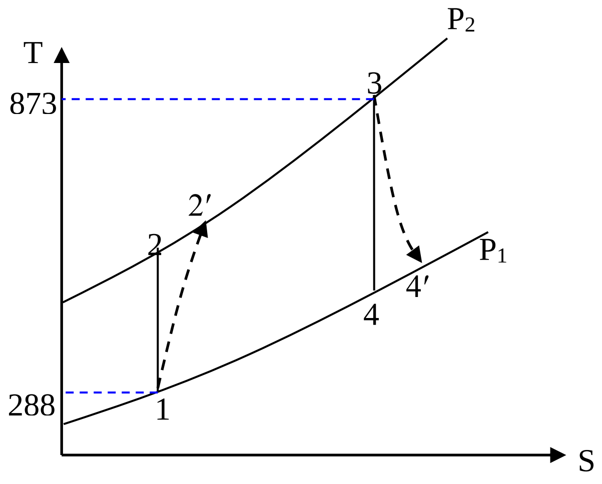
$$\text{Turbine isentropic efficiency } (\eta_{isent.T}) = \frac{\text{Actual work output}}{\text{Isentropic work output between the same Pressure ratio}}$$

$$= \frac{Cp_g\,(T_3 - T_{4'})}{Cp_g\,(T_3 - T_4)} = \frac{(T_3 - T_{4'})}{(T_3 - T_4)}$$

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work out put}} = \frac{W\ net}{W\ T}$$

EX:- A gas turbine unit has a pressure ratio of 6/1 and a maximum temperature of 600°C . The isentropic efficiency of the compressor and turbine are 0.82 and 0.85 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 15 kg /sec. Take Cp = 1.005 kJ /kg.K and γ = 1.4 for the compression process, and take Cp = 1.11 kJ /kg.K and γ = 1.333 for the expansion process.

Sol:-



$T_2/ T_1= (p_2 / p_1)^{(\gamma-1)/\gamma}$

Where $T_1 = (15 + 273) = 288$ K ,

$T_2 / 288 = (6)^{(1.4 - 1) / 1.4}$

$T_2 = 480.5$ K

$$\text{Compressor isentropic efficiency } (\eta_{isent.C}) = \frac{(T_2 - T_1)}{(T_{2'} - T_1)} = \frac{(480.5 - 288)}{(T_{2'} - 288)} = 0.82$$

$T_{2'} = 522.8$ K

For the compressor: work input = $Cp_a\,(T_{2'} - T_1)$

$= 1.005\,(522.8 - 288) = 236$ kJ/kg

$T_3 / T_4 = (p_3 / p_4)^{(\gamma - 1) / \gamma}$

Where $T_3 = (600 + 273) = 873$ K ,

$873 / T_4 = (6)^{(1.333 - 1) / 1.333}$

$T_4 = 558$ K

$$\text{Turbine isentropic efficiency } (\eta_{isent.T}) = \frac{(T_3 - T_{4'})}{(T_3 - T_4)} = \frac{(873 - T_{4'})}{(873 - 558)} = 0.85$$

$T_{4'} = 605.3$ K

For the turbine: work output = $Cp_g\,(T_3 - T_{4'})$

$= 1.11\,(873 - 605.3)$

$= 297.2$ kJ/kg

Net work out put = work turbine – work compressor

= 297.2 - 236

= 61.2kJ/kg

Power in kilowatts = mass flow rate * Net work out put

= 15 * 61.2

= 918 kW

EX: - Calculate the thermal efficiency and the work ratio of the plant in last example assuming that Cp for the combustion process is 1.11 kJ/ kg.K .

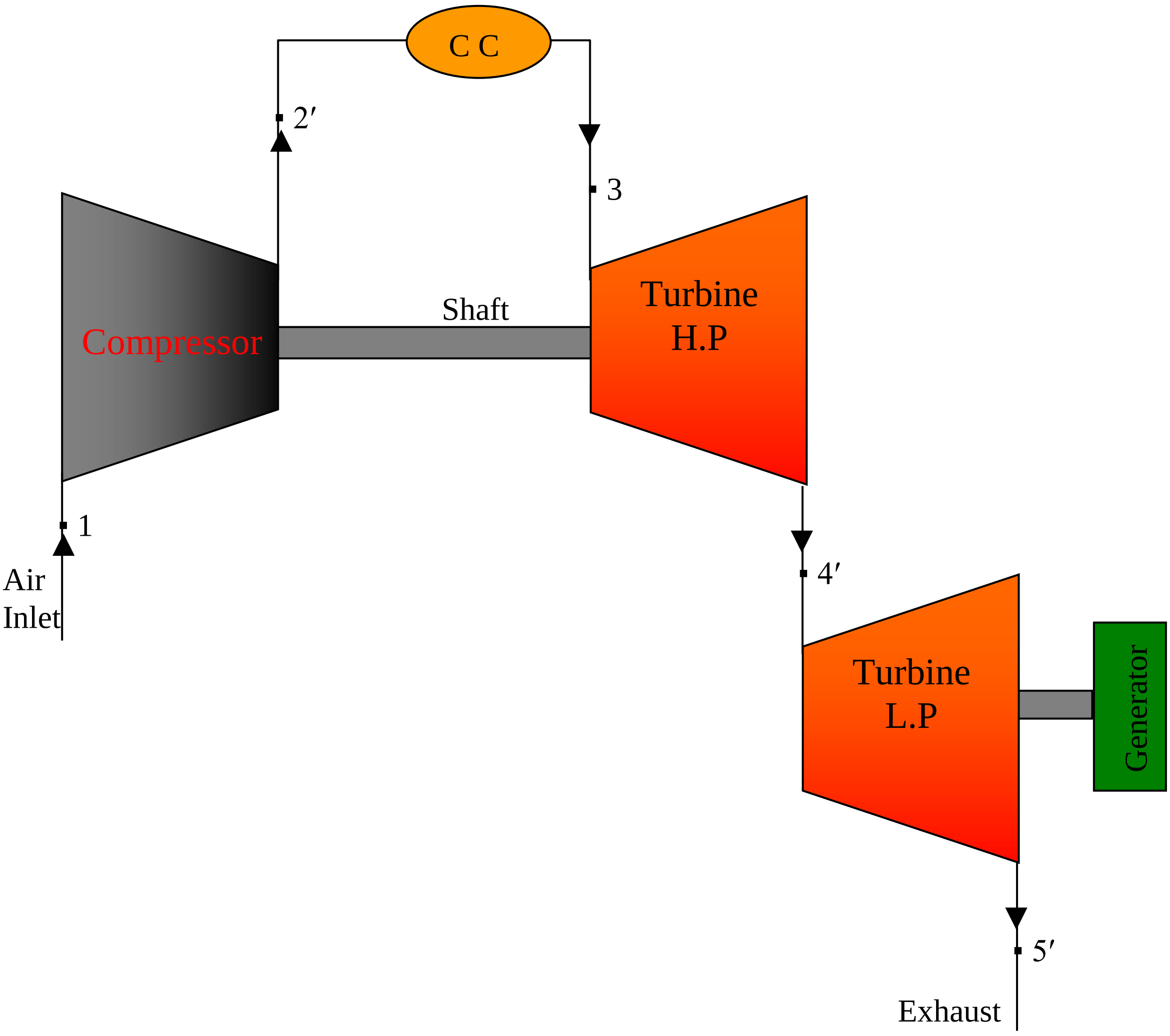
Sol:-

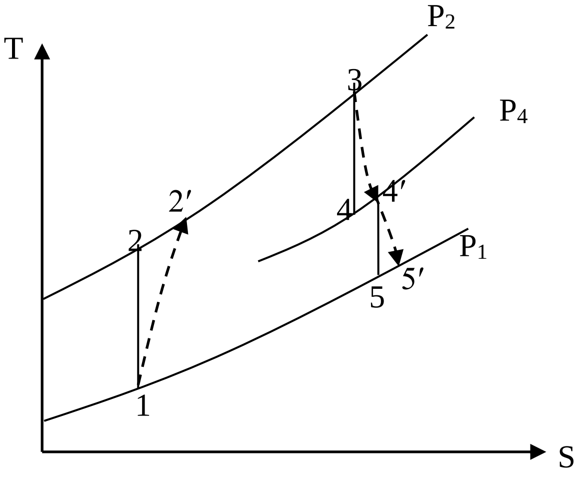
Heat supplied = $Cp_g(T_3 - T_{2'}) = 1.11(873 - 522.8) = 388.7$ kJ/kg

$$\text{Thermal efficiency} = \frac{\text{Net work out put}}{\text{Heat supplied}} = \frac{61.2}{388.7} = 0.157 = 15.7\%$$

$$\text{Work ratio} = \frac{W\ net}{W\ T} = \frac{61.2}{297.2} = 0.206$$

It is sometimes more convenient to have two separate turbines, one of which drives the compressor while the other provides the power out put. The first, or high – pressure turbine, is then known as the compressor turbine, and the second, or low pressure turbine, is called the power turbine. Assuming that each turbine has its own isentropic efficiency, neglecting kinetic energy changes, we have





Work from H.P. turbine = Work in put to compressor

$$Cp_g\ (T_3 - T_{4'}) = Cp_a\ (T_{2'} - T_1)$$

(Where Cp_g = and Cp_a are the specific heats at constant pressure of the gases in the turbine and the air in the compressor respectively).
The net work output is then given by the L.P. turbine,

$$\text{Net work output} = Cp_g\ (T_{4'} - T_{5'})$$

EX:- A gas turbine unit takes air at 17°C and 1.01 bar and the pressure ratio is 8 /1 . The compressor is driven by the H.P. turbine and the L.P. turbine drives a separate power shaft. The isentropic efficiencies of the compressor and the H.P. and L.P. turbines are 0.8, 0.85, and 0.83 respectively. Calculate the pressure and temperature of the gases entering the power turbine, the net power developed by the unit per kg/sec mass flow, the work ratio and the thermal efficiency of the unit. The maximum cycle temperature is 650°C . For the compression process take Cp = 1.005 kJ/kgK and γ = 1.4 ; for the combustion process , and for the expansion process take
Cp = 1.15 kJ/kgK and γ = 1.333. Neglect the mass of fuel .

Sol:-

$T_2/ T_1= (p_2 / p_1)^{(\gamma-1) / \gamma}$

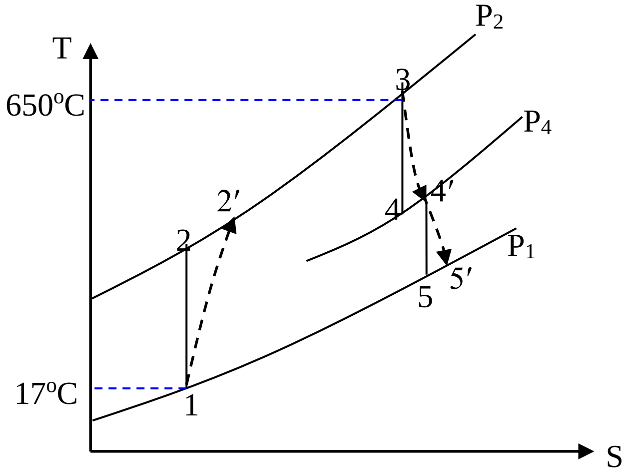
Where $T_1 = (17 + 273) = 290$ K ,

$T_2/ 290= (8)^{(1.4 -1) / 1.4}$

$T_2 = 525$ K

Compressor isentropic efficiency $(\eta_{\text{isent.C}}) = \dfrac{(T_2 - T_1)}{(T_{2'} - T_1)} = \dfrac{(525 - 290)}{(T_{2'} - 290)} = 0.8$

$T_{2'} = 584.1\text{K}$



For the compressor: work input = $Cp_a\,(T_{2'} - T_1)$

$= 1.005\,(584.1 - 290) = 295.6$ kJ/kg

$W_c = W_{\text{.H.P.T}}$

$W_{\text{.H.P.T}} = 295.6 = Cp_g\,(T_3 - T_{4'}) = 1.15\,(923 - T_{4'})$

$T_{4'} = 666$ K

$\eta_{\text{isent.H.P.T}} = \dfrac{T_3 - T_{4'}}{T_3 - T_4} = \dfrac{923 - 666}{923 - T_4} = 0.85$, $T4 = 620.6$ K

$p_3 / p_4 = (T_3 / T_4)^{\gamma/(\gamma-1)}$ or , $T_3 / T_4 = (p_3 / p_4)^{(\gamma-1)/\gamma}$

$p_3 / p_4 = (923 / 620.5)^{1.333/(1.333-1)} = 4.9$

$p_4 = p_3 / 4.9 = p_2 / 4.9 = (1.01 * 8) / 4.9 = 1.65$ bar

$$\frac{p_4}{p_5} = \frac{p_4}{p_1} * \frac{p_2}{p_3} = \frac{p_4}{p_3} * \frac{p_2}{p_1} = \frac{1}{4.9} * 8 = 1.63$$

$T_{4'} / T_5 = (p_4 / p_5)^{(\gamma-1)/ \gamma}$

$666 / T_5 = (1.63)^{(1.333-1)/ 1.333}$

$T_5 = 589.5$ K

$$\eta_{isent.L.P.T} = \frac{T_{4'} - T_{5'}}{T_{4'} - T_5} = \frac{666 - T_{5'}}{666 - 588.5} = 0.83,\ T_{5'} = 602.5 \text{ K}$$

Work output from L.P. turbine $= Cp_g (T_{4'} - T_{5'})$

$= 1.15 (666 - 602.5)$

$= 73$ kJ/kg

Net power output = 1 * 73 = 73 kW

$$\text{Work ratio} = \frac{W \text{ net}}{\sum W\ T} = \frac{73}{295.6 + 73} = 0.198$$

Heat supplied $= Cp_g (T_3 - T_{2'})$

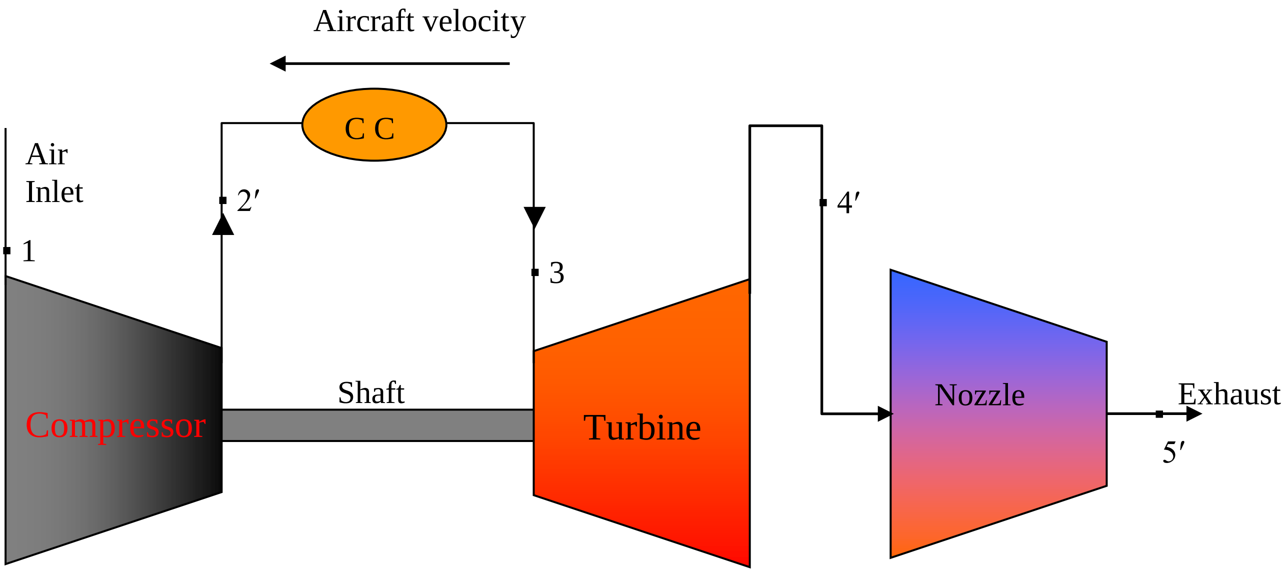
$= 1.15 (923 - 584.1)$

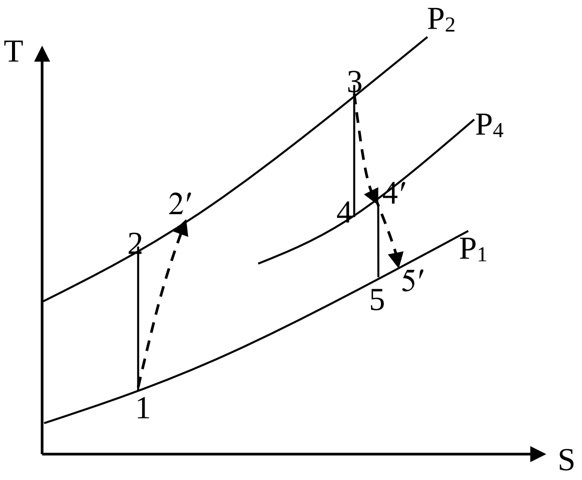
$= 389.74$ kJ / kg

$$\text{Thermal efficiency} = \frac{\text{Net work out put}}{\text{Heat supplied}} = \frac{73}{389.74} = 0.167 = 18.7\%$$

Application of gas turbine:-

In a jet engine the propulsion nozzle takes the place of the low pressure stage turbine, the aircraft is powered by the reactive thrust of the jet of gases leaving the nozzle , and this high velocity jet is obtained at the expense of the enthalpy drop from 4′ to 5′ . The turbine develops just enough work to drive the compressor and overcome mechanical losses.

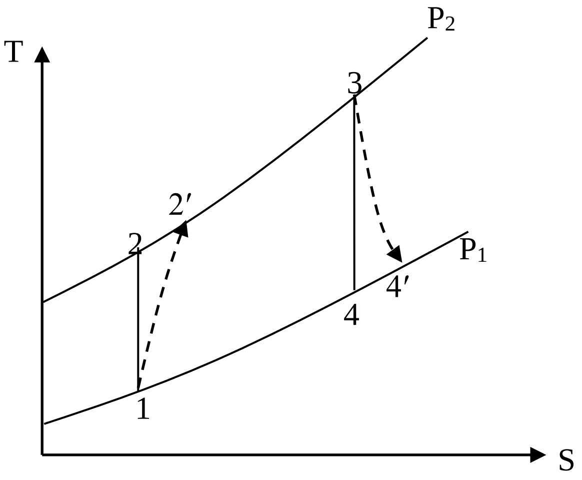


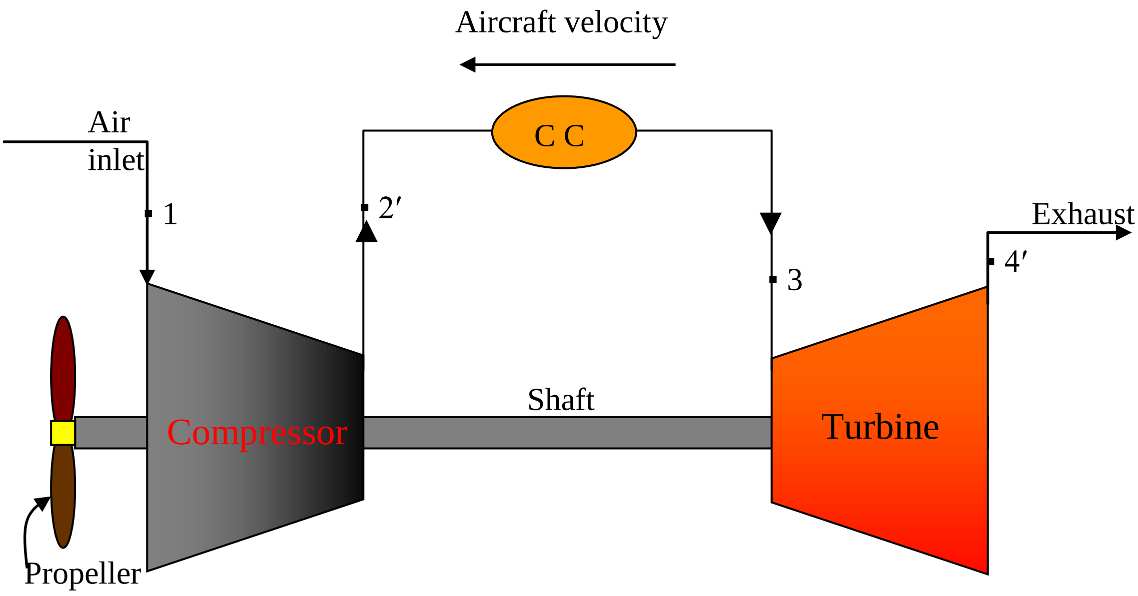


In a turbo – prop engine the turbine drives the compressor and also the airscrew, or propeller, the net work available to drive the propeller is given by ,

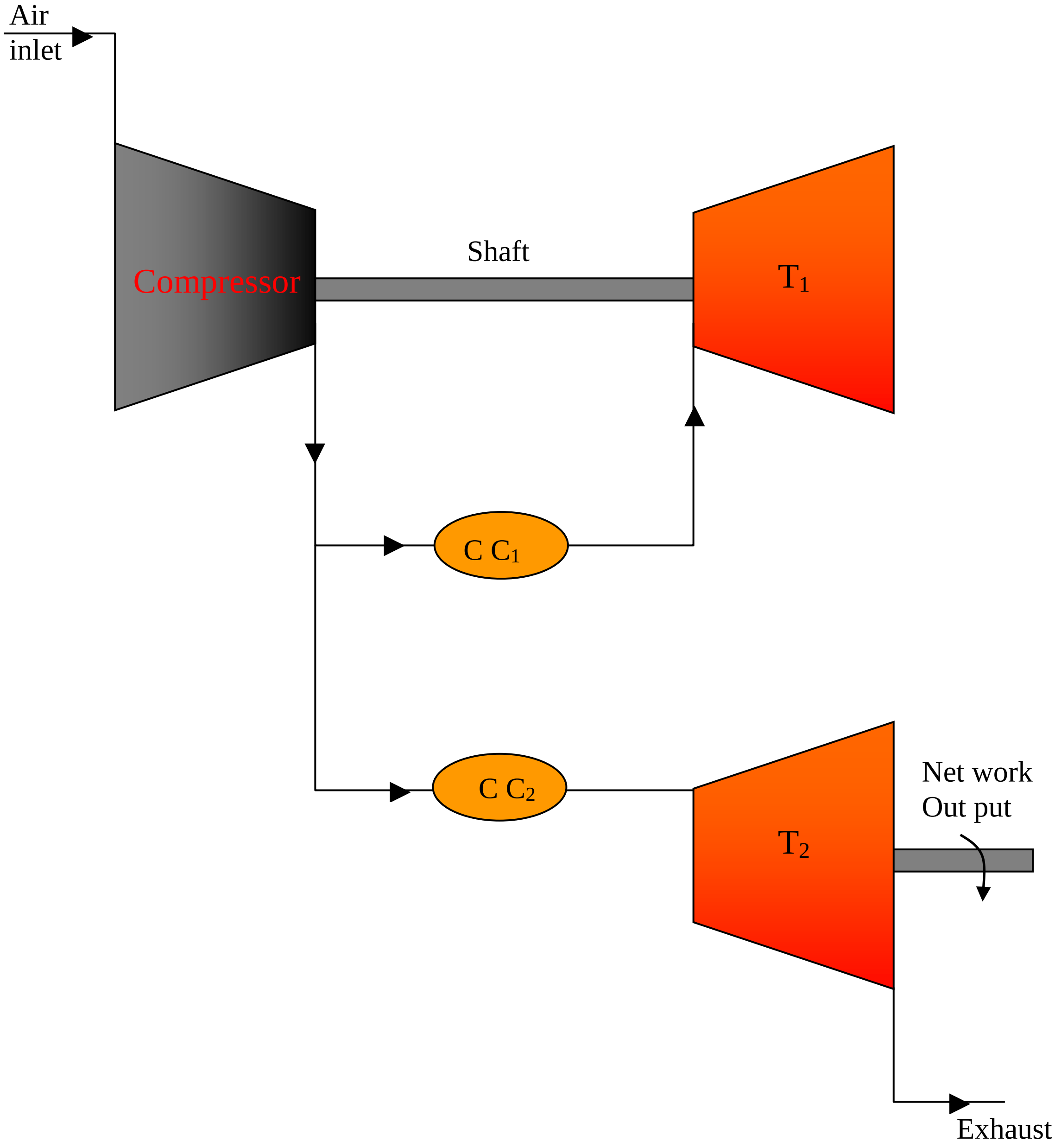
Net work = $Cp_g\ (T_3 - T_{4'}) - Cp_a\ (T_{2'} - T_1)$

(Neglecting mechanical losses)





The parallel flow unit, the air flow is split into two streams after the compression process is completed. Some air is then passed to a combustion chamber which supplies hot gases to the turbine driving the compressor, while the rest of the air is passed to a second combustion chamber and from thence to the power turbine.



In this system each turbine expands the gases received by it through the full pressure ratio. Parallel flow turbines are not as common as the more conventional series flow, and will not be considered in any detail.

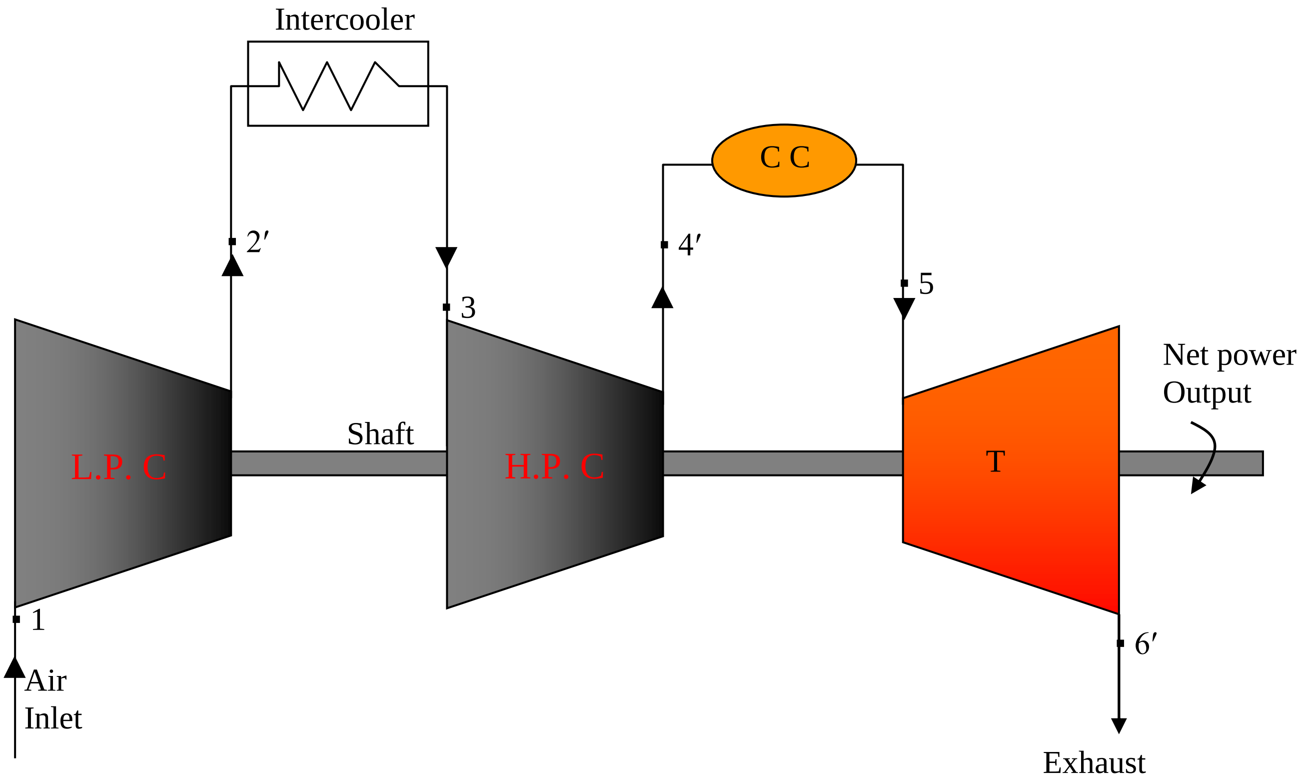
Modifications to the basic gas turbine cycle

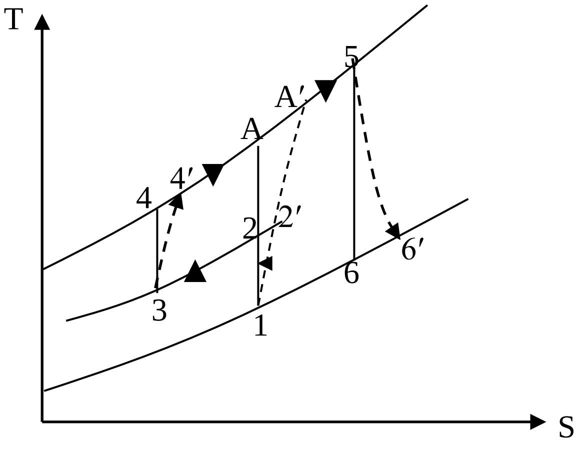
The work ratio and the thermal efficiency of the basic gas turbine cycle are low. These can be improved by increasing the isentropic efficiencies of the compressor and turbine, and this is a matter of blade design and manufacture.

In a practical cycle with irreversibilities in the compression and expansion processes the thermal efficiency depends on the maximum cycle temperature as well as on the pressure ratio. The maximum cycle temperature is limited by metallurgical consideration. The temperature of the gases entering the turbine can be raised, provided a means of blade cooling, it is important to have as high a work ratio as possible, and methods of increasing the work ratio, such as intercooling between compressor stages, and reheating between turbine stages.

(A) Intercooling

When the compression is performed in two stages with an intercooler between the stages, then the work input for a given pressure ratio and mass flow is reduced.





The actual cycle process is: 1− 2′ L.P. compressor,

2′ − 3 the intercooler,

3 – 4′ high pressure compressor

4′ − 5 in the combustion chamber

5 −6′ in the turbine

The ideal cycle for this arrangement is 1−2−3−4−5−6; the compression process without intercooling is shown as 1− A′ in the actual case, and 1−A in ideal isentropic case,

The work input with intercooling is given by

Work input = $W_{L.PC} + W_{H.PC} = \sum W_C$

Work input (with intercooling) = $C_p (T_{2'} - T_1) + C_p (T_{4'} - T_3)$

The work input with no intercooling is given by

Work input (no intercooling) = $C_p (T_{2'} - T_1) + C_p (T_{A'} - T_{2'})$

Comparing between two equation will give that the (W_{in}) with intercooling is less than the work input with no intercooling when $C_p (T_{4'} - T_3)$ is less than $C_p (T_{A'} - T_{2'})$. The work input is minimum when the pressure ratio in each stage is the same $p_2 / p_1 = p_4 / p_3$, and when the temperature of the air is cooled in the intercooler to inlet temperature to the unit ($T_3 = T_1$)

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{\sum W_T - \sum W_C}{\sum W_T}$$

When work of compression is reduced then the work ratio is increasing

$$\eta_{th} = \frac{\text{Net work}}{\text{Heat supplied in (C C)}}$$

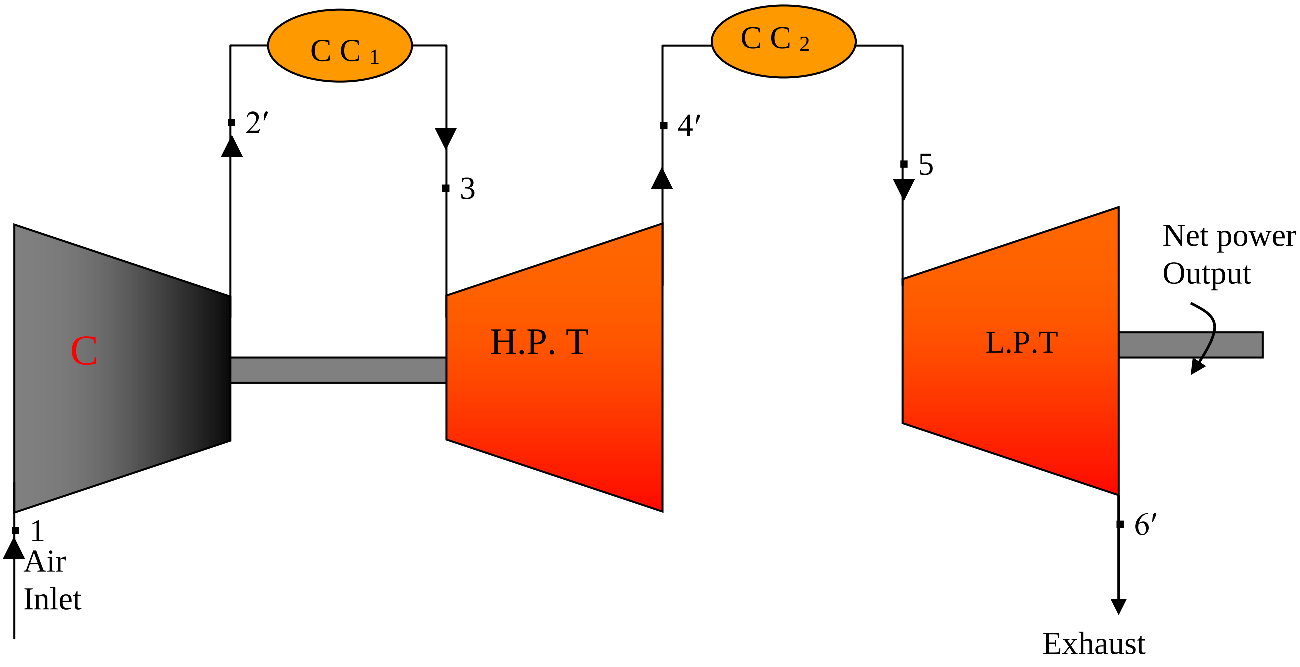
Heat supplied (with intercooling) = $Cp_g(T_5 - T_{4'})$

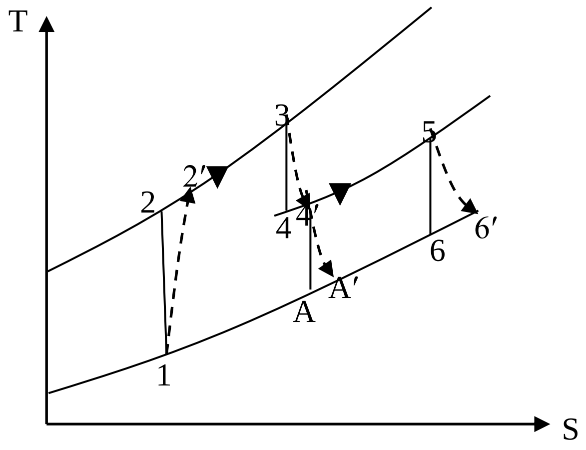
Heat supplied (no intercooling) = $Cp_g(T_5 - T_{A'})$

Hence the heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in the heat to be supplied causes the thermal efficiency to decrease.

(B) Reheat

The expansion process can be performed in two separate turbine stages, the H.P. turbine driving the compressor and the L.P. turbine providing the useful power output. The work output of L.P. turbine can be increased by raising the temperature at inlet to this stage. This can be done by placing a second combustion chamber between the two turbine stages in order to heat the gases leaving the H.P turbine.





The line 4′ - A′ represents the expansion in L.P. turbine if reheating is not used

If $\eta_{\text{shaft H.P.T} \longrightarrow \text{comp}} = 100\ \%$ therefore,

$W_{H.P.T} = W_{comp}$

$Cp_a.(T_{2'} - T_1) = Cp_g.(T_3 - T_{4'})$

The net work output = $W_{L.P.T}$

W_{net} (reheat) = $Cp_g.(T_5 - T_{6'})$

W_{net} (no reheat) = $Cp_g.(T_{4'} - T_{A'})$

Since pressure lines diverge to the right on T-S diagram, it can be seen that the temperature difference.$(T_5 - T_{6'})$ is always greater than .$(T_{4'} - T_{A'})$, so that reheating increases the net work output .

$$\text{Work ratio} = \frac{\text{Work of expansion} - \text{work of compression}}{\text{Work of expansion}}$$

$$\text{Work ratio} = 1 - \frac{\text{Work of compression}}{\text{Work of expansion}}$$

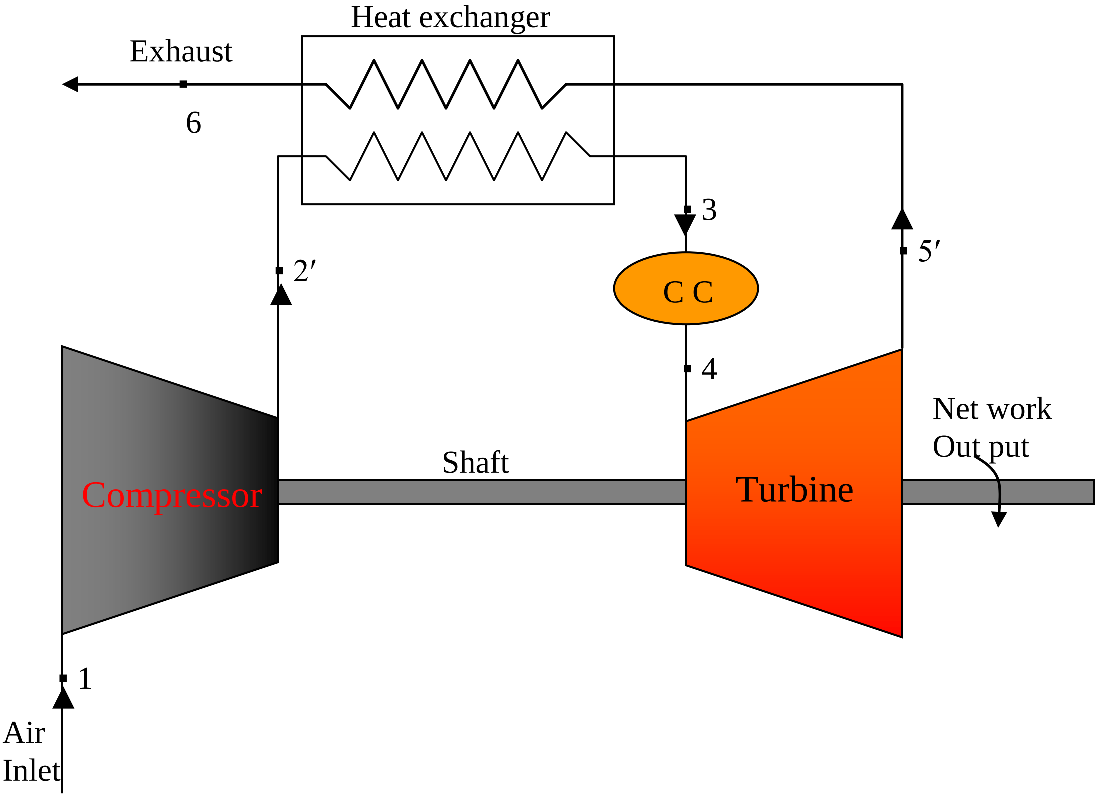
Work of expansion is increased and the work of compression is unchanged, then the work ratio is increased.

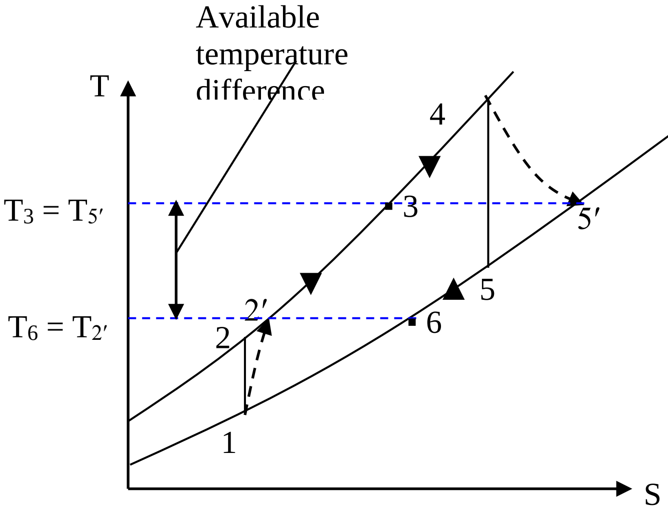
Heat supplied = $Cp_g\,(T_3 - T_{2'}) + Cp_g\,(T_5 - T_{4'})$

The exhaust temperature of the gases leaving the L.P. turbine is much higher when reheating is used (i.e . $T_{6'}$ as compared with $T_{A'}$) and a heat exchanger can be used to enable some of the energy of the exhaust gases to be used.

(C) Heat exchanger

The exhaust gases leaving the turbine at the end of expansion are still at a high temperature, if these gases are allowed to pass into the atmosphere, then this represents a loss of available energy. Some of this energy can be recovered by passing the gases from the turbine through a heat exchanger, where heat transferred from the gases is used to heat the air leaving the compressor.





In the ideal heat exchanger the air would be heated from $T_{2'}$ to $T_3 = T_{5'}$, and the gases would be cooled from $T_{5'}$ to $T_6 = T_{2'}$, in practice this is impossible. If no heat is lost from the heat exchanger to the atmosphere, then the heat given up by the gases must be exactly equal to the heat taken up by the air,

$$\dot{m}_a Cp_a.(T_3 - T_{2'}) = \dot{m}_g Cp_g.(T_{5'} - T_6)$$

A more convenient way of assessing the performance of the performance of the heat exchanger is to use a thermal ratio, defined as

$$\text{Thermal ratio} = \frac{\text{Temperature rise of the air}}{\text{Maximum temperature difference available}}$$

$$\text{Thermal ratio} = \frac{T_3 - T_{2'}}{T_{5'} - T_{2'}}$$

When a heat exchanger is used then the heat to be supplied in the combustion chamber is reduced, assuming that the maximum cycle temperature is unchanged. The net work output is unchanged and hence the thermal efficiency increased.

Heat supplied by the fuel (without heat exchanger) = $Cp_g(T_4 - T_{2'})$

Heat supplied by the fuel (with heat exchanger) = $Cp_g(T_4 - T_3)$

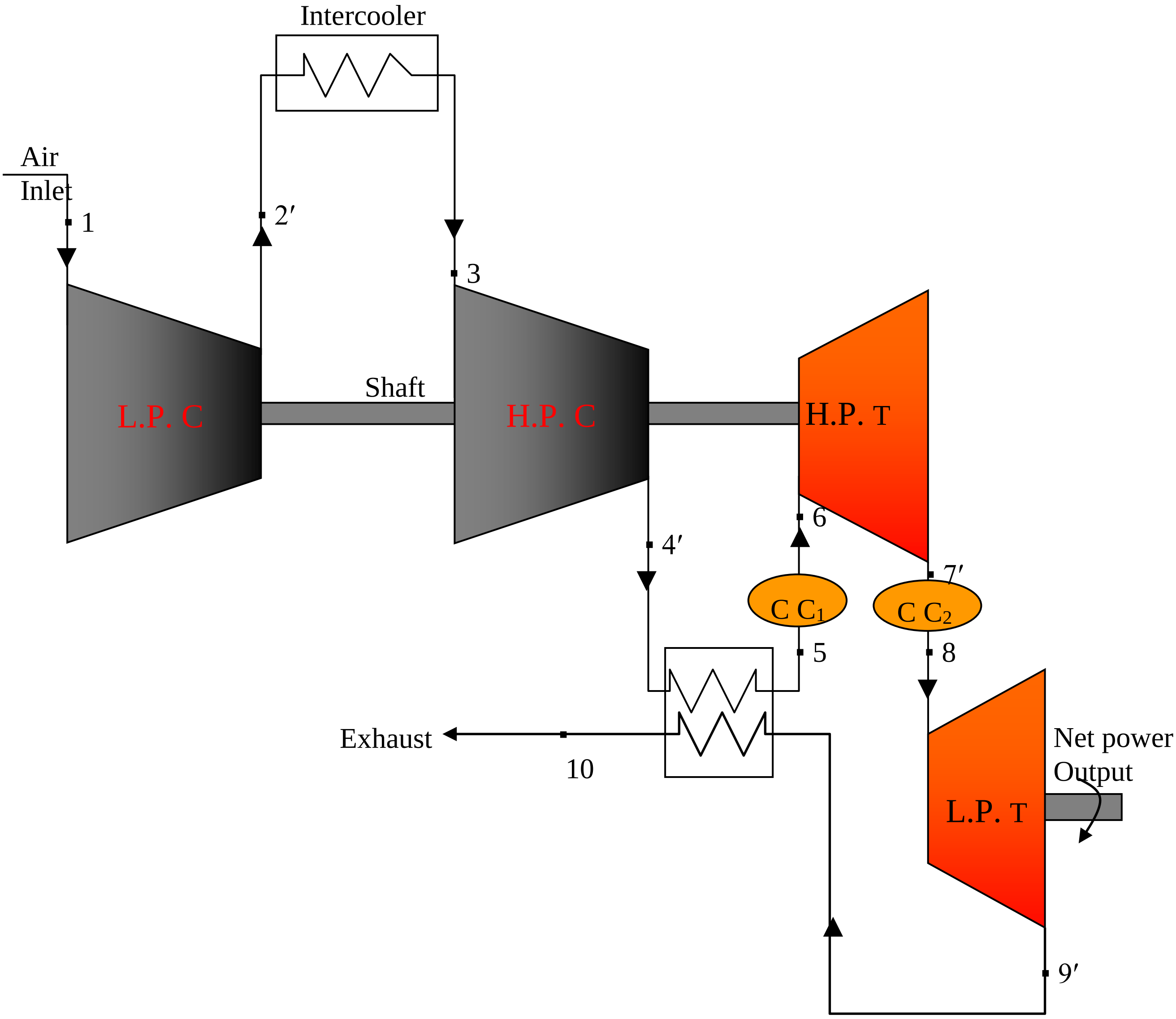
A heat exchanger can only be used if there is a sufficiently large temperature difference between the gases leaving the turbine, and the air leaving the compressor.

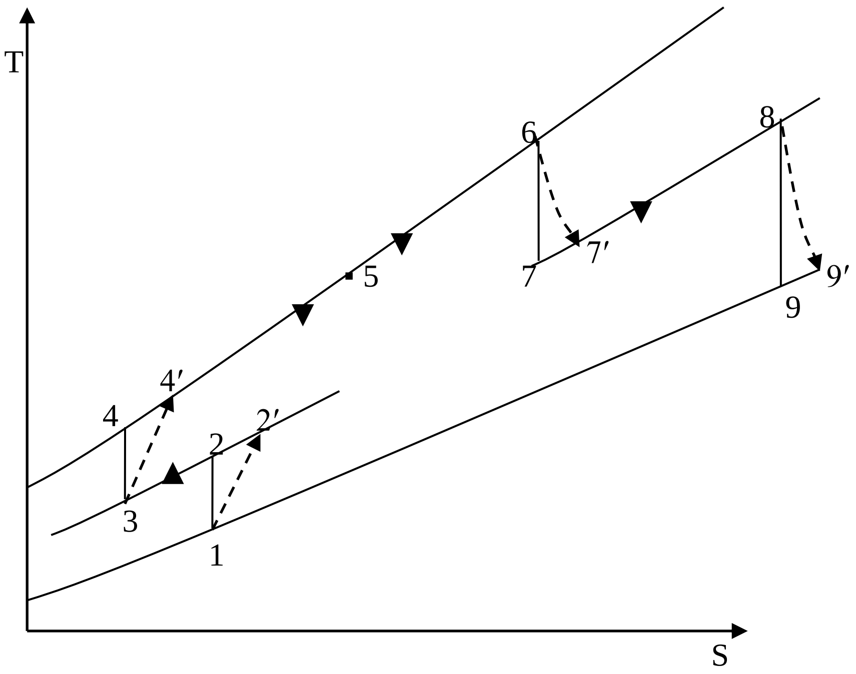
In aircraft gas turbines, where the power / weight ratio is much more important than thermal efficiency or long life, the heat exchanger is not used. In large gas turbine units for marine propulsion or industrial power, a heat exchanger is usually used.

EX:- A 500 kW gas turbine generating set operates with two compressor stages with intercooling between stages ; the overall pressure ratio is 9/1 . A high – pressure turbine is used to drive the compressors, and a low – pressure turbine drives the generator. The temperature of the gases at entry to the high – pressure turbine is 650ºC and the gases are reheated to650ºC after expansion in the first turbine. The exhaust gases leaving the low – pressure turbine are passed through a heat exchanger to heat the air leaving the high – pressure stage compressor. The compressors have equal pressure ratios and intercooling is complete between stages. The air inlet temperature to the unit is 15 ºC . The isentropic efficiency of each compressor stage is 0.8 , and the isentropic efficiency of each turbine stage is 0.85 ; the heat exchanger thermal ratio is 0.75 . A mechanical efficiency of 98% can be assumed for both the power shaft and the compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy,

calculate the thermal efficiency and work ratio of the plant, and the mass flow in kg/sec. For air take Cp = 1.005 kJ/kg.K and $\gamma = 1.4$, for the gases in the combustion chamber and in the turbines and heat exchanger take Cp = 1.15 kJ/kg .K and $\gamma = 1.333$. Neglect the mass of the fuel.

Sol:-





$T_2 / T_1 = (p_2 / p_1)^{(\gamma - 1)/\gamma}$

$T_2 / 288 = (3)^{(1.4 - 1)/1.4}$

$T_2 = 394$ K

$$\eta_{c.L.P.compressor} = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{394 - 288}{T_{2'} - 288} = 0.8,\ T_{2'} = 420.5 \text{ K}$$

Work input per compressor stage = $Cp_a (T_{2'} - T_1)$

$= 1.005 (420.5 - 288)$

$= 133.1$kJ/kg

$$\text{Work output of H.P. turbine} = \frac{2 * 133.1}{0.98} = 272 \text{ kJ/kg}$$

$272 = Cp_g (T_6 - T_{7'})$

$272 = 1.15 (923 - T_{7'})$

$T_{7'} = 686.5$ K

$$\eta_{T.H.P.turbine} = \frac{T_6 - T_{7'}}{T_6 - T_7} = \frac{923 - 686.5}{923 - T_7} = 0.85,\ T_7 = 645\text{ K}$$

$(p_6 / p_7) = (T_6 / T_7)^{\gamma/(\gamma - 1)}$

$(p_6 / p_7) = (923 / 645)^{1.333/(1.333 - 1)} = 4.19$

$p_8 / p_9 = (9 / 4.19) = 2.147$

$T_8 / T_9 = (p_8 / p_9)^{(\gamma - 1)/\gamma}$

$923 / T_9 = (2.147)^{(1.333 - 1)/1.333}$

$T_9 = 762.6$ K

$$\eta_{T.H.P.turbine} = \frac{T_8 - T_{9'}}{T_8 - T_9} = \frac{923 - T_{9'}}{923 - 762.6} = 0.85,\ T_{9'} = 786.7\text{ K}$$

Net work output $= Cp_g (T_8 - T_{9'}) * 0.98$

$= 1.15 (923 - 786.7) * 0.98$

$= 153.7$ kJ/ kg

Thermal ratio of heat exchanger $= \frac{T_5 - T_{4'}}{T_{9'} - T_{4'}} = 0.75$

$T_5 - 420.5 = 0.75 (786.7 - 420.5)$

$T_5 = 695.2$ K

Heat supplied $= Cp_g (T_6 - T_5) + Cp_g (T_8 - T_{7'})$

$= 1.15 (923 - 695.2) + 1.15 (923 - 686.5)$

$= 534$ kJ/kg

Thermal efficiency $= \frac{W}{Q} = \frac{153.7}{534} = 0.288 = 28.8\ \%$

Gross work of the plant = work of H.P. turbine + work of L.P. turbine

$$= 272 + 153.7 / 0.98 = 429 \text{ kJ/kg}$$

Therefore,

$$\text{Work ratio} = \frac{\text{Net work out put}}{\text{Gross work output}} = \frac{153.7}{429} = 0.358$$

The electrical output is 5000 kW. Let the mass flow be $m^{\cdot}$ kg / sec. then

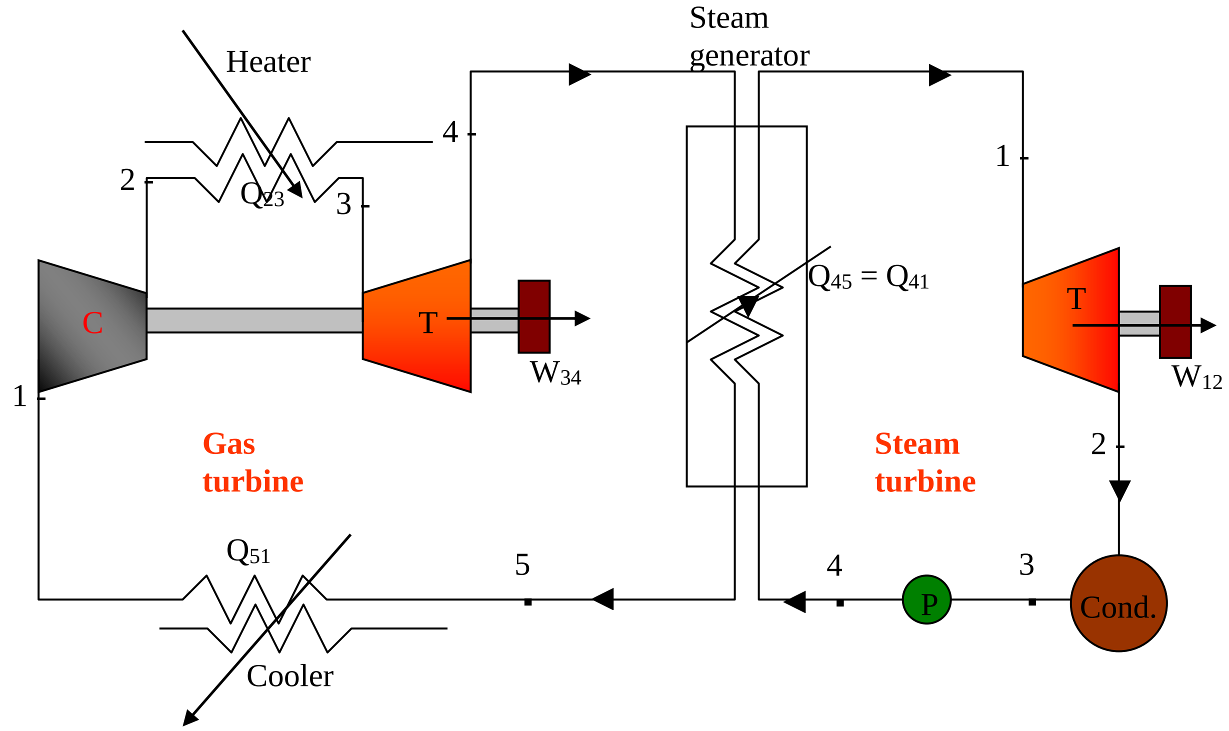
$5000 = m^{\cdot} * 153.7$

Rate of flow of air, $m^{\cdot} = 32.6$ kg / sec

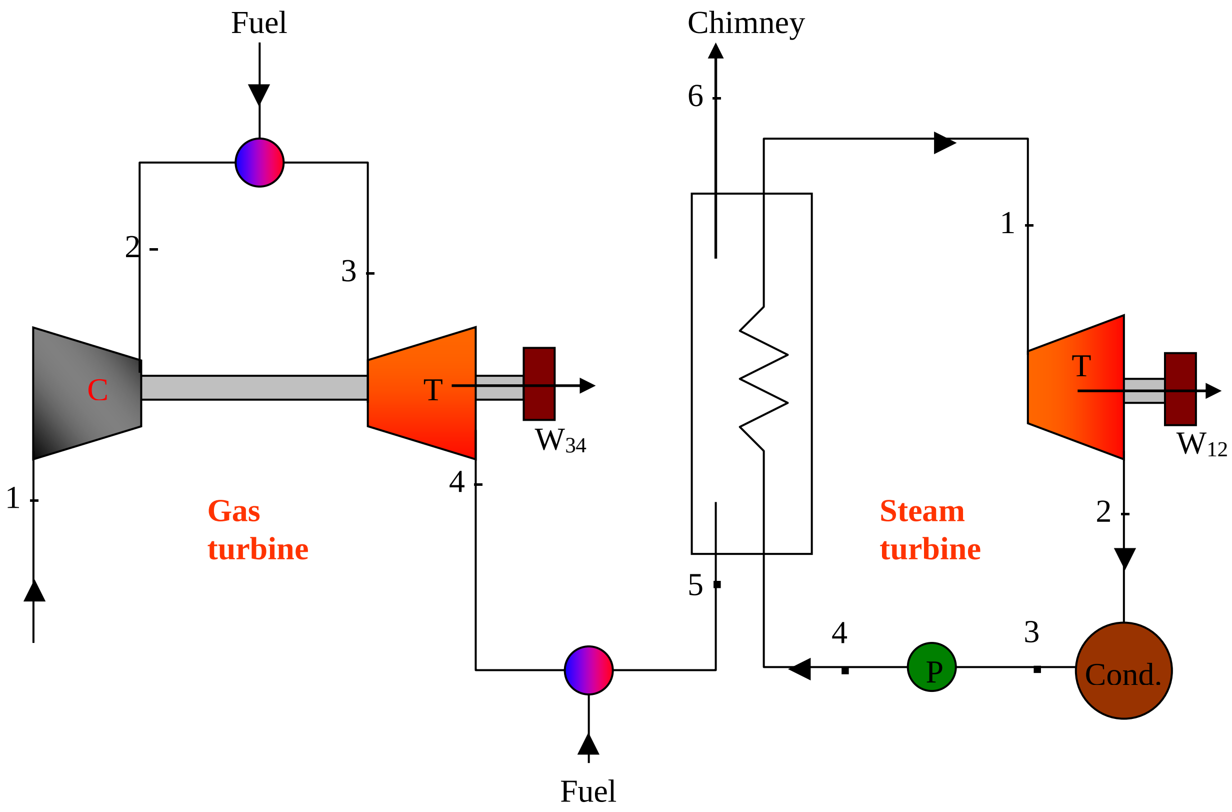
Combined gas – steam cycles:-

The gas and steam turbines as basic combination, the gas turbine is the higher temperature unit and the gas leaving the turbine is at a sufficiently high temperature to be used as a source of heat for the production of steam at a suitable pressure and temperature.

The simplest form of energy transfer from one cycle to the other is by means of a recuperative heat exchanger in which a closed cycle gas turbine and a steam turbine are combined.



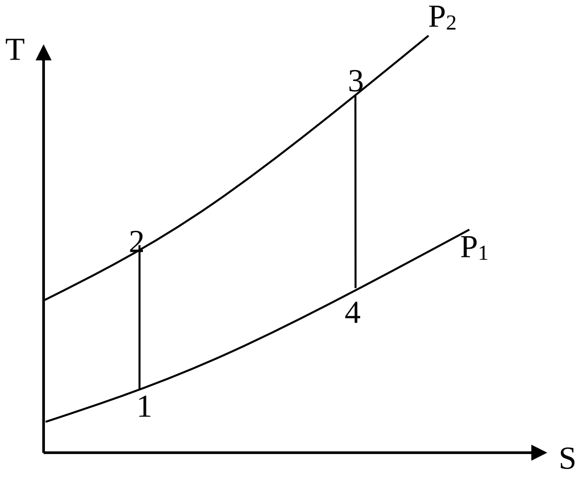
Another characteristic of the gas turbine, its high air – to fuel ratio, can be exploited if, with an open cycle gas turbine, the exhaust gas, with its high oxygen content, is used as the inlet gas to the steam generator where the combustion of additional fuel takes place. And the combination allows nearer equality between the power outputs of the two units than is obtained with the simple recuperative heat exchanger. For a given total power output the energy input is reduced and the installed cost of a gas turbine per unit of power output is about one quarter of that of the steam turbine . The disadvantage of the combined cycles up to now has been greater complexity, the possible loss of flexibility and reliability and the different fuel requirements between the two , the gas turbine requiring a higher quality fuel , advanced developments have made the combined cycle a more serious proposition , particularly for the middle load range , and include low capital cost , lower fuel inputs , quick construction and running reliability with minimum supervision , good cold starring characteristics , independent operation of the gas turbine and the increased ability to use lower grade fuel . Added to this are the considerable advantage with respect to atmospheric pollution.



EX:- A combined power plant consists of a gas turbine unit and a steam turbine unit , the exhaust gas from the turbine being the supply gas to the steam generator at which a further supply of fuel is burned in the gas . The pressure ratio for the gas turbine is 8 to 1; the inlet air temperature is 15°C and the maximum cycle temperature is 800 °C. Combustion at the steam generator raises the gas temperature to 800 °C and the gas leaves the generator at 100°C. The steam pressure at supply is at 60bar and 600 °C and the condenser pressure is 0.05 bar. Calculate the flow rates of air and steam required for a total power output of 190 MW and the thermal efficiency of the combined plant. Assume ideal cycles for the two units. What would be the overall air/fuel ratio? Take Cp

for the combustion gases as 1.11 kJ /kg.K and γ = 1.333 and neglect the effect of the mass flow rate of fuel on the air flow. Take the calorific value of the fuel as 43300kJ/kg.

Sol:-
Gas turbine



$T_2 / T_1 = (p_2 / p_1)^{(\gamma - 1)/\gamma}$

$(T_1 = 15 + 273 = 288K)$

$T_2 / 288 = (8)^{(1.4 - 1)/1.4}$

$T_2 = 522$ K

$T_4 / T_3 = (p_2 / p_1)^{(\gamma - 1)/\gamma}$

$(T_3 = 800 + 273 = 1073K)$

$T_4 / 1073 = (8)^{(1.333 - 1)/1.333}$

$T_4 = 638$ K

$W = Cp_g (T_3 - T_4) - Cp_a (T_2 - T_1)$

$= 1.11 (1073 - 638) - 1.005 (522 - 288)$

$= 249$ kJ/kg

Energy supplied to the gas, $Q_{23} = Cp_g (T_3 - T_2)$

$= 1.11 (1073 - 522)$

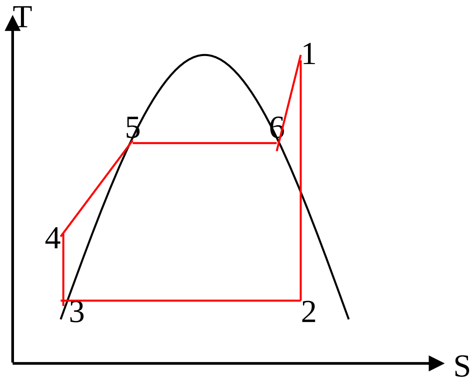
= 612 kJ/kg

Energy supplied to the gas, $Q_{45} = Cp_g (T_5 – T_4)$

= 1.11 (800 – 365)

=483 kJ/kg

Steam turbine



p_1= 60 bar , t_1= 600°C

h_1= 3657 kJ/kg ,& S_1= 7.166 kJ/kg

p_2 = 0.05bar & S_2= S_1= 7.166 kJ/kg

$s_{f2} < s_2 < s_{g2}$, therefore wet

$s_2 = s_{f2} + x_2\, s_{fg2}$

$7.166 = 0.476 + x_2\, 7.918$

$x_2 = 0.845$

$h_2 = h_{f2} + x_2\, h_{fg2}$

$h_2 = 138 + 0.845 * 2423$

h_2 = 2185.4 kJ/kg

p_3 = 0.05bar & sat. liq.

h_3 = 138
Neglect feed pump

$h_4 = h_3 = 138$

$W = h_1 - h_2 = 3657 - 2185.4 = 1471.6$ kJ/kg

Change in enthalpy of combustion gas $= 1.11\,(800 - 100) = 777$ kJ/kg

Increase in enthalpy of steam $= h_1 - h_3$

$= 3657 - 138$

$= 3519$ kJ/kg

If $\dot{m}_a$ and $\dot{m}_s$ are the mass flow rates of air and steam then an energy balance for the steam generator gives

$\dot{m}_a * 777 = \dot{m}_s\ 3519$

$\dot{m}_a = 4.53\ \dot{m}_s$

The total power output = 190 MW (given)

$\dot{m}_a * 249 + \dot{m}_s * 1471.6 = 190 * 10^6$

$(4.53\ \dot{m}_s) * 249 + \dot{m}_s * 1471.6 = 190 * 10^6$

$\dot{m}_s = 7.42 * 10^4$ kg / sec

$\dot{m}_a = 4.53 * 7.42 * 10^4 = 33.6 * 10^4$ kg / sec

Fuel energy input $= 33.6*10^4 * 612 + 33.6*10^4 * 483$

$= 33.6*10^4 * 1095 = 368$ MW

$$\text{Thermal efficiency} = \frac{\text{total work output}}{\text{Fuel energy input}}$$

$= (190 / 368) * 100 = 51.6\%$

If the flow rate of the fuel is $\dot{m}_f$ then

$\dot{m}_f * 43300 = \dot{m}_a * 1095$

$\frac{\dot{m}_a}{\dot{m}_f} = 39.5$, overall air / fuel ratio = 39.5 / 1